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FRINGE WAVES IN AN IMPEDANCE HALF-PLANE

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Abstract—The uniform expressions of scalar fringe waves which are based on the physical theory of diffraction (PTD) were obtained for the impedance half plane in terms of the Fresnel integrals. Asymptotic and uniform forms of the fringe fields were compared. The radiated fields of the fringe expressions were analyzed numerically.

1. INTRODUCTION

Physical optics (PO) is based on the integration of surface currents which are induced by the incident fields. This concept is valid and accurate for metallic scatterer surfaces and gives exact geometric optics (GO) waves in the high frequencies. There are lots of studies in the literature about the application of the PO method [1–3]. The edge diffracted fields which are found from the contribution of the PO surface integral are not correct. In addition PO currents just consider the illuminated part of the scatterer and ignore the shadow regions. The physical theory of diffraction (PTD) method was proposed by Ufimtsev to overcome the first defeat [4]. The PTD consists of two main parts. The first part is the uniform part which is obtained from the PO and the second part is the contribution of the non-uniform or fringe part. The fringe part is the result of the fringe currents. It is pointed out that the fringe currents are obtained by dividing the induced surface currents into two parts. The fields, radiated by the fringe currents, can be found by subtracting of uniform part from the exact solution. The method is widely used in the literature for investigation of the scattering problems [5–8]. Using the uniform fringe fields instead of the non-uniform version is more reliable. Non-uniform fringe fields can be obtained by the subtraction of asymptotic

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PO fields from asymptotic exact solutions. Similarly, uniform fringe fields can be obtained by subtraction of uniform PO fields from uniform exact solutions of the related geometry. Umul showed in a study by making comparison that the values used by Ufimtsev were quite large and not giving the correct results for the fringe fields. According to Umul's work, approximate expressions are not valid in the transition regions. Therefore, the solutions that are represented by the Fresnel functions must be used for the scattering problems [9]. Syed and Volakis derived PTD formulation for investigation of radar cross section (RCS) and they presented their formulation for impedance and coated structures [10]. Although PTD results for different impedance structures were presented by Syed and Volakis, fringe field expressions were not studied analytically and numerically. In addition, the work does not include the uniform version of the fringe field expressions and results were analyzed in terms of the total fields. In the literature, there are lots of applications on the diffraction analysis for impedance surfaces such as land-sea transition, solar cell panels on satellites and edges of high performance antennas [11–14]. The aim of this paper is to investigate the uniform fringe field expressions for impedance half-plane. Although the diffraction from a half-plane was studied by many researchers according to our knowledge, there is no study on the application of the PTD method to an impedance-half plane. Moreover, uniform expressions of fringe fields were not examined for an impedance half-plane. It is first examined in this study. Additionally, uniform and non-uniform expressions were also derived and compared numerically. Differences were examined according to the distribution of fringe field.

The time factor of $\exp(j\omega t)$ is assumed and suppressed throughout the paper where ω is the angular frequency.

2. THEORY

In this section the PO expression for the half plane geometry will be obtained from the Huygens-Kirchhoff integral with the impedance boundary condition. The geometry of the half plane is given in Figure 1. The half plane is lying on the surface $\{x \in [0, \infty],\, y = 0,\, z \in [-\infty, \infty]\}$.

We consider the E-polarization case for the incident field in this study. The method can also be applicable for the H-polarization case. The half-plane is illuminated by the plane-wave of

$$u_i = u_0 e^{jk(x \cos \phi_0 + y \sin \phi_0)} \tag{1}$$

where the u_0 is the complex amplitude factor and u_i is any component

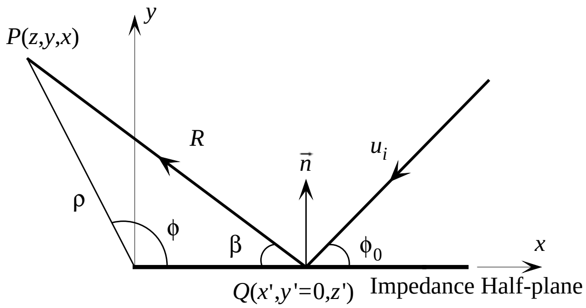

Figure 1. The geometry of the half-plane.

of the electric field. The total field can be written as

$$u_t = u_i + u_r \tag{2}$$

where u_r is the reflected field from the surface. The PO takes the reflected fields as a geometric optics (GO) fields so the reflected field can be described as

$$u_r = \Gamma u_0 e^{jk(x\cos\phi_0 - y\sin\phi_0)} \tag{3a}$$

where Γ is the reflection coefficient and equal to

$$\Gamma = \frac{\sin\phi_0 - \sin\theta}{\sin\phi_0 + \sin\theta} \tag{3b}$$

according to [15]. When taking into account the reflection coefficient which is given in Equation (3b), the angle θ determines the surfaces polarization case. When $\sin\theta$ take the values between one and infinity electric polarization case is observed and if $\sin\theta$ take the values between zero and one magnetic polarization case is observed.

Equation (2) is rearranged according to Equation (1) and Equations (3a), (3b) and redefined on the scattering surface as

$$u_t\big|_{y'=0} = (1+\Gamma)\, u_0 e^{jkx'\cos\phi_0}. \tag{4}$$

The impedance boundary condition for the Cartesian coordinates is defined as

$$u_t\big|_s = \frac{1}{jk\sin\theta}\left.\frac{\partial u_t}{\partial n}\right|_s \tag{5}$$

where s is the $y = 0$ plane, n the unit normal vector of the scattering surface, $\sin\theta$ equal to $\frac{Z_0}{Z}$, and Z_0 the impedance of the vacuum and Z is the impedance of the scatterer. According to Equation (4) and Equation (5) derivative of the field expression is written as

$$\left.\frac{\partial u_t}{\partial y'}\right|_{y'=0} = \frac{2jku_0\sin\theta\sin\phi_0}{\sin\phi_0 + \sin\theta} e^{jkx'\cos\phi_0}. \tag{6}$$

For the 2-D case if the geometry is symmetric according to the z' coordinates a further expression of the Huygens-Kirchhoff integral can be given by

$$u\left(P\right) = \frac{1}{4\pi}\int\limits_{l'}\left(u\frac{\partial G}{\partial n} - G\frac{\partial u}{\partial n}\right)dl' \tag{7}$$

where P is the observation point and G the Green's function. The 2-D Green's function is given as

$$G = \frac{\pi}{j}H_0^{(2)}\left(kR\right). \tag{8}$$

The PO integral is constructed as

$$u_{PO}\left(P\right) = \frac{1}{4\pi}\int\limits_{x'=0}^{\infty}\left(\frac{2u_0\sin\phi_0}{\sin\phi_0+\sin\theta}e^{jkx'\cos\phi_0}\frac{\partial G}{\partial y'} - \frac{2jku_0\sin\theta\sin\phi_0}{\sin\phi_0+\sin\theta}e^{jkx'\cos\phi_0}G\right)dx' \tag{9}$$

according to Equations (4), (6), (7) and the geometry of the problem. The term $\frac{\partial G}{\partial y'}$ can be found from the chain rule as

$$\frac{\partial G}{\partial y'} \cong \frac{\pi}{j}\frac{\partial H_0^{(2)}\left(kR\right)}{\partial R}\frac{\partial R}{\partial y'} \tag{10}$$

where R is the ray path and equal to $[(x-x')^2+(y-y')^2]^{1/2}$. The derivative operation of the ray path according to the normal vector is written as

$$\left.\frac{\partial R}{\partial y'}\right|_{y'=0} = -\frac{y}{R} \tag{11}$$

where $-\frac{y}{R}$ is written as $-\sin\beta$ from the geometry of the problem. Equation (9) is obtained

$$\frac{\partial G}{\partial y'} \cong \frac{k\pi}{j}H_1^{(2)}\left(kR\right)\sin\beta \tag{12}$$

with inserted Equation (11) into Equation (10) and the derivative operation $\frac{\partial H_0^{(2)}(kR)}{\partial R}$ is equal to $-kH_1^{(2)}(kR)$. Hence the PO integral which is given in Equation (9) is reconstructed as

$$u_{PO}(P)=\frac{1}{2}\frac{ku_0\sin\phi_0}{\sin\phi_0+\sin\theta}\int\limits_{x'=-\infty}^{\infty}(\sin\beta-\sin\theta)e^{jkx'\cos\phi_0}H_1^{(2)}(kR_1)dx' \tag{13}$$

where R_1 is the ray path and equal to $[(x-x')^2+y^2]^{1/2}$. Debby asymptotic expansion of the Hankel function is written as

$$H_1^{(2)}(kR_1) \approx \sqrt{\frac{2}{\pi k R_1}} e^{-jkR_1} e^{j\frac{3\pi}{4}} \tag{14}$$

where $kR_1 \gg 1$. As a result, Equation (13) yields

$$u_{PO}(P) = \frac{ku_0 \sin\phi_0}{\sin\phi_0 + \sin\theta} \frac{e^{j\frac{\pi}{4}}}{\sqrt{2\pi}} \int_{x'=0}^{\infty} (\sin\beta - \sin\theta)\, e^{jkx'\cos\phi_0} \frac{e^{jkR_1}}{\sqrt{kR_1}} dx' \tag{15}$$

where R_1 is equal to $[(x-x')^2+y^2]^{1/2}$. The integral expression can be transformed into

$$u_{PO}(P) = \frac{ku_0 \sin\phi_0}{\sin\phi_0 + \sin\theta} \frac{e^{j\frac{\pi}{4}}}{\sqrt{2\pi}} \int_{x'=0}^{\infty} \frac{1}{\sin\phi_0} \left[\cos\frac{\beta+\phi_0}{2} \sin\frac{\beta-\phi}{2} - \cos\frac{\beta-\phi_0}{2} \sin\frac{\beta+\phi}{2}\right] (\sin\beta - \sin\theta) e^{jkx'\cos\phi_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} dx' \tag{16}$$

by using [16] for separately investigation of the incident and reflected diffracted fields with using the trigonometric relation of $\sin(a-b) = \sin a \cos b - \cos a \sin b$. The incident and reflected diffracted fields can be written as

$$u_{PO}^{id}(P) = \frac{ku_0}{\sin\phi_0 + \sin\theta} \frac{e^{j\frac{\pi}{4}}}{\sqrt{2\pi}} \int_{x'=0}^{\infty} (\sin\beta - \sin\theta) \cos\frac{\beta+\phi_0}{2} \times \sin\frac{\beta-\phi}{2} e^{jkx'\cos\phi_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} dx' \tag{17a}$$

and

$$u_{PO}^{rd}(P) = -\frac{ku_0}{\sin\phi_0 + \sin\theta} \frac{e^{j\frac{\pi}{4}}}{\sqrt{2\pi}} \int_{x'=0}^{\infty} (\sin\beta - \sin\theta) \cos\frac{\beta-\phi_0}{2} \times \sin\frac{\beta+\phi}{2} e^{jkx'\cos\phi_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} dx' \tag{17b}$$

respectively. The asymptotic evaluation of Equation (17a) can be found by using the edge point technique. The edge point technique is given as

$$\int_{x}^{\infty} f(\alpha)\, e^{jkg(\alpha)} d\alpha \cong -\frac{1}{jk} \frac{f(x)}{g'(x)} e^{jkg(x)} \tag{18}$$

where the detailed explanation can be found in [16]. The phase function of Equation (17a) is written as

$$g\left(x'\right) = x' \cos\phi_0 - R_1 \tag{19}$$

The first derivative of the phase function can be found as

$$g'\left(x'\right) = \cos\phi_0 + \frac{x - x'}{R_1}. \tag{20}$$

The stationary phase values of the phase functions are found as

$$g\left(x_e\right) = -\rho \tag{21}$$

and

$$g'\left(x_e\right) = \cos\phi_0 + \frac{x}{R_1} \tag{22}$$

where $\frac{x}{R_1}$ is written as $-\cos\beta_e$ from the geometry of the problem and keep in mind that β_e which is the reflection angle's value at the edge is equal to $\pi - \phi$. The amplitude function can be written as

$$f\left(x_e\right) = \left(\sin\beta_e - \sin\theta\right)\cos\frac{\beta_e + \phi_0}{2}\sin\frac{\beta_e - \phi_0}{2}\frac{1}{\sqrt{k\rho}} \tag{23}$$

at the edge. Hence the asymptotic incident diffracted PO field is obtained as

$$u^{id}_{PO}(P) = -\frac{u_0}{\sin\phi_0 + \sin\theta}\frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}}\frac{(\sin\phi - \sin\theta)\sin\frac{\phi-\phi_0}{2}\cos\frac{\phi+\phi_0}{2}}{\cos\phi_0 + \cos\phi}\frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{24}$$

with using the edge point method. Equation (24) is simplified as

$$u^{id}_{PO}\left(P\right) = -\frac{u_0}{\sin\phi_0 + \sin\theta}\frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}}\frac{\left(\sin\phi - \sin\theta\right)\sin\frac{\phi-\phi_0}{2}}{2\cos\frac{\phi-\phi_0}{2}}\frac{e^{-jk\rho}}{\sqrt{k\rho}} \tag{25}$$

with using the trigonometric relation of $\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$. The diffracted field, in Equation (25) is not uniform since it approaches to infinity at the shadow and reflection boundaries. The uniform theory is based on the asymptotic relation of

$$\text{sign}\left(x\right) F\left[|x|\right] \cong \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{\pi}}\frac{e^{-jx^2}}{x} \tag{26}$$

for $x \gg 1$ [17, 18]. The sign(x) is the signum function equal to 1 for $x > 0$ and -1 for $x < 0$. The $F[x]$ is the Fresnel function and can be defined by the integral of

$$F[x] = \frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}}\int_x^\infty e^{-jy^2}\,dy. \tag{27}$$

In order to obtain the uniform version of the diffracted fields two new parameter are introduced as

$$\xi_- = -\sqrt{2k\rho}\cos\frac{\phi-\phi_0}{2} \tag{28}$$

$$\xi_+ = -\sqrt{2k\rho}\cos\frac{\phi-\phi_0}{2} \tag{29}$$

for using instead of x in Equation (26). Hence the uniform version of Equation (25) can be obtained as

$$u_{PO}^{id}(P) = \frac{u_0}{\sin\phi_0+\sin\theta}(\sin\phi-\sin\theta)\sin\frac{\phi-\phi_0}{2}e^{jk\rho\cos\phi-\phi_0}\mathrm{sign}(\xi_-)F\left[|\xi_-|\right]. \tag{30}$$

The same procedure is valid for the reflected diffracted field so the uniform version of the reflected diffracted field is directly written as

$$u_{PO}^{rd}(P) = -\frac{u_0}{\sin\phi_0+\sin\theta}(\sin\phi-\sin\theta) \sin\frac{\phi+\phi_0}{2}e^{jk\rho\cos(\phi-\phi_0)}\mathrm{sign}(\xi_+)F\left[|\xi_+|\right] \tag{31}$$

If the scattering geometry is symmetric according to the z' coordinate, the generalized PO integral for an arbitrary impedance surface is written as

$$u_M(P) = -\frac{k}{2}\int_{l'} u(Q)_i\, q(\alpha,\pi-\beta)\, H_0^{(2)}(kR)\, dl' \tag{32}$$

from [19] where α is the incident angle and β the reflection angle. According to Figure 1, Equation (32) can be rewritten as

$$u_M(P) = \frac{-ku_0}{2}\int_{x'=-\infty}^{\infty} e^{jkx'\cos\phi_0} q(\phi_0,\pi-\beta)\, H_0^{(2)}(kR_1)\, dx' \tag{33}$$

where $q(\phi_0,\pi-\beta)$ is the compositions of the Maliuzhinetz function and equal to

$$q(\phi_0,\pi-\beta) = \frac{\psi(\phi)}{\psi(\pi-\alpha)}\sin\frac{\alpha}{2}\left[\psi(-\phi)\left(\sin\frac{\phi}{2}-\cos\frac{\alpha}{2}\right) +\psi(2\pi-\phi)\left(\sin\frac{\phi}{2}+\cos\frac{\alpha}{2}\right)\right] \tag{34}$$

where $\psi(x)$ is written as

$$\psi(x) = \psi_\pi\left(x+\frac{3\pi}{2}-\theta\right)\psi_\pi\left(x+\frac{\pi}{2}+\theta\right) \times\psi_\pi\left(x-\frac{\pi}{2}-\theta\right)\psi_\pi\left(x-\frac{3\pi}{2}+\theta\right) \tag{35}$$

and the term $\psi_\pi(x)$ is the Maliuzhinetz function and can be described as

$$\psi_\pi(x) = \exp\left(-\frac{1}{8\pi}\int_0^x \frac{\pi\sin\eta - 4\pi\cos\frac{\pi}{4}\sin\frac{\eta}{2} + 2\eta}{\cos\eta}d\eta\right). \tag{36}$$

Hence using Debby asymptotic form of the $H_0^{(2)}(kR)$ which is equal to

$$H_0^{(2)}(kR) = \sqrt{\frac{2}{\pi}}e^{j\frac{\pi}{4}}\frac{e^{-jkR}}{\sqrt{kR}} \tag{37}$$

the scattering integral takes the form

$$u_M(P) = -u_0 k\frac{e^{j\frac{\pi}{4}}}{\sqrt{2\pi}}\int_{x'=0}^{\infty} e^{jkx'\cos\phi_0} q(\phi_0, \pi-\beta)\frac{e^{-jkR_1}}{\sqrt{kR_1}}dx' \tag{38}$$

where R_1 is the ray path and equal to $[(x-x')^2+y^2]^{1/2}$. The phase function and the amplitude function of Equation (38) are written as

$$g(x') = x'\cos\phi_0 - R_1 \tag{39}$$

$$f(x') = q(\phi_0, \pi-\beta)\frac{1}{\sqrt{kR_1}}. \tag{40}$$

The first derivative of the phase function is derived as

$$g'(x') = \cos\phi_0 - \frac{dR_1}{dx'} \tag{41}$$

where $\frac{dR_1}{dx'}$ can be found $\frac{x-x'}{R_1}$ and this expression is written as $-\cos\beta$ from the geometry of the problem. The edge point contribution of Equation (38) can be obtained as

$$u_M(P) = u_0\frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}}q(\phi_0, \pi-\beta_e)\frac{1}{\sqrt{k\rho}}\frac{1}{\cos\phi_0 - \cos\beta_e}e^{-jk\rho} \tag{42}$$

with using the edge point technique which is given in Equation (18). The reflection angle β takes the β_e value in the edges and equal to $\pi-\phi$. Equation (42) is rewritten as

$$u_M(P) = u_0\frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}}\frac{q(\phi_0, \phi)}{\cos\phi_0 + \cos\phi}\frac{e^{-jk\rho}}{\sqrt{k\rho}}. \tag{43}$$

Equation (43) can be separated into incident and reflected diffracted part with using the trigonometric relation which is previously given.

After the trigonometric separation operation Equation (43)'s uniform version can be written as

$$u_M^{rd}(P) = -\frac{q(\phi_0, \phi)}{\sin\phi_0} e^{jk\rho\cos(\phi+\phi_0)} \sin\frac{\phi+\phi_0}{2} \operatorname{sign}(\xi_+) F[|\xi_+|] \quad (44)$$

and

$$u_M^{id}(P) = \frac{q(\phi_0, \phi)}{\sin\phi_0} e^{jk\rho\cos(\phi-\phi_0)} \sin\frac{\phi-\phi_0}{2} \operatorname{sign}(\xi_-) F[|\xi_-|] \quad (45)$$

with using Equation (26) and Equation (27). According to the PTD, fringe fields can be obtained with subtracting the PO fields from the exact solution so the total fringe field expression can be obtained as

$$u_f(P) = u_M(P) - u_{PO}(P) \quad (46)$$

where the $u_M(P)$ is the exact edge diffracted fields expression from the impedance half plane and formed as

$$u_M(P) = u_M^{id}(P) + u_M^{rd}(P) \quad (47)$$

with addition of Equations (44), (45). Also $u(P)_{PO}$ is the edge contribution of the PO method and formed as

$$u_{PO}(P) = u_{PO}^{id}(P) + u_{PO}^{rd}(P) \quad (48)$$

with addition of Equations (30), (31). The expressions of the incident and reflected diffracted fringe fields can be written as

$$u_f^{id}(P) = u_M^{id}(P) - u_{PO}^{id}(P) \quad (49a)$$

and

$$u_f^{rd}(P) = u_M^{rd}(P) - u_{PO}^{rd}(P) \quad (49b)$$

respectively taking into account Equations (30), (31), (44) and Equation (45). Equation (46) is valid for obtaining the asymptotic fringe field expression. In the beginning asymptotic PO contribution can be obtained from the evaluation of Equation (15) according to Equation (18). After this evaluation asymptotic PO field can be found as

$$u_{PO}^a(P) = -\frac{u_0\sin\phi_0}{\sin\phi_0+\sin\theta} \frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}} \frac{\sin\phi-\sin\theta}{\cos\phi_0+\cos\phi} \frac{e^{-jk\rho}}{\sqrt{k\rho}}. \quad (50)$$

The asymptotic fringe field expression can written as

$$u_f^a(P) = \frac{u_0}{\cos\phi_0+\cos\phi} \frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}} \left[q(\phi_0,\phi) + \frac{\sin\phi_0(\sin\phi-\sin\theta)}{\sin\phi_0+\sin\theta}\right] \frac{e^{-jk\rho}}{\sqrt{k\rho}} \quad (51)$$

with using the subtraction of Equation (50) from Equation (43).

3. NUMERICAL ANALYSIS

In this section fringe field expressions will be analyzed numerically. In order to investigate the far field radiation, the observation distance is taken reasonably away from the scatterer. The high frequency asymptotic techniques can be used under the condition of $k\rho \gg 1$ where ρ and k are the observation distance and wave numbers respectively. The wave number k is equal to $\frac{2\pi}{\lambda}$ where λ is the wavelength. The observation distance ρ was taken as 6λ in our computations. In this case, the high frequency condition is found as $12\pi \gg 1$. Hence, it is satisfied by the distance 6λ. The angle of incidence ϕ_0 was taken as $60°$ and the $\sin\theta$ term was taken as 4. Although in trigonometry the function $\sin\theta$ is defined in the range of $[-1, 1]$, in the literature for impedance boundary condition, it can take values from zero to infinity. It should not be confused with trigonometry. It has been used in this way in the literature by Maliuzhinetz [20]. Figure 2 shows the diffracted fringe fields whose expressions were given in Equations (46), (49a) and Equation (49b). It can be seen that a minor lobe occurs between the reflection and shadow boundaries. Amplitudes at the reflection (120°) and shadow (240°) boundaries are zero. The major radiation is observed in the illuminated and the shadow regions. The amplitude variations of the incident diffracted fringe fields go to zero at the 240° whereas the amplitude variation of the reflected diffracted fringe fields go to zero at the 120°. Furthermore, it can be observed from Figure 2 that incident diffracted fringe fields concentrate in the shadow regions so compensate the incident fields

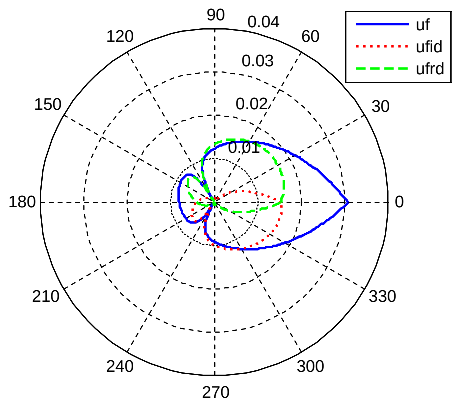

Figure 2. Total diffracted, incident diffracted and reflected diffracted fringe fields.

in the illuminated regions. Similarly, reflected diffracted fringe fields concentrate in the illuminated regions so compensate the reflected fields in the shadow regions.

Figure 3 shows that the uniform and asymptotic fringe fields. The asymptotic fringe field was computed according to the expression given in Equation (51). Although the amplitudes are equal in the illuminated and shadow regions, they are different in the transition regions. The amplitude of asymptotic fringe field is different than zero at the 120° and 240° on contrary to the uniform fringe field. It is a predicted scene for the asymptotic fringe fields for an impedance structures. The reason of the enhancement at 120° is the result of the asymptotic expression which is given in Equation (51). This behavior was explained in Figure 4. According to the PTD, asymptotic expression of fringe field can be obtained by the subtraction of PO asymptotic expression from exact asymptotic solution. The values of asymptotic expressions become infinity at the transition regions and cause an uncertainty in the result of the subtraction process. However, it yields finite field values. Although the fields compensate each other in all direction of observation, the fields cannot compensate at the reflection boundary. Therefore, the enhancement at 120° is observed. Figure 4 shows the variation of the asymptotic fringe field amplitude with respect to the observation angle. In Figure 4, the amplitude takes a reasonable value at the 120° (reflection boundary). This behavior is related to the reflection coefficient. Incident diffracted fields do not depend on the surface reflection coefficient. Therefore, there is no amplitude change at the 240°.

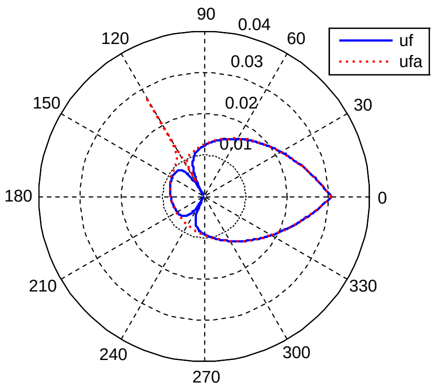

Figure 3. Uniform and nonuniform fringe fields.

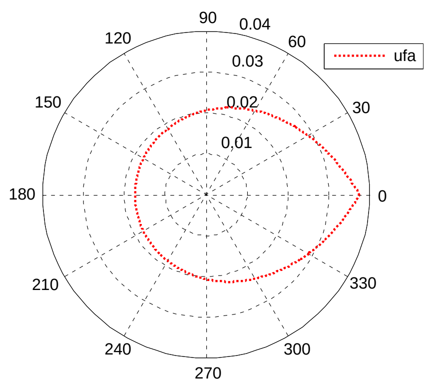

Figure 4. The asymptotic fringe field when $\sin\theta \to \infty$.

In the reflection coefficient when $\sin\theta$ approaches to infinity in Equation (51), surface acts as a perfectly conducting surface. It can be observed from Figure 4 that reflected diffracted fringe field amplitude go to equal point with respect to the incident diffracted fringe field. Hence, at the 120° and 240° amplitudes take equal values. The amplitude values for all plots were found as expected because diffracted fields compensates the deficiencies of GO fields in the transition regions and takes half of the amplitude value of GO fields.

4. CONCLUSION

In this study, uniform and asymptotic fringe field expressions were derived for a half plane with the impedance boundary conditions by using the method of PTD. First of all, the contributions of the PO fields were derived by integrating the fields in the given surface. Then, the field expression which had been obtained from the surface integral was subtracted from the exact solution in order to obtain the asymptotic form of the fringe fields. The asymptotic fringe fields were transformed into the uniform fringe fields by using the method which is given in Refs. [17, 18].

The derived uniform fringe fields were found to be more reliable than the asymptotic forms because asymptotic expressions gives wrong field values in the transition regions. It was observed that the separated fringe fields compensate the reflected and incident fields in the shadow regions. Numerical analysis showed that the uniform fringe fields are in harmony with the theory. We observed that amplitudes of the asymptotic fringe field are finite for all direction of observation and nearly consistent with the uniform fringe fields except for the reflection and shadow boundaries. In this respect, when compared to Ufimtsev's works, it appears that Ufimtsev's amplitude values were exaggerated [4, 21]. In Chapter 4, Figure 4.2 of [4], amplitude values of asymptotic fringe field components take equal value with GO fields [4]. However, it is noticeable that diffracted fields amplitude values have to be half of the GO amplitude values in the shadow and reflection regions. The more rigorous expressions were presented and numerically analyzed in this work.

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