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THEORY OF A STRIP LOOP ANTENNA LOCATED ON THE SURFACE OF AN AXIALLY MAGNETIZED PLASMA COLUMN

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Abstract—We study the current distribution and input impedance of a circular loop antenna in the form of an infinitesimally thin, perfectly conducting narrow strip coiled into a ring. The antenna is located on the surface of an axially magnetized plasma column surrounded by a homogeneous isotropic medium. The current in the antenna is excited by a time-harmonic voltage creating an electric field with the azimuthal component in a gap of small angular opening on the strip surface. The emphasis is placed on the solution of the integral equations for the azimuthal harmonics of the antenna current in the case where the magnetoplasma inside the column is nonresonant. The properties of the kernels of the integral equations are discussed and the current distribution in the antenna is obtained. It is shown that the presence of a magnetized plasma column can significantly influence the electrodynamic characteristics of the antenna compared with the case where it is located in the surrounding medium or a homogeneous plasma medium the parameters of which coincide with those inside the column.

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1. INTRODUCTION

Electrodynamic characteristics of loop antennas in a magnetoplasma were studied in many works [1–10]. The interest in the subject is stimulated by wide application of such antennas for performing various experiments in laboratory and space plasmas [5, 8, 11–16]. In earlier theoretical papers, loop antennas with given current distribution in a homogeneous magnetoplasma were considered (see, e.g., [1–3, 5–8] and references therein). The problem of the current distribution and input impedance of a loop antenna located in a homogeneous magnetized plasma was studied in [4] within the framework of the transmission line theory. Then a rigorous solution to this problem was found in [9, 10] by using an integral equation method.

Recently, with reference to certain laboratory and space experiments [12, 13, 15], enhanced attention has been paid to the antenna characteristics in the presence of magnetic-field-aligned plasma structures capable of guiding the excited electromagnetic waves. However, until the present time, the influence of such plasma structures on the current distribution and input impedance of a loop antenna has not been examined within the framework of a rigorous approach.

In this article, using an integral equation method, we solve the problem of the current distribution and input impedance of a circular loop antenna located on the surface of an axially magnetized uniform plasma column. It is assumed that the column is surrounded by a homogeneous isotropic medium. The main attention is focused on studying the antenna characteristics in a nonresonant band of a magnetoplasma, for which the diagonal elements of the plasma dielectric tensor have identical signs.

Our article is organized as follows. In Section 2, we present the formulation of the problem and write the basic equations. Section 3 contains the derivation of integral equations for the antenna current. These equations are solved in Section 4. The power radiated from the antenna with the found current distribution is discussed in Section 5. In Section 6, numerical results are reported. Section 7 presents our conclusions along with suggestions for future work. In Appendix A, we give expressions for the field coefficients used in the analysis of the integral equations for the antenna current. Appendix B describes the contributions of eigenmodes guided by the plasma column to the antenna-excited field.

2. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Consider an antenna in the form of an infinitesimally thin, perfectly conducting narrow strip of half-width d, which is coiled into a circular loop of radius a ($d \ll a$). The antenna is located coaxially on the surface of a uniform plasma column surrounded by a homogeneous isotropic medium and aligned with an external static magnetic field $\mathbf{B}_0$ (see Fig. 1), which is parallel to the z axis of a cylindrical coordinate system (ρ, ϕ, z). The plasma inside the column is described by the dielectric tensor

$$\boldsymbol{\varepsilon} = \epsilon_0 \begin{pmatrix} \varepsilon & -ig & 0 \\ ig & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix}, \tag{1}$$

where ϵ_0 is the permittivity of free space. For a monochromatic field with a time dependence of $\exp(i\omega t)$, the elements ε, g, and η of tensor (1) in the case of a two-component cold collisionless magnetoplasma can be written as [8, 17]

$$\begin{aligned} \varepsilon &= \frac{\left(\omega^2 - \omega_{\mathrm{LH}}^2\right)\left(\omega^2 - \omega_{\mathrm{UH}}^2\right)}{\left(\omega^2 - \omega_H^2\right)\left(\omega^2 - \Omega_H^2\right)}, \\ g &= \frac{\omega_p^2 \omega_H \omega}{\left(\omega^2 - \omega_H^2\right)\left(\omega^2 - \Omega_H^2\right)}, \\ \eta &= 1 - \frac{\omega_p^2}{\omega^2}, \end{aligned} \tag{2}$$

where Ω_H, ω_H, ω_{LH}, ω_p, and ω_{UH} are the ion and electron gyrofrequencies, the lower hybrid frequency, the electron plasma frequency, and the upper hybrid frequency, respectively. The homogeneous isotropic medium surrounding the plasma column has the dielectric permittivity $\varepsilon_{\mathrm{out}} = \epsilon_0 \varepsilon_a$.

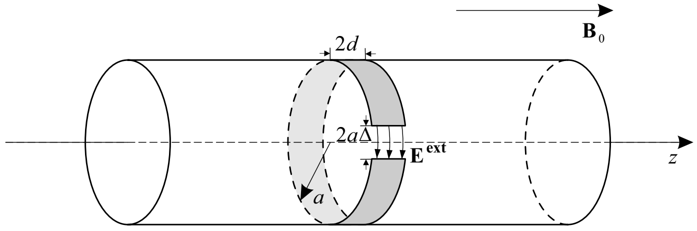


Figure 1. Geometry of the problem.

The antenna is excited by a time-harmonic given voltage which creates an electric field with a single azimuthal component $E_\phi^{\rm ext}$ that is nonzero for $\rho = a$ and $|z| < d$ in a narrow angular interval (gap) $|\phi - \phi_0| \le \Delta \ll \pi$:

$$\begin{aligned} E_\phi^{\rm ext}(a,\phi,z) \;=\; & \frac{V_0}{2a\Delta}[U(\phi-\phi_0+\Delta) - U(\phi-\phi_0-\Delta)] \\ & \times [U(z+d) - U(z-d)]\,. \end{aligned} \tag{3}$$

Here, $V_0 = \text{const}$ is a constant amplitude of the given voltage supplied to the gap, Δ is the angular half-width of the gap centered at $\phi = \phi_0$, and U is a Heaviside function. The quantity $E_\phi^{\rm ext}$ can be written as

$$E_\phi^{\rm ext} = \sum_{m=-\infty}^{\infty} A_m \exp(-im\phi), \tag{4}$$

where

$$A_m = \frac{V_0}{2\pi a}\frac{\sin(m\Delta)}{m\Delta}\exp(im\phi_0). \tag{5}$$

The density $\mathbf{J}$ of the electric current excited on the antenna by field (3) can be sought in the form

$$\mathbf{J} = \boldsymbol{\phi}_0 I(\phi,z)\delta(\rho-a), \tag{6}$$

where $|z| < d$, δ is a Dirac function, and $I(\phi,z)$ is the surface current density which admits the following representation:

$$I(\phi,z) = \sum_{m=-\infty}^{\infty} \mathcal{I}_m(z)\exp(-im\phi). \tag{7}$$

To find $I(\phi,z)$, we express the azimuthal (E_ϕ) and longitudinal (E_z) components of the electric field excited by current (6) in terms of unknown quantities $\mathcal{I}_m(z)$ and then use the boundary conditions on the surface of the plasma column ($\rho = a$ and $-\infty < z < \infty$) along with the following boundary conditions on the antenna surface ($\rho = a$ and $|z| < d$):

$$E_\phi + E_\phi^{\rm ext} \;=\; 0, \tag{8}$$

$$E_z \;=\; 0. \tag{9}$$

The described procedure makes it possible to obtain integral equations for the quantities $\mathcal{I}_m(z)$ and thus reduce the problem of the antenna current distribution to solving the corresponding integral equations.

3. DERIVATION OF INTEGRAL EQUATIONS FOR THE ANTENNA CURRENT

To derive integral equations for the antenna current, we should first obtain expressions for the field components corresponding to the current density in form (7). To do this, we rewrite the unknown current density (7) as

$$I(\phi, z) = \sum_{m=-\infty}^{\infty} \exp(-im\phi) \frac{k_0}{2\pi} \int_{-\infty}^{\infty} \mathcal{I}_m(p) \exp(-ik_0 pz) dp, \qquad (10)$$

where $k_0 = \omega/c$ is the free-space wave number (c is the speed of light in free space). Then we represent the field excited by this unknown current in a similar form:

$$\begin{bmatrix} \mathbf{E}(\mathbf{r}) \\ \mathbf{H}(\mathbf{r}) \end{bmatrix} = \sum_{m=-\infty}^{\infty} \exp(-im\phi) \frac{k_0}{2\pi} \int_{-\infty}^{\infty} \begin{bmatrix} \mathbf{E}_m(\rho, p) \\ \mathbf{H}_m(\rho, p) \end{bmatrix} \exp(-ik_0 pz) dp. \qquad (11)$$

Recall that in the case considered, the magnetic field $\mathbf{H}$ is related to the magnetic induction $\mathbf{B}$ by $\mathbf{H} = \mathbf{B}/\mu_0$, where μ_0 is the magnetic permeability of free space.

Since the antenna current is zero in the spatial regions $\rho < a$ and $\rho > a$, the electric and magnetic fields should satisfy the source-free Maxwell equations in these regions. It is convenient to express the radial and azimuthal field components of $\mathbf{E}_m(\rho, p)$ and $\mathbf{H}_m(\rho, p)$ in terms of the longitudinal components $E_{z,m}(\rho, p)$ and $H_{z,m}(\rho, p)$, which satisfy the following system of equations for $\rho < a$:

$$\hat{L}_m E_{z,m} - k_0^2 \frac{\eta}{\varepsilon} \left(p^2 - \varepsilon\right) E_{z,m} = -ik_0^2 \frac{g}{\varepsilon} p Z_0 H_{z,m}, \qquad (12)$$

$$\hat{L}_m H_{z,m} - k_0^2 \left(p^2 + \frac{g^2 - \varepsilon^2}{\varepsilon}\right) H_{z,m} = ik_0^2 \frac{g}{\varepsilon} \eta p Z_0^{-1} E_{z,m}, \qquad (13)$$

where

$$\hat{L}_m = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} - \frac{m^2}{\rho^2}$$

and $Z_0 = (\mu_0/\epsilon_0)^{1/2}$ is the impedance of free space. For the ambient region $\rho > a$, one should put $\varepsilon = \eta = \varepsilon_a$ and $g = 0$ in Equations (12) and (13).

The solutions of these equations must be regular on the column axis ($\rho = 0$) and satisfy the radiation condition at infinity. At the column surface $\rho = a$, the quantities $E_{\phi,m}(\rho, p)$, $E_{z,m}(\rho, p)$,

$H_{\phi,m}(\rho,\, p)$, and $H_{z,m}(\rho,\, p)$ should satisfy the boundary conditions

$$\begin{aligned} E_{\phi,m}(a-0,p) &= E_{\phi,m}(a+0,p), \\ E_{z,m}(a-0,p) &= E_{z,m}(a+0,p), \\ H_{\phi,m}(a-0,p) &= H_{\phi,m}(a+0,p), \\ H_{z,m}(a-0,p) &= H_{z,m}(a+0,p) + \mathcal{I}_m(p). \end{aligned} \tag{14}$$

It can be shown upon solution of the field equations that in the source-free regions, the components of $\mathbf{E}_m(\rho, p)$ and $\mathbf{H}_m(\rho, p)$ are given by the following expressions:
(a) for $\rho < a$,

$$\begin{aligned} E_{\rho,m}(\rho,p) &= -\sum_{k=1}^{2} B_{mk}\left[\frac{n_k p + g}{\varepsilon} J_{m+1}(k_0 q_k \rho) + \alpha_k m \frac{J_m(k_0 q_k \rho)}{k_0 q_k \rho}\right], \\ E_{\phi,m}(\rho,p) &= i\sum_{k=1}^{2} B_{mk}\left[J_{m+1}(k_0 q_k \rho) + \alpha_k m \frac{J_m(k_0 q_k \rho)}{k_0 q_k \rho}\right], \\ E_{z,m}(\rho,p) &= \frac{i}{\eta}\sum_{k=1}^{2} B_{mk} n_k q_k J_m(k_0 q_k \rho), \\ H_{\rho,m}(\rho,p) &= -iZ_0^{-1}\sum_{k=1}^{2} B_{mk}\left[p J_{m+1}(k_0 q_k \rho) - n_k \beta_k m \frac{J_m(k_0 q_k \rho)}{k_0 q_k \rho}\right], \\ H_{\phi,m}(\rho,p) &= -Z_0^{-1}\sum_{k=1}^{2} B_{mk} n_k \left[J_{m+1}(k_0 q_k \rho) - \beta_k m \frac{J_m(k_0 q_k \rho)}{k_0 q_k \rho}\right], \\ H_{z,m}(\rho,p) &= -Z_0^{-1}\sum_{k=1}^{2} B_{mk} q_k J_m(k_0 q_k \rho); \end{aligned} \tag{15}$$

(b) for $\rho > a$,

$$\begin{aligned} E_{\rho,m}(\rho,p) &= C_m m \frac{H_m^{(2)}(k_0 q \rho)}{k_0 q \rho} - D_m \frac{p}{\varepsilon_a}\left[H_{m+1}^{(2)}(k_0 q \rho) - m \frac{H_m^{(2)}(k_0 q \rho)}{k_0 q \rho}\right], \\ E_{\phi,m}(\rho,p) &= iC_m\left[H_{m+1}^{(2)}(k_0 q \rho) - m \frac{H_m^{(2)}(k_0 q \rho)}{k_0 q \rho}\right] - iD_m \frac{p}{\varepsilon_a} m \frac{H_m^{(2)}(k_0 q \rho)}{k_0 q \rho}, \\ E_{z,m}(\rho,p) &= \frac{i}{\varepsilon_a} D_m q H_m^{(2)}(k_0 q \rho), \\ H_{\rho,m}(\rho,p) &= -iZ_0^{-1} C_m p\left[H_{m+1}^{(2)}(k_0 q \rho) - m \frac{H_m^{(2)}(k_0 q \rho)}{k_0 q \rho}\right] \end{aligned} \tag{16}$$

$$+iZ_0^{-1}D_m m\frac{H_m^{(2)}(k_0q\rho)}{k_0q\rho},$$

$$H_{\phi,m}(\rho,p) = Z_0^{-1}C_m pm\frac{H_m^{(2)}(k_0q\rho)}{k_0q\rho} - Z_0^{-1}D_m\Big[H_{m+1}^{(2)}(k_0q\rho) - m\frac{H_m^{(2)}(k_0q\rho)}{k_0q\rho}\Big],$$

$$H_{z,m}(\rho,p) = -Z_0^{-1}C_m q H_m^{(2)}(k_0q\rho).$$

Here, J_m is a Bessel function of the first kind of order m, $H_m^{(l)}$ a Hankel function of the lth kind of order m, and B_{mk}, C_m, and D_m are undetermined coefficients ($k = 1$, 2 and $l = 1$, 2). Other quantities in Equations (15) and (16) are given by the expressions

$$\begin{aligned}
n_k &= -\frac{\varepsilon}{pg}\left[p^2 + q_k^2(p) + \frac{g^2}{\varepsilon} - \varepsilon\right],\\
\alpha_k &= \left[p^2 + q_k^2(p) - \varepsilon\right]g^{-1} - 1, \quad \beta_k = pn_k^{-1} + 1,\\
q_k(p) &= \frac{1}{\sqrt{2}}\left\{\varepsilon - \frac{g^2}{\varepsilon} + \eta - \left(\frac{\eta}{\varepsilon} + 1\right)p^2 - \left(\frac{\eta}{\varepsilon} - 1\right)\right.\\
&\quad \left.\times (-1)^k\left[\left(p^2 - P_b^2\right)\left(p^2 - P_c^2\right)\right]^{1/2}\right\}^{1/2},\\
P_{b,c} &= \left\{\varepsilon - (\eta + \varepsilon)\frac{g^2}{(\eta - \varepsilon)^2} + \frac{2\chi_{b,c}}{(\eta - \varepsilon)^2}\right.\\
&\quad \left.\times\left[\varepsilon g^2\eta\left(g^2 - (\eta - \varepsilon)^2\right)\right]^{1/2}\right\}^{1/2},\\
\chi_b &= -\chi_c = -1,
\end{aligned} \tag{17}$$

and

$$q(p) = (\varepsilon_a - p^2)^{1/2}. \tag{18}$$

It is evident that the field components (15) are regular on the column axis. The radiation condition at infinity requires that the branch of function (18) should be chosen so as to ensure the inequality

$$\mathrm{Im}q(p) < 0. \tag{19}$$

The coefficients B_{mk}, C_m, and D_m, which are obtained by substituting field expressions (15) and (16) into the boundary conditions (14), are presented in Appendix A. These coefficients contain the factor $\mathcal{I}_m(p)$, which is given by the Fourier integral

$$\mathcal{I}_m(p) = \int_{-d}^{d} \mathcal{I}_m(z') \exp(ik_0 p z') dz'. \tag{20}$$

Using Equation (20), we first perform integration over p in (11) in order to obtain the azimuthal and longitudinal components of the electric field at $\rho = a$, which will be needed in what follows. It turns out that for evaluation of the corresponding integrals, it is more convenient to distort the integration path in the complex p plane so as to enclose all poles and branch singularities of the integrands and then choose the quantity q defined by Equation (18) as an integration variable. The integrands of Equation (11) have the branch points $p = \pm\varepsilon_a^{1/2}$, from which the corresponding branch cuts go along the lines $\operatorname{Im} q(p) = 0$ in the complex p plane. In addition, these integrands may have poles at some points $p = \pm p_{m,n}$, which are the normalized (to k_0) propagation constants of eigenmodes guided by the column. The quantities m and n are the azimuthal and radial indices of the eigenmodes, respectively ($m = 0, \pm 1, \pm 2, \ldots$ and $n = 1, 2, \ldots$). The location of the above-mentioned singularities is shown qualitatively in Fig. 2. The figure also shows the distorted integration path enclosing all the singularities located in the lower half of the complex p plane for $z - z' > 0$. For $z - z' < 0$, the integration path should be distorted to go in the upper half of the complex p plane. Since the integral over the semicircle of infinite radius is zero (see the path Γ_∞ in Fig. 2 for the case $z - z' > 0$), the field is determined only by the sign-reversed residues of the poles at $p = p_{m,n}$ and the integral around the branch cut, i.e., along the contour Γ. To avoid misunderstanding, it is worth mentioning that the poles, branch cuts, and branch points are slightly displaced off the real or imaginary axis in Fig. 2, because a minor dissipative loss is assumed to be introduced to the outer region $\rho > a$. This makes it possible to easily clarify the mutual location of the integration paths and all the singularities of the integrand in the complex p plane. However, in the resulting expressions, the dissipative loss is put equal to zero throughout.

When performing the above-described calculations, we observe that the quantity q runs all real values from $-\infty$ to ∞ during the integration along Γ. Passing to integration over only the positive real values of q reduces the integrals for the azimuthal and longitudinal components of the electric field at the boundary $\rho = a$ to the following form:

$$E_\phi(a, \phi, z) = \sum_{m=-\infty}^{\infty} \exp(-im\phi) \int_{-d}^{d} K_m(z - z') \mathcal{I}_m(z') dz', \tag{21}$$

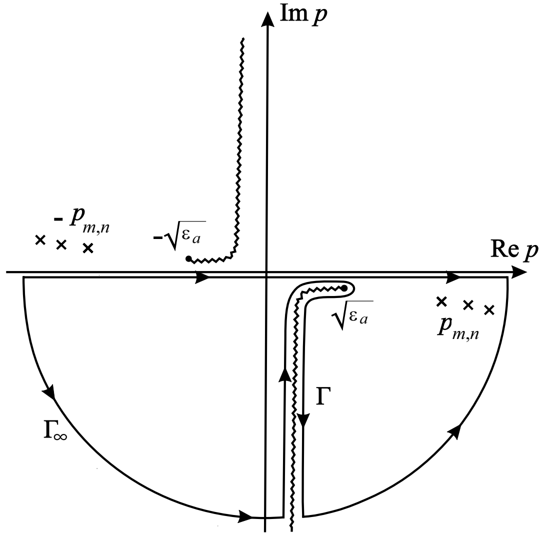


Figure 2. Paths of integration in the complex p plane.

$$E_z(a,\phi,z) = \sum_{m=-\infty}^{\infty} \exp(-im\phi) \int_{-d}^{d} k_m(z-z')\mathcal{I}_m(z')dz'. \tag{22}$$

Here,

$$\begin{aligned} K_m(\zeta) &= \sum_n \frac{2\pi a}{N_{m,n}} E^2_{\phi;m,n}(a) \exp\left(-ik_0 p_{m,n}|\zeta|\right) \\ &+ \frac{ik_0}{2\pi} \int_0^\infty \frac{q}{p(q)} \sum_{l=1}^{2} \sum_{k=1}^{2} \frac{B^{(l)}_{mk}}{\Delta^{(l)}_m} \left[J_{m+1}(Q_k) + \alpha_k m \frac{J_m(Q_k)}{Q_k} \right] \\ &\times \exp(-ik_0 p(q)|\zeta|)dq, \end{aligned} \tag{23}$$

$$\begin{aligned} k_m(\zeta) &= \mathrm{sgn}\zeta \left\{ \sum_n \frac{2\pi a}{N_{m,n}} E_{\phi;m,n}(a) E_{z;m,n}(a) \exp\left(-ik_0 p_{m,n}|\zeta|\right) \right. \\ &+ \frac{i}{2\pi a\eta} \int_0^\infty \frac{q}{p(q)} \sum_{l=1}^{2} \sum_{k=1}^{2} \frac{B^{(l)}_{mk}}{\Delta^{(l)}_m} n_k Q_k J_m(Q_k) \\ &\left. \times \exp(-ik_0 p(q)|\zeta|)dq \right\}, \end{aligned} \tag{24}$$

where

$$Q_k = k_0 a q_k(p(q)), \quad Q = k_0 a q, \quad p(q) = \left(\varepsilon_a - q^2\right)^{1/2}, \tag{25}$$

$E_{\phi;m,n}(\rho)$ and $E_{z;m,n}(\rho)$ are functions describing the distributions over the transverse coordinate ρ of the azimuthal and longitudinal electric-field components of eigenmodes guided by the column, respectively, and $N_{m,n}$ is the norm of an eigenmode with the propagation constant $p_{m,n}$ [8, 18–20]. Note that the function $p(q)$ has the meaning of the normalized propagation constant of the normal mode in the background medium for the dimensionless transverse wave number $q = k_\perp/k_0$. It is assumed that $\operatorname{Im} p(q) < 0$.

Expressions for the quantities $B_{mk}^{(l)}$ and $\Delta_m^{(l)}$ in Equations (23) and (24) are written as

$$\begin{aligned}
B_{m1}^{(l)} =& Z_0 \frac{k_0 a}{Q_1 J_m(Q_1)} \left[\frac{\eta}{\varepsilon_a} n_2 \mathcal{J}_m^{(2)} \mathcal{H}_m^{(l)} + \frac{\eta}{\varepsilon_a} p \frac{m}{Q^2} \tilde{\mathcal{J}}_m^{(2)} \right. \\
&\left. - n_2 \left(\left(\mathcal{H}_m^{(l)}\right)^2 - \frac{p^2}{\varepsilon_a} \frac{m^2}{Q^4} \right) \right], \\
B_{m2}^{(l)} =& Z_0 \frac{k_0 a}{Q_2 J_m(Q_2)} \left[-\frac{\eta}{\varepsilon_a} n_1 \tilde{\mathcal{J}}_m^{(1)} \mathcal{H}_m^{(l)} - \frac{\eta}{\varepsilon_a} p \frac{m}{Q^2} \mathcal{J}_m^{(1)} \right. \\
&\left. + n_1 \left(\left(\mathcal{H}_m^{(l)}\right)^2 - \frac{p^2}{\varepsilon_a} \frac{m^2}{Q^4} \right) \right],
\end{aligned} \tag{26}$$

$$\begin{aligned}
\Delta_m^{(l)} = (-1)^l \Bigg\{ & n_2 \left[\frac{\eta}{\varepsilon_a} \mathcal{J}_m^{(1)} \mathcal{J}_m^{(2)} - \left(\mathcal{J}_m^{(1)} + \frac{\eta}{\varepsilon_a} \mathcal{J}_m^{(2)} \right) \mathcal{H}_m^{(l)} \right] \\
& - n_1 \left[\frac{\eta}{\varepsilon_a} \tilde{\mathcal{J}}_m^{(1)} \tilde{\mathcal{J}}_m^{(2)} - \left(\tilde{\mathcal{J}}_m^{(2)} + \frac{\eta}{\varepsilon_a} \tilde{\mathcal{J}}_m^{(1)} \right) \mathcal{H}_m^{(l)} \right] \\
& + (n_2 - n_1) \left[\left(\mathcal{H}_m^{(l)}\right)^2 - \frac{p^2}{\varepsilon_a} \frac{m^2}{Q^4} \right] \\
& + p \frac{\eta}{\varepsilon_a} \frac{m}{Q^2} \left[\mathcal{J}_m^{(1)} + \tilde{\mathcal{J}}_m^{(1)} - \mathcal{J}_m^{(2)} - \tilde{\mathcal{J}}_m^{(2)} \right] \Bigg\},
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
&\mathcal{J}_m^{(1)} = \frac{J_{m+1}(Q_1)}{Q_1 J_m(Q_1)} + m \frac{\alpha_1}{Q_1^2}, \quad \mathcal{J}_m^{(2)} = \frac{J_{m+1}(Q_2)}{Q_2 J_m(Q_2)} - m \frac{\beta_2}{Q_2^2}, \\
&\tilde{\mathcal{J}}_m^{(1)} = \frac{J_{m+1}(Q_1)}{Q_1 J_m(Q_1)} - m \frac{\beta_1}{Q_1^2}, \quad \tilde{\mathcal{J}}_m^{(2)} = \frac{J_{m+1}(Q_2)}{Q_2 J_m(Q_2)} + m \frac{\alpha_2}{Q_2^2}, \\
&\mathcal{H}_m^{(l)} = \frac{H_{m+1}^{(l)}(Q)}{Q H_m^{(l)}(Q)} - \frac{m}{Q^2}, \qquad k = 1, 2, \quad l = 1, 2.
\end{aligned} \tag{28}$$

It is to be emphasized that all the quantities entering the integrands in Equations (23) and (24) and containing p are calculated for $p =$

$p(q)$. The details of derivation of Equations (26) and (27) are briefly described in Appendix A.

Let us dwell in greater detail on the calculation of the terms that describe the contributions of eigenmodes to Equations (23) and (24). According to [8, 19], the quantity $\Delta_m^{(2)}$, regarded as a function of p, can be used to determine the propagation constants of eigenmodes. To this end, one should solve the equation

$$\Delta_m^{(2)}(p) = 0, \tag{29}$$

the roots of which are the propagation constants $p = p_{m,n}$ of eigenmodes. The eigenmode fields can be obtained as $\mathbf{E}_{m,n}(\rho) = \tilde{\mathbf{E}}_m(\rho, p_{m,n})$ and $\mathbf{H}_{m,n}(\rho) = \tilde{\mathbf{H}}_m(\rho, p_{m,n})$, where $\tilde{\mathbf{E}}_m(\rho, p) = \mathbf{E}_m(\rho, p)\Delta_m^{(2)}(p)\mathcal{I}_m^{-1}(p)$ and $\tilde{\mathbf{H}}_m(\rho, p) = \mathbf{H}_m(\rho, p)\Delta_m^{(2)}(p)\mathcal{I}_m^{-1}(p)$ [8]. The norms $N_{m,n}$ of the eigenmodes are calculated as follows [8, 20]:

$$N_{m,n} = 2\pi \int_0^\infty \left[\mathbf{E}_{m,n}(\rho) \times \mathbf{H}_{-m,-n}^{(T)}(\rho) - \mathbf{E}_{-m,-n}^{(T)}(\rho) \times \mathbf{H}_{m,n}(\rho)\right] \cdot \mathbf{z}_0 \rho d\rho. \tag{30}$$

Here, the negative sign of the subscript n stands to denote eigenmodes propagating in the negative direction of the z axis, for which $p_{m,-n} = -p_{m,n}$, and the superscript (T) denotes fields taken in an auxiliary ("transposed") medium described by the transposed dielectric tensor $\boldsymbol{\varepsilon}^T$. The details of calculation of the eigenmode contribution to the antenna field are given in Appendix B.

Using the boundary conditions (8) and (9) for the tangential components of the electric field on the antenna surface and allowing for Equations (4), (5), (21), and (22), we can obtain integral equations for $\mathcal{I}_m(z)$. Equation (8) yields

$$\int_{-d}^{d} K_m(z - z')\mathcal{I}_m(z')dz' = -A_m, \tag{31}$$

whereas Equation (9) gives

$$\int_{-d}^{d} k_m(z - z')\mathcal{I}_m(z')dz' = 0. \tag{32}$$

It is assumed in Equations (31) and (32) that $m = 0, \pm 1, \pm 2, \ldots$ and $|z| < d$.

The behavior of the solutions of the obtained integral equations is determined by the properties of their kernels. It will be shown below that in the case of a sufficiently narrow strip where the conditions

$$\begin{gathered} d \ll a, \;\; d \ll a|\eta/\varepsilon|^{1/2}, \\ (k_0 d)^2 \max\{|\varepsilon_a|, |\varepsilon|, |g|, |\eta|\} \ll 1 \end{gathered} \tag{33}$$

hold, the properties of the kernels make it possible to obtain approximate solutions of Equations (31) and (32). In what follows, we present a method for obtaining such solutions in the case of a nonresonant plasma where $\operatorname{sgn}\varepsilon = \operatorname{sgn}\eta$. The case of a resonant plasma, which corresponds to the relation $\operatorname{sgn}\varepsilon \neq \operatorname{sgn}\eta$, is much more complex, and its consideration falls beyond the scope of this paper.

4. SOLUTION OF INTEGRAL EQUATIONS FOR THE ANTENNA CURRENT

We start our analysis of integral Equations (31) and (32) from studying the properties of their kernels (23) and (24) in the case of a nonresonant plasma in the column. We represent the kernels of the integral equations as

$$\begin{aligned} K_m(\zeta) &= K_m^{(s)}(\zeta) + F_m(\zeta), \\ k_m(\zeta) &= k_m^{(s)}(\zeta) + f_m(\zeta). \end{aligned} \tag{34}$$

Here,

$$\begin{aligned} K_m^{(s)}(\zeta) \;&=\; -iZ_0\frac{k_0^2 a}{2}\int_0^\infty J_{m+1}^2(k_0 a q)\exp(-k_0 q|\zeta|)dq \\ &\quad +iZ_0\frac{m^2}{2a}\frac{\varepsilon_a}{\varepsilon\eta}\int_0^\infty \frac{J_m^2(k_0 a q)}{V_m(q)}\exp\left(-k_0\sqrt{\frac{\varepsilon}{\eta}}q|\zeta|\right)dq, \end{aligned} \tag{35}$$

$$\begin{aligned} k_m^{(s)}(\zeta) \;&=\; \operatorname{sgn}(\zeta)Z_0\frac{k_0}{2}m\frac{\varepsilon_a}{\varepsilon\eta}\sqrt{\frac{\varepsilon}{\eta}}\int_0^\infty \frac{qJ_m^2(k_0 a q)}{V_m(q)} \\ &\quad \times\exp\left(-k_0\sqrt{\frac{\varepsilon}{\eta}}q|\zeta|\right)dq, \end{aligned} \tag{36}$$

where

$$V_m(q) = \sin^2\left(k_0 a q - \frac{\pi m}{2} - \frac{\pi}{4}\right) + \frac{\varepsilon_a^2}{\varepsilon\eta}\cos^2\left(k_0 a q - \frac{\pi m}{2} - \frac{\pi}{4}\right). \tag{37}$$

The integrands of Equations (35) and (36) are obtained by making the limiting transition $q \to \infty$ in the corresponding integrands of kernels (23) and (24) and changing the integration variable in the $k = 2$ terms of the resulting kernels in accordance with the relation $\sqrt{\eta/\varepsilon}q \to q$. In the derivation of Equations (34)–(37), we made use of the fact that for $q \to \infty$ and $\operatorname{sgn}\varepsilon = \operatorname{sgn}\eta$,

$$q_1 = q\,,\;\; q_2 = \sqrt{\frac{\eta}{\varepsilon}}\,q\,,\;\; p = -iq\,,\;\; n_1 = -i\frac{g}{\eta-\varepsilon}\frac{\eta}{q},\;\; n_2 = -i\frac{\eta-\varepsilon}{g}q,$$

$$\alpha_1 = -1 + \frac{g}{\eta-\varepsilon},\;\; \alpha_2 = \frac{\eta-\varepsilon}{g\varepsilon}q^2,\;\; \beta_1 = \frac{\eta-\varepsilon}{g\eta}q^2,\;\; \beta_2 = 1 + \frac{g}{\eta-\varepsilon}.$$

The quantities $F_m(\zeta)$ and $f_m(\zeta)$ are written as

$$\begin{aligned}F_m(\zeta) \;=\; & \sum_n \frac{2\pi a}{N_{m,n}} E^2_{\phi;m,n}(a)\exp\left(-ik_0 p_{m,n}|\zeta|\right)\\ & +\frac{ik_0}{2\pi}\int_0^\infty\Bigg\{\frac{q}{p(q)}\sum_{l=1}^{2}\sum_{k=1}^{2}\frac{B^{(l)}_{mk}}{\Delta^{(l)}_m}\left[J_{m+1}(Q_k)+\alpha_k m\frac{J_m(Q_k)}{Q_k}\right]\\ & \times\exp\left(-ik_0 p(q)|\zeta|\right)\\ & + Z_0 k_0 a\pi\left[J^2_{m+1}(k_0aq)-\frac{m^2}{(k_0a)^2}\frac{\varepsilon_a}{\varepsilon\eta}\sqrt{\frac{\eta}{\varepsilon}}\frac{J^2_m(k_0aq\sqrt{\eta/\varepsilon})}{V_m(q\sqrt{\eta/\varepsilon})}\right]\\ & \times\exp\left(-k_0q|\zeta|\right)\Bigg\}dq, \end{aligned}\tag{38}$$

$$\begin{aligned}f_m(\zeta) \;=\; & \operatorname{sgn}\zeta\Bigg\{\sum_n\frac{2\pi a}{N_{m,n}}E_{\phi;m,n}(a)E_{z;m,n}(a)\exp\left(-ik_0p_{m,n}|\zeta|\right)\\ & +\frac{k_0}{2}\int_0^\infty\Bigg[\frac{i}{\pi k_0 a\eta}\frac{q}{p(q)}\sum_{l=1}^{2}\sum_{k=1}^{2}\frac{B^{(l)}_{mk}}{\Delta^{(l)}_m}n_kQ_kJ_m(Q_k)\\ & \times\exp(-ik_0p(q)|\zeta|)\\ & -Z_0m\frac{\varepsilon_a}{\varepsilon\eta}\sqrt{\frac{\eta}{\varepsilon}}\frac{qJ^2_m(k_0aq\sqrt{\eta/\varepsilon})}{V_m(q\sqrt{\eta/\varepsilon})}\exp\left(-k_0q|\zeta|\right)\Bigg]dq\Bigg\}. \end{aligned}\tag{39}$$

It is worth noting that the quantities $F_m(\zeta)$ and $f_m(\zeta)$ are determined by the terms corresponding to eigenmodes in (23) and (24), as well as by the integrals over q, the integrands of which are given by the differences of the respective quantities entering rigorous formulas for the kernels $K_m(\zeta)$ and $k_m(\zeta)$ and those entering relations (35) and (36).

It is easily shown that the quantities (35) and (36) tend to infinity for $\zeta = 0$, whereas the quantities $F_m(\zeta)$ and $f_m(\zeta)$ have no singularities at this point. Thus, formulas (34) give representations of the kernels $K_m(\zeta)$ and $k_m(\zeta)$ as the sums of the terms $K^{(s)}_m(\zeta)$ and $k^{(s)}_m(\zeta)$, which comprise singular parts, and the nonsingular terms $F_m(\zeta)$ and $f_m(\zeta)$. The latter ones can then be taken at $\zeta = 0$ under conditions (33).

Expression (35) can be reduced to the form

$$\begin{aligned}K^{(s)}_m(\zeta) \;=\; & iZ_0\frac{k_0}{2\pi}\Bigg\{\left[\ln\frac{|\zeta|}{2a}+\psi\left(m+\frac{3}{2}\right)+\gamma\right]\\ & +\frac{m^2}{(k_0a)^2}\frac{\varepsilon_a}{\varepsilon\eta}\left(b_m\ln\frac{|\zeta|}{2a}+c_m\right)\Bigg\}, \end{aligned}\tag{40}$$

where $\gamma = 0,5772\ldots$ is Euler's constant, $\psi(z) = d\ln\Gamma(z)/dz$ is the logarithmic derivative of a gamma function, and b_m and c_m are coefficients which are independent of ζ and obtainable only numerically in the general case. In the special case $\varepsilon_a^2 = \varepsilon\eta$, they admit the rigorous analytical representation

$$b_m = -1, \;\; c_m = \frac{1}{2}\ln\frac{\eta}{\varepsilon} - \psi\Big(m + \frac{1}{2}\Big) - \gamma.$$

As a result, integral Equation (31) takes the form

$$\begin{aligned} &\int_{-d}^{d} \mathcal{I}_m(z')\ln\frac{|z-z'|}{2a}dz' \\ &= i\frac{2\pi A_m}{Z_0 k_0}\frac{(k_0a)^2\varepsilon\eta\varepsilon_a^{-1}}{m^2 b_m + (k_0a)^2\varepsilon\eta\varepsilon_a^{-1}} - S_m\int_{-d}^{d}\mathcal{I}_m(z')dz', \end{aligned} \tag{41}$$

where

$$\begin{aligned} S_m \;=\; &\frac{1}{m^2 b_m + (k_0a)^2\varepsilon\eta\varepsilon_a^{-1}}\Big\{m^2 c_m + (k_0a)^2\varepsilon\eta\varepsilon_a^{-1} \\ &\times\left[\psi\left(m+\frac{3}{2}\right) + \gamma - i\frac{2\pi}{Z_0k_0}F_m(0)\right]\Big\}. \end{aligned} \tag{42}$$

In turn, Equation (32) under conditions (33) is transformed to the following form:

$$\int_{-d}^{d} m\frac{\mathcal{I}_m(z')}{z-z'}dz' = 0. \tag{43}$$

When deriving Equation (43), we took into account that the quantity $k_m^{(s)}(\zeta)$ in (36) is proportional to the derivative of the second term of $K_m^{(s)}(\zeta)$ in (35) with respect to ζ. In addition, we allowed for the relation $f_m(0) = 0$. Although this relation follows formally from the properties of the function $\operatorname{sgn}\zeta$, it can also be justified on physical grounds if we approximately put $z = 0$ in the nonsingular part $f_m(z - z')$ of the corresponding integral equation, which is always possible for a sufficiently narrow strip. Then, using the evenness of the function $\mathcal{I}_m(z)$, we have for $z = 0$

$$\int_{-d}^{d} f_m(z-z')\mathcal{I}_m(z')dz' = 0.$$

It can be shown that the solutions of Equations (41) and (43) are the main terms of the asymptotics of solutions to initial integral Equations (31) and (32) under conditions (33) and $d \ll 2a\Delta \ll a$ (see [10] for details). Here, we restrict ourselves to analysis of Equations (41) and (43).

It is a straightforward matter to verify that the solution to Equation (41) with the logarithmic kernel automatically satisfies singular integral Equation (43) with the Cauchy kernel [10]. This fact allows us to consider only Equation (41) in what follows. The solution to Equation (41) can be found using the techniques discussed in [10] and has the form

$$\mathcal{I}_m(z) = \frac{2i}{Z_0 k_0 \sqrt{d^2 - z^2}} \frac{A_m \delta_m}{\ln(4a/d) - S_m}, \tag{44}$$

where

$$\delta_m = -\frac{(k_0 a)^2 \varepsilon \eta \varepsilon_a^{-1}}{m^2 b_m + (k_0 a)^2 \varepsilon \eta \varepsilon_a^{-1}}. \tag{45}$$

Substituting (44) into (7), we obtain the following formula for the linear current density $I(\phi, z)$:

$$\begin{aligned} I(\phi, z) \;=\; & \frac{iV_0}{Z_0 \pi k_0 a \sqrt{d^2 - z^2}} \sum_{m=-\infty}^{\infty} \frac{\sin(m\Delta)}{m\Delta} \\ & \times \frac{\delta_m}{\ln(4a/d) - S_m} \exp[-im(\phi - \phi_0)]. \end{aligned} \tag{46}$$

It is seen from the formulas obtained that the surface current density tends to infinity near the edges of a perfectly conducting strip. Such behavior of the current density corresponds to the well-known Meixner condition at the edge [21]. Note that despite the divergence of $I(\phi, z)$ for $|z| \to d$, the total current $I_\Sigma(\phi)$ in the cross section $\phi = \text{const}$, which is determined by the formula

$$I_\Sigma(\phi) = \int_{-d}^{d} I(\phi, z) dz,$$

is finite. Upon calculation of $I_\Sigma(\phi)$, one can find the input impedance $Z = R + iX$ of the antenna by the formula $Z = V_0 / I_\Sigma(\phi_0)$.

5. POWER RADIATED

We now derive an expression for the total power radiated from the antenna with the obtained current in the presence of a cylindrical plasma column immersed in an isotropic medium. With allowance for the guided eigenmodes, the expression for the total radiated power in the case of a loss-free open guide takes the form

$$P_\Sigma = \sum_{m, \pm n} P_{m, \pm n} + P_{\text{rad}}, \tag{47}$$

where $P_{\rm rad}$ is the power radiated to the surrounding medium and $P_{m,\pm n}$ the power going to an eigenmode with the indices m and $\pm n$. As in the above, the signs "+" and "−" of the subscript n stand to denote the eigenmodes propagating from the source region in the positive and negative directions of the z axis, respectively. The quantities $P_{m,\pm n}$ are calculated in a standard way [8, 18, 20]. The power $P_{\rm rad}$ can be found by integrating the time-averaged Poynting vector over the surface of a cylinder of radius $\rho = \rho_a \geq a$:

$$P_{\rm rad} = \frac{1}{2}\int_{-\infty}^{\infty} dz \int_0^{2\pi} {\rm Re}[E_\phi(\mathbf{r})H_z^*(\mathbf{r}) - E_z(\mathbf{r})H_\phi^*(\mathbf{r})]\Big|_{\rho=\rho_a} \rho d\phi. \quad (48)$$

Here, the asterisk denotes complex conjugation. Integrals on the right-hand side of Equation (48) can be reduced to a more convenient form by using the representations

$$E_{\phi,z}(\mathbf{r}) = \sum_{m=-\infty}^{\infty} e^{-im\phi}\frac{k_0}{2\pi}\int_{-\infty}^{\infty} E_{\phi,z;m}(\rho,p)e^{-ik_0pz}dp, \quad (49)$$

$$H_{\phi,z}^*(\mathbf{r}) = \sum_{\tilde{m}=-\infty}^{\infty} e^{i\tilde{m}\phi}\frac{k_0}{2\pi}\int_{-\infty}^{\infty} H_{\phi,z;\tilde{m}}^*(\rho,\tilde{p})e^{ik_0\tilde{p}z}d\tilde{p}. \quad (50)$$

Substituting representations (49) and (50) into Equation (48), we make use of the well-known formulas

$$\frac{1}{2\pi}\int_0^{2\pi} e^{-i(m-\tilde{m})\phi}d\phi = \delta_{m,\tilde{m}}, \quad \frac{k_0}{2\pi}\int_{-\infty}^{\infty} e^{-ik_0(p-\tilde{p})z}dz = \delta(p-\tilde{p}),$$

where $\delta_{m,\tilde{m}}$ is the Kronecker delta. Then, allowing for the Wronskians of cylindrical functions and the fact that the quantity q is purely imaginary for $p > \varepsilon_a^{1/2}$ and $p < -\varepsilon_a^{1/2}$, we obtain after some algebra

$$P_{\rm rad} = (Z_0\pi)^{-1}\sum_{m=-\infty}^{\infty}\int_{-\sqrt{\varepsilon_a}}^{\sqrt{\varepsilon_a}} \left(|C_m|^2 + \varepsilon_a^{-1}|D_m|^2\right)dp. \quad (51)$$

Recall that the coefficients C_m and D_m in (51) are given in Appendix A.

6. NUMERICAL RESULTS

The results of the preceding sections can be used to analyze the current distribution and input impedance of the antenna. We now present some numerical results illustrating the behavior of these characteristics. Calculations have been performed for the following values of parameters: the plasma density inside the column is equal to

$N = 10^{11}\,\text{cm}^{-3}$, the external static magnetic field $B_0 = 200\,\text{G}$, and the relative dielectric permittivity of the background medium is equal to $\varepsilon_a = 1$, which corresponds to free space. With these values, which can easily be realized under laboratory conditions, the plasma has $\omega_p = 1.78 \times 10^{10}\,\text{s}^{-1}$ and $\omega_H = 3.51 \times 10^9\,\text{s}^{-1}$. Since presenting results for all nonresonant frequency ranges would take up much space, we will dwell on the case where the angular frequency ω lies in the range $\omega_H < \omega < \omega_{\text{UH}}/\sqrt{2}$. Here, the upper hybrid frequency is given by $\omega_{\text{UH}} = (\omega_p^2 + \omega_H^2)^{1/2}$. The quantities Ω_H and ω_{LH}, which are much lower than ω in this case, can approximately be put equal to zero in (2). Note that in the chosen frequency range, the nonresonant plasma column with $\varepsilon < 0$ and $\eta < 0$ is capable of guiding an axisymmetric eigenmode of the surface type [22]. In addition, nonsymmetric eigenmodes of the surface type can exist at such frequencies.

It is assumed that the midpoint of the region to which the given voltage is supplied has the azimuthal coordinate $\phi_0 = 0$. The antenna radius a coincides with the column radius and ranges from 0.4 to 2.5 cm, $d/a = 0.02$, and $\Delta = 0.05\,\text{rad}$.

Since the current distribution of an antenna with a sufficiently large radius a can be influenced by the presence of eigenmodes guided by the column, we briefly discuss their dispersion properties and field structures. As an example, Fig. 3 shows the dispersion curves $p_{m,n}(\omega)$ of three surface modes with the azimuthal indices $m = 0, \pm 1$ for $a = 2.5\,\text{cm}$ and the above-mentioned plasma parameters inside the column. For the chosen values of parameters, it is these modes that notably affect the current distribution, whereas the influence of other modes is negligible. Recall that the mode propagation constants $p = p_{m,n}$ are roots of Equation (29). Since no more than one surface

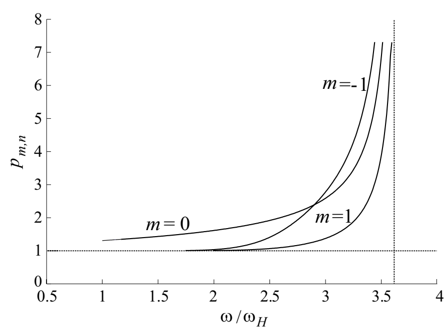


Figure 3. Dispersion curves of eigenmodes with the indices $m = 0, \pm 1$ and $n = 1$ for $N = 10^{11}\,\text{cm}^{-3}$, $B_0 = 200\,\text{G}$, $\varepsilon_a = 1$, and $a = 2.5\,\text{cm}$.

mode exists for a fixed azimuthal index m in the considered frequency range, the radial index of each mode is taken equal to $n = 1$. It is seen that the propagation constant of the axisymmetric eigenmode with the azimuthal index $m = 0$ tends to infinity as ω approaches the boundary frequency $\tilde{\omega} = \omega_{\mathrm{UH}}/\sqrt{2}$. The propagation constants of the modes with the indices $m = \pm 1$ tend to infinity at some other frequencies, which lie in the vicinity of $\tilde{\omega}$.

Figure 4 shows the field components of the axisymmetric ($m = 0$) eigenmode as functions of the radial coordinate ρ normalized to a at the angular frequency $\omega = 6.6 \times 10^9\,\mathrm{s}^{-1}$ if other parameters are the same as in Fig. 3. In this case, the diagonal elements of the dielectric tensor amount to $\varepsilon = -9.2$ and $\eta = -6.3$. The propagation constant of this eigenmode is equal to $p_{m,n} = 1.56$. It is seen that the mode demonstrates surface behavior and is of the quasi-TM type.

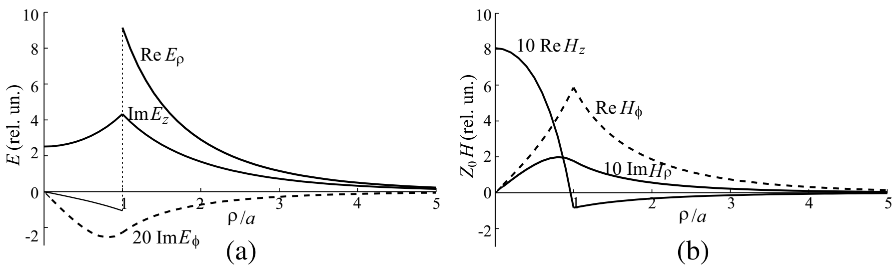
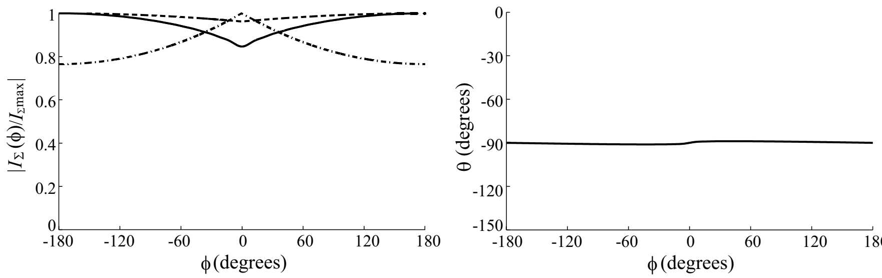


Figure 4. Fields (a) $\mathbf{E}_{m,n}(\rho)$ and (b) $\mathbf{H}_{m,n}(\rho)$ of an eigenmode with the indices $m = 0$ and $n = 1$ for $N = 10^{11}\,\mathrm{cm}^{-3}$, $B_0 = 200\,\mathrm{G}$, $\varepsilon_a = 1$, $a = 2.5\,\mathrm{cm}$, and $\omega = 6.6 \times 10^9\,\mathrm{s}^{-1}$.

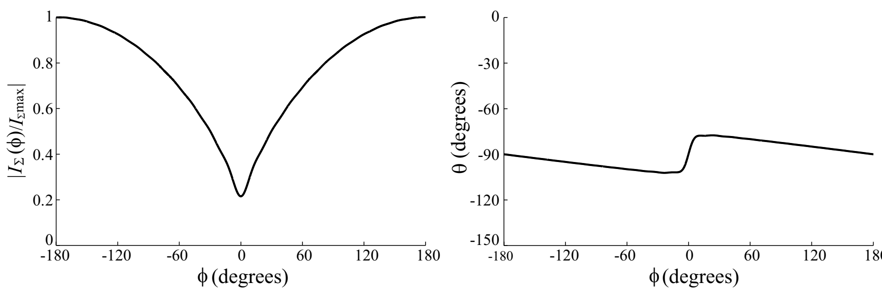
For relatively small values of a, the current distribution turns out to be similar to that of the same antenna in free space. This is illustrated by Fig. 5, which shows the absolute value $|I_\Sigma(\phi)|$, normalized to its maximum $|I_{\Sigma\max}|$, and the phase $\theta(\phi) = \arctan(\mathrm{Im}I_\Sigma(\phi)/\mathrm{Re}I_\Sigma(\phi))$ of the antenna current as functions of the azimuthal angle ϕ for $a = 0.4\,\mathrm{cm}$ (other parameters are the same as in Fig. 4). In addition, the left-hand plot of Fig. 5 shows the current distributions of the same antenna located in free space and a homogeneous magnetoplasma the parameters of which coincide with those of the plasma medium inside the column. We do not show the phase distributions of the antenna current in a homogeneous plasma or free space, because they almost coincide with the curve on the right-hand plot of Fig. 5.

For a larger radius of the antenna and the plasma column, we observe a greater difference between the current distributions of the



(a) (b)

Figure 5. Normalized amplitude (a) and phase (b) of the antenna current as functions of the angle ϕ if the antenna is located on the surface of a plasma column (solid line), in free space (dashed line), and in a homogeneous magnetoplasma (dashed-dotted line) for $a = 0.4\,\mathrm{cm}$, $d/a = 0.02$, $\Delta = 0.05\,\mathrm{rad}$, $\phi_0 = 0$, $N = 10^{11}\,\mathrm{cm}^{-3}$, $B_0 = 200\,\mathrm{G}$, and $\omega = 6.6 \times 10^9\,\mathrm{s}^{-1}$.



(a) (b)

Figure 6. Normalized amplitude (a) and phase (b) of the antenna current as functions of the angle ϕ for $a = 1\,\mathrm{cm}$, $d/a = 0.02$, $\Delta = 0.05\,\mathrm{rad}$, $\phi_0 = 0$, $N = 10^{11}\,\mathrm{cm}^{-3}$, $B_0 = 200\,\mathrm{G}$, and $\omega = 6.6 \times 10^9\,\mathrm{s}^{-1}$.

antenna placed on the surface of the plasma column and the same antenna located in free space or a homogeneous magnetoplasma. For example, Figs. 6 and 7 present the absolute values and phases of the antenna current for $a = 1\,\mathrm{cm}$ and $a = 1.5\,\mathrm{cm}$, respectively.

Figure 8 corresponds to the radius $a = 1.7\,\mathrm{cm}$. In this case, the current distribution still resembles qualitatively that in the previous case.

The amplitude and phase distributions of the antenna current for a comparatively large radius $a = 2.5\,\mathrm{cm}$ are shown in Fig. 9. In this case, three surface eigenmodes (with the indices $m = 0, \pm 1$ and

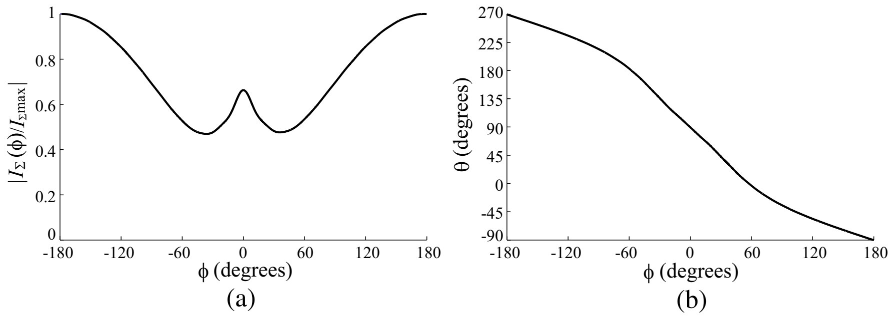


Figure 7. The same as in Fig. 6, but for $a = 1.5\,\text{cm}$.

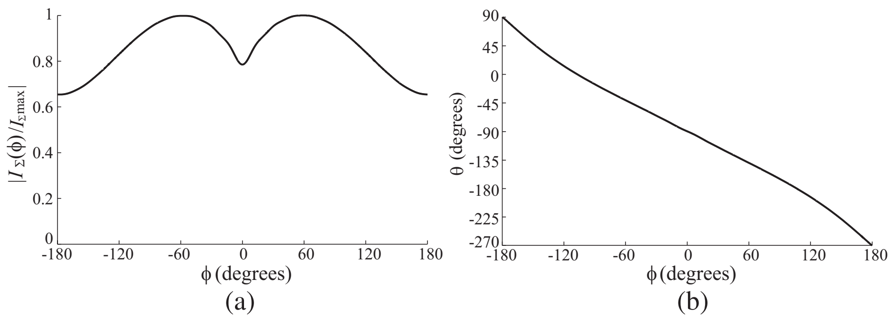


Figure 8. The same as in Fig. 6, but for $a = 1.7\,\text{cm}$.

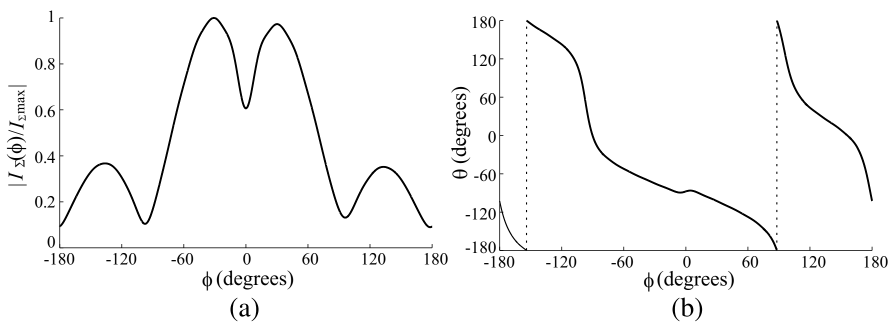


Figure 9. The same as in Fig. 6, but for $a = 2.5\,\text{cm}$.

$n = 1$) significantly affect the current distribution. If a loop antenna of the same radius were located in free space, it would have a current distribution close to that of a half-wave circular loop since $k_0a = 0.55$. The presence of the plasma column evidently leads to an essentially different current distribution of the antenna.

The distributions of Figs. 5–9 demonstrate some asymmetry about the midpoint of the region to which the excitation voltage is supplied. This asymmetry, which is the most pronounced in the dependences $\theta(\phi)$, is stipulated by the gyrotropy of the plasma inside the column. The reversal of the direction of the external magnetic field leads to a change in the current distribution. This change is described by the replacement $I_\Sigma(\phi) \to I_\Sigma(-\phi)$.

The obtained results can also be applied for finding the input impedance $Z = R + iX$ of the antenna. Note that the real part $R = \mathrm{Re}\, Z = \mathrm{Re}(V_0/I_\Sigma(\phi_0))$ of the input impedance, i.e., the input radiation resistance of the antenna, is completely determined by the nonsingular terms of the integral equation kernels in the case considered. Since the nonsingular parts $F_m(\zeta)$ were approximated by their values $F_m(0)$ at $\zeta = 0$ during the solution of the integral equations for the antenna current, the quantity R should be refined for relatively large values of a. A close examination shows that the most accurate result for R is obtained if we calculate the total radiated power P_Σ (see Section 5) for the found current distribution $I_\Sigma(\phi)$ and then put $R = 2P_\Sigma/|I_\Sigma(\phi_0)|^2$. It is interesting to mention that for the chosen parameters, the values of $R = \mathrm{Re}(V_0/I_\Sigma(\phi_0))$ and $R = 2P_\Sigma/|I_\Sigma(\phi_0)|^2$ almost coincide if $a < 1$ cm. For $1 < a < 2.5$ cm the difference between these quantities increases with increasing a, although the ratio of $R = \mathrm{Re}(V_0/I_\Sigma(\phi_0))$ to $R = 2P_\Sigma/|I_\Sigma(\phi_0)|^2$ remains a factor of order unity. Thus, in what follows we take $R = 2P_\Sigma/|I_\Sigma(\phi_0)|^2$ as the input radiation resistance. The input reactance of the antenna is determined by the imaginary part of the impedance Z and is calculated using the standard formula $X = \mathrm{Im}(V_0/I_\Sigma(\phi_0))$.

As an example, Table 1 presents R and X for the same plasma parameters in the column and its radii as in Figs. 5–9. In addition, we give the relative contributions of both the continuous-spectrum waves ($R_{\mathrm{cs}} = 2P_{\mathrm{rad}}/|I_\Sigma(\phi_0)|^2$) and the eigenmodes ($R_{\mathrm{mod}} = R - R_{\mathrm{cs}}$) to the quantity R. One can see a significant contribution of eigenmodes to R

Table 1. X, R, and the relative contributions of the continuous-spectrum waves and the eigenmodes to the radiation resistance.

a, cm	X, Ω	R, Ω	R_{cs}/R	R_{mod}/R
0.4	173.96	0.018	0.833	0.167
1	1361.80	6.021	0.925	0.075
1.5	−281.24	1.367	0.979	0.021
1.7	160.10	1.638	0.988	0.012
2.5	465.47	8.972	0.597	0.403

for a moderately large antenna radius a.

It is evident from the obtained numerical results that the presence of a column filled with a nonresonant magnetoplasma can significantly affect the electrodynamic characteristics of the loop antenna compared with the cases where it is located in free space [23] or a homogeneous magnetoplasma the parameters of which coincide with those inside the column. Recall that in the latter case, the current distribution decays exponentially along the strip conductor of the antenna with distance from the region to which the given voltage is supplied if $\varepsilon < 0$ and $\eta < 0$ [10].

7. CONCLUSION

In this article, we have obtained the solution to the problem of the current distribution of a loop antenna in the form of an infinitesimally thin, perfectly conducting narrow strip located on the surface of an axially magnetized plasma column. The found solution describes the distribution of the surface-current density both along and across the strip and makes it possible to study the electrodynamic characteristics of the antenna as functions of its parameters as well as the parameters of the plasma column and the surrounding medium. It is important that the developed method of solution, which has been used in the case where the plasma inside the column is nonresonant, can be extended to a more complex case of a resonant magnetoplasma when an infinite number of propagating quasielectrostatic eigenmodes exist on the plasma column. This case, which falls beyond the scope of this work, will be considered separately.

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APPENDIX A. COEFFICIENTS IN THE FIELD REPRESENTATION

The coefficients B_{mk}, C_m, and D_m in Equations (15) and (16), which ensure the fulfillment of the boundary conditions (14), are written as follows:

$$B_{m1} = \frac{\mathcal{I}_m(p)}{\Delta_m^{(2)}(p)} \frac{Z_0 k_0 a \chi^{(1)}}{Q_1 J_m(Q_1)} \left[\frac{\eta}{\varepsilon_a} n_2 \mathcal{J}_m^{(2)} \mathcal{H}_m^{(2)} + \frac{\eta}{\varepsilon_a} p \frac{m}{Q^2} \tilde{\mathcal{J}}_m^{(2)} - n_2 \left(\left(\mathcal{H}_m^{(2)}\right)^2 - \frac{p^2}{\varepsilon_a} \frac{m^2}{Q^4} \right) \right], \tag{A1}$$

$$B_{m2} = \frac{\mathcal{I}_m(p)}{\Delta_m^{(2)}(p)} \frac{Z_0 k_0 a \chi^{(2)}}{Q_2 J_m(Q_2)} \left[\frac{\eta}{\varepsilon_a} n_1 \tilde{\mathcal{J}}_m^{(1)} \mathcal{H}_m^{(2)} + \frac{\eta}{\varepsilon_a} p \frac{m}{Q^2} \mathcal{J}_m^{(1)} - n_1 \left(\left(\mathcal{H}_m^{(2)}\right)^2 - \frac{p^2}{\varepsilon_a} \frac{m^2}{Q^4} \right) \right], \tag{A2}$$

$$C_m = \frac{\mathcal{I}_m(p)}{\Delta_m^{(2)}(p)} \frac{Z_0 k_0 a}{Q H_m^{(2)}(Q)} \left[\left(n_1 \tilde{\mathcal{J}}_m^{(2)} - n_2 \mathcal{J}_m^{(1)} \right) \mathcal{H}_m^{(2)} + \frac{\eta}{\varepsilon_a} \left(\tilde{\mathcal{J}}_m^{(1)} - \mathcal{J}_m^{(2)} \right) p \frac{m}{Q^2} + \frac{\eta}{\varepsilon_a} \left(n_2 \mathcal{J}_m^{(1)} \mathcal{J}_m^{(2)} - n_1 \tilde{\mathcal{J}}_m^{(1)} \tilde{\mathcal{J}}_m^{(2)} \right) \right], \tag{A3}$$

$$D_m = \frac{\mathcal{I}_m(p)}{\Delta_m^{(2)}(p)} \frac{Z_0 k_0 a}{Q H_m^{(2)}(Q)} \left[\eta \left(\tilde{\mathcal{J}}_m^{(1)} - \mathcal{J}_m^{(2)} \right) \mathcal{H}_m^{(2)} + \left(n_1 \tilde{\mathcal{J}}_m^{(2)} - n_2 \mathcal{J}_m^{(1)} \right) p \frac{m}{Q^2} \right], \tag{A4}$$

where $\chi^{(1)} = -\chi^{(2)} = 1$, and other notations are defined in Section 3.

Since the azimuthal and longitudinal components of the electric field are continuous on the surface of the plasma column, either the coefficients (A1) and (A2) or coefficients (A3) and (A4) can be used to obtain the field components (21) and (22) at $\rho = a$. To derive the integral equations for the antenna current, we find it more convenient to utilize coefficients (A1) and (A2). Substituting them into the expressions for the azimuthal and longitudinal electric-field components at $\rho = a$, making use of the transformations described in Section 3, and allowing for the relationship $\Delta_m^{(2)}(e^{-i\pi}q) = -\Delta_m^{(1)}(q)$, one can arrive at kernels (23) and (24), the integrands of which contain the quantities given by Equations (26) and (27).

APPENDIX B. CONTRIBUTION OF EIGENMODES TO THE ANTENNA FIELD

As is shown in [8, 18, 20], to determine the contribution of eigenmodes guided by the plasma column to the total field which is excited by the antenna current, one should either calculate residues of the poles at $p = p_{m,n}$ in the complex p plane for the integrals in (11), or apply an eigenfunction expansion technique. Although both approaches yield identical results, the latter one is simpler in the case considered.

Using the results of [8, 18, 20], the contributions of eigenmodes to the electric and magnetic fields in the region $|z| < d$, which are hereafter denoted as $\mathbf{E}_{\rm mod}(\mathbf{r})$ and $\mathbf{H}_{\rm mod}(\mathbf{r})$, respectively, are written as

$$\begin{bmatrix}\mathbf{E}_{\rm mod}(\mathbf{r})\\ \mathbf{H}_{\rm mod}(\mathbf{r})\end{bmatrix} = \sum_{m=-\infty}^{\infty}\sum_{s=-}^{+}\sum_{n_s} a_{m,n_s}(z)\begin{bmatrix}\mathbf{E}_{m,n_s}(\mathbf{r})\\ \mathbf{H}_{m,n_s}(\mathbf{r})\end{bmatrix}. \tag{B1}$$

Here, $s = \pm$, $n_+ = n$, $n_- = -n$, and $a_{m,n_s}(z)$ are the expansion coefficients of the modal fields which are defined as follows:

$$\begin{bmatrix}\mathbf{E}_{m,n_\pm}(\mathbf{r})\\ \mathbf{H}_{m,n_\pm}(\mathbf{r})\end{bmatrix} = \begin{bmatrix}\mathbf{E}_{m,n_\pm}(\rho)\\ \mathbf{H}_{m,n_\pm}(\rho)\end{bmatrix}\exp(-im\phi \mp ik_0 p_{m,n} z). \tag{B2}$$

The expansion coefficients in (B1) can be found in the form [8, 18]

$$a_{m,n_s}(z) = \frac{1}{N_{m,n}}\int_{(z_s^{(1)},z_s^{(2)})}\mathbf{J}(\mathbf{r}')\cdot\mathbf{E}^{(T)}_{-m,-n_s}(\mathbf{r}')d\mathbf{r}', \tag{B3}$$

where the prime denotes the integration variables $\mathbf{r}' = (\rho', \phi', z')$ and the notation $(z_s^{(1)}, z_s^{(2)})$ designates the interval of integration with respect to z':

$$z_s^{(1)} = \begin{cases} -d & \text{for} \quad s=+, \\ z & \text{for} \quad s=-, \end{cases} \qquad z_s^{(2)} = \begin{cases} z & \text{for} \quad s=+, \\ d & \text{for} \quad s=-. \end{cases} \tag{B4}$$

We substitute the electric current in the form of Equations (6) and (7) into Equation (B3) and take into account the relationship $E^{(T)}_{\phi;-m,-n}(a) = E_{\phi;m,n}(a)$ [8]. After some algebra, we obtain the azimuthal and longitudinal components of the modal electric field $\mathbf{E}_{\rm mod}$ (see Equation (B1)) at $\rho = a$. Then we arrive at formulas for the modal terms that are summed over n in Equations (23) and (24).

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