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COMPUTATION OF THE RESONANT FREQUENCY AND QUALITY FACTOR OF LOSSY SUBSTRATE INTEGRATED WAVEGUIDE RESONATORS BY METHOD OF MOMENTS

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Abstract—This paper presents a technique for the efficient and accurate determination of resonant frequencies and quality factors of Substrate Integrated Waveguide (SIW) resonators. To consider resonators of a general shape the SIW structure is modelled as a parallel plate waveguide populated with metalized via holes. The field into the SIW cavity is found by solving the scattering problem for the set of vias into the parallel plate. Resonances are determined searching for the complex frequencies for which the determinant of the system of equations pertinent to the scattering is zero. To speed up the search, a first guess for the resonance frequency is found using an estimate of the minimum singular value of the system of equations. A Muller search in the complex plane is later used to accurately determine both frequencies and quality factors. Results relevant to resonators of various shapes are presented and compared with results obtained with a commercial code.

1. INTRODUCTION

Since its advent, Substrate Integrated Waveguides (SIW) technology has been exploited to realize several passive devices at the frequency of the microwaves and of the millimeter waves. Many examples like filters [1, 2], power dividers [3, 4], antennas [5, 6] and other realizations have been presented in the recent literature. Many of these devices are based on the working principles of resonators. The design of a resonant cavity based device relies on the accurate determination of the

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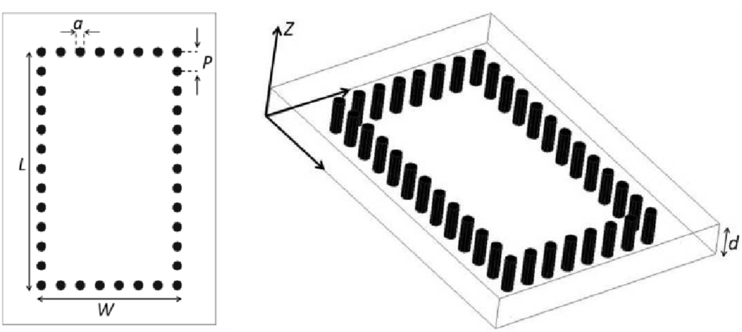
resonance frequency and the quality factor of the resonator. In the case of SIW resonating structure, these parameters could be determined by modeling the SIW cavity as a fully walled structure. However, as it known, in the full wall approximation the power leaking out the vias cageis neglected and both resonance frequencies and quality factors found may be inaccurate. For this reason, for a careful characterization of the SIW resonators, one has to resort instead to commercial codes based on full wave techniques [7].

Recently, an alternative analysis technique has been proposed in which the SIW structures are considered as an ensemble of metallic vias placed into a parallel plate waveguide. This method has been applied in [8, 9] with the simplifying hypothesis that only the TEM mode is present in the structure. In [10] a full modal expansion has been considered by using the dyadic Green's function of the parallel plate, which was calculated as series of vector wave functions [11]. The presence of the vias was included solving the scattering problem of a set of metallic cylinders inserted into an infinite parallel plate waveguide. The method was also extended to the analysis of SIW arrays of slots [12]. SIW resonators were also characterized with the help of the method described above. A preliminary analysis of lossless circular SIW resonators was presented in [13]. Resonance frequencies were found by considering the frequencies for which the determinant of the system of the equations relevant to the scattering from vias is zero. This is usually achieved setting up a search in the complex plane which can be time consuming if an adequate starting point is not available. In [14, 15] it was proposed to take the frequencies corresponding to the minima of the singular values of the matrix relevant to the system of equations and to use them as starting points of a Muller search on the complex plane. In this paper we apply the same methodto lossy resonators. To take into account the power dissipated on the metallic plates and into the dielectric slab the vector eigen functions used in [13] are modified following [16]. The finite conductivity of the metallic posts is also considered as in [16] where only the contribution of the TM (to z) modes is considered to the scattering. In fact, as shown in [10], the fundamental mode of excitation is always TM. Furthermore, since vias are made of good conductor one can consider that the polarization of the scattered field does not depart much from the one of the perfectly conducting case, so when the exciting field is TM (TE) the scattered TE (TM) field component is negligible.

In what follows, the eigen functions used for the lossy resonators, shown in [16] are briefly reviewed together with the treatment of the scattering from the finite conductivity vias. Later the algorithm to locate the complex resonance frequencies will be described. The method allows the analysis of resonators of any shape but firstly results on rectangular SIW resonators will be presented, this geometry being of a common use and showing resonant frequencies and quality factor that can be easily compared with the ones of conventional metallic waveguides. To show results of more generally shaped resonators, the case of an hexagonal resonator will be also presented. Results will be compared to data obtained with HFSS FEM-based eigen solver [7] showing very good agreement.

2. VECTOR WAVE-FUNCTIONS OF THE LOSSY PARALLEL PLATES WAVEGUIDE

In [13] the characteristics of lossless circular SIW resonators were determined by expanding the electric field in terms of the cylindrical vector wave-functions of the parallel plate waveguides as in [10]. The presence of the vias fence was included considering the field scattered from the metallic cylinders expanded in terms of the cylindrical wave functions and enforcing the condition that the electric field tangent on each cylinder was zero. The system of equations derived with this procedure was then used to determine the resonant frequencies of the cavity. To consider lossy resonators, one could use the same set of functions of the lossless case and to include losses with a perturbational approach. However, in [16] the rigorous derivation of the dyadic Green's function of lossy SIW structures was presented considering losses on the top and the bottom plates, on the conducting vias and into the dielectric. Following the same way of reasoning used in [16, 17], the TM to z (see Figure 1) vector wave functions, when losses into dielectric and finite conductivity bottom and top plates are considered,



(a) (b)

Figure 1. (a) Geometry and (b) coordinate system for the SIW cavity.

are determined as:

$$\mathbf{M_n}(k_{\rho m}, k_{zm}, |\boldsymbol{\rho} - \boldsymbol{\rho}_l|, Z) = \left(\nabla \times \hat{\mathbf{Z}}\right) H_n^{(2)}(k_{\rho m}|\boldsymbol{\rho} - \boldsymbol{\rho}_l|) \times e^{-jn\phi}\left(e^{-jk_{zm}z} + e^{jk_{zm}(z-2d)}R_{\mathrm{TM}}\right) \quad (1)$$

Notice that functions (1) represent magnetic field into the parallel plates. In the previous expressions $H_n^{(2)}(k_{\rho m}|\boldsymbol{\rho} - \boldsymbol{\rho}_l|)$ are Hankel functions of second kind and

$$R_{\mathrm{TM}} = \frac{\varepsilon_c k_{zm} - \varepsilon k_{2zm}}{\varepsilon_c k_{zm} + \varepsilon k_{2zm}} \quad (2)$$

with $k = \omega\sqrt{\mu_0(\varepsilon' - j\varepsilon'')}$, $k_{2zm} = \sqrt{k_c^2 - k_{\rho m}^2}$, $k_z = \sqrt{k^2 - k_{\rho m}^2}$, $k_c = \omega\sqrt{\mu_0\varepsilon_c}$ and $\varepsilon_c = -j\sigma/\omega$. Quantity $k_{\rho m}$ is the transverse propagation constant of the modes which propagate into the lossy parallel plate waveguide and it is calculated as residues of [17]

$$\frac{N(k_{\rho m})}{D(k_{\rho m})} = \frac{k}{2}(-1)^n \frac{(1 + R_{\mathrm{TM}})e^{-jk_{zm}d}}{k_{\rho m}k_{zm}\left(1 - R_{\mathrm{TM}}^2 e^{-2jk_{zm}d}\right)} \quad (3)$$

3. SCATTERING FROM METALLIC VIAS OF FINITE CONDUCTIVITY

Contrary to what happens in the perfectly conducting case, the scattering by cylinders of finite conductivity would require considering both TE and TM components irrespective of the polarization of the impinging field. However, considering that vias are made of good conductor, the polarization of the scattered field does not depart much from the one of the perfectly conducting case, so when the exciting field is TM (TE) the scattered TE (TM) field component is negligible. In the case considered in this paper only the TM polarized impinging and scattered fields will be considered. The field scattered by metallic vias is determined as in [13, 15] but with the following impedance boundary conditions on the cylinders surface in place of the perfect conductor condition:

$$\hat{\boldsymbol{\rho}} \times \nabla \times \mathbf{H} = -j\omega\varepsilon_r\varepsilon_0 Z_s \mathbf{H} \quad (4)$$

with

$$Z_s = (1 + j)\sqrt{\frac{\omega\mu_0}{2\sigma}} \quad (5)$$

The field scattered from vias is expressed in general as series of outgoing TM vector wave functions as follows

$$\mathbf{H}_{sCyl} = \sum\nolimits_l \sum\nolimits_{n,m} \mathbf{M}_n(k_{\rho m}, k_{zm}, \boldsymbol{\rho} - \boldsymbol{\rho}_{l,z}) A_{m,n,l}^{\mathrm{TM}} \quad (6)$$

where (see Figure 2) l is an index spanning over the cylinders, m and n are relevant to vertical and angular dependencies, $\boldsymbol{\rho}_l$ is the position of the center of the cylinder l, and $A^{\mathrm{TM}}_{m,n,l}$ are unknown coefficients to be determined.

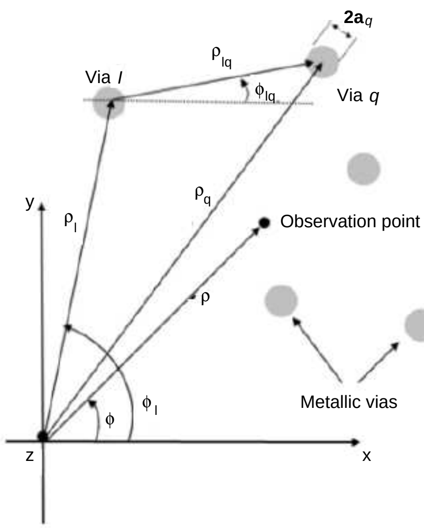


Figure 2. Reference system for the scattering problem.

For any cylinder q the following equations apply:

$$\Gamma^{\mathrm{TM}}_{q,r,m} = \sum\nolimits_{l\neq q} \sum\nolimits_{n} L^{\mathrm{TM}}_{q,r,m,l,n} A^{\mathrm{TM}}_{m,n,l} + A^{\mathrm{TM}}_{m,r,q} \tag{7}$$

$$\begin{aligned} L^{\mathrm{TM}}_{q,r,m,l,n} &= T^{\mathrm{TM}}_{r,m,q} H^{(2)}_{n-r}(k_{\rho m}\rho_{lq}) e^{-j(n-r)\phi_{lq}} \\ \Gamma^{\mathrm{TM}}_{q,r,m} &= -T^{\mathrm{TM}}_{r,m,q} v^{\mathrm{TM}}_{r,m,q} \end{aligned} \tag{8}$$

In the previous equations the following quantities have been defined

$$\begin{aligned} T^{\mathrm{TM}}_{r,m,q} &= -\frac{J_r(k_{\rho m} a_q) + Z^{J,\mathrm{TM}}_m}{H^{(2)}_r(k_{\rho m} a_q) + Z^{H,\mathrm{TM}}_m} \\ Z^{J,\mathrm{TM}}_m &= \frac{j\omega\varepsilon_r\varepsilon_0}{k_{\rho m}} Z_s J'_r(k_{\rho m} a_q) \\ Z^{H,\mathrm{TM}}_m &= \frac{j\omega\varepsilon_r\varepsilon_0}{k_{\rho m}} Z_s H^{(2)\prime}_r(k_{\rho m} a_q) \end{aligned} \tag{9}$$

and a_q is the radius of cylinder q. Notice that no sum over m appears in Equation (7). In fact, as shown in [10, 12], system (7) has to be set up and solved for each mode along z considered. The solution will correspond to the resonant TM mode of order m along z. System (7) is better cast in the following matrix form

$$\mathbf{L}^{\mathrm{TM}}\mathbf{A}^{\mathrm{TM}} = \mathbf{\Gamma}^{\mathrm{TM}} \tag{10}$$

In the previous formulas $v_{r,m,q}^{M}$ are excitation coefficients that depends on the sources [10]. As resonances are the frequencies at which system has solutions for $\mathbf{\Gamma}^{\mathrm{TM}}$ identically zero, the knowledge of $v_{r,m,q}^{M}$ is unessential. As an example the common method to locate resonances is to find the complex frequencies for which the determinant of matrix $\mathbf{L}^{M,N}$ is zero (i.e., for which $\mathbf{L}^{M,N}$ is singular). However, determinant is not easy to calculate with enough accuracy due the finite precision of numerical computations. An effective technique which make uses of the matrix singular value decomposition (SVD) has been proposed in [14] and applied to SIW resonator in [13, 15]. The determinant of the system can be expressed as

$$\det\left(\mathbf{L}^{\mathrm{TM}}\right) = \prod_{j=1}^{N} \sigma_j \tag{11}$$

where σ_j are the matrix singular values. When one of the σ_j, which are real positive numbers, is zero the matrix $\mathbf{L}^{\mathrm{TM}}$ is singular. In [13], it has been shown that an estimate of the resonance frequencies can be found evaluating the minima of the last singular value σ_N as a function of the complex frequency in a certain frequency range. The algorithm is based on the **QR** decomposition of the matrix. In fact, **R** is an upper triangular which retains the singular values. The smallest singular value can be estimated considering the element on the main diagonal of matrix **R** having the smallest absolute value. As in [13] the search span over real frequencies only. The estimated frequency is used as initial guess for a Muller search routine in the complex plane. Once the complex frequency of resonance $\omega_r + j\omega_j$ is located the quality factor is determined as

$$Q = \frac{\omega_r}{2\omega_j} \tag{12}$$

4. RESULTS

The method presented in the previous sections has been used to implement a MATLAB code to locate the resonance frequency of SIW cavities. The accuracy of the method has been tested simulating a rectangular structure with both the HFSS and the MATLAB code.

A rectangular cavity $24\,\mathrm{mm} \times 14\,\mathrm{mm}$ was considered (Figure 1). Vias radius was $a = 0.4\,\mathrm{mm}$ and their separation (center to center) was $p = 2\,\text{mm}$. The layer between the conducting plates has $\varepsilon_r = 3.5$, $\tan\delta = 0.0035$ and thickness $d = 0.5\,\text{mm}$. Only the first mode along z is considered due to the thin substrate considered. In Figure 3 is presented a plot of the minima of the singular values as function of the frequency for the first two modes. As it can be observed, the curve is free of spurious solution. The values shown in Figure 3 are initial guesses but, as shown in Table 1, they are very close to the ones predicted by HFSS. Notice that the MATLAB code took about 5 sec. to produce the data shown in Figure 3. Frequencies in Table 1 have been used as initial values for a Muller search.

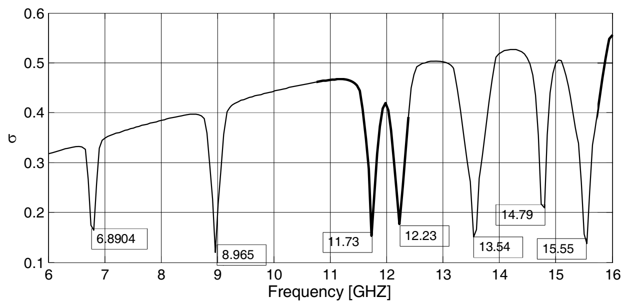


Figure 3. Minimum singular values vs. frequency for a cavity with $L = 24\,\text{mm}$, $W = 14\,\text{mm}$, $p = 2\,\text{mm}$, $a = 0.4\,\text{mm}$, $d = 0.5\,\text{mm}$, $\varepsilon_r = 3.5$, $\tan\delta = 0.0035$, $\sigma = 5.8e7$. Minima locate resonant frequencies.

Table 1. Resonant frequencies given by HFSS and taken from Figure 3.

HFSS freq. [GHz]	**6.71**	**8.87**	**11.62**	**12.10**	**13.42**	**14.62**	**15.38**
This paper freq. [GHz]	6.894	8.965	11.73	12.23	13.54	14.79	15.55

In Table 2 are reported the complex frequencies found with Muller method at resonance for the same cavity. Real values don't differ from the initial values significantly. With respect to HFSS, resonant frequencies are slightly higher but the difference is within 0.1%. Running time of the Muller search was less than 1 sec. per resonant frequency on a PC with a CPU running at 2.4 GHz and with 4 MB RAM. In Table 2 are also presented the relevant quality factors. In all the cases Q factors are in a good agreement even if the MATLAB underestimate Q with respect to HFSS. For a further analysis only the first mode which correspond to TE_{101} mode of the

Table 2. Resonant frequencies and Q factors given by the Muller method. Initial values are reported in Table 1. Results from HFSS are also shown.

This paper freq. [GHz]	6.78	8.964	11.734	12.21	13.55	14.76	15.52
Q this paper	190	198.7	205.6	208.6	210.2	212.1	213.3
Q HFSS	191.65	202.76	212.94	212.7	217.06	222.1	222.73

rectangular cavity has been considered. Notice, that in this case TE_{101} refer to the notation common to rectangular waveguide in which modes TE are with respect to the direction on which propagation occurs. For this mode, resonant frequencies and quality factors have been evaluated considering dielectric layers of increasing thickness. Results are reported in Table 3. Also shown in Figure 4 is the plot of the electric field showing that the mode is correctly identified as TE_{101}.

The method has been tested against a more complex resonating structure taken from [18]. An hexagonal SIW cavity was first

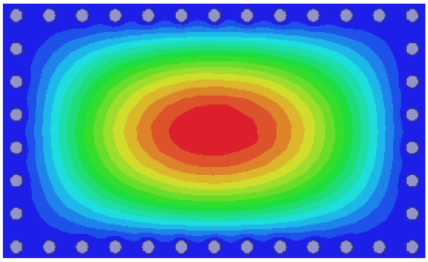


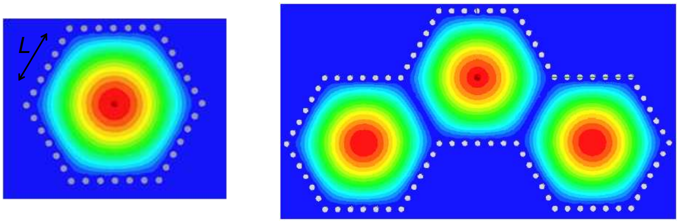
Figure 4. Plot of the electric field of the TE_{101} mode. Propagation direction is along the shorter side.

Table 3. Resonant frequency and quality factor of the rectangular cavity as in Figure 3.

d	**Freq. this paper [GHz]**	**Freq. HFSS [GHZ]**	**Q this paper**	**Q HFSS**
0.5 mm	6.78	6.71	190.1	193.5
1 mm	6.78	6.72	224.3	229.4
1.5 mm	6.78	6.72	238.6	245
2 mm	6.78	6.72	246.5	253.2

considered and then a three cavities resonator was also analyzed. The two structures are displayed in Figure 5 where the plot of the electric field of the first resonant mode is also shown. Resonant frequency and Q factor of the first mode are shown in Tables 4 and 5.

As a further result a comparison between measured unloaded Q, for a rectangular resonator presented in [19], and the method in this paper are reported in Table 6. In this case also a good agreement is observed.



(a) (b)

Figure 5. Plots of the electric field into the hexagonal resonators as in [18] $L = 6$ mm, $p = 1.2$ mm, $a = 0.6$ mm, $d = 0.75$ mm, $\varepsilon_r = 3$, $\tan\delta = 0.0035$, $\sigma = 5.8e7$. Dimensions p, a, d, are described in Figure 1.

Table 4. Resonant frequency and quality factor of the single cavity resonator shown in Figure 5(a).

	HFSS	This paper
Resonant frequency	10.13 GHz	10.11 GHz
Quality factor	255.4	250.5

Table 5. Resonant frequency and quality factor of the three cavities resonator shown in Figure 5(b).

	HFSS	This paper
Resonant frequency	10.07 GHz	10.04 GHz
Quality factor	258.2	268.3

Table 6. Resonant frequency and quality factor of the rectangular resonator shown in [19] with $L = 12.5$ mm, $W = 12.7$ mm, $p = 0.65$ mm, $a = 0.4$ mm, $d = 0.508$ mm, $\varepsilon_r = 2.2$, $\tan\delta = 0.0009$, $\sigma = 5.8e7$.

	Measurement [19]	HFSS	This paper
Resonant frequency	11.498 GHZ	11.383 GHz	11.452 GHz
Quality factor	537	505	500

5. CONCLUSIONS

In this paper, an efficient semi-analytical method to find resonances of lossy SIW cavity has been presented. The method is based on the expansion of the field inside the cavity in terms of cylindrical vector wave functions. The presence of vias is taken into consideration considering the field scattered by the metallic cylinders. The method presented is efficient and accurate and results compare well with the ones obtained with HFSS.

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