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A NEW QUALITY INDICATOR FOR WIDEBAND UNTERMINATION BY USING REFLECTIVE STANDARDS

José Fayos-Fernández*, Antonio Lozano-Guerrero, and Juan Monzó-Cabrera

Departamento de Tecnologías de la Información y las Comunicaciones,
Universidad Politécnica de Cartagena, Plaza del Hospital 1, Cartagena
30202, Spain

Abstract—In this paper a new quality indicator for two-tier calibration procedures that use only reflection standards is presented and applied to coaxial-to-waveguide transitions. The quality indicator is based on the algebraic conditioning of the system of equations solved for obtaining transition characteristics. The study has been carried out in a wide bandwidth as a difference with previous works. The obtained results indicate that a threshold value for this indicator around 10% can be established. For values below this limit the error grows to unacceptable values. Additionally, it has been shown that an exponential relationship between quality indicator and the error can be predicted.

1. INTRODUCTION

Coaxial-to-waveguide transitions are employed in waveguide measurements such as permittivity estimation, characterization of waveguide devices, etcetera, since vector network analyzers (VNA) employ coaxial interfaces. Therefore, it is necessary to have a good characterization of these structures in order to perform precise measurements in waveguide technology. However, the measurement of transition scattering parameters is not straightforward and these must be characterized from measurements made at the VNA reference plane when known standards are embedded in the waveguide port of the transition. This process is referred to as unterminating [1].

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* Corresponding author: José Fayos-Fernández (jose.fayos@upct.es).

There are several procedures for unterminating depending on the standards types and error minimization procedures. For instance open-short-load (OSL) or thru-reflect-line (TRL) standards may be used [1–3] for unterminating. However, several studies based on iterative approaches [1, 4] show that it is possible to use redundant standards in order to increase accuracy versus conventional calibration procedures.

Freeware software packages such as MultiCal and StatistiCAL developed at the National Institute of Standards and Technology (NIST) implement calibration algorithms based on different studies such as [1, 2, 4–7] that perform both one and two-tier deembedding. The two-tier calibration procedure can be used to electrically characterize probe heads or other components such as coaxial-to-waveguide transitions and can handle up to 40 different standard types.

In [8] is presented a calibration procedure specifically applied to coaxial-to-waveguide transitions known as the three-cavity technique. It makes use of three cavities and their input reflection coefficients for obtaining three linear equations with which the two-port scattering matrix of the transition can be calculated, by impressing the incident wave in the coaxial line only. The main drawback of this method is a restriction regarding the phases of the reflection coefficient of the short-circuited waveguide sections, which must not have 360° differences at a given frequency. The authors suggest using phase differences of 120° and 240°. In [9], a two-tier inverse technique for characterizing coaxial to waveguide transitions based on the use of genetic algorithms and the gradient descent method is described and compared to different well-known calibration techniques providing good results. In [10] this procedure is extended and both the transitions and the device under test are simultaneously characterized.

None of previous works, however, provide estimations or indications of how good the calibration procedure is and no clue is provided about the selection criteria for the different standards. In fact, this is a key issue that has not been studied in depth although some attempts have been made. Bauer and Penfield [2], for instance, show in a DC study that the calibration precision is better as the number of standards grows. Hoer [11] studies the best length for a precision transmission line for calibrating network analyzers and concludes that lines with phases near 180° or multiples should be avoided in order to achieve a well-conditioned equation system. Williams [12] uses the normalized variance of the unterminating procedure as indicator for the calibration quality as a function of the number of thrus and shorts and concludes that the best calibration process is achieved when the maximum number of available shorts is used. Maury et al. [13] recommend that for line-reflect-line (LRL) calibration procedures lines should show phase differences between 30° and 150°. All these studies are carried out at single frequencies or narrow bandwidths.

However, the selection of standard number or phases is not clear. For instance, Adalev et al. [14] show that using a greater number of standards does not guarantee that accuracy is increased. On the contrary, Marks [4] shows that the additional information provided by redundant standards minimizes the effects of random errors, such as those caused by imperfect connector repeatability. The resulting method exhibits improvements in both accuracy and bandwidth over conventional methods.

Commercial calibration kits, which consist of 3 reflection standards typically, might be used for unterminating coaxial-to-waveguide transitions. Nevertheless, adding some extra standards for the untermination process might increase the calibration quality [4].

In this paper, we present a simple indicator for standard selection and we relate this indicator to calibration errors and final accuracy. We exclusively use reflection standards in order to calibrate coaxial-to-waveguide transitions and we assess the calibration quality in terms of the algebraic conditioning of the system of equations solved for obtaining transition characteristics. As a difference from previous works, we carry out this study in a 1 GHz bandwidth in order to evaluate the utility of the proposed calibration quality indicator as a function of frequency.

In addition, in the result section a comparison of the system of equations quality when using a commercial calibration kit standalone and when using it supplemented with extra standards is presented.

2. CALIBRATION PROCEDURE AND QUALITY ESTIMATION

The objective of the calibration procedure is to accurately estimate the scattering parameters of the coaxial-to-waveguide transition. This kind of devices can be modeled as a 2-port microwave network. In this study, it is assumed that the port 1 is linked to its coaxial interface, whereas port 2 is to its waveguide interface. Let us represent the transition through its scattering parameter matrix:

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{1}$$

This investigation used a collection of waveguide short standards with different lengths that were connected to the port two of the transition. This means that measurements were only available at the coaxial port. Let us call Γ_i to the ith reflection coefficient measured at port one when the ith short standard is applied to the waveguide port of the transition and Γ_{L_i} as the theoretical reflection coefficient of the ith short standard with $i \in [1, N]$. N is the number of standards used during the calibration process. The reflection coefficients of the shorted guides can be computed as $\Gamma_{L_i} = e^{-j2\beta L_i}$, where β is the waveguide propagation constant and L_i are the lengths of the shorted guides. Figure 1 summarizes this explanation.

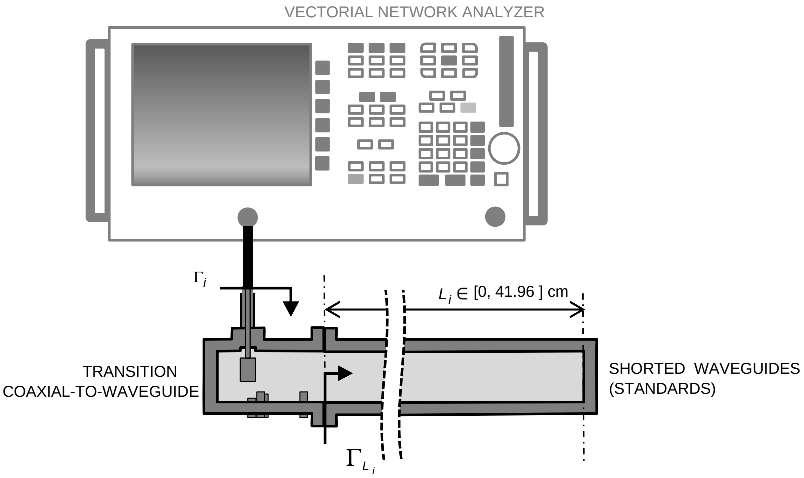


Figure 1. Scheme showing the different magnitudes employed during calibration procedure.

By expanding the methodology employed in [8], one can express the equations relating Γ_i, Γ_{L_i} and $[S]$ as expressed in Equation (2) when N shorts are employed as standards:

$$\begin{bmatrix} 1 & \Gamma_1\Gamma_{L_1} & \Gamma_{L_1} \\ 1 & \Gamma_2\Gamma_{L_2} & \Gamma_{L_2} \\ 1 & \Gamma_3\Gamma_{L_3} & \Gamma_{L_3} \\ \vdots & \vdots & \vdots \\ 1 & \Gamma_N\Gamma_{L_N} & \Gamma_{L_N} \end{bmatrix} \begin{bmatrix} S_{11} \\ S_{22} \\ S_{12}S_{21} - S_{11}S_{22} \end{bmatrix} = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \vdots \\ \Gamma_N \end{bmatrix} \tag{2}$$

Supposing that each short provides an independent equation in (2), the system can be easily solved by algebraic procedures when its matrix has a full rank and $N = 3$. Nevertheless, when $N > 3$ the system of equations is overdetermined, and it must be solved by applying Least Square Method (LSM) as implemented in Matlab$^{\text{TM}}$ software.

In order to assess the quality of the calibration process, it was used the 2-norm condition number of the first matrix in Equation (2) as:

$$C = cond \begin{bmatrix} 1 & \Gamma_1\Gamma_{L_1} & \Gamma_{L_1} \\ 1 & \Gamma_2\Gamma_{L_2} & \Gamma_{L_2} \\ 1 & \Gamma_3\Gamma_{L_3} & \Gamma_{L_3} \\ \vdots & \vdots & \vdots \\ 1 & \Gamma_N\Gamma_{L_N} & \Gamma_{L_N} \end{bmatrix}$$

$$= \left\| \begin{bmatrix} 1 & \Gamma_1\Gamma_{L_1} & \Gamma_{L_1} \\ 1 & \Gamma_2\Gamma_{L_2} & \Gamma_{L_2} \\ 1 & \Gamma_3\Gamma_{L_3} & \Gamma_{L_3} \\ \vdots & \vdots & \vdots \\ 1 & \Gamma_N\Gamma_{L_N} & \Gamma_{L_N} \end{bmatrix} \right\| * \left\| inv \begin{bmatrix} 1 & \Gamma_1\Gamma_{L_1} & \Gamma_{L_1} \\ 1 & \Gamma_2\Gamma_{L_2} & \Gamma_{L_2} \\ 1 & \Gamma_3\Gamma_{L_3} & \Gamma_{L_3} \\ \vdots & \vdots & \vdots \\ 1 & \Gamma_N\Gamma_{L_N} & \Gamma_{L_N} \end{bmatrix} \right\| \quad (3)$$

In Equation (3) the operator $\| \; \|$ indicates the 2-norm of the matrix, and *inv* is the inverse matrix operator. C yields a value of 1 for well-conditioned systems or it will tend to infinity as the condition system gets worse toward an undetermined system. Therefore, it seems more convenient to express the quality of the system of equations as a percentage from 0% to 100%. The following relationship was chosen in order to estimate the quality of the solved system of equations at each frequency point:

$$Q_i(f_i) = 100/C(f_i)\ (\%) \quad (4)$$

where f_i is the ith frequency point where the quality indicator is computed. The procedure of solving the system of Equation (2) and obtaining the quality indicator was simultaneously applied to all the frequency points of the measurement by using vector operations and no iterative procedures were employed. The Equation (5) was used for averaging values of the quality indicator within the considered bandwidth [2, 3] GHz with k sampled frequencies.

$$Q_{average} = \frac{\sum\limits_{i=f_1}^{f_k} Q_i(f_i)}{k} \quad (5)$$

For ill-conditioned systems the quality indicator will tend to 0%, and for well-conditioned systems it will approach to 100%. This indicator system must be carried out for each frequency point within the considered bandwidth. It is difficult to estimate a threshold value for Q_i in order to decide whether the calibration is precise enough or not. However, a limit value of 10% was set due to multiple observations carried out with the experimental data. Consequently, the calibration process is considered having a low quality for $Q < 10\%$.

For validation purposes, the error was computed at each frequency point as shown in Equation (6):

$$Error(f_i) = \left|\Gamma_n^{VNA}(f_i) - \Gamma_n^{CAL}(f_i)\right| \tag{6}$$

where $Error(f_i)$ is the error computed for the ith frequency point, $\Gamma_n^{VNA}(f_i)$ the reflection coefficient of the nth standard chosen for the validation measured at the coaxial port of the transition, and $\Gamma_n^{CAL}(f_i)$ the computed reflection coefficient calculated as the scattering parameters of the unterminated transition cascaded to the chosen standard.

The average error within the considered bandwidth with k sampled frequencies has been computed as:

$$Error_{average} = \frac{\sum\limits_{i=f_i}^{f_k} Error(f_i)}{k} \tag{7}$$

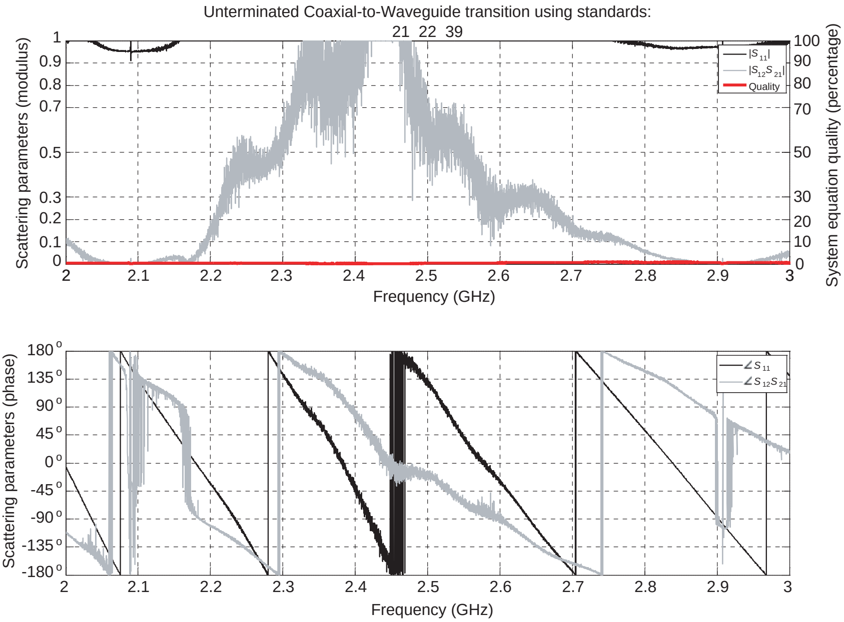


Figure 2. Extracted S-parameters for the coaxial-to-waveguide transition using a bad combination of standards for its untermination.

3. EXPERIMENTAL SETUP

A Rohde&Schwarz ZVA 67 VNA has been used in order to measure the scattering matrix frequency behavior of the coaxial-to-waveguide transitions when using different standards. The reflection standards were built as a combination of a shortplate or any of two offset short-circuited waveguides (lengths of 1.82 and 5.46 cm) with four different waveguide sections available for building up the transmission standards. As a result, up to 48 different reflection standards with physical lengths ranging from 0 cm up to 41.96 cm were available for short-circuiting the waveguide port of each transition as shown in Figure 1. The complete list of the short-circuited waveguide sections used in the calibration process and their physical lengths are provided in Appendix A.

In each measurement, 10001 points were sampled homogeneously distributed within the [2, 3] GHz frequency range, which yields a 100 kHz frequency resolution. Therefore, it was ensured that waveguide components worked only with the TE_{10} main mode.

A Rohde & Schwarz ZV-Z32 PC 3.5 fixed matched calibration kit

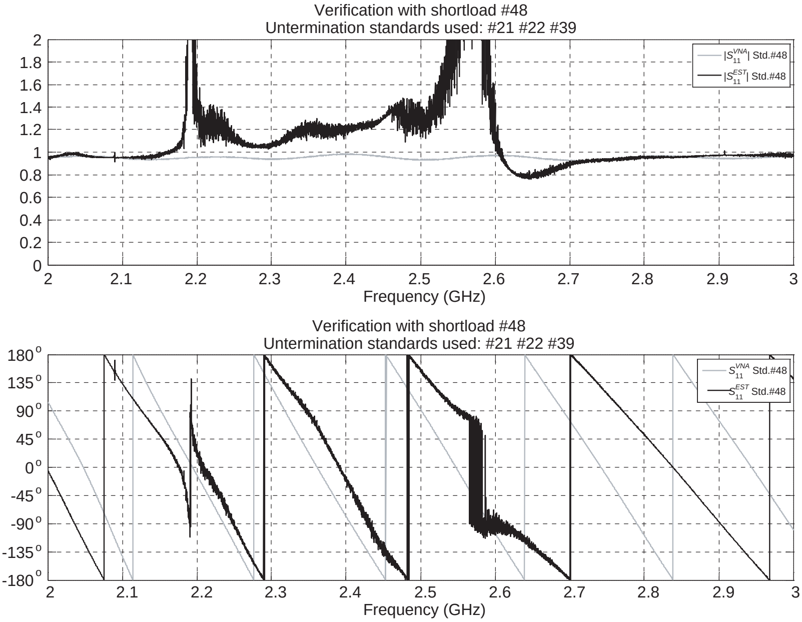


Figure 3. Verification of the untermined transition using a bad combination of standards using standard #48 as load.

was employed in order to calibrate the VNA at port 1 of the transition. The coaxial-to-waveguide transition that was used to carry out the study belongs to a Continental Microwave WCK340-HP waveguide calibration kit [15]. All the waveguide sections used to build the offset short standards accomplished the WR-340 standard.

4. RESULTS

Figure 2 shows a calibration procedure using shorts {#21, #22, #39}. The obtained quality indicator of the system of equations Q_i is near zero for all frequencies, and both the phase and magnitude values show a noisy and erroneous behavior. In this case, the phase values and phase slopes for shorts #21 and #22 were very similar and therefore the equation system had not enough information to properly carry out the characterization of the transition.

Figure 3 shows the comparison of the measured reflection coefficient of a validation standard that was not used during the calibration procedure and the reflection coefficient of that standard computed by using the obtained values of the scattering parameters of the transition. The standard #48 was chosen for the procedure

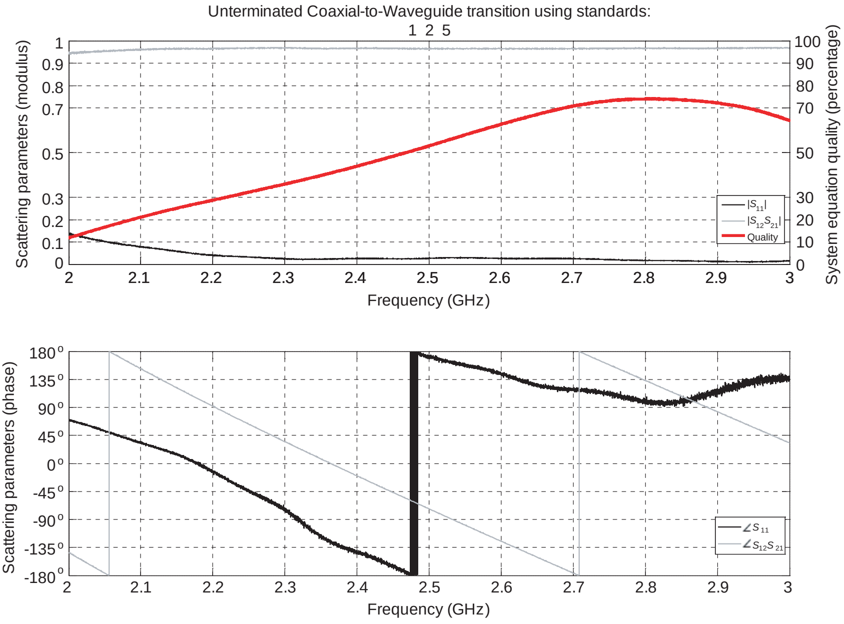


Figure 4. Extracted S-parameters for the coaxial-to-waveguide transition using a good combination of standards for its untermination.

validation. It can be observed that the transition characterization is completely erroneous: magnitude and phase values were not correctly computed due to an imprecise calibration procedure.

Figure 4 shows a calibration procedure using shorts {#1, #2, #5}. In this case, the calibration quality indicator stays above the 10% threshold along the bandwidth, so the calibration process can be considered precise. In fact, noisy values for the $[S]$ matrix of the transition are no longer observed.

This is confirmed in Figure 5, where the magnitude of the measured reflection coefficient for validation standard #48 is compared to the concatenation of the scattering matrix of the transition obtained with the calibration process and the theoretical value of the reflection coefficient of the validation standard Γ_{L48}. The agreement is complete both in magnitude and phase for the considered bandwidth.

Therefore, it can be observed that there is a strong correlation between the quality indicator proposed in this paper and the precision of the calibration procedure. For values of Q greater than 10% the calibration process could be considered of acceptable, whereas erroneous calibrations are expected to occur for values lower than this threshold.

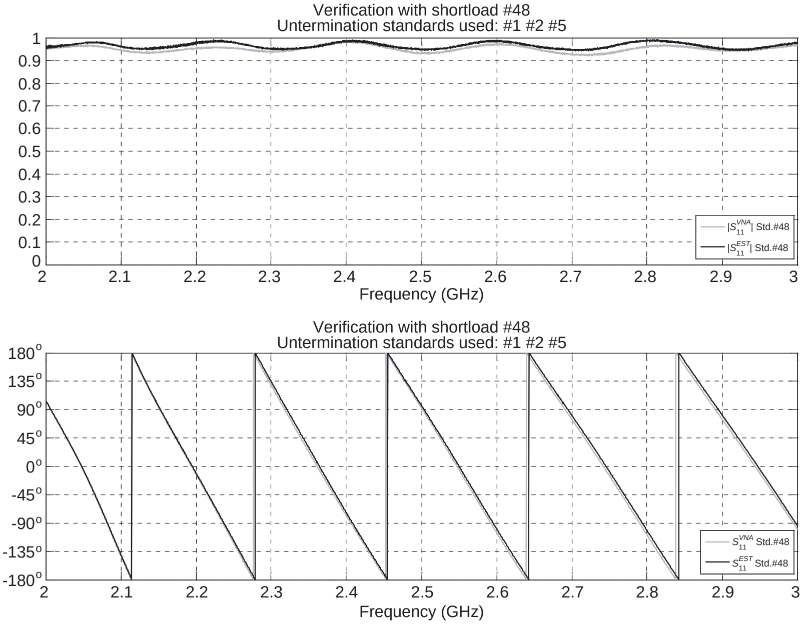
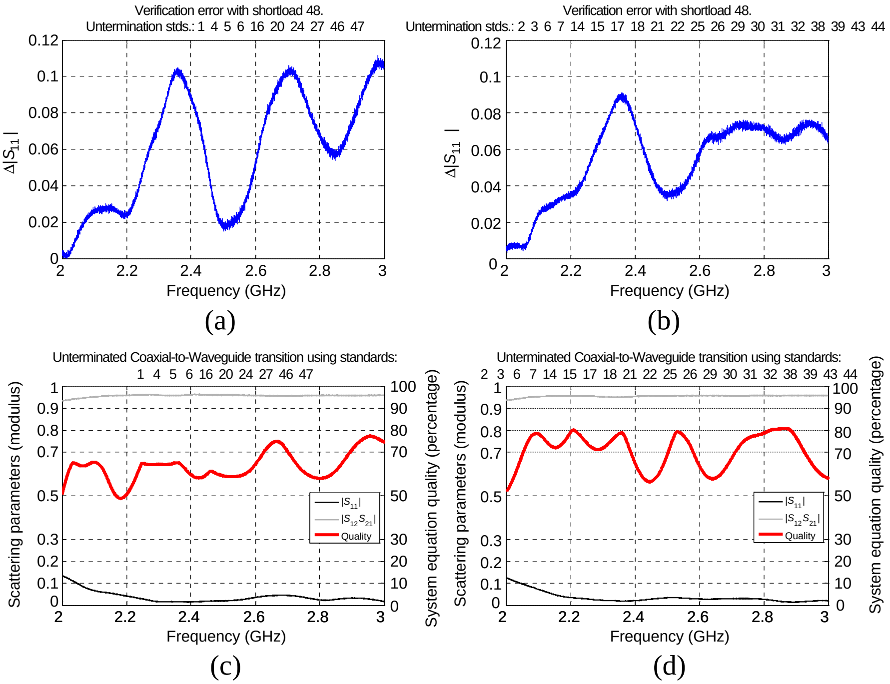


Figure 5. Verification of the untermined transition using a good combination of standards using standard #48 as load.



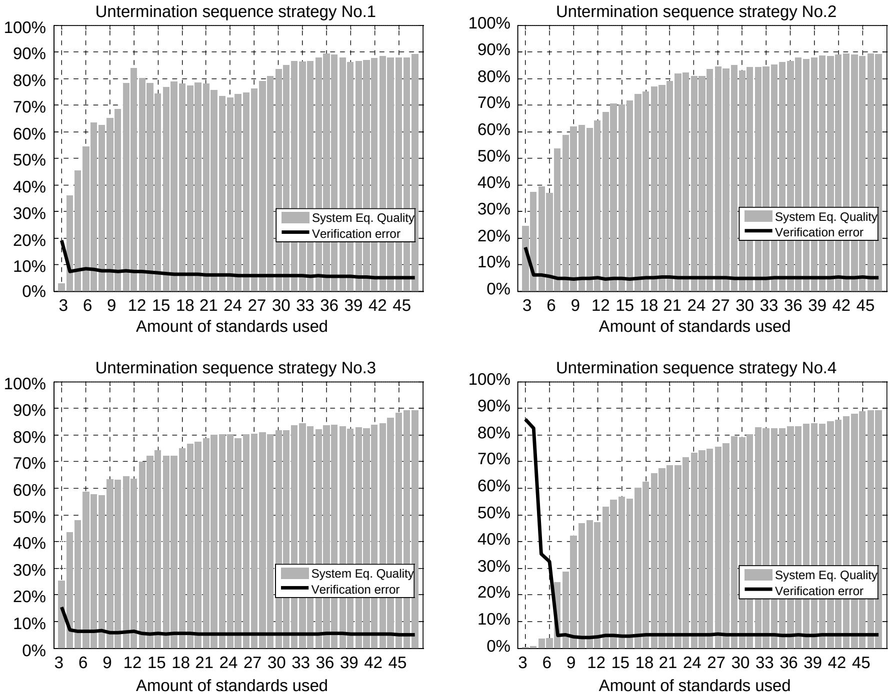
(a) (b)

(c) (d)

Figure 6. Comparison of (a), (b) validations and (c), (d) system qualities of two different calibration processes by using (a), (c) 10 and (b), (d) 20 standards.

Figure 6 shows a comparison for two calibration procedures where 10 and 20 shorts are employed. In the first case, shown in the left side of the figure, 10 shorts with minimum phase coincidences within the considered bandwidth were chosen, which are {#1 #4 #5 #6 #16 #20 #24 #27 #46 #47}. That case is compared to a calibration procedure where shorts {#2 #3 #6 #7 #14 #15 #17 #18 #21 #22 #25 #26 #29 #30 #31 #32 #38 #39 #43 #44} were employed, shown at the right side of the Figure 6. It can be observed that both calibration procedures have very similar values for the quality indicator. In fact, the error for both procedures when comparing to the validation standard number #48 is very similar for both cases. Moreover, it can be stated that using a greater number of standards does not necessarily guarantee that accuracy is increased.

Figure 7 shows the frequency-averaged quality indicator $Q_{average}$ and the average error $Error_{average}$ as a function of the number of standards by following different short selection strategies. In the first selection strategy, the standards were sequentially collected in


Untermination sequence strategy No.1

Untermination sequence strategy No.2

Untermination sequence strategy No.3

Untermination sequence strategy No.4

Figure 7. Frequency-averaged quality indicator $Q_{average}$ and average error $Error_{average}$ as a function of the number of standards for different standard selection strategies. Sequence strategy number 1: {1 2 3 4 ... 47}, sequence strategy number 2: {9 18 47 27 37 17 23 34 25 46 39 2 40 13 1 16 4 5 31 11 35 42 36 19 45 15 8 32 41 33 28 10 38 3 22 14 24 30 26 12 29 6 21 20 7 43 44}, sequence strategy number 3: {19 24 1 4 47 27 5 6 16 37 45 46 41 42 9 36 10 34 35 40 11 12 13 17 18 28 29 26 25 2 3 32 33 31 30 7 8 44 43 14 15 39 38 20 23 21 22}, sequence strategy number 4: {22 21 23 20 38 39 15 14 43 44 8 7 30 31 33 32 3 2 25 26 29 28 18 17 13 12 11 40 35 34 10 36 9 42 41 46 45 37 16 6 5 27 47 4 1 24 19}.

groups of 3, 4, ... up to the 47 available standards in ascending length order. In sequence strategy number 2, the standards were collected from a randomly ordered sequence. For the untermination sequence strategy number 3, those shorts with less phase coincidences within the considered bandwidth were collected first and those with more phase coincidences last. For sequence strategy number 4, the selection strategy of shorts was the reverse version of strategy number 3. The calibration procedure was compared to standard #48 in all cases.

Several common behaviors are observed in all cases. For instance,

$Q_{average}$ increases as the number of standards used for the calibration rises as a general trend. However, it cannot be stated that this is a monotonic behavior from the results observed when using the untermination strategy number 1. In this figure it is noticeable that using 12 standards gives a better conditioning for the system of equations rather than using greater amounts starting from 13 to 30 shorts.

Additionally, it can also be observed that the more standards used during the calibration process the lower the average error $Error_{average}$ is. However, it is observed that the reduction of error with the number of standards is not linear but follows an exponential behavior. Furthermore, for quality indicator values near or below 10% the error grows substantially. For higher values of $Q_{average}$ the error reduces at a much lower rate when the number of standards used grows.

An interesting conclusion that can be derived from Figure 7 is that the selection criterion of the shorts, when the number of standards is low, is more critical than in the case of using more standards. In fact, selection strategy number 3, where the phase coincidences are minimized, provides higher $Q_{average}$ values and lower $Error_{average}$ values than the other selection strategies for low numbers of standards. Therefore, it seems that a good selection criterion would be to minimize phase coincidences among shorts within the considered bandwidth. For higher number of standards phase coincidences grow and the error and quality indicator improve very slowly.

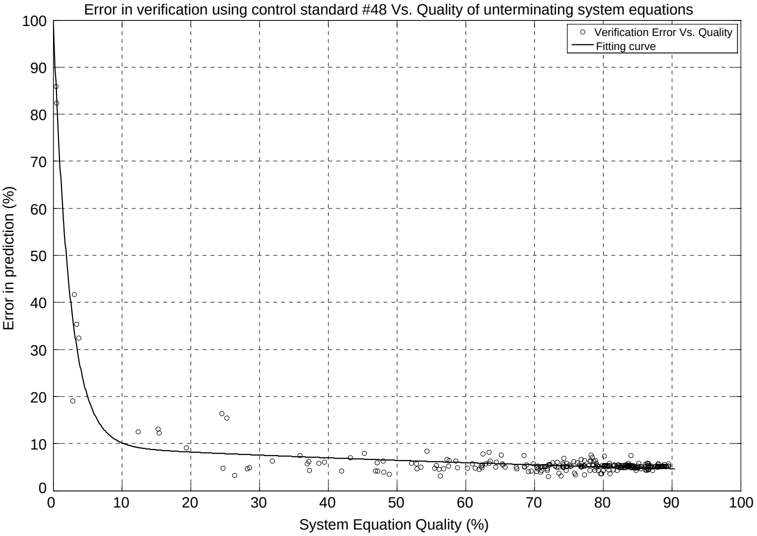


Figure 8. Exponential relationship between $Error_{average}$ and $Q_{average}$.

Finally, Figure 8 shows $Error_{average}$ as a function of $Q_{average}$ for all the data obtained in the previous selection sequences. An exponential function of the type $f(x) = a \cdot e^{bx} + c \cdot e^{dx}$ has been fitted to experimental data in order to model the relationship between $Error_{average}$ and $Q_{average}$. The coefficients obtained for the fitting are $a = 91.7$, $b = -0.4252$, $c = 9.584$ and $d = -0.007967$.

Commercial waveguide calibration kits are designed for a specific operation bandwidth. Nevertheless, such a wideband could be expanded conveniently by adding more calibration standards. Figure 9 shows a practical use of this research for demonstrating that supplementing the Continental Microwave WCK-340 Waveguide Calibration Kit (identified in this paper as standards #1, #2 and #6) with two extra standards (standards #11 and #13) improves the untermination quality, while it broadens its operational bandwidth covering the full monomode waveguide frequency range [1.74, 3.47] GHz.

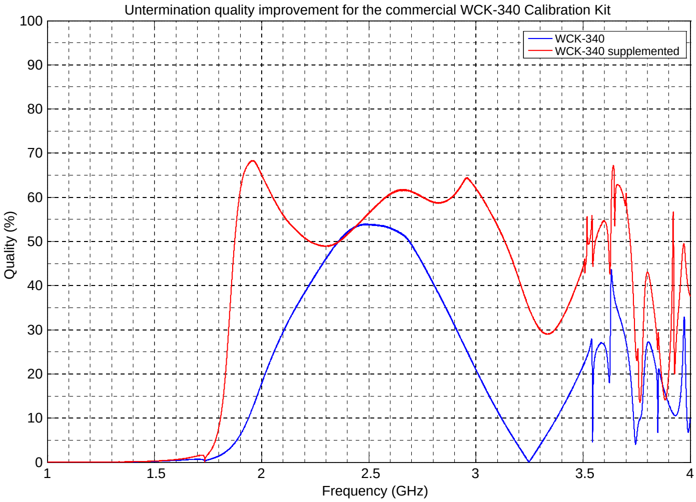


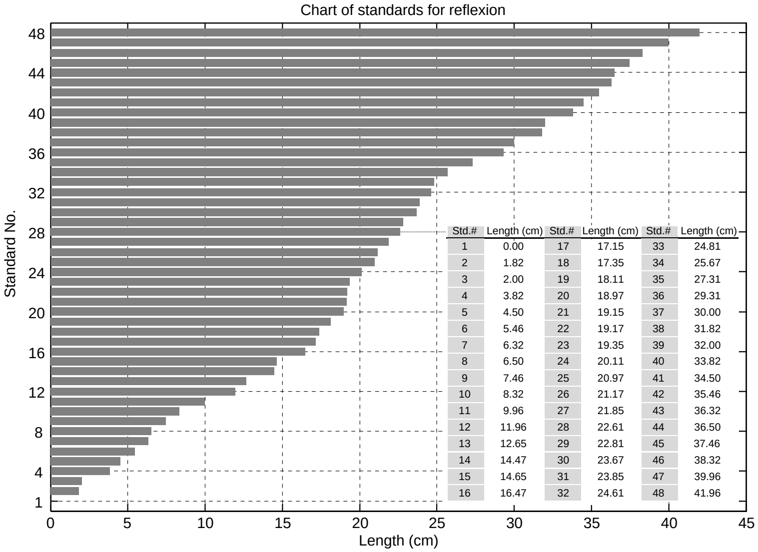
Figure 9. Example of a destermination quality improvement of a commercial waveguide calibration kit by supplementing it with two additional standards.

5. CONCLUSIONS

The relationship between the conditioning of the systems of equations and the calibration error when using short standards in order to calculate the scattering parameters of coaxial-to-waveguide transitions has been studied. From the obtained results, an exponential relationship between the calibration quality indicator and the calibration error has been established. A threshold value for this quality indicator has been set to 10%, below this limit the calibration error grows exponentially.

The usage of redundant standards has increased the quality indicator and reduced the error when comparing with a validating standard as a general trend. For lower number of standards, nevertheless, the selection criterion is of utmost importance. In this last case, it has also been shown that choosing those shorts with less number of phase coincidences within the considered bandwidth provides better quality indicators and lower error values.

APPENDIX A.



Std.#	Length (cm)	Std.#	Length (cm)	Std.#	Length (cm)
1	0.00	17	17.15	33	24.81
2	1.82	18	17.35	34	25.67
3	2.00	19	18.11	35	27.31
4	3.82	20	18.97	36	29.31
5	4.50	21	19.15	37	30.00
6	5.46	22	19.17	38	31.82
7	6.32	23	19.35	39	32.00
8	6.50	24	20.11	40	33.82
9	7.46	25	20.97	41	34.50
10	8.32	26	21.17	42	35.46
11	9.96	27	21.85	43	36.32
12	11.96	28	22.61	44	36.50
13	12.65	29	22.81	45	37.46
14	14.47	30	23.67	46	38.32
15	14.65	31	23.85	47	39.96
16	16.47	32	24.61	48	41.96

Figure A1. Number and physical length of the short standards used in the calibration process.

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