

ANALYSIS OF ELECTROMAGNETIC CYLINDRICAL WAVE INTERACTION WITH INHOMOGENEOUS PLANAR MEDIA

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Abstract—An analytical method based on combination of Fourier transform and Taylor's series expansion is presented for analyzing interaction of electromagnetic cylindrical waves with inhomogeneous planar layered media. In the proposed method, constitutive parameters and Fourier transformed electric and magnetic fields of inhomogeneous layer are expressed using Taylor's series expansion. The validity of the method is verified by considering some special types of inhomogeneous media and comparing the obtained results by the presented method with those of other reported methods. The results showed that when Fourier transform combined with Taylor's series expansion, they could provide a powerful technique for analyzing such problems.

1. INTRODUCTION

Considerable researches have been performed on the application of inhomogeneous media in the problems of electromagnetic wave propagation, scattering and radiation. Inhomogeneous media are described by the constitutive parameters varying with spatial variables and are efficiently used in various microwave devices [1–3]. Exact solution of the wave equation in inhomogeneous media is known for only a few particular profiles; and due to this, the scattering from inhomogeneous media has been intensively investigated and several approaches for analyzing such problems have been presented [4–17].

In most of the previous works, the analysis of reflection and transmission problems involving inhomogeneous planar layered

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media illuminated by plane waves have been discussed. In addition, the analytical formulation of line source radiation near homogeneous planar or cylindrical structures has been discussed in the literature [18–21], but investigating of the line source radiation near the inhomogeneous slab is an almost untouched topic in such electromagnetic problems.

In the present work, an efficient method to analyze of linear antenna scattering by planar inhomogeneous media is introduced. This approach is based on the use of Taylor's series expansion. The validation of the proposed method is discussed along with example calculations. Briefly, Section 2 describes Fourier transforms of the fields. In Section 3, applicability of the Taylor's series expansion approach in the analysis of the problem is presented. The accuracy of the proposed method is verified in Section 4.

2. FOURIER TRANSFORMED FIELDS

The geometry of the scattering problem is shown in Figure 1, where a line source is located above a grounded inhomogeneous slab with the thickness of $(h - d)$. The general wave function can be expressed as follows

$$\psi(x, z) = \int_{-\infty}^{+\infty} X(k_x, z) e^{jk_x x} dk_x \quad (1)$$

where $\psi(x, z)$ can be each component of the electromagnetic field, and $X(k_x, z)$ is an analytic function. The integration is over any path in the eigenvalues domain k_x . We know that solutions for unbounded region often require continuous spectra. The wave function defined in (1) may be used to construct Fourier integral. The Fourier transform pair is

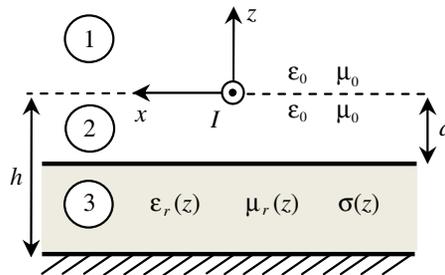


Figure 1. A typical inhomogeneous planar media exposed to an incident cylindrical wave.

defined as

$$\psi(x, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{\psi}(k_x, z) e^{jk_x x} dk_x, \quad (2)$$

$$\tilde{\psi}(k_x, z) = \int_{-\infty}^{+\infty} \psi(x, z) e^{-jk_x x} dx. \quad (3)$$

The electric current line source embedded in y direction excites the fields E_y , H_x and H_z , and there is no cross-polarized component of the incident electric field in total electric field. The geometry has been divided into three regions. The region occupying the space away from the line source, i.e., $z > 0$ is named as Region 1, and the region between the line source and the inhomogeneous layer, i.e., $-d < z < 0$ is named as Region 2. These two regions are free space. Inhomogeneous layer occupying $-h < z < -d$ is taken as Region 3. The electric fields of three regions can be written in the spectral domain as

$$\tilde{E}_{y1} = A(k_x) e^{-jk_{0z} z} \quad (4)$$

$$\tilde{E}_{y2} = B(k_x) e^{+jk_{0z} z} + C(k_x) e^{-jk_{0z} z} \quad (5)$$

$$\tilde{E}_{y3} = f(k_x, z) \quad (6)$$

where A , B , and C are unknown coefficients in term of k_x . k_{0z} is z -component of the free space wave vector, and $f(k_x, z)$ is the Fourier transformed solution of wave equation in the inhomogeneous layer. Using Faraday's law, the magnetic fields in the regions are

$$\tilde{H}_{x1} = -\frac{A(k_x) k_{0z}}{\omega \mu_0} e^{-jk_{0z} z} \quad (7)$$

$$\tilde{H}_{x2} = -\frac{k_{0z}}{\omega \mu_0} \left(-B(k_x) e^{+jk_{0z} z} + C(k_x) e^{-jk_{0z} z} \right) \quad (8)$$

$$\tilde{H}_{x3} = g(k_x, z). \quad (9)$$

As a word of caution, k_{0z} is double-valued, and we must choose the correct root, so that the fields remain finite as $z \rightarrow \infty$. Therefore

$$k_{0z} = \begin{cases} \sqrt{k_0^2 - k_x^2} & k_0 > |k_x| \\ -j\sqrt{k_x^2 - k_0^2} & k_0 < |k_x| \end{cases}. \quad (10)$$

3. TAYLOR'S SERIES EXPANSION APPROACH

Consider an inhomogeneous medium with constitutive parameters $\varepsilon(z) = \varepsilon_0 \varepsilon_r(z)$, $\mu(z) = \mu_0 \mu_r(z)$, and $\sigma(z)$. Assuming $\partial/\partial x = +jk_x$ and

$\partial/\partial y = 0$, and by eliminating \tilde{H}_{z3} from Fourier transformed Maxwell's equations, the differential equations describing inhomogeneous layer can be written as

$$\frac{\partial \tilde{E}_{y3}}{\partial z} = Z(z) \tilde{H}_{x3} \quad (11)$$

$$\frac{\partial \tilde{H}_{x3}}{\partial z} = (Y(z) + k_x^2 Z(z)) \tilde{E}_{y3} \quad (12)$$

in which

$$Z(z) = j\omega\mu_0\mu_r(z) \quad (13)$$

$$Y(z) = j\omega\varepsilon_0\varepsilon_r(z) + \sigma(z). \quad (14)$$

Observe that solving the above equations analytically is a hard and challenging problem. Thus, the analysis of inhomogeneous media using Taylor's series expansion is presented here. Taylor's series expansions of electromagnetic parameters of the inhomogeneous slab which occupies the region $-h \leq z \leq -d$ could be written as

$$\varepsilon_r(z) = \sum_{n=0}^{\infty} Ep_n \left(\frac{z+h}{h-d} \right)^n \quad (15)$$

$$\mu_r(z) = \sum_{n=0}^{\infty} Mu_n \left(\frac{z+h}{h-d} \right)^n \quad (16)$$

$$\sigma(z) = \sum_{n=0}^{\infty} \sigma_n \left(\frac{z+h}{h-d} \right)^n \quad (17)$$

$$Z^{-1}(z) = \sum_{n=0}^{\infty} Zi_n \left(\frac{z+h}{h-d} \right)^n \quad (18)$$

where Ep_n , Mu_n , σ_n and Zi_n are known coefficients. Also, we can write

$$Z(z) = \sum_{n=0}^{\infty} Z_n \left(\frac{z+h}{h-d} \right)^n; \quad Z_n = j\omega\mu_0 Mu_n \quad (19)$$

$$Y(z) = \sum_{n=0}^{\infty} Y_n \left(\frac{z+h}{h-d} \right)^n; \quad Y_n = j\omega\varepsilon_0 Ep_n + \sigma_n \quad (20)$$

Moreover, Fourier transformed electric and magnetic fields of the inhomogeneous medium are expressed by using Taylor's series expansion as follows

$$\tilde{E}_{y3} = \sum_{n=0}^{\infty} Ey_n \left(\frac{z+h}{h-d} \right)^n \quad (21)$$

$$\tilde{H}_{x3} = \sum_{n=0}^{\infty} Hx_n \left(\frac{z+h}{h-d} \right)^n \tag{22}$$

where Ey_n and Hx_n are unknown functions of k_x . By substituting (21) and (22) in (11) and (12) one can write

$$\frac{1}{h-d} \sum_{n=0}^{\infty} (n+1)Ey_{n+1} \left(\frac{z+h}{h-d} \right)^n = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} Z_p Hx_q \left(\frac{z+h}{h-d} \right)^{p+q} \tag{23}$$

$$\begin{aligned} & \frac{1}{h-d} \sum_{n=0}^{\infty} (n+1)Hx_{n+1} \left(\frac{z+h}{h-d} \right)^n \\ &= \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} (Y_p + k_x^2 Z_i p) Ey_q \left(\frac{z+h}{h-d} \right)^{p+q} \end{aligned} \tag{24}$$

Equating the coefficients terms with the same power in two sides of (23) and (24), the following recursive relations are obtained

$$Ey_{n+1} = \frac{h-d}{n+1} \left[\sum_{p=0}^n Z_{n-p} Hx_p \right] \tag{25}$$

$$Hx_{n+1} = \frac{h-d}{n+1} \sum_{p=0}^n (Y_{n-p} + k_x^2 Z_i p) Ey_p \tag{26}$$

where $n = 0, 1, 2, \dots$. Furthermore, the boundary conditions enforcing the tangential electric and magnetic fields at the boundaries of the structure are

$$\tilde{E}_{y1} = \tilde{E}_{y2} \text{ at } z = 0 \Rightarrow A = B + C \tag{27}$$

$$\tilde{H}_{x1} - \tilde{H}_{x2} = J_y \text{ at } z = 0 \Rightarrow -A - B + C = I\omega\mu_0/k_z \tag{28}$$

$$\tilde{E}_{y2} = \tilde{E}_{y3} \text{ at } z = -d \Rightarrow e^{-jk_z d} B + e^{+jk_z d} C = \sum_{n=0}^{\infty} Ey_n \tag{29}$$

$$\tilde{H}_{x2} = \tilde{H}_{x3} \text{ at } z = -d \Rightarrow (k_z/\omega\mu_0) \left(e^{-jk_z d} B - e^{+jk_z d} C \right) = \sum_{n=0}^{\infty} Hx_n \tag{30}$$

$$\tilde{E}_{y3} = 0 \text{ at } z = -h \Rightarrow Ey_0 = 0, \tag{31}$$

where inhomogeneous layer is assumed to be terminated at $z = -h$ by a perfect electric conductor (PEC) plane. Notice that the termination of inhomogeneous layer may be other boundary conditions such as perfect magnetic conductor, and impedance surface.

Truncating Taylor's series expansions at N , (25) and (26) for $n = 0, 1, 2, \dots, N - 1$ along (27)–(31) will be a $(2N + 5)$ equations to

find $(2N + 5)$ unknown coefficients. Here, the inverse matrix method is used to solve the system of coupled equations. After finding the unknown coefficients of Taylor's series expansions, the fields in the spectral domain are obtained. Afterward, the fields in the frequency domain can be calculated by taking inverse Fourier transform using numerical integration techniques.

Taylor's series approach is convenient for expansion of any continuous function. Notice that the necessary condition for the convergence of the solutions is the capability of expressing all electromagnetic parameters of the inhomogeneous layer by a converged Taylor's series expansion at all points on the region $-h \leq z \leq -d$.

4. NUMERICAL EXAMPLES AND RESULTS

4.1. Example 1

In the first example, A homogeneous lossless slab with constitutive parameters of $\varepsilon_r = 10 \exp(z)$, $\mu_r = 1$ is considered. Assuming $d = 1.3\lambda_0$, $h = 1.5\lambda_0$, the excitation frequency $f = 1$ GHz, and $I = 1$ mA, the exact solution may be obtained by solving the wave equation in the spectral domain as presented in Appendix A. The amplitude of y -component of electric field in the inhomogeneous slab obtained from the exact solution and the presented method with $N = 20$ are compared in Figure 2(a). Furthermore, Figure 2(b) shows the far-zone field of the line source near the assumed inhomogeneous layer. Observe that the obtained solution from the presented method is in the excellent agreement with that of exact solutions.

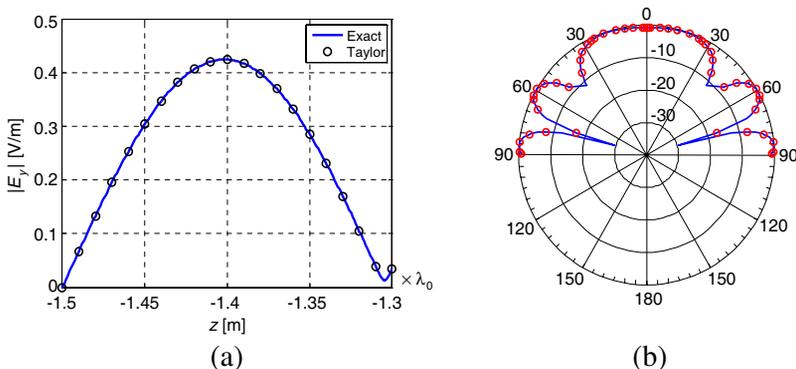


Figure 2. (a) Amplitude of the electric field in the inhomogeneous layer, (b) the far-zone field of the line source near the assumed PEC backed inhomogeneous layer.

4.2. Example 2

In the second example, consider the problem of radiation of an electric line source above an inhomogeneous layer with constitutive parameters $\epsilon_r(z) = 6/(1 + z)$, $\mu_r(z) = 2 \exp(z)$, and with assuming excitation frequency $f = 1$ GHz and $I = 1$ mA. This problem does not have any straightforward exact solution. In the first case, we assume that the electric line source is located at a large distance compared to the wavelength from the slab, e.g., $d = 20\lambda_0$ and $h = 20.2\lambda_0$. The amplitude of electric field in the inhomogeneous layer region obtained from the proposed method with $N = 20$ is shown in Figure 3(a). In order to ensure accuracy of the results, we use an approximate method. It is clear that the wave radiated from the line source at large distances is TEM^ρ , which ρ is the radial distance from the line source. Therefore, the y -component of electric field radiated from line source at the upper

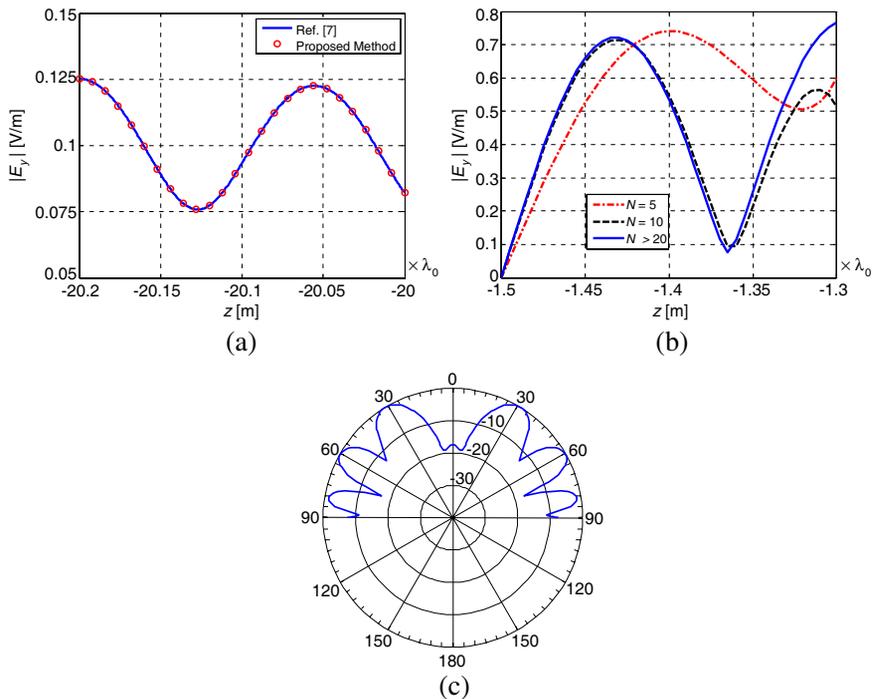


Figure 3. Amplitude of electric field in the inhomogeneous layer where an electric line source is located at (a) a large distance from the slab, and (b) near the slab, (c) the far-zone field of the line source near the assumed PEC backed inhomogeneous layer.

interface of the slab and free space (i.e., $z = -d$) could approximately be written as the follows [22]:

$$E_y \approx -\eta_0 I \sqrt{\frac{jk_0}{8\pi}} \frac{e^{-jk_0 d}}{\sqrt{d}} \quad (32)$$

where η_0 is the intrinsic impedance of free space. Since the incident wave on the slab is an approximated plane wave, we can use the discussed method in [6] for the computation of electric field in the slab. The findings are compared with results of the proposed method in Figure 3(a). Observe that there is an excellent agreement between the results.

In the second case, we assume $d = 1.3\lambda_0$ and $h = 1.5\lambda_0$. Clearly, in this case, the approximated method cannot be applied. Assuming $N = 5$, $N = 10$, and $N > 20$ the proposed method is applied to the problem and the unknown coefficients of the truncated Taylor's series are evaluated. Observe that with assuming $N > 20$, the solution is fully converged. The amplitude of y -component of electric field in the inhomogeneous region obtained from the presented method is shown in Figure 3(b). The far-zone field of electric line source near the assumed inhomogeneous layer is presented in Figure 3(c).

The proposed method is a systematic approach allowing one to simply implement it in a programming language supporting matrix and numerical manipulations. The consumed time for the discussed examples is less than few minutes using a computer with Intel Core (TM) I3 CPU and MATLAB program.

5. CONCLUSIONS

In this paper, an analytical method was presented to analyze scattering and wave propagation in the problems involving electric line sources near inhomogeneous planar layered media. The discussed approach is based on using Taylor's series expansion for all electromagnetic parameters and Fourier transformed electric and magnetic fields of the inhomogeneous medium. Solving such complex problems by the proposed method leads to finding of the solution for a simple system of linear equations. A special example with exact solution was considered and the exact results were compared with those of proposed method for showing the high accuracy of the approach. In the future, it is expected that the inhomogeneous planar layers can be optimally designed to achieve desired radiation in a specified frequency range.

APPENDIX A.

In this section, the exact Fourier transformed electric and magnetic fields of inhomogeneous media with $\varepsilon(z) = \varepsilon_{r0} \exp(z)$, $\mu = 1$ and $\sigma = 0$ are discussed. Using (11) and (12), the following second order differential equation for \tilde{E}_{y3} is obtained

$$\frac{\partial^2 \tilde{E}_{y3}(k_x, z)}{\partial z^2} - (k_x^2 - k_0^2 \varepsilon_{r0} \exp(z)) \tilde{E}_{y3}(k_x, z) = 0. \quad (\text{A1})$$

The general solution of this differential equation can be expressed as the following

$$\tilde{E}_{y3}(k_x, z) = C_1 J_{2k_x}(2k_0 \sqrt{\varepsilon_{r0}} \exp(z/2)) + C_2 Y_{2k_x}(2k_0 \sqrt{\varepsilon_{r0}} \exp(z/2)), \quad (\text{A2})$$

where the function $J_a(b)$ and $Y_a(b)$ is Bessel functions of the first and second kind respectively, with order a and argument b . Using Faraday's law, one can obtain an expression for $\tilde{H}_{y3}(k_x, z)$. The Fourier transformed electric fields of the regions 2 and 3 can be written in the as (4) and (5). Using (A2), (4), (5), (7), (8) and the boundary conditions enforcing the tangential electric and magnetic fields at the boundaries of the structure, the unknown coefficients and then the electric and magnetic fields in the inhomogeneous region are completely determined. The results are shown in Figure 2.

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