Progress In Electromagnetics Research B, Vol. 51, 247–268, 2013

FIELDS IN FRACTIONAL PARALLEL PLATE D′B′, DB′ AND D′B WAVEGUIDES

Samina Gulistan*, **Farhat Majeed, and Aqeel A. Syed**

Department of Electronics, Quaid-i-Azam University, Islamabad 45320, Pakistan

Abstract—D′B′, DB′ and D′B boundary conditions are used to investigate the resulting field patterns inside a parallel plate waveguide. The D′B′ boundary conditions are accomodated by assigning the behavior of perfect magnetic conductor (PMC) for transverse electric mode (TE) and that of perfect electric conductor (PEC) for transverse magnetic (TM) mode, to the boundary, respectively. Likewise, DB′ boundary conditions are incorporated by assuming the behavior of boundary as PMC for both the TE mode and TM mode. Finally D′B boundary conditions are realized by assigning PEC characteristic to the boundary for both TE and TM modes. A general wave propagating inside the parallel plate waveguide is assumed and decomposed into TE and TM modes for the purpose of analysis. Fractional curl operator has been used to study the fractional parallel plate D′B′, DB′ and D′B waveguides for different values of fractional parameter α. Behavior of the field patterns in the waveguides are studied with respect to the fractional parameter α describing the order of the fractionalization.

1. INTRODUCTION

Fractional calculus is a branch of mathematical analysis which deals with the differentiation and integration operators, of arbitrary real (non-integer) or complex order [1]. It has been demonstrated that these mathematical operators are useful mathematical tools in various disciplines of science and engineering including Electromagnetic theory [2–5]. Fractionalization of ordinary derivative and integral operators motivated the researchers in electromagnetics to explore the potential of fractionalization of other operators in the field [6–25]. Engheta proposed a recipe to fractionalize the curl operator, describing

Received 22 March 2013, Accepted 22 April 2013, Scheduled 28 April 2013

* Corresponding author: Samina Gulistan (samina1025@gmail.com).

the differential form of Maxwell's equations [24]. He regarded the new solutions as intermediate between two dual solutions. In an isotropic, homogeneous, and source free medium described by wave number k and impedance η, the new set of solutions to the source-free Maxwell equations may be obtained by using the following relations [24]

$$\begin{aligned} \mathbf{E}_{\mathrm{fd}} &= \left[\frac{1}{(ik)^{\alpha}}\mathrm{curl}^{\alpha}\mathbf{E}\right] \\ \eta\mathbf{H}_{\mathrm{fd}} &= \left[\frac{1}{(ik)^{\alpha}}\mathrm{curl}^{\alpha}(\eta\mathbf{H})\right] \end{aligned} \tag{1}$$

From Eqs. (1) it can be seen that for $\alpha = 0$, ($\mathbf{E}_{\mathrm{fd}}$, $\eta\mathbf{H}_{\mathrm{fd}}$) reduces to the original solutions whereas ($\mathbf{E}_{\mathrm{fd}}$, $\eta\mathbf{H}_{\mathrm{fd}}$) gives dual to the original solution to the Maxwell equations for $\alpha = 1$. Therefore for all values of α between zero and unity, ($\mathbf{E}_{\mathrm{fd}}$, $\eta\mathbf{H}_{\mathrm{fd}}$) provides the new set of solutions which can effectively be regarded as intermediate solutions. These solutions are also called the fractional dual fields as expressed with the subscript fd. Naqvi and Rizvi extended Engheta's work on fractional curl operator by determining sources corresponding to the fractional dual solutions to the Maxwell equations. Results of their work show that surface impedance of a planar reflector, an intermediate between PEC and PMC, is anisotropic in nature [26]. Naqvi et al. further studied fractional dual solutions to the Maxwell equations for reciprocal, homogenous, and lossless chiral medium [27]. Lakhtakia pointed out that any fractional operator that commutes with curl operator may yield fractional solutions [23]. Naqvi and Abbas studied the role of complex and higher order fractional curl operators in electromagnetic wave propagation [28]. They also studied the fractional dual solutions in double negative (DNG) medium [29]. Veliev et al. extended the work on the fractional curl operator by finding the reflection coefficients and surface impedance corresponding to fractional dual planar surfaces with planar impedance surface as the original problem [30]. The work on this topic entered into new era when concepts of fractional transmission lines, fractional waveguides, and fractional resonators were introduced [31–40]. Modelling of transmission of electromagnetic plane wave through a chiral slab using fractional curl operator and fractional dual solutions in bi-isotropic medium are also available [41, 42]. After the introduction of nihility concept by Lakhtakia [43], Tretyakov et al. incorporated the nihility conditions to chiral medium and proposed another metamaterial termed as chiral nihility metamaterial [44, 45]. Study of nihility/chiral nihility metamaterials is a topic of current research by several researchers [46–57]. Naqvi contributed many research articles on chiral nihility and fractional dual solutions in chiral nihility metamaterial [51–

57].

In computational electromagnetics, special attention has been paid to newly introduced DB and D′B′ boundary conditions. A DB boundary requires that the normal components of electric and magnetic flux densities vanish at a DB interface [58–65], i.e.,

$$\begin{aligned} \hat{\mathbf{n}} \cdot \mathbf{D} &= 0 \\ \hat{\mathbf{n}} \cdot \mathbf{B} &= 0 \end{aligned} \tag{2}$$

where $\hat{\mathbf{n}}$ is normal vector to the interface. A D′B′ boundary is defined such that the derivatives of the normal components of the flux densities become zero, i.e.,

$$\begin{aligned} \partial_z D_z &= 0 \\ \partial_z B_z &= 0 \end{aligned}$$

These conditions are in contrast with traditional boundary conditions, like PEC (perfect electric conductor) or PMC (perfect magnetic conductor) boundary conditions, which restrict the freedom of the tangential field components only. It has been noted that PEC and PMC boundary conditions are special cases of DB or D′B′ boundary conditions. Another pair of boundary conditions, namely, DB′ and D′B can also be introduced along similar lines. All boundary conditions stated above are mathematical concepts. From the practical point of view, they can be realized in terms of physical structures, very precisely in many cases. In electromagnetics the PEC boundary corresponds to an interface of an ideal conducting material, which can be approximated by metals. In [66], it was shown that the DB boundary can be realized by an interface of uniaxial anisotropic medium, whose normal permittivity and permeability parameters become zero simultaneously. Such a uniaxial medium was named as zero axial parameter (ZAP) medium in [64]. Realization of the D′B′ boundary conditions is shown in [67], where it is suggested that the planar D′B′ boundary is realized by transforming a DB boundary, by means of a wave guiding quarter wave transformer. Such a device is a quarter wave slab of uniaxial medium with infinitely large axial parameter. It has been observed that DB interface behaves like perfect reflector for the right handed circularly polarized (RHCP) and left handed circularly polarized (LHCP) incident fields [65]. Moreover, when field is reflected from top and bottom of chiral nihility coated DB interface, it keeps on rotating its plane of polarization and it appears as a circularly polarized field inside the core of the waveguide [68, 69]. Fractional dual solutions to the Maxwell equations for fields inside a parallel plate DB waveguide have been discussed by Hussain et al. [70].

In the present work, to complete the study of set of boundary conditions requiring vanishing of the normal components of the flux densities D and B (DB boundary) or their normal derivatives (D′B′, DB′, and D′B boundary), we discuss fractional dual solutions to the Maxwell equations for fields inside a parallel plate D′B′, DB′ and D′B waveguide. A variety of field configurations (electric and magnetic) can be obtained by applying either any of the D′B′, DB′, D′B boundary conditions or the fractionalization operator. Such configurations may be required for some particular applications, e.g., couplers. So if any one desire to get some particular field pattern in any experiment or in some device, this work can serve the purpose. In Section 2 behavior of waves along a guiding structure is discussed. In Section 3 fractional dual solutions of D′B′, DB′, D′B are derived. Section 4 deals with results and discussions and paper has been concluded in Section 5.

2. GENERAL BEHAVIOUR OF WAVES ALONG A GUIDING STRUCTURE

Consider a waveguide consisting of two parallel plates one located at $y = 0$, other at $y = b$ and separated by a dielectric medium having constitutive parameters ϵ and μ. The plates are assumed to be of infinite extent and the direction of propagation is taken along positive z-axis as shown in Figure 1. Electric and magnetic fields propagating in the source free dielectric region must satisfy the following homogeneous vector Helmholtz equations

$$\nabla^2\mathbf{E}(x,y,z) + k^2\mathbf{E}(x,y,z) = 0 \tag{3a}$$

$$\nabla^2\mathbf{H}(x,y,z) + k^2\mathbf{H}(x,y,z) = 0 \tag{3b}$$

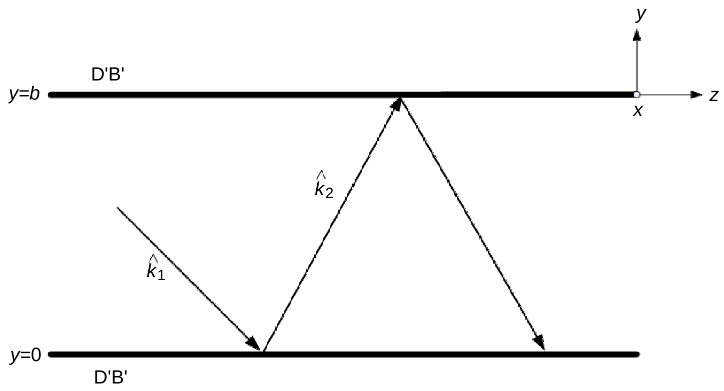


Figure 1. Plane wave representation of the fields inside the waveguide.

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator and $k = \omega\sqrt{\mu\epsilon}$ is the wave number. By taking $z-$dependance as $\exp(i\beta z)$, Eqs. (3a) and (3b) reduce to two dimensional vector Helmholtz equations as

$$\nabla^2_{xy}\mathbf{E}(x,y) + h^2\mathbf{E}(x,y) = 0 \tag{4a}$$

$$\nabla^2_{xy}\mathbf{H}(x,y) + h^2\mathbf{H}(x,y) = 0 \tag{4b}$$

where $h^2 = k^2 - \beta^2$, β is the propagation constant. Since propagation direction is along z-axis and the waveguide dimensions are taken to be infinite in xz-plane, so x-dependence can be ignored in Eqs. (4a) and (4b). Under this condition, it will take the form of ordinary, second order differential equation as

$$\frac{d^2\mathbf{E}(y)}{dy^2} + h^2\mathbf{E}(y) = 0 \tag{5a}$$

$$\frac{d^2\mathbf{H}(y)}{dy^2} + h^2\mathbf{H}(y) = 0 \tag{5b}$$

In general, for the waveguide problems, the Helmholtz equation is solved for the axial field components only. The transverse field components can be obtained by using axial components of the fields and Maxwell equations. So scalar Helmholtz equations for the axial components can be written as

$$\frac{d^2E_z}{dy^2} + h^2E_z = 0 \tag{5c}$$

$$\frac{d^2H_z}{dy^2} + h^2H_z = 0 \tag{5d}$$

General solution of the above equations is

$$E_z = a_n\cos(hy) + b_n\sin(hy) \tag{6a}$$

$$H_z = c_n\cos(hy) + d_n\sin(hy) \tag{6b}$$

where a_n, b_n, c_n and d_n are constants and can be found from the boundary conditions. Using Maxwell curl equations, the transverse components can be expressed in terms of longitudinal components (E_z, H_z), i.e.,

$$E_x = \frac{1}{h^2}\left(i\beta\frac{\partial E_z}{\partial x} + ik\frac{\partial \eta H_z}{\partial y}\right) \tag{7a}$$

$$E_y = \frac{1}{h^2}\left(i\beta\frac{\partial E_z}{\partial y} - ik\frac{\partial \eta H_z}{\partial x}\right) \tag{7b}$$

$$H_x = \frac{1}{h^2}\left(i\beta\frac{\partial H_z}{\partial x} - \frac{ik}{\eta}\frac{\partial E_z}{\partial y}\right) \tag{7c}$$

$$H_y = \frac{1}{h^2}\left(i\beta\frac{\partial H_z}{\partial y} + \frac{ik}{\eta}\frac{\partial E_z}{\partial x}\right) \tag{7d}$$

where $\eta = \sqrt{\frac{\mu}{\epsilon}}$ is impedance of the medium inside the guide.

3. FRACTIONAL DUAL WAVEGUIDES

A wave of general polarization propagating in positive z-direction through a parallel plate waveguide can be written as a linear sum of the transverse electric (TE^z) and transverse magnetic (TM^z) modes. A D′B′ boundary can be simulated as the boundary which behaves like perfect electric conductor (PEC) for (TM^z) and perfect magnetic conductor (PMC) for (TE^z) modes. Therefore fields inside a parallel plate D′B′ waveguide may be obtained by linear superposition of two canonical solutions which are transverse electric (TE^z) mode solution for PMC waveguide and transverse magnetic (TM^z) mode solution for PEC waveguide.

3.1. D′B′ Waveguide

3.1.1. Case 1: Transverse Electric (TE^z) Mode Propagation through a PMC Waveguide

Let us first consider that (TE^z) mode is propagating through a PMC waveguide. For this mode, axial component of the electric field becomes zero and the corresponding transverse components can be found by using Eqs. (7a)–(7d).

$$E_x = \left(\frac{ik}{h}\right)[-c_n\sin(hy) + d_n\cos(hy)] \tag{8a}$$

$$H_y = \left(\frac{i\beta}{h}\right)[-c_n\sin(hy) + d_n\cos(hy)] \tag{8b}$$

$$E_y = 0 \tag{8c}$$

$$H_x = 0 \tag{8d}$$

Using boundary conditions for PMC boundary, that is, $H_{x,z} = 0|_{y=0,b}$, we get solutions as

$$E_x = \left(\frac{ik}{h}\right)[D_n\cos(hy)] \tag{9a}$$

$$\eta H_y = \left(\frac{i\beta}{h}\right)[D_n\cos(hy)] \tag{9b}$$

$$\eta H_z = [D_n \sin(hy)] \tag{9c}$$
$$E_y = 0 \tag{9d}$$
$$H_x = 0 \tag{9e}$$
$$\text{where} \quad D_n = d_n \eta \quad h = \frac{n\pi}{b} \quad n = 1, 2, 3 \dots$$

By taking again the z-dependance $\exp(i\beta z)$ and writing Eqs. (9) in exponential form. Electric and magnetic fields inside the dielectric region will be obtained as sum of two plane waves given as

$$\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} \tag{10a}$$
$$\eta\mathbf{H} = \eta\mathbf{H_1} + \eta\mathbf{H_2} \tag{10b}$$

where (E_1, H_1) are the electric and magnetic fields associated with one plane wave, and (E_2, H_2) are the electric and magnetic fields associated with the second plane wave. These fields are given as following

$$\mathbf{E}_1 = \left(\frac{D_n}{2}\right)\left(\frac{ik}{h}\hat{\mathbf{x}}\right)\exp(ihy + i\beta z) \tag{11a}$$
$$\eta\mathbf{H}_1 = \left(\frac{D_n}{2}\right)\left(\frac{\hat{z}}{i} + \frac{i\beta}{h}\hat{\mathbf{y}}\right)\exp(ihy + i\beta z) \tag{11b}$$
$$\mathbf{E}_2 = \left(\frac{D_n}{2}\right)\left(\frac{ik}{h}\hat{\mathbf{x}}\right)\exp(-ihy + i\beta z) \tag{11c}$$
$$\eta\mathbf{H}_2 = \left(\frac{D_n}{2}\right)\left(\frac{-\hat{z}}{i} + \frac{i\beta}{h}\hat{\mathbf{y}}\right)\exp(-ihy + i\beta z). \tag{11d}$$

This situation is shown in Figure 1.

Once we have obtained electric and magnetic fields inside the dielectric region in terms of two plane waves, recipe for fractionalization [24, 32] can be applied to get the fractional dual solutions as

$$\begin{aligned}\mathbf{E}^{\mathrm{TE}}_{\mathrm{PMCfd}} = D_n\left(\frac{k}{h}\right)\Bigg[& i\cos\left(\frac{\alpha\pi}{2}\right)\cos\left(hy + \frac{\alpha\pi}{2}\right)\hat{\mathbf{x}} \\ & -\frac{\beta}{k}\sin\left(\frac{\alpha\pi}{2}\right)\sin(hy + \frac{\alpha\pi}{2})\hat{\mathbf{y}} \\ & -i\frac{h}{k}\sin\left(\frac{\alpha\pi}{2}\right)\cos\left(hy+\frac{\alpha\pi}{2}\right)\hat{\mathbf{z}}\Bigg]\exp\left[i\left(\beta z-\frac{\alpha\pi}{2}\right)\right]\end{aligned} \tag{12a}$$

$$\begin{aligned}\eta\mathbf{H}^{\mathrm{TE}}_{\mathrm{PMCfd}} = D_n\left(\frac{k}{h}\right)\Bigg[& \sin\left(\frac{\alpha\pi}{2}\right)\sin\left(hy + \frac{\alpha\pi}{2}\right)\hat{\mathbf{x}} \\ & +i\frac{\beta}{k}\cos\left(\frac{\alpha\pi}{2}\right)\cos\left(hy + \frac{\alpha\pi}{2}\right)\hat{\mathbf{y}} \\ & +\frac{h}{k}\cos\left(\frac{\alpha\pi}{2}\right)\sin\left(hy + \frac{\alpha\pi}{2}\right)\hat{\mathbf{z}}\Bigg]\exp\left[i\left(\beta z-\frac{\alpha\pi}{2}\right)\right]\end{aligned} \tag{12b}$$

3.1.2. Case 2: Transverse Magnetic (TM^z) Mode Propagation through a PEC Waveguide

Similar to the treatment done in Case 1, using Eqs. (7a)–(7d) results for transverse magnetic mode propagating through a PEC waveguide can be written as,

$$\mathbf{E}^{\mathrm{TM}}_{\mathrm{PECfd}} = B_n\left(\frac{k}{h}\right)\left[\sin\left(\frac{\alpha\pi}{2}\right)\sin\left(hy+\frac{\alpha\pi}{2}\right)\hat{\mathbf{x}}\right.$$
$$+i\frac{\beta}{k}\cos\left(\frac{\alpha\pi}{2}\right)\cos\left(hy+\frac{\alpha\pi}{2}\right)\hat{\mathbf{y}}$$
$$\left.+\frac{h}{k}\cos\left(\frac{\alpha\pi}{2}\right)\sin\left(hy+\frac{\alpha\pi}{2}\right)\hat{\mathbf{z}}\right]\exp\left[i\left(\beta z-\frac{\alpha\pi}{2}\right)\right] \quad (13a)$$

$$\eta\mathbf{H}^{\mathrm{TM}}_{\mathrm{PECfd}} = B_n\left(\frac{k}{h}\right)\left[-i\cos\left(\frac{\alpha\pi}{2}\right)\cos\left(hy+\frac{\alpha\pi}{2}\right)\hat{\mathbf{x}}\right.$$
$$+\frac{\beta}{k}\sin\left(\frac{\alpha\pi}{2}\right)\sin\left(hy+\frac{\alpha\pi}{2}\right)\hat{\mathbf{y}}$$
$$\left.+i\frac{h}{k}\sin\left(\frac{\alpha\pi}{2}\right)\cos\left(hy+\frac{\alpha\pi}{2}\right)\hat{\mathbf{y}}\right]\exp\left[i\left(\beta z-\frac{\alpha\pi}{2}\right)\right] \quad (13b)$$

Fractional dual solutions for the D′B′ waveguide can be written by taking linear sum of the fractional dual fields of the above two cases as

$$\mathbf{E}_{\mathrm{fd}} = \mathbf{E}^{TE}_{\mathrm{PMCfd}} + \mathbf{E}^{TM}_{\mathrm{PECfd}}$$
$$\eta\mathbf{H}_{\mathrm{fd}} = \eta\mathbf{H}^{TE}_{\mathrm{PMCfd}} + \eta\mathbf{H}^{TM}_{\mathrm{PECfd}}$$

which give

$$\mathbf{E}_{\mathrm{fd}} = \left(\frac{k}{h}\right)\exp\left[i\left(\beta z-\frac{\alpha\pi}{2}\right)\right]\left[(B_nS_\alpha S_{y+\alpha}+iD_nC_\alpha C_{y+\alpha})\hat{\mathbf{x}}\right.$$
$$+\frac{\beta}{k}(iB_nC_\alpha C_{y+\alpha}-D_nS_\alpha S_{y+\alpha})\hat{\mathbf{y}}$$
$$\left.+\frac{h}{k}(B_nC_\alpha S_{y+\alpha}-iD_nS_\alpha C_{y+\alpha})\hat{\mathbf{z}}\right] \quad (14a)$$

$$\eta\mathbf{H}_{\mathrm{fd}} = \left(\frac{k}{h}\right)\exp\left[i\left(\beta z-\frac{\alpha\pi}{2}\right)\right]\left[(D_nS_\alpha S_{y+\alpha}-iB_nC_\alpha C_{y+\alpha})\hat{\mathbf{x}}\right.$$
$$+\frac{\beta}{k}(B_nS_\alpha S_{y+\alpha}+iC_\alpha C_{y+\alpha})\hat{\mathbf{y}}$$
$$\left.+\frac{h}{k}(iB_nS_\alpha C_{y+\alpha}+D_nC_\alpha S_{y+\alpha})\hat{\mathbf{z}}\right] \quad (14b)$$

with

$$S_\alpha = \sin\left(\frac{\alpha\pi}{2}\right) \quad S_{y+\alpha} = \sin\left(hy + \frac{\alpha\pi}{2}\right)$$
$$C_\alpha = \cos\left(\frac{\alpha\pi}{2}\right) \quad C_{y+\alpha} = \cos\left(hy + \frac{\alpha\pi}{2}\right)$$

B_n, D_n are the constants to be determined from the initial conditions.

3.2. DB′ and D′B waveguides

In DB′ waveguide, the DB′ boundary behaves like PMC boundary for the both modes, i.e., (TE^z) and (TM^z). After solving on similar lines as for D′B′ waveguide, fractional dual solutions for the DB′ waveguide can be written as

$$\begin{aligned}
\mathbf{E}_{\text{fd}} = \left(\frac{k}{h}\right) \Bigg[& \left(iD_n C_\alpha C_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right] \right. \\
& \left. -A_n S_\alpha C_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{x}} \\
& -\left(\frac{\beta}{k}\right) \left(D_n S_\alpha S_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right] \right. \\
& \left. +iA_n C_\alpha S_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{y}} \\
& +\left(\frac{h}{k}\right) \left(-iD_n S_\alpha C_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right] \right. \\
& \left. +A_n C_\alpha C_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{z}} \Bigg]
\end{aligned} \tag{15a}$$

$$\begin{aligned}
\eta\mathbf{H}_{\text{fd}} = \left(\frac{k}{h}\right) \Bigg[& \left(D_n S_\alpha S_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right] \right. \\
& \left. +iA_n C_\alpha S_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{x}} \\
& +\left(\frac{\beta}{k}\right) \left(iD_n C_\alpha C_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right] \right. \\
& \left. -A_n S_\alpha C_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{y}} \\
& +\left(\frac{h}{k}\right) \left(D_n C_\alpha S_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right] \right. \\
& \left. +iA_n S_\alpha S_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{z}} \Bigg]
\end{aligned} \tag{15b}$$

In D′B waveguide, the D′B boundary behaves like PEC boundary for the both modes, i.e., (TE^z) and (TM^z). After solving along similar lines as for D′B′ waveguide, fractional dual solutions for the D′B waveguide can be written as

$$\begin{aligned}\mathbf{E}_{\rm fd} = \left(\frac{k}{h}\right) \Bigg[& \left(B_n S_\alpha S_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right]\right. \\ & \left. - i C_n C_\alpha S_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{x}} \\ & + \left(\frac{\beta}{k}\right)\left(i B_n C_\alpha C_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right]\right. \\ & \left. + C_n S_\alpha C_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{y}} \\ & + \left(\frac{h}{k}\right)\left(B_n C_\alpha S_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right]\right. \\ & \left. - i D_n S_\alpha S_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{z}} \Bigg] \end{aligned} \tag{16a}$$

$$\begin{aligned}\eta\mathbf{H}_{\rm fd} = \left(\frac{k}{h}\right) \Bigg[& \left(-i B_n C_\alpha C_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right]\right. \\ & \left. - C_n S_\alpha C_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{x}} \\ & + \left(\frac{\beta}{k}\right)\left(B_n S_\alpha S_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right]\right. \\ & \left. - i C_n C_\alpha S_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{y}} \\ & + \left(\frac{h}{k}\right)\left(i B_n S_\alpha C_{y+\alpha} \exp\left[i\left(\beta z - \frac{\alpha\pi}{2}\right)\right]\right. \\ & \left. + C_n C_\alpha C_{y+\alpha} \exp\left[i\left(\beta z + \frac{\alpha\pi}{2}\right)\right]\right) \hat{\mathbf{z}} \Bigg] \end{aligned} \tag{16b}$$

The fields given in Eqs. (14a)–(16b) are plotted in Figures 2, 3 and 4 by varying values of α between $[0,1]$ at an observation point $(hy, \beta z) = (\pi/4, \pi/4)$.

From Figures 2, 3 and 4 it can be seen that principle of duality is being satisfied by fractional dual fields, i.e., for $\alpha = 0$

$$E_{\rm fdx} = E_{\rm x}, \quad \eta H_{\rm fdx} = \eta H_{\rm x}$$
$$E_{\rm fdy} = E_{\rm y}, \quad \eta H_{\rm fdy} = \eta H_{\rm y}$$
$$E_{\rm fdz} = E_{\rm z}, \quad \eta H_{\rm fdz} = \eta H_{\rm z}$$

and for $\alpha = 1$

$$E_{\rm fdx} = \eta H_{\rm x}, \quad \eta H_{\rm fdx} = -E_{\rm x}$$
$$E_{\rm fdy} = \eta H_{\rm y}, \quad \eta H_{\rm fdy} = -E_{\rm y}$$
$$E_{\rm fdz} = \eta H_{\rm z}, \quad \eta H_{\rm fdz} = -E_{\rm z}$$

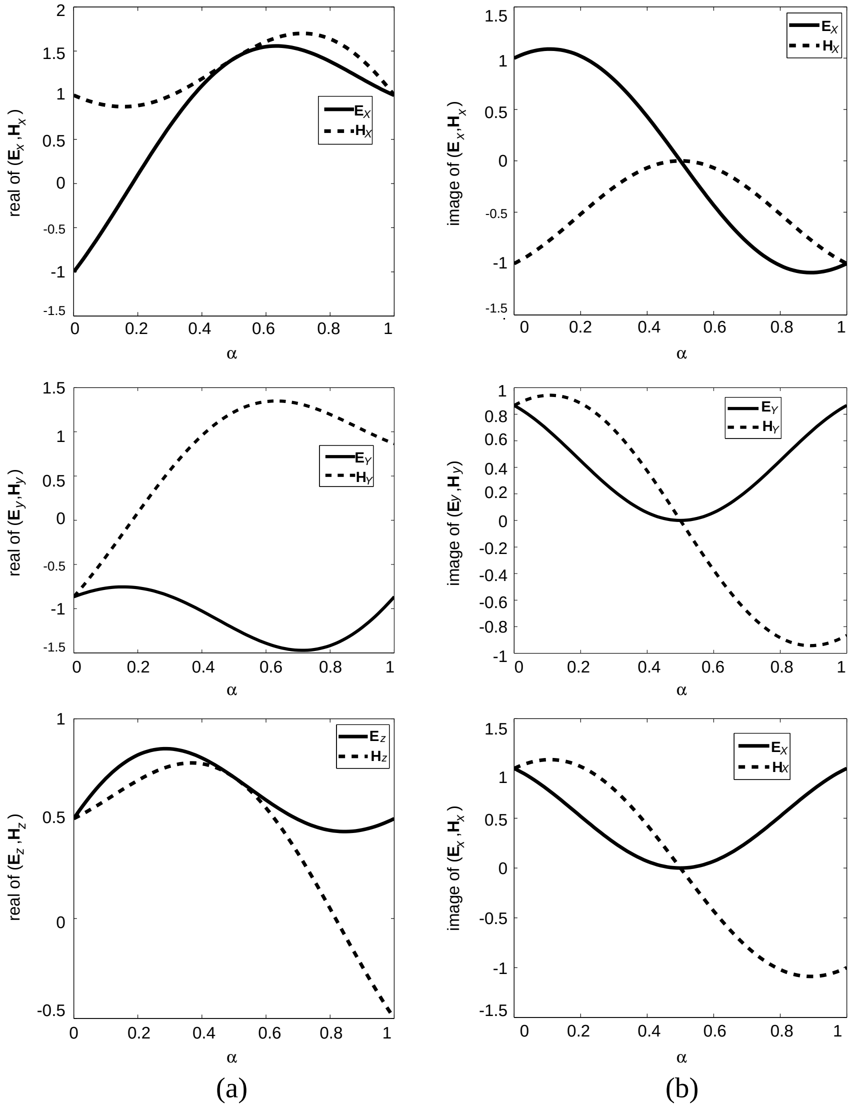
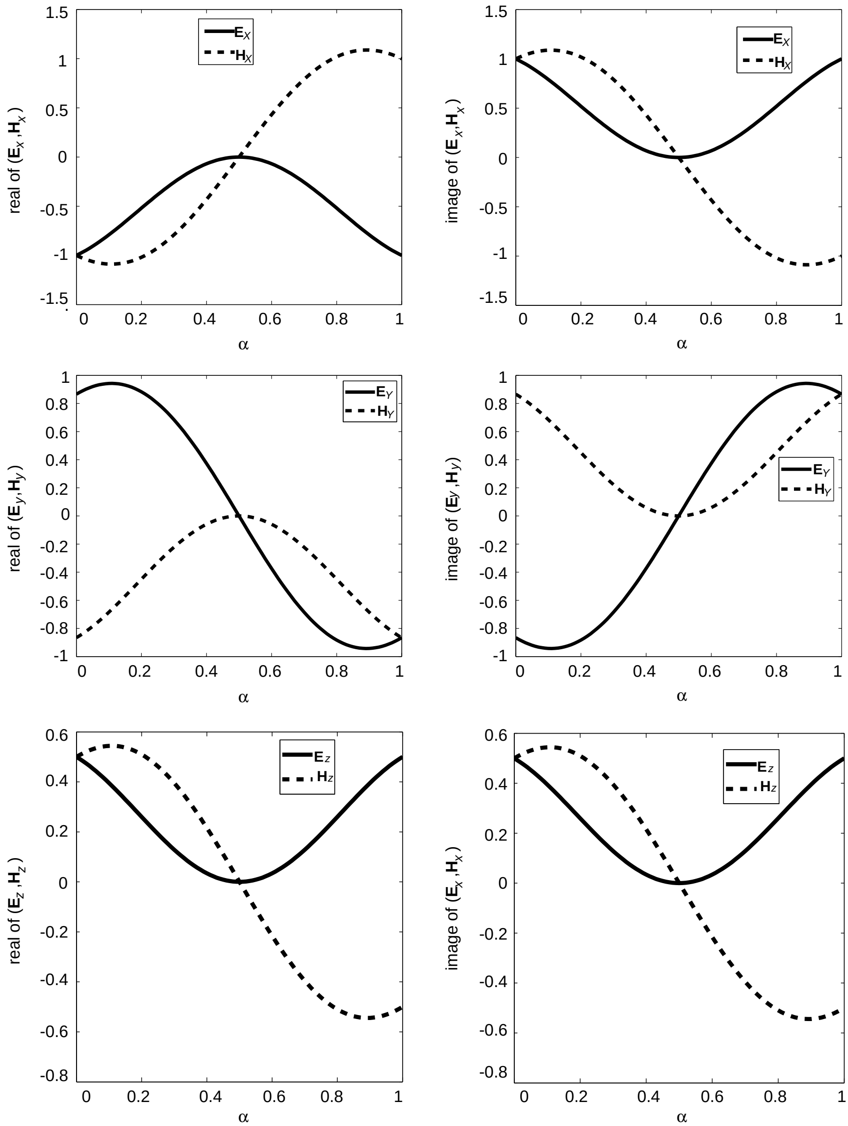


Figure 2. Plots of fractional dual fields for D′B′ waveguide, (a) real parts, (b) imaginary parts.

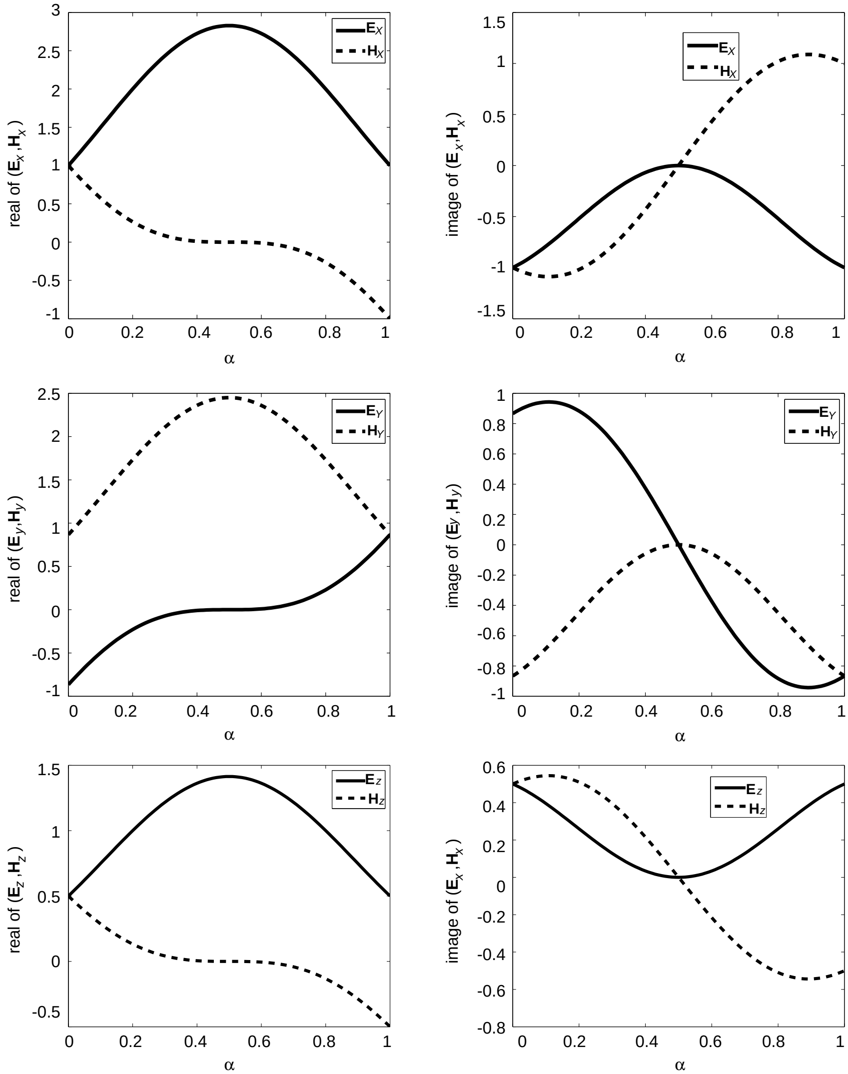
4. RESULTS AND DISCUSSION

In order to analyze the behavior of fractional fields inside the waveguides, plots of electric and magnetic field lines in the yz-plane are presented and are shown in Figures 5, 6 and 7. We have taken yz-



(a) (b)

Figure 3. Plots of fractional dual fields for DB′ waveguide, (a) real parts, (b) imaginary parts.



(a) (b)

Figure 4. Plots of fractional dual fields for D′B waveguide, (a) real parts, (b) imaginary parts.

plane as an observation plane. The instantaneous field expressions are obtained by multiplying the phasor vector expressions with $\exp(j\omega t)$ and taking the real part of the product. Equation that describe the behaviour of fractional fields at a given time t can be found from the following relation.

$$\frac{dy}{E_{\rm fdy}} = \frac{dz}{E_{\rm fdz}} \tag{17}$$

Field lines behavior is obtained by integrating above equation. These plots are for the mode propagating through the guide at an angle $\pi/6$ so that $\beta/k = \cos(\pi/6)$, $h/k = \sin(\pi/6)$. Initial conditions for both the modes are taken same. Electric as well as magnetic field plots for waveguides are shown by solid lines. From Figure 5 we see that there is no tangential component of the electric or the magnetic field for $\alpha = 0$.

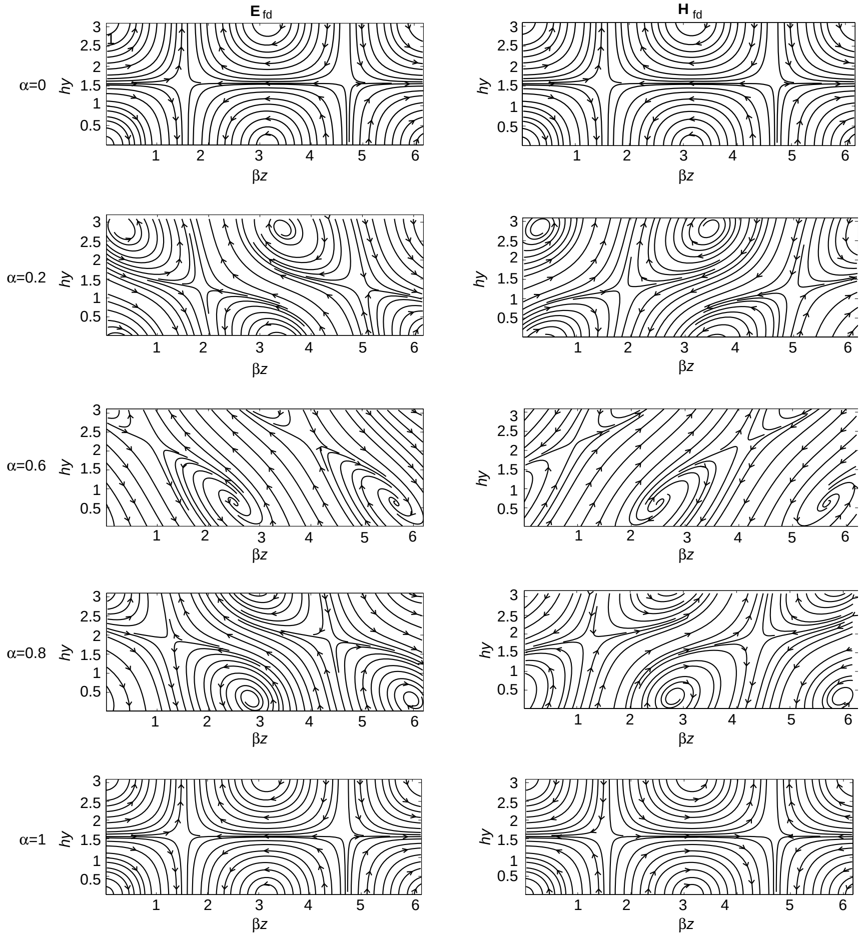


Figure 5. Field lines in yz-plane at different values of α; for D′B′ waveguide.

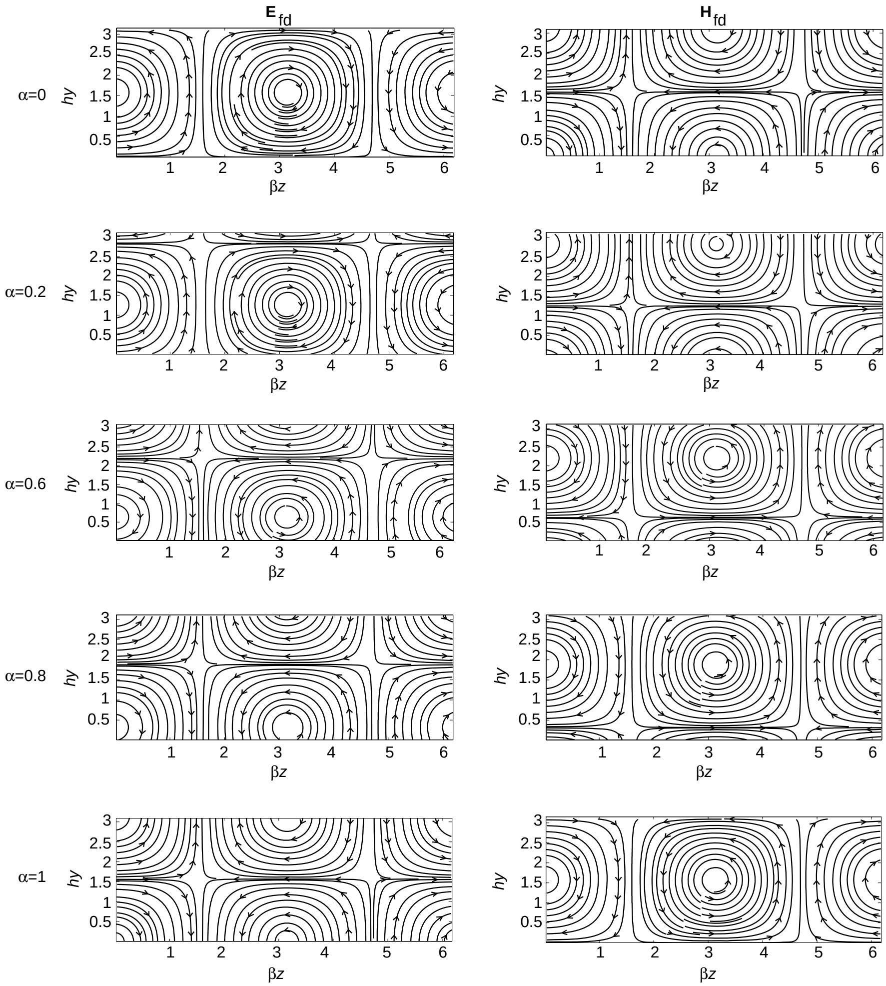


Figure 6. Field lines in yz-plane at different values of α; for DB$'$ waveguide.

This is because for D$'$B$'$ waveguide, the plates of the guide behave as perfect magnetic conductors for transverse electric components while they behave as perfect electric conductor for transverse magnetic mode. For the DB$'$ waveguide at $\alpha = 0$, there is no normal component of the electric filed at the guide surface while magnetic field has no tangential component. This is because for DB$'$ waveguide, the plates of the guide behave as perfect magnetic conductors for both, the transverse electric mode and transverse magnetic mode. For the D$'$B waveguide

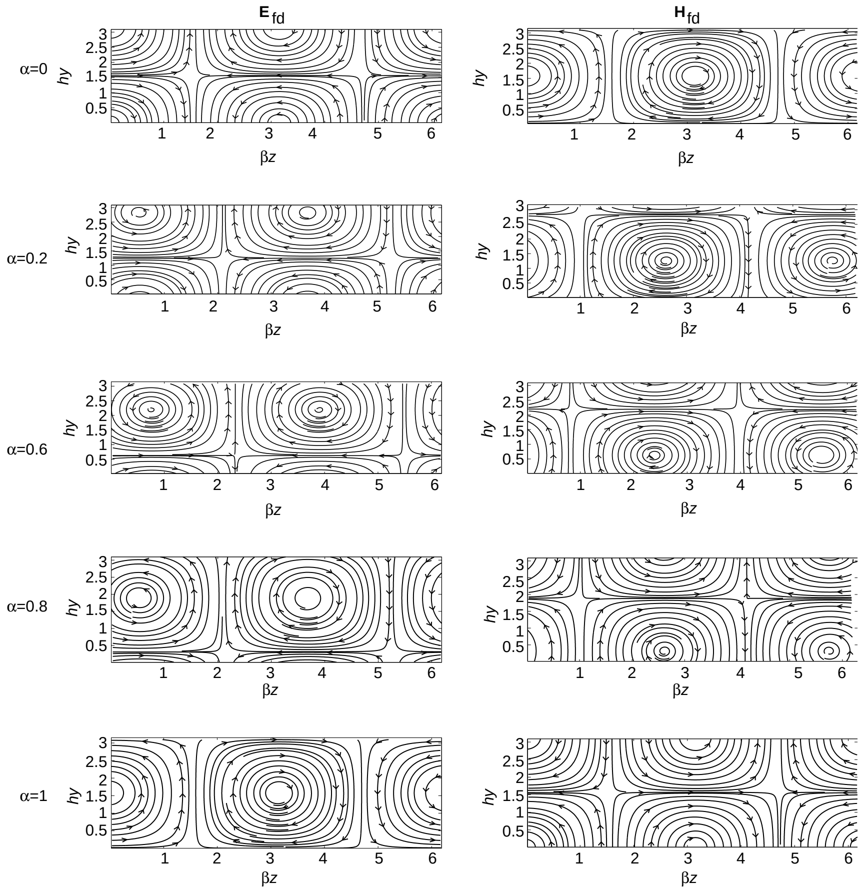


Figure 7. Field lines in yz-plane at different values of α; for D′B waveguide.

at $\alpha = 0$, there is no tangential component of the electric filed at the guide surface while magnetic field has no normal component. This is because for D′B waveguide, the plates of the guide behave as perfect electric conductors for both, the transverse electric mode and transverse magnetic mode. For all three cases at $\alpha = 1$, we can see clearly that electric field lines have attain the shape of magnetic field lines of $\alpha = 0$, and magnetic field lines have attain the shape of electric field lines of $\alpha = 0$ with opposite direction of arrows, i.e., solutions corresponds to dual waveguides. While for $0 < \alpha < 1$, the electric and magnetic field distributions corresponds to fractional dual waveguides.

5. CONCLUSIONS

Fractional dual solutions to the Maxwell equations for the fields inside a parallel plate D′B′, DB′ and D′B waveguides are derived using fractional curl operator. The purpose of this work was to complete the study of set of boundary conditions requiring vanishing of the normal components of the flux densities D and B (DB boundary) or their normal derivatives (D′B′, DB′ and D′B boundary). Electric and magnetic field distributions for limiting value of α corresponds to D′B′, DB′ or D′B waveguide and dual waveguide, while for $0 < \alpha < 1$ distributions of fractional dual fields are obtained. This work can serve the purpose to get a variety of field distributions.

REFERENCES

1. Oldham, K. B. and J. Spanier, *The Fractional Calculus,* Academic Press, New York, 1974.
2. Hilfer, R., *Applications of Fractional Calculus in Physics,* World Scientific, 2000.
3. Podlubny, I., *Fractional Differential Equations, Mathematics in Science and Engineering,* Vol. 198, XV–XXIV, Academic Press, 1999.
4. Das, S., *Functional Fractional Calculus for System Identification and Controls,* Springer-Verlag, 2008.
5. Debnath, L., "Recent applications of fractional calculus to science and engineering," *International Journal of Mathematics and Mathematical Sciences*, Vol. 54, 3413–3442, 2003.
6. Engheta, N., "Note on fractional calculus and the image method for dielectric spheres," *Journal of Electromagnetic Waves and Applications*, Vol. 9, No. 9, 1179–1188, 1995.
7. Engheta, N., "Use of fractional calculus to propose some fractional solution for the scalar Helmholtzs equation," *Progress In Electromagnetics Research*, Vol. 12, 107–132, 1996.
8. Engheta, N., "Electrostatic fractional image methods for perfectly conducting wedges and cones," *IEEE Transactions on Antennas and Propagation*, Vol. 44, 1565–1574, 1996.
9. Engheta, N., "On the role of fractional calculus in electromagnetic theory," *IEEE Antennas and Propagation Magazine*, Vol. 39, 35–46, 1997.
10. Engheta, N., "Phase and amplitude of fractional-order intermediate wave," *Microwave and Optical Technology Letters*, Vol. 21, 338–343, 1999.

11. Engheta, N., "Fractional paradigm in electromagnetic theory," *Frontiers in Electromagnetics*, Chapter 12, 523–552, D. H. Werner and R. Mittra, Eds., IEEE Press, 1999.
12. Tarasov, V. E., "Universal electromagnetic waves in dielectric," *J. Phys.: Condens. Matter*, Vol. 20, 175–223, 2008.
13. Tarasov, V. E., "Fractional integro-differential equations for electromagnetic waves in dielectric media," *Teoret. Mat. Fiz.*, Vol. 158, 419–424, 2009.
14. Musliha, S. I. and D. Baleanu, "Fractional multipoles in fractional space," *Nonlinear Analysis: Real World Applications*, Vol. 8, 198–203, 2007.
15. Baleanua, D., A. K. Golmankhanehb, and A. K. Golmankhaneh, "On electromagnetic field in fractional space," *Nonlinear Analysis: Real World Applications*, Vol. 11, 288–292, 2010.
16. Zubair, M., M. J. Mughal, and Q. A. Naqvi, "The wave equation and general plane wave solutions in fractional space," *Progress In Electromagnetics Research Letters*, Vol. 19, 137–146, 2010.
17. Zubair, M., M. J. Mughal, Q. A. Naqvi, and A. A. Rizvi, "Differential electromagnetic equations in fractional space," *Progress In Electromagnetics Research*, Vol. 114, 255–269, 2011.
18. Zubair, M., M. J. Mughal, and Q. A. Naqvi, "An exact solution of the cylindrical wave equation for electromagnetic field in fractional dimensional space," *Progress In Electromagnetics Research*, Vol. 114, 443–455, 2011.
19. Zubair, M., M. J. Mughal, and Q. A. Naqvi, "On electromagnetic wave propagation in fractional space," *Nonlinear Analysis: Real World Applications,* Vol. 12, 2844–2850, 2011.
20. Zubair, M., M. J. Mughal, and Q. A. Naqvi, "An exact solution of the spherical wave equation in d-dimensional fractional space," *Journal of Electromagnetic Waves and Applications*, Vol. 25, No. 10, 1481–1491, 2011.
21. Engheta, N., "On fractional paradigm and intermediate zones in Electromagnetism: I. Planar observation," *Microwave and Optical Technology Letters*, Vol. 22, 236–241, 1999.
22. Engheta, N., "On fractional paradigm and intermediate zones in Electromagnetism: II. Cylindrical and spherical observations," *Microwave and Optical Technology Letters*, Vol. 23, 100–103, 1999.
23. Lakhtakia, A., "A representation theorem involving fractional derivatives for linear homogeneous chiral media," *Microwave and Optical Technology Letters*, Vol. 28, 385–386, 2001.
24. Engheta, N., "Fractional curl operator in electromagnetics,"

Microwave and Optical Technology Letters, Vol. 17, 86–91, 1998.

25. Ozaktas, H. M., Z. Zalevsky, and M. A. Kutay, "The fractional Fourier transform with applications," *Optics and Signal Processing*, Wiley, New York, 2001.
26. Naqvi, Q. A. and A. A. Rizvi, "Fractional dual solutions and corresponding sources," *Progress In Electromagnetics Research*, Vol. 25, 223–238, 2000.
27. Naqvi, Q. A., G. Murtaza, and A. A. Rizvi, "Fractional dual solutions to Maxwell equations in homogeneous chiral medium," *Optics Communications*, Vol. 178, 27–30, 2000.
28. Naqvi, Q. A. and M. Abbas, "Complex and higher order fractional curl operator in electromagnetics," *Optics Communications*, Vol. 241, 349–355, 2004.
29. Naqvi, Q. A. and M. Abbas, "Fractional duality and metamaterials with negative permittivity and permeability," *Optics Communications*, Vol. 227, 143–146, 2003.
30. Veliev, E. I., M. V. Ivakhnychenko, and T. M. Ahmedov, "Fractional boundary conditions in plane waves diffraction on a strip," *Progress In Electromagnetics Research*, Vol. 79, 443–462, 2008.
31. Hussain, A. and Q. A. Naqvi, "Fractional curl operator in chiral medium and fractional nonsymmetric transmission line," *Progress In Electromagnetics Research*, Vol. 59, 199–213, 2006.
32. Hussain, A., S. Ishfaq, and Q. A. Naqvi, "Fractional curl operator and fractional waveguides," *Progress In Electromagnetics Research*, Vol. 63, 319–335, 2006.
33. Hussain, A., M. Faryad, and Q. A. Naqvi, "Fractional curl operator and fractional chiro-waveguide," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 8, 1119–1129, 2007.
34. Faryad, M. and Q. A. Naqvi, "Fractional rectangular waveguide," *Progress In Electromagnetics Research*, Vol. 75, 383–396, 2007.
35. Hussain, A. and Q. A. Naqvi, "Perfect electromagnetic conductor (PEMC) and fractional waveguide," *Progress In Electromagnetics Research*, Vol. 73, 61–69, 2007.
36. Maab, H. and Q. A. Naqvi, "Fractional surface waveguide," *Progress In Electromagnetics Research C*, Vol. 1, 199–209, 2008.
37. Hussain, A. and Q. A. Naqvi, "Fractional rectangular impedance waveguide," *Progress In Electromagnetics Research*, Vol. 96, 101–116, 2009.
38. Maab, H. and Q. A. Naqvi, "Fractional rectangular cavity

resonator," *Progress In Electromagnetics Research B*, Vol. 9, 69–82, 2008.

39. Hussain, A., M. Faryad, and Q. A. Naqvi, "Fractional waveguides with impedance walls," *Progress In Electromagnetics Research C*, Vol. 4, 191–204, 2008.
40. Hussain, A. and Q. A. Naqvi, "Fractional rectangular impedance waveguide," *Progress In Electromagnetics Research*, Vol. 96, 101–116, 2009.
41. Naqvi, S. A., Q. A. Naqvi, and A. Hussain, "Modelling of transmission through a chiral slab using fractional curl operator," *Optics Communications*, Vol. 266, 404–406, 2006.
42. Naqvi, S. A., M. Faryad, Q. A. Naqvi, and M. Abbas, "Fractional duality in homogeneous bi-isotropic medium," *Progress In Electromagnetics Research*, Vol. 78, 159–172, 2008.
43. Lakhtakia, A., "An electromagnetic trinity from "Negative permittivity and negative permeability," *Int. Journal of Infrared and Millimeter Waves*, Vol. 22, 1731–1734, 2001.
44. Tretyakov, S., I. Nefedov, A. Sihvola, and S. Maslovski, "A metamaterial with extreme properties: The chiral nihility," *Progress In Electromagnetics Research Symposium*, 468, Honolulu, Hawaii, USA, Oct. 13–16, 2003.
45. Tretyakov, S., I. Nefedov, A. Sihvola, S. Maslovski, and C. Simovski, "Waves and energy in chiral nihility," *Journal of Electromagnetic Waves and Applications*, Vol. 17, No. 5, 695–706, 2003.
46. Tretyakov, S. A., I. S. Nefedov, and P. Alitalo, "Generalized field transforming metamaterials," *New Journal of Physics*, Vol. 10, 115028, 2008.
47. Cheng, Q., T. J. Cui, and C. Zhang, "Waves in planar waveguide containing chiral nihility metamaterial," *Optics Communications*, Vol. 276, 317–321, 2007.
48. Zhang, C. and T. J. Cui, "Negative reflections of electromagnetic waves in chiral media," *Appl. Phys. Lett.*, Vol. 91, 194101, 2007.
49. Dong, J. F. and C. Xu, "Surface polaritons in planar chiral nihility meta-material waveguides," *Optics Communications*, Vol. 282, 3899–3904, 2009.
50. Naqvi, A., "Comments on waves in planar waveguide containing chiral nihility metamaterial," *Optics Communications*, Vol. 284, 215–216, 2011.
51. Naqvi, Q. A., "Fractional dual solutions to the Maxwell equations in chiral nihility medium," *Optics Communications*, Vol. 282,

2016–2018, 2009.
52. Naqvi, Q. A., "Planar slab of chiral nihility metamaterial backed by fractional dual/PEMC interface," *Progress In Electromagnetics Research*, Vol. 85, 381–391, 2009.
53. Naqvi, Q. A., "Fractional dual solutions in grounded chiral nihility slab and their effect on outside fields," *Journal of Electromagnetic Waves and Applications*, Vol. 23, Nos. 5–6, 773–784, 2009.
54. Naqvi, A., S. Ahmed, and Q. A. Naqvi, "Perfect electromagnetic conductor and fractional dual interface placed in a chiral nihility medium," *Journal of Electromagnetic Waves and Applications*, Vol. 24, Nos. 14–15, 1991–1999, 2010.
55. Illahi, A. and Q. A. Naqvi, "Study of focusing of electromagnetic waves reflected by a PEMC backed chiral nihility reflector using Maslov's method," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 7, 863–873, 2009.
56. Naqvi, Q. A., "Fractional dual interface in chiral nihility medium," *Progress In Electromagnetics Research Letters*, Vol. 8, 135–142, 2009.
57. Naqvi, A., A. Hussain, and Q. A. Naqvi, "Waves in fractional dual planar waveguides containing chiral nihility metamaterial," *Journal of Electromagnetic Waves and Applications*, Vol. 24, Nos. 11–12, 1575–1586, 2010.
58. Lindell, I. V. and A. H. Sihvola, "Zero axial parameter (ZAP) sheet," *Progress In Electromagnetics Research*, Vol. 89, 213–224, 2009.
59. Lindell, I. V. and A. H. Sihvola, "Uniaxial IB-medium interface and novel boundary conditions," *IEEE Transactions on Antennas and Propagation*, Vol. 57, 694–700, 2009.
60. Lindell, I. V. and A. Sihvola, "Circular waveguide with DB boundary conditions," *IEEE Trans. on Micro. Theory and Tech.*, Vol. 58, 903–909, 2010.
61. Lindell, I. V., H. Wallen, and A. Sihvola, "General electromagnetic boundary conditions involving normal field components," *IEEE Ant. and Wirel. Propag. Lett.*, Vol. 8, 877–880, 2009.
62. Sihvola, A., H. Wallen, and P. Yla-Oijala, "Scattering by DB spheres," *IEEE Ant. and Wirel. Propag. Lett.*, Vol. 8, 542–545, 2009.
63. Lindell, I. V. and A. Sihvola, "Electromagnetic boundary and its realization with anisotropic metamaterial," *Phys. Rev. E*, Vol. 79, 026604, 2009.
64. Lindell, I. V. and A. Sihvola, "Zero-axial-parameter (ZAP)

medium sheet," *Progress In Electromagnetics Research*, Vol. 89, 213–224, 2009.

65. Naqvi, A., F. Majeed, and Q. A. Naqvi, "Planar dB boundary placed in a chiral and chiral nihility metamaterial," *Progress In Electromagnetics Research Letters*, Vol. 21, 41–48, 2011.
66. Lindell, I. V. and A. Sihvola, "Electromagnetic boundary conditions defined in terms of normal field components," *IEEE Transactions on Antennas and Propagation*, Vol. 58, No. 4, Apr. 2010.
67. Lindell, I. V., A. H. Sihvola, L. Bergamin, and A. Favaro, "Realization of the D$'$B$'$ boundary condition," *IEEE Ant. and Wirel. Propag. Lett.*, Vol. 10, 2011.
68. Abbas, S. S., "Fractional electromagnetics for chiral and bi-isotropic media," Thesis, Department of Electronics, Quaid-i-Azam University, Pakistan, 2012.
69. Gulistan, S., A. A. Syed, and Q. A. Naqvi, "Fields in fractional dual DB waveguides containing chiral nihility metamaterials," *Journal of Electromagnetic Waves and Applications*, Vol. 26, No. 16, 2130–2141, 2012.
70. Hussain, A., S. A. Naqvi, A. Illahi, A. A. Syed, and Q. A. Naqvi, "Fields in fractional parallel plate DB waveguides," *Progress In Electromagnetics Research*, Vol. 125, 273–294, 2012.