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W-L_1-SRACV ALGORITHM FOR DIRECTION-OF-ARRIVAL ESTIMATION

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Abstract—This paper presents an effective weighted-L_1-sparse representation of array covariance vectors (W-L_1-SRACV) algorithm which exploits compressed sensing theory for direction-of-arrival (DOA) estimation of multiple narrow-band sources impinging on the far field of a uniform linear array (ULA). Based on the sparse representation of array covariance vectors, a weighted L_1-norm minimization is applied to the data model, in which the weighted vector can be obtained by taking advantage of the orthogonality between the noise subspace and the signal subspace. By searching the sparsest coefficients of the array covariance vectors simultaneously, DOAs can be effectively estimated. Compared with the previous works, the proposed method not only has a super-resolution but also improves the robustness in low SNR cases. Furthermore, it can effectively suppresses spurious peaks which will disturb the correct judgment of real signal peak in the signal recovery processing. Simulation results are shown to demonstrate the efficacy of the presented algorithm.

1. INTRODUCTION

Array signal processing has been an important study field in the past few decades [1], which plays a fundamental role in many applications, such as electromagnetic [2], acoustic and seismic sensing, etc.. Direction-of-arrival (DOA) estimation, which is also called spatial spectra estimation, is one of the crucial applications of the array signal processing. The existing popular DOA estimation methods can be mainly classified into three types: beamforming [3, 4], Capon's method [5] and subspace algorithms based on covariance matrix analysis. Beamforming spectrum suffers from the Rayleigh resolution

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limit, which is independent of the SNR. While, Capon's method and subspace algorithms, such as multiple signal classification (MUSIC) [6], estimation of signal parameters via rotational invariance techniques (ESPRIT) [7], and weighted subspace fitting (WSF) [8], are able to resolve sources with super-resolution within a Rayleigh cell. Provided that SNR is reasonably high, the sources are not highly correlated and the number of snapshots is sufficient.

Recently, the DOA estimation problem by exploiting sparse representation [9] has gained much attention. Gorodnitsky [10] uses a recursive weighted minimum-norm algorithm called focal underdetermined system solver (FOCUSS) to enforce sparsity in the problem of DOA estimation, but it can only be used for single snapshot. Cotter [11] combines multiple measurement vectors (MMV) and matching pursuit (MP) to solve the joint-sparse recovery problem in DOA estimation, however its angle resolution is unattractive. Instead of approximating L_0-norm with the L_1-norm, Hyder [12] proposes to make use of a class of Gaussian functions to deal with the $L_{2,0}$-norm minimization problem. However, it does not satisfy numerical stability, as matrix inversion is inevitable in every iteration. In [13], a sparse representation model is proposed, which is based on the L_1-norm penalty in time domain after the singular value decomposition (SVD) of the data matrix. It converts the DOA estimation into a problem of sparse signal recovery, and then solves it in a second order cone (SOC) framework. Based on the fact that DOAs of incoming signals are usually very sparse relative to the whole spatial domain, a new sparse-representation-based DOA estimation method [14], referred to as L_1-sparsely representing array covariance vectors (L_1-SRACV) is proposed by exploiting compressed sensing theory [15]. It is carried out by L_1-norm minimization for it is a convex problem. However, L_1-norm minimization has a drawback that larger coefficients of signal are punished more heavily than smaller coefficients, unlike the more impartial punishment of the L_0-norm. This leads to the degradation of signal recovery performance based on direct L_1-norm minimization.

In this paper, an effective weighted-L_1-sparsely representing array covariance vectors (W-L_1-SRACV) algorithm based on compressed sensing theory for DOA estimation is formulated. Via the sparse representation of array covariance vectors, weighted L_1-norm minimization is introduced to the proposed approach. By taking advantage of the orthogonality between the noise subspace and the signal subspace, the weighted vector can be obtained. Through searching the sparsest weighted coefficients of the array covariance vectors in an overcomplete basis, DOAs can be estimated effectively. The proposed method can not only effectually enforce the sparsity, but also successfully suppress the spurious peaks in signal recovery processing. Furthermore, it can effectively improve the robustness in low SNR cases.

The outline of the paper is as follows. Section 2 briefly introduces the data model. The presented DOA estimation method is discussed in detail in Section 3. Section 4 presents several simulation results to verify the performance of the proposed method. Section 5 provides a concluding remark to summarize the paper.

2. DATA MODEL

Assume that K uncorrelated narrowband far-field signals $u_k(t)$ impinge on a uniform linear array (ULA) consisting of M ($M > K$) elements from distinct direction angles θ_k with power σ_k^2, $k = 1, \ldots, K$, where the distance between adjacent elements is equal to half of the wavelength. The $M \times 1$ array output $\mathbf{y}(t)$ at time t can be expressed as

$$\begin{aligned} \mathbf{y}(t) &= [y_1(t), \ldots, y_M(t)]^T = \sum_{k=1}^{K} \mathbf{a}(\theta_k) u_k(t) + \mathbf{n}(t) \\ &= \mathbf{A}\mathbf{u}(t) + \mathbf{n}(t), \qquad t = 1, \ldots, N \end{aligned} \tag{1}$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K)]$ is the $M \times K$ array manifold matrix, in which $\mathbf{a}(\theta_k) = [1, e^{-j\pi \sin\theta_k}, \ldots, e^{-j(M-1)\pi \sin\theta_k}]^T$ is the $M \times 1$ steering vector. $\mathbf{u}(t) = [u_1(t), \ldots, u_K(t)]^T$ is a zero-mean signal vector, $\mathbf{n}(t) = [n_1(t), \ldots, n_K(t)]^T$ with $n_k(t)$ denoting the additive noise of the kth sensor, where $n_k(t)$ is a complex Gaussian random process with zero-mean and equal covariance $\sigma^2 \mathbf{I}_M$, N is the number of data samples and the superscript $(\cdot)^T$ stands for the transpose operation.

From the data model (1), we can obtain the following $M \times M$ array covariance matrix

$$\mathbf{R} = E\left\{\mathbf{y}(t)\mathbf{y}^H(t)\right\} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma^2 \mathbf{I}_M \tag{2}$$

where $\mathbf{R}_s = E\{\mathbf{u}(t)\mathbf{u}^H(t)\} = \mathrm{diag}\{\sigma_1^2, \ldots, \sigma_K^2\}$ is the signal covariance matrix, in which $\mathrm{diag}\{\sigma_1^2, \ldots, \sigma_K^2\}$ denotes a diagonal matrix with diagonal entries $\sigma_1^2, \ldots, \sigma_K^2$ and $\mathbf{I}_M$ represents an $M \times M$ identity matrix. In addition, the operator $(\cdot)^H$ and $E\{\cdot\}$ indicate conjugate transpose and expectation, respectively.

3. THE PROPOSED W-L_1 SRACV ALGORITHM

3.1. Background of Compressed Sensing

In most previous works, the DOA estimation is based on the Nyquist sampling: the sampling rate must be at least twice the maximum frequency present in the signal. Compressed sensing theory is a novel data collection and coding theory under the condition that signal is sparse or compressible. It has been shown that when the measurement matrix satisfies certain random properties, the original signal can be reconstructed even when the number of observation is far less than the number of Nyquist rate samples.

Compressed sensing considers the sparse reconstruction problem of estimating an (approximately) sparse vector $\mathbf{x}$ with a finite length N from an observed vector of measurements $\mathbf{y}$ with a finite length M based on the following linear model ("measurement equation")

$$\mathbf{y} = \mathbf{\Phi x} + \mathbf{z} \tag{3}$$

where $\mathbf{\Phi}$ is an $M \times N$ known measurement matrix, and each of its M rows is usually orthogonal. $\mathbf{z}$ is an $M \times 1$ unkown vector that accounts for measurement noise and modeling errors. The reconstruction is subject to the constraint that $\mathbf{x}$ is (approximately) K-sparse, that is, at most K of its entries are not (approximately) zero and $K \ll N$.

In domain $\mathbf{\Psi}$, $\mathbf{x}$ can be represented as $\mathbf{x} = \sum_{i=1}^{N} \psi_i s_i = \mathbf{\Psi s}$, where $\mathbf{x}$ and $\mathbf{s}$ are the $N \times 1$ column vectors, respectively. The basis matrix $\mathbf{\Psi} = [\psi_1, \psi_2, \ldots, \psi_N]$. Thus, (3) can be written as

$$\mathbf{y} = \mathbf{\Phi x} = \mathbf{\Phi \Psi s} = \mathbf{A s} \tag{4}$$

$\mathbf{x}$ is thus transformed, or down sampled, to an $M \times 1$ vector $\mathbf{y}$. The coefficient vector $\mathbf{s}$ can be solved by the following optimization

$$\min_s \|\mathbf{s}\|_1, \quad \text{s.t.} \quad \mathbf{A s} = \mathbf{y} \tag{5}$$

where $\| \cdot \|_1$ means L_1 norm.

3.2. Sparse Representation of DOA Estimation Problem

Consider each column (i.e., the array covariance vector) of $\mathbf{R}$ in (2). Introduce an overcomplete basis $\mathbf{\Phi}$ in terms of all possible source locations. Let $\{\varphi_1, \ldots, \varphi_Q\}$ be a sampling grid of all source locations of interest, e.g., from -90° to 90° with 1° intervals. The number of potential source locations Q will typically be much greater than that of sources K or even the number of sensors M. Construct a matrix composed of steering vectors corresponding to each potential source location as its columns: $\mathbf{\Phi} = [\mathbf{a}(\varphi_1), \ldots, \mathbf{a}(\varphi_Q)] \in \mathbb{C}^{M \times Q}$. In this framework, $\mathbf{\Phi}$ is known and does not depend on the actual source locations $\boldsymbol{\theta}$. We can reform the mth column of $\mathbf{R}$ as [14]

$$\mathbf{r}_m = E\left[\mathbf{y}(t) y_m^*(t)\right] = \mathbf{\Phi s}_m + \sigma^2 \mathbf{e}_m, \quad m = 1, 2, \ldots, M \tag{6}$$

where $(\cdot)^*$ denotes the complex conjugate operation, $\mathbf{\Phi}$ the $M \times Q$ array manifold matrix as presented above, $\mathbf{s}_m$ the $Q \times 1$ representation coefficient vector according to the above overcomplete basis, and the error term $\mathbf{e}_m$ is an $M \times 1$ vector with 1 in the mth entry and 0 elsewhere. An ideal $\mathbf{s}_m$ should be the vector with all elements zeros except for the K elements associated with the K basis vectors, i.e., an ideal $\mathbf{s}_m$ has a sparse framework related to the DOAs of the signals. Then, the model (6) can be rewritten in matrix form as follow

$$\mathbf{R} = \mathbf{\Phi}\mathbf{S} + \sigma^2\mathbf{I}_M \tag{7}$$

where $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_M]$. It is obvious that the ideal $\{\mathbf{s}_m\}_{m=1}^{M}$ should share the identical sparse structure, i.e., the nonzero elements of each ideal $\mathbf{s}_m$ should appear in the same row of $\mathbf{S}$. By bringing in a vector $\mathbf{s}^\circ = [s_1^\circ, s_2^\circ, \ldots, s_Q^\circ]$, where the qth element s_q° equals to the L_2-norm of the qth row of $\mathbf{S}$, i.e., $s_q^\circ = \|(\mathbf{S})_{q\cdot}\|_2 = (\sum_{m=1}^{M} s_{qm}^2)^{1/2}$, we find that knowledge of the ideal $\{\mathbf{s}_m\}_{m=1}^{M}$ sharing a specific sparse structure can be coherently described by $\mathbf{s}^\circ$ with the same sparse structure. Thus, seeking a sufficiently sparse $\mathbf{s}^\circ$ will make $\{\mathbf{s}_m\}_{m=1}^{M}$ consistently fit $\{\mathbf{R}_m\}_{m=1}^{M}$ as sparsely as possible in a manner such that all the elements in a row of $\mathbf{S}$ tend to be zero or nonzero simultaneously. As a result, DOA estimation based on (7) can be equivalent to find a sufficiently sparse $\mathbf{s}^\circ$, supposing that the error term $\sigma^2\mathbf{I}_M$ is well suppressed. When $\mathbf{s}^\circ$ is scored, DOAs can be determined from its sparse framework by plotting it on the grid of direction samples.

3.3. DOA Estimation

According to the sparse recovery theory, DOA estimation based on (7) can be described as the following constrained optimization problem

$$\min_{\mathbf{S}} \|\mathbf{s}^\circ\|_1 \quad \text{s.t.} \quad \mathbf{R} = \mathbf{\Phi}\mathbf{S} + \sigma^2\mathbf{I}_M \tag{8}$$

In practical application, the unknown $\mathbf{R}$ can be consistently estimated by $\hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N}[\mathbf{y}(t)\mathbf{y}^H(t)] = \mathbf{R} + \Delta\mathbf{R}$ where $\Delta\mathbf{R} = \hat{\mathbf{R}} - \mathbf{R}$ is the estimated error whose vectorized form satisfies

$$\mathrm{vec}(\Delta\mathbf{R}) \sim \mathrm{AsN}\left(\mathbf{0}, \frac{1}{N}\mathbf{R}^T \otimes \mathbf{R}\right) \tag{9}$$

where $\mathrm{vec}(\cdot)$ denotes the stack operation, placing in order, the columns of a matrix on the top of one another, $\mathrm{AsN}(\mu, \sigma^2)$ the asymptotic normal distribution with mean μ and variance σ^2, and $\otimes$ the Kronecker matrix product. If $\hat{\mathbf{R}}$ takes place of $\mathbf{R}$ directly, the equality constrain in (8) collapses due to $\Delta\mathbf{R}$.

From (9), it can be inferred that

$$\mathbf{J}\mathrm{vec}\left[\hat{\mathbf{R}} - \mathbf{\Phi}\mathbf{S} - \sigma^2\mathbf{I}_M\right] \sim \mathrm{AsN}(\mathbf{0}, \mathbf{I}_{M^2}) \tag{10}$$

where $\mathbf{J} = \sqrt{N}\mathbf{R}^{-\frac{T}{2}} \otimes \mathbf{R}^{-\frac{1}{2}}$. Then, we can obtain the following expression

$$\left\|\mathbf{J}\mathrm{vec}\left[\hat{\mathbf{R}} - \mathbf{\Phi}\mathbf{S} - \sigma^2\mathbf{I}_M\right]\right\|_2^2 \sim \mathrm{As}\chi^2\left(M^2\right) \tag{11}$$

where $\mathrm{As}\chi^2(M^2)$ signifies the asymptotic chi-square distribution with M^2 degrees of freedom. We recommend a parameter ξ in (11) such that

$$\left\|\mathbf{J}\mathrm{vec}\left[\hat{\mathbf{R}} - \mathbf{\Phi}\mathbf{S} - \sigma^2\mathbf{I}_M\right]\right\|_2^2 \leq \xi \tag{12}$$

with a high probability p, where p is large and is close to 1. In general, it is enough to set $p = 0.999$ to determine the value of ξ, which makes the above inequality robustly hold for an arbitrary array.

After the above eduction and analysis, we can get a robust and more controllable expression for DOA estimation as follow [14]

$$\min_{\mathbf{S}} \|\mathbf{s}^{\circ}\|_1 \quad \text{s.t.} \quad \|\mathbf{z} - \mathbf{\Psi}\mathrm{vec}(\mathbf{S})\|_2 \leq \sqrt{\xi} \tag{13}$$

where $\mathbf{z} = \hat{\mathbf{J}}\mathrm{vec}(\hat{\mathbf{R}} - \hat{\sigma}^2\mathbf{I}_M)$, $\mathbf{\Psi} = \hat{\mathbf{J}}[\mathbf{I}_M \otimes \mathbf{\Phi}]$, $\hat{\mathbf{J}} = \sqrt{N}\hat{\mathbf{R}}^{-\frac{T}{2}} \otimes \hat{\mathbf{R}}^{-\frac{1}{2}}$, $\hat{\sigma}$ is the estimation of σ. Note that an unnormalized overcomplete basis relative to $\mathrm{vec}(\mathbf{S})$ has an infaust effect on computing its correct sparse structure, so, before the calculation of (13), a renormalization procedure is needed for each column of $\mathbf{\Psi}$ which is normalized to 1 in the L_2-norm as done in [16].

The merits of the L_1-SRACV method lie in its superior resolution and explicit error-suppression criterion that makes it statistically robust even in low SNR cases. However, L_1-SRACV has one big drawback, that is, it can not be obtained exact result in the recovery processing due to the direct L_1-norm minimization.

3.4. Weighted L_1-Norm Minimization

The L_1-SRACV algorithm enforces sparsity by the direct L_1-norm penalty. However, the direct L_1-norm minimization can not obtain exact result in the recovery precessing. To solve the single measurement vector (SMV) problem, Candes devised an iterative reweighted approach of L_1-norm minimization that larger weights are assigned to the entries of the recovered signal whose indices are outside of the signal support [17]. The iterative L_1-norm reweighting is given as

$$w_i^{(p+1)} = \left[x_i^{(p+1)} + \epsilon\right]^{-1} \tag{14}$$

where x_i denotes the ith entry of the recovered signal and w_i the corresponding weighted value, $\epsilon(> 0)$ is an application-dependent parameter and it must be carefully designed, p is the iteration count number.

Now, the idea of iterative reweighted L_1-norm minimization is expanded from the SMV problem to the MMV problem. This idea can be achieved by utilizing the orthogonality between the noise subspace and the signal subspace spanned by the array manifold matrix [3].

By taking advantage of the eigendecomposition on the array covariance matrix $\mathbf{R}$, the following equation can be obtained

$$\mathbf{R} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H = [\mathbf{U}_S\ \mathbf{U}_N]\mathbf{\Sigma}[\mathbf{U}_S\ \mathbf{U}_N]^H \tag{15}$$

where $\mathbf{\Sigma}$ is a diagonal matrix which is composed of the eigenvalues of the array covariance matrix, $\mathbf{U}$ consists of the corresponding eigenvectors, $\mathbf{U_S}$ is the signal subspace, which is the first K columns of $\mathbf{U}$ and $\mathbf{U_N}$ is the noise subspace, which is the last $M-K$ columns of $\mathbf{U}$. It is easy to know that

$$\mathbf{A}^H\mathbf{U}_N = \mathbf{0} \in \mathbb{C}^{K\times(M-K)} \tag{16}$$

Considering the relationship between the overcomplete basis matrix $\mathbf{\Phi}$ and the array manifold matrix $\mathbf{A}$, we can rewrite $\mathbf{\Phi} = [\mathbf{A}\ \ \mathbf{B}]$, where $\mathbf{B} \in \mathbb{C}^{M\times(Q-K)}$.

Utilizing the property in (16), we have the following equation

$$\mathbf{\Phi}^H\mathbf{U}_N = \left[\mathbf{U}_N^H\mathbf{A}\quad \mathbf{U}_N^H\mathbf{B}\right]^H = \left[\mathbf{0}^H\ \ \mathbf{D}^H\right]^H \tag{17}$$

where $\mathbf{D}_i^{(l_2)} > 0$, $\mathbf{D}_i^{(l_2)}$ denotes the ith entry of $\mathbf{D}^{(l_2)}$, $\mathbf{D}^{(l_2)}$ is the column vector that denotes the L_2-norm of each row of $\mathbf{D}$. In practical application, we have to substitute the sample data matrix $\hat{\mathbf{R}}$ for $\mathbf{R}$. Substituting $\hat{\mathbf{U}}_N$ for $\mathbf{U}_N$ yields that

$$\mathbf{\Phi}^H\hat{\mathbf{U}}_N = \left[\hat{\mathbf{U}}_N^H\mathbf{A}\ \ \hat{\mathbf{U}}_N^H\mathbf{B}\right]^H = \left[\mathbf{W}_A^H\ \ \mathbf{W}_B^H\right]^H = \mathbf{W} \tag{18}$$

The weighted vector can be expressed as

$$\mathbf{w}^{(l_2)} = \left[\mathbf{W}_A^{(l_2)T}\ \ \mathbf{W}_B^{(l_2)T}\right]^T \tag{19}$$

when the snapshot $N \to \infty$, then $\mathbf{W}_A^{(l_2)} \to \mathbf{0}^{(l_2)} \in \mathbb{R}^{K\times 1}$ and $\mathbf{W}_B^{(l_2)} \to \mathbf{D}^{(l_2)} \in \mathbb{R}^{(Q-K)\times 1}$ and then the entries of $\mathbf{W}_A^{(l_2)}$ are smaller than those of $\mathbf{W}_B^{(l_2)}$.

Define

$$\mathbf{G} = \operatorname{diag}\left\{\mathbf{w}^{(l_2)}\right\}. \tag{20}$$

Consequently, we can employ $\mathbf{G}$ as a weighted matrix to achieve the idea that the nonzero entries whose indices are inside of the row support of the jointly sparse signals are punished by smaller weights, and the other entries whose indices are more likely to be outside of the row support of the jointly sparse signals are punished by larger weights. Finally, we can formulate the weighted L_1-norm minimization for sparse signal reconstruction

$$\min_{\mathbf{S}} \|\mathbf{G}\mathbf{s}^{\circ}\|_1 \quad \text{s.t.} \quad \|\mathbf{z} - \boldsymbol{\Psi}\text{vec}(\mathbf{S})\|_2 \leq \sqrt{\xi} \tag{21}$$

Equation (21) can be calculated by SOC programming software packages such as CVX. The DOA estimates are then obtained by plotting $\mathbf{s}^{\circ}$, solved from (21).

The procedure of the proposed W-L_1-SRACV method is concluded as follows:

(1) Collect received data and estimate the covariance matrix $\hat{\mathbf{R}}$.
(2) Compute the eigendecomposition of $\hat{\mathbf{R}}$ and obtain the noise subspace $\hat{\mathbf{U}}_N$.
(3) Construct the overcomplete basis matrix $\boldsymbol{\Phi}$.
(4) Obtain the weighted matrix $\mathbf{G}$ by (20).
(5) Get $\mathbf{z}$ and $\boldsymbol{\Psi}$ by numeration.
(6) Estimate DOA by calculating (21).

4. SIMULATION

In this section, several simulation results are presented to illustrate the effectiveness of the proposed method. Assume the uniform linear array (ULA) have $\mathbf{M} = 8$ sensors whose separation distances are half a wavelength. Note that the extension of the presented algorithm to an arbitrary array is straightforward.

In the first simulation, we compare the spectrum of the three algorithm: MUSIC, the L_1-SRACV algorithm and the presented approach. Consider four uncorrelated equal power signals that arrive from $[-35°, -30°, -10°, 20°]$ impinging on the array. The direction grid is set to have 181 points sampled from $-90°$ to $90°$ with $1°$ intervals, and the number of snapshots is 256, the signal-to-noise-ratio (SNR) is 5 dB. From Figure 1, we can see that both L_1-SRACV and the proposed approach have higher resolution than that of MUSIC. It is to note that the L_1-SRACV has serious spurious peaks, while the presented method has no spurious peak and show better performance than MUSIC and the L_1-SRACV. The main reason for existing spurious peaks is that: firstly, we usually substitute L_1-norm for L_0-norm in the recovery processing, since the L_0-norm optimization is an NP hard problem; secondly, the problem we want to solve is

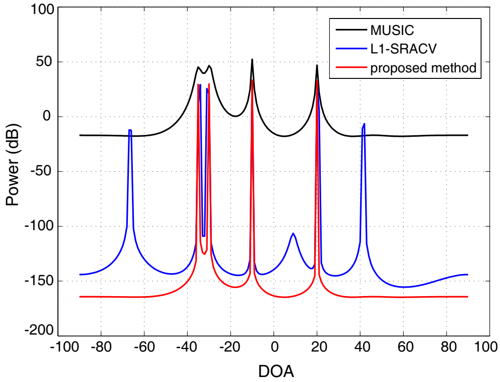

Figure 1. Spatial spectra for MUSIC, L_1-SRACV and proposed method.

not absolutely sparse when there exists noise in practical application; in addition, the coherence of the columns is usually so high in the overcomplete basis matrix that it is hard to satisfy the restricted isometry property (RIP). The proposed method can suppress spurious peaks well because of the orthogonality between the noise subspace and the array manifold matrix.

In the following simulation, the root-mean-square-error (RMSE) of DOA estimation versus angel separation between L_1-SRACV and the proposed method is presented. And it is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{100K}\sum_{n=1}^{100}\sum_{k=1}^{K}\left(\hat{\theta}_k(n) - \theta_k\right)^2} \tag{22}$$

where $\hat{\theta}_k(n)$ is the estimation of θ_k for the nth Monte Carlo trial and K the number of signals. Assume that two uncorrelated signals impinge on the array from $\theta_1 = -30^\circ$ and $\theta_2 = -30^\circ + \Delta\theta$, respectively, where $\Delta\theta$ is varied from 3° to 10° in 1° steps. The SNR is set to 5 dB and the snapshot is 512. The RMSE versus angel separation in Figure 2 is obtained via 100 independent Monte Carlo trials for each angel spacing. It is presented that the proposed algorithm tends to become unbiased when $\Delta\theta$ is greater than about 5°, while the unbiased angle spacing is about 9° for the L_1-SRACV approach.

Figure 3 shows a comparison of the RMSE of DOA estimation versus SNR between the L_1-SRACV approach and the proposed method. In this simulation, two uncorrelated sources at -10° and 10° are considered. The number of snapshot is taken as $N = 256$, and all the results are averaged over 100 Monte Carlo runs for each

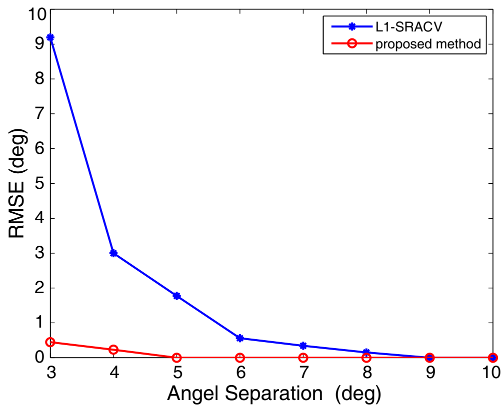

Figure 2. RMSE of the DOA estimation versus angel separation between L_1-SRACV and the proposed method.

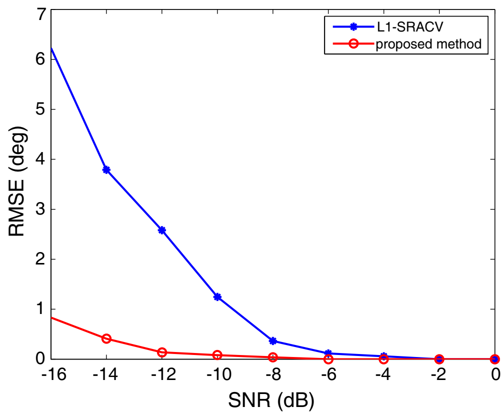

Figure 3. RMSE of the DOA estimation versus SNR between L_1-SRACV and the proposed method.

SNR. It is easy to know from Figure 3 that the proposed method show better performance than the L_1-SRACV algorithm, especially in low SNR cases.

5. CONCLUSION

In this paper, a W-L_1-SRACV algorithm is developed for DOA estimation, in which the sparse representation of array covariance vectors and weighted L_1-norm are exploited. The weighted vector is obtained by exploiting the orthogonality between the noise subspace and the signal subspace spanned by array manifold matrix. DOAs can be effectively estimated simultaneously by searching the sparsest coefficients of the array covariance vectors. Compared with the previous works, the proposed method can offer a number of advantages, such as smaller estimation error, higher resolution, etc.. Furthermore, the spurious peaks can be successfully suppressed in signal recovery processing.

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