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CLOSED-FORM PDF FOR MULTIUSER TR-UWB SYSTEMS UNDER GAUSSIAN NOISE AND IMPULSIVE INTERFERENCE

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Abstract—In most existing transmitted-reference ultra-wideband (TR-UWB) communication systems, receivers use the standard Gaussian approximation (SGA) for multiuser interference (MUI). It is an assumption used in most conventional multiuser systems, where the MUI tends to a Gaussian process by the central limit theorem, and convergence is relatively fast with respect to the number of users. However, for TR-UWB systems which are developed for short-range applications, we have a small number of active users. In this case, significant performance degradation is found in TR-UWB receivers due to the impreciseness of SGA. In this paper, we show that the Middleton class-A model is a more appropriate statistical model for MUI modeling in TR-UWB systems than the often used SGA. A closed-form expression for the probability density function (PDF) of the TR-UWB system under MUI, Gaussian noise and impulsive alpha-stable interference is developed. All these analytical results are confirmed by numerical simulations.

1. INTRODUCTION

Ultra-wideband (UWB) technology which uses narrow pulses having duration on the order of nanosecond was approved by the federal communications commission (FCC) in February 2002 for short-range, high speed wireless systems. To avoid interference with the co-existing applications, the radiated power should not exceed −41 dBm/MHz within the band from 3.1 GHz to 10.6 GHz [1, 2]. Because of

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these tight restrictions on the transmitted power spectral density (PSD) of UWB systems and since the transmit pulse energy is dispersed over the large number of multipath components passing through the channel, it is essential to design an efficient receiver which can collect most of the signal energy. The rake receiver has been usually addressed in the literature for its capability to harness multipath energy in UWB systems [3–5]. However, there are some challenges in implementing this receiver. Including, practical complexity due to the precise timing synchronization and the perfect channel estimation requirements. Also, the need for using usually a large number of fingers (branches) to get an acceptable performance increase the receiver complexity. To overcome such difficulties, Hoctor and Tomilson [6] proposed in 2002 a promising and more practical structure called transmitted reference (TR) UWB system. In TR-UWB systems, pulses are transmitted in pairs for each frame, where the first unmodulated pulse acts as the reference for detecting the second data modulated pulse. At the receiving stage, using a simple autocorrelation receiver (AcR) can detect the data signal without complexity. In order to improve the TR-UWB receiver in multiple-access environment many schemes are proposed [7–9]. In these studies the MUI is modeled by a Gaussian process. It is an assumption used in most multiuser narrowband and wide band communication systems, where the MUI tends to a Gaussian process by the central limit theorem, and convergence is relatively fast with respect to the number of users. However, UWB communication systems are developed for short-range applications. In this case we have a small number of active users at close range area, and the empirical PDF of the MUI is more heavy-tailed than the Gaussian PDF. Therefore, conventional receivers which are optimal for Gaussian noise, are not optimal for UWB applications.

In this paper, we consider a TR-UWB communication system in multiuser scenario with Gaussian noise and impulsive interference. We show the impulsiveness behavior of MUI and we present a statistical model of MUI more appropriate than the SGA. Finally we derive an accurate PDF model of the MUI, noise and impulsive interference. The following notations are used in this paper. $\lfloor\cdot\rfloor$ is the floor operator, $\delta(\cdot)$ is Dirac delta function and $u(\cdot)$ is the unit step function. $\otimes$ is the convolution product, $\|\cdot\|$ denotes the norm, $F\{\cdot\}$ and $F^{-1}\{\cdot\}$ are the Fourier transform and the inverse Fourier transform, respectively. $\Re\mathrm{e}[\cdot]$ is the real part, and $\mathrm{erfc}(\cdot)$ presents the complex complementary error function.

2. THE TR-UWB SYSTEM MODEL

We consider a conventional TR-UWB communication system in a multiple access scenario disturbed by both Gaussian noise and impulsive interference. The transmitted signal of the u-th user can be written as

$$s^{(u)}(t) = \sqrt{E_w} \sum_{j=-\infty}^{+\infty} d_j^{(u)} \left[w_{tr}\left(t - jT_f - c_j^{(u)} T_c\right) + b_{\lfloor j/N_s \rfloor}^{(u)} w_{tr}\left(t - jT_f - c_j^{(k)} T_c - T_d\right)\right] \quad (1)$$

where E_w is the transmitted pulse energy, $d_j^{(u)}$ is a pseudorandom sequence of values ± 1, T_f is the frame duration, T_c is the chip duration, T_d is time separation between data and reference pulses and $b_{\lfloor j/N_s \rfloor}^{(u)} \in \{+1, -1\}$ denote the sequence of *iid* data symbols. The monocycle waveform $w_{tr}(t)$ is modeled as a second derivative Gaussian pulse given by

$$w_{tr}(t) = N_F \left[1 - 4\pi \left(\frac{t - t_d}{\tau_w} \right)^2 \right] \exp\left(-2\pi \left(\frac{t - t_d}{\tau_w} \right)^2 \right) \quad (2)$$

where t_d corresponds to the location of the pulse center in time, and τ_w is the parameter that determines the temporal width of the pulse. The parameter N_F is the normalization factor yielding a unit energy pulse. The received UWB signal can be written as

$$\begin{aligned} r(t) &= \sum_{u=1}^{N_u} \alpha_u s^{(u)}(t - \tau_u) + N_G(t) + I_{Imp}(t) \\ &= s_D(t) + \underbrace{I_{\text{MUI}}(t) + N_G(t) + I_{Imp}(t)}_{N\text{: Overall Noise}} \end{aligned} \quad (3)$$

where α_u and τ_u model the attenuation and asynchronous delay of the u-th user over the propagation path to the receiver. $s_D(t)$ is the signal of the desired user, $I_{\text{MUI}}(t) = \sum_{u=2}^{N_u} \alpha_u s^{(u)}(t - \tau_u)$ is the MUI of $(N_u - 1)$ TR-UWB users, $N_G(t)$ models a zero-mean additive white Gaussian noise, and $I_{Imp}(t)$ is the impulsive interference.

3. THE MUI COMPONENT MODELING

In TR-UWB systems each user can use N_f frames for the transmission of one bit. Within each frame two pulses are transmitted, the first unmodulated pulse acts as the reference for detecting the second data modulated pulse. We assume that the interferences due to each pulse are independent such as in [10]. The total PDF of each user is obtained by convolving N_f PDFs of interfering pulses.

$$p_X^{N_f}(x) = \underbrace{p_X^{Data}(x) \otimes p_X^{Ref}(x) \otimes p_X^{Data}(x) \ldots \otimes p_X^{Ref}(x)}_{N_f\ times} \tag{4}$$

where $p_X^{Data}(x) = p_X^{Ref}(x) = p_X(x)$. The PDF of MUI can be obtained by the convolution of the PDFs of the interfering users

$$\begin{aligned} p_X^{\mathrm{MUI}}(x) &= \underbrace{p_X^{N_f}(x) \otimes p_X^{N_f}(x) \otimes \ldots \otimes p_X^{N_f}(x)}_{(N_u-1)\ times} \\ &= \underbrace{p_X(x) \otimes p_X(x) \otimes \ldots \otimes p_X(x)}_{N_f(N_u-1)\ times} \end{aligned} \tag{5}$$

The exact calculation of $p_X(x)$ expression is unwieldy. To avoid this difficulty, we use the good approximation given in [10]

$$p_X(x)\lambda_1\delta(x) + \lambda_2\left[u(x+\beta_p) - u(x-\beta_p)\right] \tag{6}$$

The parameters $\lambda_1 = 1 - 2\lambda_2\beta_p$ and $\lambda_2 = (3\sigma_p^2/2\beta_p^3)$ are selected such that the variance and the mean of the interference do not change. $\beta_p = \int_0^{T_w+T_d} w_{tr}(t)v(t)dt$ determines the correlator output for each pulse, T_w is the pulse width and $v(t)$ is the receiver's template signal. $\sigma_p^2 = \int_{-\infty}^{+\infty} x^2 p_X(x)dx = \int_{-\beta_p}^{\beta_p} \lambda_2 x^2 dx = 2\lambda_2\beta_p^3/3$ is the variance of the interference, caused by a single pulse (more details in [10]).

Let $n = N_f(N_u - 1)$ and substituting (6) in (5) the derived n-th order convolution of $p_X(x)$ is given by

$$\begin{aligned} p_X^{\mathrm{MUI}}(x) = \lambda_1^n\delta(x) + \sum_{m=1}^{n}\binom{n}{m}\lambda_1^{n-m}\lambda_2^m\frac{\beta_p^{m-1}}{(m-1)!} \\ \times \underbrace{\sum_{\mu=0}^{m}(-1)^v\binom{m}{\mu}\left(\frac{x}{\beta_p}+m-2\mu\right)^{m-1}u\left(\frac{x}{\beta_p}+m-2\mu\right)}_{K(x)} \end{aligned} \tag{7}$$

Furthermore, we can exploit the Gaussian approximation given in [11] for the term $K(x)$ of the above equation for large values of m, as follows

$$K(x) \approx \frac{2^m\,(m-1)!\beta_p}{\sqrt{2\pi\sigma_m^2}}\ \exp\left(-\frac{x^2}{2\sigma_m^2}\right) \tag{8}$$

where $\sigma_m^2 = m\beta_p^2/3$. In applied mathematics, the Dirac delta function $\delta(x)$ is often replaced by a Gaussian PDF with variance tending to zero

$$\delta(x) \approx \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{x^2}{2\sigma_0^2}\right), \quad (\sigma_0^2 \to 0) \tag{9}$$

Substituting (8) and (9) in (7) we obtain

$$p_X^{\mathrm{MUI}}(x) = \sum_{m=0}^{n} \binom{n}{m} (1-2\beta_p\lambda_2)^{n-m} (2\beta_p\lambda_2)^m \times \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{x^2}{2\sigma_m^2}\right) \tag{10}$$

The above expression shows that the $p_X^{\mathrm{MUI}}(x)$ can be described by weighted sum of zero-mean Gaussians with increasing variance. The weights present a binomial distribution with the random parameter $(2\beta_p\lambda_2)$. If $n > 20$ and $2\beta_p\lambda_2 < 0.05$ the binomial distribution converges towards the Poisson distribution [12].

$$\binom{n}{m} (1-2\beta_p\lambda_2)^{n-m} (2\beta_p\lambda_2)^m \approx \frac{A^m e^A}{m!} \tag{11}$$

Substituting (11) in (10) we obtain

$$P_X^{\mathrm{MUI}}(x) = \sum_{m=0}^{n} \frac{A^m e^A}{m!} \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{x^2}{2\sigma_m^2}\right) \tag{12}$$

This expression correspond to Middleton class-A model [13], where the parameter $A = 2N_f\beta_p\lambda_2(N_u - 1)$ is called the impulsive index. It describes the impulsiveness of the MUI, a small value of A implies a highly impulsive MUI.

4. MIDDLETON CLASS-A MODEL

The Middleton class-A (MCA) model has been found to provide good fits to a variety of noise and interference measurements [17]. The PDF $f_{MCA}(x)$ of the MCA model is defined as an infinite weighted sum of Gaussian densities with decreasing weights for Gaussian densities with increasing variances. The $f_{MCA}(x)$ is given by

$$f_{MCA}(x) = \sum_{i=0}^{\infty} \frac{e^{-A}A^i}{i!} \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{\frac{-x^2}{2\sigma_i^2}} \tag{13}$$

The major appeal of this model is that its parameters can be directly physically interpreted. The parameter A is called the impulsive index and describes the impulsiveness of the noise. A small value of

A implies a highly impulsive interference. The variances $\sigma_i^2 = \frac{\frac{i}{A}+\gamma_A}{1+\gamma_A}$ are functions of the parameter A, where γ_A is defined as the ratio of the power in the Gaussian noise component (σ_G^2) to the power of the interfering Poisson process (σ_p^2). The $f_{MCA}(x)$ for different values of A is illustrated in Fig. 1.

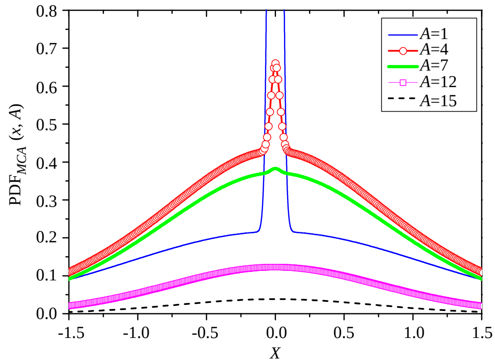

Figure 1. The PDF of MCA model for different values of the impulsive index A.

5. THE IMPULSIVE INTERFERENCE COMPONENT

In many practical UWB communication systems, the experimental measurements show the existence of the impulsive interference component. Recently, a wide range of phenomena of varying degrees of impulsivity are modeled by using the class of symmetric-alpha-stable (SαS) distributions. The SαS distribution is usually defined by the characteristic function as

$$\Phi(\omega) = \exp(-\gamma|\omega|^{\alpha}) \tag{14}$$

where $\alpha \in (0, 2]$ is the characteristic exponent, and γ is a quantity analogous to the variance called the dispersion. Unfortunately, SαS does not provide an analytic form PDF except for special cases. An approximation PDF model with less computational burden, called a simplified bi-parameter Cauchy Gaussian mixture (BCGM), is given in [14]

$$P_X^{\mathrm{S}\alpha\mathrm{S}}(x) = \frac{(1-\varepsilon)}{2\sqrt{\pi\sigma_s^2}} \exp\left(\frac{-x^2}{4\sigma_s^2}\right) + \frac{\varepsilon\sigma_s}{\pi(x^2+\sigma_s^2)} \tag{15}$$

where σ_s^2 is the variance and $\varepsilon \in [0, 1]$ is the mixture ratio, evaluated as

$$\varepsilon = \frac{2\Gamma(\rho/\alpha) - \alpha\Gamma(-\rho/2)}{2\alpha\Gamma(-\rho) - \alpha\Gamma(-\rho/2)} \tag{16}$$

in which $\Gamma(\cdot)$ denotes the Gamma function, and the parameter $\rho < \alpha$ is fixed at $-1/4$ as in [14]. Fig. 2 shows the SαS impulsive interference of 10^4 samples for different values of $\alpha : \alpha = 1$ (Cauchy distribution), $\alpha = 1.5$ (Lévy distribution) and $\alpha = 2$ (Gaussian distribution).

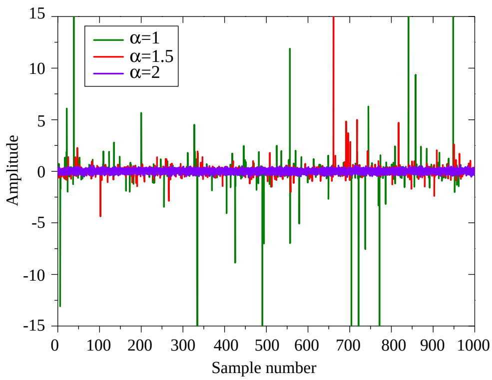

Figure 2. The SαS impulsive interference, for different value of $\alpha : \alpha = 1$ (Cauchy distribution), $\alpha = 1.5$ (Lévy distribution) and $\alpha = 2$ (Gaussian distribution).

6. ANALYTICAL PDF OF THE OVERALL NOISE

Since $I_{\text{MUI}}(t)$, $N_G(t)$ and $I_{Imp}(t)$ are independent random variables, the PDF of the overall noise can be found by convolving the distributions of the variables. The PDF of the overall noise can be written as

$$\begin{aligned} P_X^N(x) &= P_X^{\text{MUI}}(x) \otimes P_X^{GN}(x) \otimes P_X^{\text{S}\alpha\text{S}}(x) \\ &= [T_1(x) \otimes T_2(x)] + [T_1(x) \otimes T_3(x)] \end{aligned} \tag{17}$$

where $P_X^{\text{MUI}}(x)$, $P_X^{GN}(x)$ and $P_X^{\text{S}\alpha\text{S}}(x)$ are the PDFs of $I_{\text{MUI}}(t)$, $N_G(t)$ and $I_{Imp}(t)$, respectively. $T_1(x) \otimes T_2(x)$ presents the PDF of the sum of Gaussian random variables and $T_1(x) \otimes T_3$ denotes the PDF of the sum of Gaussian and Cauchy random variables. $T_1(x)$, $T_1(x) \otimes T_2(x)$ and

$T_1(x) \otimes T_3(x)$ are given by Equations (18), (19) and (20), respectively.

$$P_X^N(x) = \underbrace{\sum_{m=0}^{n} \frac{A^m e^A}{m!} \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{x^2}{2\sigma_m^2}\right) \otimes \frac{1}{\sqrt{2\pi\sigma_g^2}} \exp\left(\frac{-x^2}{2\sigma_g^2}\right)}_{T_1(x)}$$

$$\otimes \left[\underbrace{\frac{(1-\varepsilon)}{2\sqrt{\pi\sigma_s^2}} \exp\left(\frac{-x^2}{4\sigma_s^2}\right)}_{T_2(x)} + \underbrace{\frac{\varepsilon\sigma_s}{\pi(x^2+\sigma_s^2)}}_{T_3(x)} \right] \tag{18}$$

$$T_1(x) = \sum_{m=0}^{n} \frac{A^m e^A}{m!} F^{-1} \left[F\left\{ \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{x^2}{2\sigma_m^2}\right) \right\} \times F\left\{ \frac{1}{\sqrt{2\pi\sigma_g^2}} \exp\left(\frac{-x^2}{2\sigma_g^2}\right) \right\} \right]$$

$$= \sum_{m=0}^{n} \frac{A^m e^A}{m!} \frac{1}{\sqrt{2\pi(\sigma_m^2+\sigma_g^2)}} \exp\left(-\frac{x^2}{2(\sigma_m^2+\sigma_g^2)}\right) \tag{19}$$

$$T_1(x) \otimes T_2(x) = \sum_{m=0}^{n} \frac{A^m e^A}{m!} F^{-1} \left[F\left\{ \frac{1}{\sqrt{2\pi(\sigma_m^2+\sigma_g^2)}} \exp\left(-\frac{x^2}{2\left(\sigma_m^2+\sigma_g^2\right)}\right) \right\} \right] \times F^{-1} \left[F\left\{ \frac{1-\varepsilon}{2\sqrt{\pi\sigma_s^2}} \exp\left(\frac{-x^2}{4\sigma_s^2}\right) \right\} \right]$$

$$= \sum_{m=0}^{n} \frac{A^m e^A}{m!} \frac{1-\varepsilon}{\sqrt{2\pi\left(\sigma_m^2+\sigma_g^2+2\sigma_s^2\right)}} \exp\left(-\frac{x^2}{2\left(\sigma_m^2+\sigma_g^2+2\sigma_s^2\right)}\right) \tag{20}$$

$$T_1(x) \otimes T_3(x) = \sum_{m=0}^{n} \frac{A^m e^A}{m!} \frac{1}{\sqrt{2\pi(\sigma_m^2+\sigma_g^2)}} \int_{\infty}^{\infty} \exp\left(-\frac{x^2}{2(\sigma_m^2+\sigma_g^2)}\right) \times \frac{\varepsilon\sigma_s}{\pi\left((x-\tau)^2+\sigma_s^2\right)} d\tau$$

$$= \sum_{m=0}^{n} \frac{A^m e^A}{m!} \frac{\varepsilon}{\sqrt{2\pi(\sigma_m^2+\sigma_g^2)}} \Re\text{e} \left[\exp\left(-\left[\frac{x+j\sigma_s}{\sqrt{2(\sigma_m^2+\sigma_g^2)}} \right]^2 \right) \times \text{erfc}\left(\frac{\sigma_s - jx}{\sqrt{2(\sigma_m^2+\sigma_g^2)}} \right) \right] \tag{21}$$

By substituting in (17), we obtain

$$P_X^N(x) = \sum_{m=0}^{n} \frac{A^m e^A}{m!} \frac{1-\varepsilon}{\sqrt{2\pi(\sigma_m^2+\xi^2)}} \exp\left(-\frac{x^2}{2(\sigma_m^2+\xi^2)}\right) + \frac{\varepsilon}{\sqrt{2\pi(\sigma_m^2+\sigma_g^2)}} \Re\mathrm{e}\left[F\left(\frac{x+j\sigma_s}{\sqrt{2(\sigma_m^2+\sigma_g^2)}}\right)\right] \tag{22}$$

where $\xi = \sigma_g^2 + 2\sigma_s^2$ and $F(\cdot)$ is the Faddeeva function [15] defined as $F(x+jy) = \exp[-(x+jy)^2]\mathrm{erfc}(y-jx)$ for real positive x and y.

7. SIMULATION RESULTS

To verify analytical results, a TR-UWB system in a multiple access scenario with Gaussian noise and impulsive SαS interference is simulated with the following set of parameters ; $N_f = 4$, $T_f = 100\,\mathrm{ns}$, $\tau_m = 0.2877$, $T_d = 50\,\mathrm{ns}$, and $N_u = 5$. The channel impulse response (CIR) is generated according to IEEE 802.15.3a CM1 model. Fig. 3 shows the empirical PDF of the MUI component for four equal power TR-UWB interferers. The empirical PDF of the MUI is estimated from MUI data by using kernel density estimator. The Middleton class-A, the Laplacian, the SαS and the Gaussian PDFs with the same estimated variance are plotted for comparison. It is shown that the Middleton class-A model is closed to the empirical PDF of the MUI, which confirm the result in (12). Fig. 4 shows a comparison between the empirical PDF and the derived analytical PDF of the overall noise.

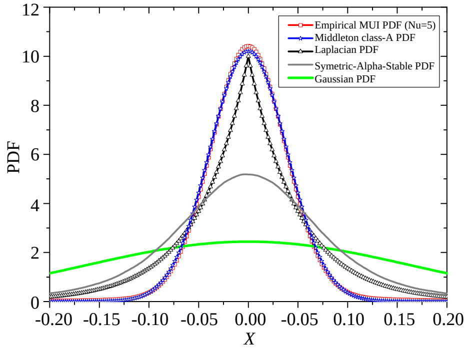

Figure 3. The empirical PDF of the MUI component for TR-UWB system, for 4 interfers. The Middleton class-A, the Laplacian, the SαS and the Gaussian PDFs are shown for comparison.

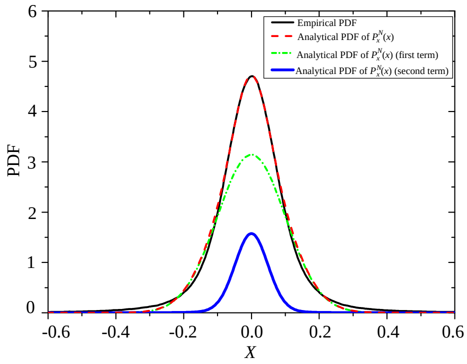

Figure 4. The derived analytical PDF of the overall noise for TR-UWB system, for 4 interfers. The empirical PDF of the overall noise is shown for comparison. $\alpha = 1.9$, $m = 10$ and $A = 0.015$.

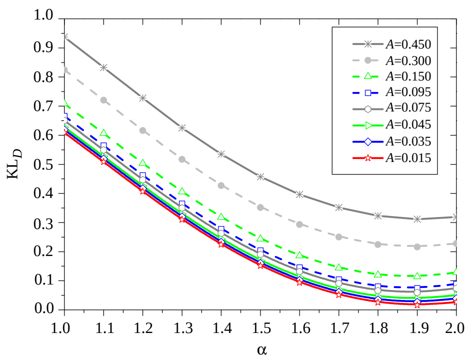

Figure 5. The Kullback-Leibler (KL) divergence versus the characteristic exponent α of SαS impulsive interference in range of [1, 2], for different values of the impulsive index A of the Middleton class-A MUI.

The first and the second terms of the analytical PDF in (22) are also, plotted. It is shown that the proposed analytical PDF is close to the empirical one, which verifies the derived $P_X^N(x)$.

In order to characterize the closeness between the analytical and the empirical PDFs, we use the Kullback-Leibler (KL) divergence defined by [16]

$$\mathrm{KL}_D = \int_{-\infty}^{+\infty} P_X^{N_{Anly}}(x) \log_2\left(\frac{P_X^{N_{Anly}}(x)}{P_X^{N_{Emp}}(x)}\right) dx \tag{23}$$

where $P_X^{N_{Anly}}(x)$ and $P_X^{N_{Emp}}(x)$ are the derived analytical PDF and the empirical PDF of the overall noise, respectively. Fig. 5 shows the KL divergence versus the characteristic exponent α of the impulsive interference for different values of the impulsive index A of the MUI. It is found that $\alpha = 1.9$ is the optimal value for the different values of A. Also, the KL divergence is proportional to A decreasing. It can be interpreted by the improvement in the binomial-Poisson approximation used in Equation (11).

8. CONCLUSION

In this paper, we have presented an accurate expression for the TR-UWB system in multiuser scenario with Gaussian noise and impulsive interference. We show that the Middleton class-A model is a more appropriate statistical model for the MUI than the generally used SGA. We succeeded to obtain an exact closed-form expression of the PDF of the sum of MUI, Gaussian noise and impulsive interference. The PDF so developed would find their applications in receivers design and improvement for UWB systems under MUI-plus-noise and impulsive interference.

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