

## NUMERICAL APPROACH ON DOPPLER SPECTRUM ANALYSIS FOR MOVING TARGETS ABOVE A TIME-EVOLVING SEA SURFACE

Conghui Qi, Zhiqin Zhao\*, and Zaiping Nie

School of Electronic Engineering, University of Electronic Science and Technology of China, China

**Abstract**—In order to analyze the Doppler spectrum of three-dimensional (3-D) moving targets above a time-evolving sea surface, a hybrid method with acceleration techniques is proposed to simulate the electromagnetic (EM) scattering from the composite moving model. This hybrid iterative method combines Kirchhoff approximation (KA) and the multilevel fast multipole algorithm (MLFMA) to solve the EM backscattering from the rough sea surface and the targets, respectively, then mutual EM coupling effects between them are taken into account through an iterative process. To overcome the vast computational cost in the iterative process, acceleration approaches which can greatly reduce the calculation time are applied. Coupling area on the sea surface is truncated according to geometrical optic principle. Then a fast far-field approximation (FAFFA) is applied to speed up the mutual interactions between the targets and the sea surface. A successive iteration method is proposed to reduce the convergence steps for the MLFMA process. The accuracy and efficiency of this hybrid method with accelerations are demonstrated. Doppler spectra of backscattering signals obtained from such numerical EM simulations are compared for different incident angles, target velocities and surface models. The broadening effects of the Doppler spectra due to the mutual EM coupling interactions are studied.

### 1. INTRODUCTION

Doppler spectra are of great value in moving target detection and remote sensing. Numerical studies of the Doppler spectra are based on accurate and efficient electromagnetic (EM) backscattering

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\* Corresponding author: Zhiqin Zhao (zqzhao@uestc.edu.cn).

simulations [1, 2]. The specific environments have great impact on the EM backscattering of the targets as well as the Doppler spectra from them [3–5]. Many EM methods have been proposed to calculate the scattering from the target including the influence induced by the background [6–8]. When a target is above a three-dimensional (3-D) sea surface at a low altitude, the EM scattering from the sea surface as well as the mutual EM coupling interactions between the sea surface and the moving target has to be considered. This is a typical composite scattering problem which has been widely studied [9, 10]. In pervious studies, there are two main types of algorithms based on integral equation method, one applies half space Green function in a specified environment [11, 12], the other uses an iterative method which calculates the EM scattering from the target and the sea surface separately, then solves the mutual EM coupling between those two parts using an iterative process [13, 14]. The former type of algorithms spends lots computation time on finding the half space Green function which varies with the movement of sea surface. As a result, these algorithms require large computer memory as well as huge computational cost in Doppler spectral simulations. The latter type of algorithms is more flexible and allows calculating EM backscattering from the sea surface and the targets by using different computational methods. Moreover, the efficiencies of these methods can be notably improved by applying some acceleration techniques. This is critical for Doppler spectral simulations for 3-D composite scattering problem.

In this paper, an iterative method is applied to solve the 3-D composite scattering problem. Different computational EM methods are applied on the targets and sea surface. Due to its flexibility and accuracy for complex target, multilevel fast multipole algorithm (MLFMA) [15] is applied to compute the EM scattering of the targets. Whereas, due to its fast computation, Kirchhoff approximation (KA) [16] is used to calculate the scattering of electrically large sea surface. However, the computational load is still a big problem due to the mutual coupling interaction process between the targets and the sea surface. Especially when there are several targets over the surface, it is rather time-consuming to solve the mutual interactions. In order to improve the efficiency of the method, coupling area of the sea surface is truncated according to geometrical optical principle [17]. Then, a fast far-field approximation (FAFFA) [18] is used to calculate the mutual EM coupling interactions between the targets and the sea surface. Also, a successive iteration method [19] is firstly proposed in this algorithm to reduce the convergence steps for MLFMA. Thus, the currents on both the sea surface and the targets will reach a stable state in short time. These techniques remarkably speed up the EM

calculation process.

By using this efficient method, Doppler spectral simulations from 3-D targets above a 3-D sea surface are performed. Quasi-stationary algorithm [20, 21] is used to obtain the time-evolving EM scattering field from the composite model. Then the Doppler spectra are analyzed by using a standard spectrum estimation technique. Due to the mutual EM coupling interactions between the targets and the sea surface, a broadening of the spectra is studied. In the simulations, the spectral broadening as well as the influence induced by incident angles and sea states is investigated.

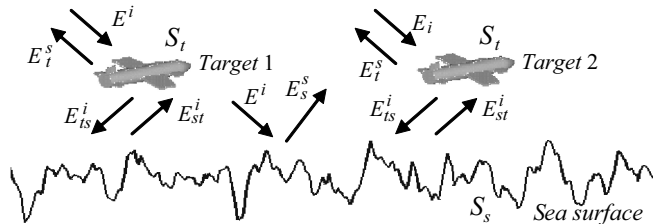
## 2. GEOMETRIC MODELS

As shown in Fig. 1, the composite model is a combination of 3-D targets with arbitrary geometrical model and a 3-D time-evolving sea surface. The time-evolving surface is generated by using spectral method. It assumes that the sea surface is a superposition of harmonics whose amplitudes are proportional to a certain sea surface roughness spectrum. Pierson-Moskowitz (PM) spectrum [22] is adopted in the simulation, which can be expressed as

$$W(K, \varphi) = \frac{\alpha}{2K^4} \exp \left\{ -\frac{\beta g^2}{K^2 U_{19.5}^4} \right\} \cos^4 \left( \frac{\varphi - \varphi_w}{2} \right), \quad (1)$$

in which,  $K$  is the spatial wave number and  $U_{19.5}$  the wind speed at a height of 19.5 m above the sea surface.  $\alpha = 0.0081$  and  $\beta = 0.74$ . The angle  $\varphi$  is with respect to  $x$  axis in  $xy$  plane, and  $\varphi_w$  is wind direction. For simplify,  $\varphi_w$  is assumed as zero in the simulation. The sea surface can be realized by Fourier transform of the sea spectrum. The spatial Fourier components at time  $t$  can be expressed as

$$S(\mathbf{K}, t) = R(\mathbf{K})\pi\sqrt{2L_xL_yW(K, \varphi)}e^{j\omega(K)t} + R(-\mathbf{K})\pi\sqrt{2L_xL_yW(K, \pi - \varphi)}e^{-j\omega(K)t}, \quad (2)$$



**Figure 1.** An illustration of the geometry of the composite model.

where  $w(K) = \sqrt{gK(1 + (K/K_m)^2)}$  is the water wave dispersion relationship,  $K_m = 363.2 \text{ rad/m}$  the maximum spatial wave number of the sea surface,  $R(\mathbf{K})$  a set of independent complex random numbers with unit variance,  $\mathbf{K} = (K_x, K_y)$  the vector spatial wave number, and  $g = 9.81 \text{ m/s}^2$  the gravity acceleration constant.

### 3. ELECTROMAGNETIC METHODS

#### 3.1. The Composite Scattering Field

The EM scattering from this composite model in Fig. 1 includes three parts. The first part is the EM scattering from the sea surface. The second part is the EM scattering from each separate targets taking account of inter-target EM coupling interactions. The third part is the coupling scattering effects between the targets and their underlying rough sea surface. The hybrid method allows calculating EM backscattering from the sea surface and the target using different algorithms. Because MLFMA can efficiently compute the EM scattering from geometrically arbitrary object with high precision, it is used to calculate the scattering of the targets. While for the electrically large sea surface, the approximate method KA is applied.

Refer to Johnson's four-path scattering model [13], the EM coupling interactions between the targets and the sea surface are computed in an iterative way. This stratagem for the 3-D composite EM scattering problem can obtain an accurate result with less computing resources. Suppose the excitation fields on the sea surface ( $S_s$ ) and the target surface ( $S_t$ ) are  $\mathbf{E}_s^i$  and  $\mathbf{E}_t^i$  separately. For the targets the induced currents are excited by the incident wave and the scattered wave from the sea surface. While for the sea surface, the induced currents are excited by the incident wave and the scattered wave from the targets. This stratagem reflects the scattering mechanism of the composite scattering between the sea surface and the targets. The updating processes are expressed as:

$$\mathbf{E}_s^{i(n)}(\mathbf{r}) = \mathbf{E}^i(\mathbf{r}) + \mathbf{E}_{st}^{s(n-1)}(\mathbf{r}), \quad \mathbf{r} \in S_s, \quad (3)$$

$$\mathbf{E}_t^{i(n)}(\mathbf{r}) = \mathbf{E}^i(\mathbf{r}) + \mathbf{E}_{ts}^{s(n-1)}(\mathbf{r}), \quad \mathbf{r} \in S_t, \quad (4)$$

where  $n = 1, 2, \dots$  denotes the  $n$ -th sea-target iteration step and  $\mathbf{E}^i$  the incident field. The interaction fields  $\mathbf{E}_{st}^s$  and  $\mathbf{E}_{ts}^s$  are the scattering fields from target to rough surface and from rough surface to target, respectively. The iteration process can be terminated when the induced currents on the sea surface and targets tend stable. The total scattering

electric field in far zone  $\mathbf{E}_{total}^s$  can be acquired through Equation (5).

$$\mathbf{E}_{total}^s = -j \frac{\omega \mu_0}{4\pi r} e^{-jkr} \left\{ \iint_{s_t} \{ \mathbf{J}_t(\mathbf{r}') - [\mathbf{J}_t(\mathbf{r}') \cdot \hat{k}_s] \hat{k}_s \} e^{jk\mathbf{r}' \cdot \hat{k}_s} ds' + \iint_{s_s} \{ \mathbf{J}_s(\mathbf{r}') - [\mathbf{J}_s(\mathbf{r}') \cdot \hat{k}_s] \hat{k}_s \} e^{jk\mathbf{r}' \cdot \hat{k}_s} ds' \right\} \quad (5)$$

where  $\hat{k}_s$  is the scattered wave vector and  $k$  the wave number of the incident wave.  $\mathbf{J}_t(\mathbf{r}')$  and  $\mathbf{J}_s(\mathbf{r}')$  are the electric currents on the surface of the targets and the sea surface, respectively.

### 3.2. Acceleration Approaches

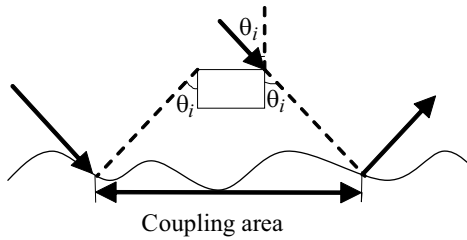
The hybrid method can solve the 3-D composite scattering problem with less computer memory. However, usually it takes a rather long time to get the EM backscattering due to the mutual EM coupling process between the targets and the sea surface. Therefore, some acceleration approaches are always very helpful. The following three acceleration approaches in this paper greatly improve the efficiency of the hybrid method in updating the mutual EM coupling fields between the electrically large sea surface and the targets.

#### 3.2.1. Reducing the Coupling Areas

The coupling process between the sea surface and the targets takes the majority of the computing time in solving the composite problem. For example, if the numbers of the unknowns of the targets and the sea surface are  $N$  and  $M_1$ , respectively, then the computation of the coupling is  $O((M_1 * N)^2)$ . In this paper, specular reflection property of the rough sea surface scattering is applied to reduce the coupling area on the sea surface. This property shows that the main scattering energy is concentrated at the specular direction which is 10 dB higher than the scatterings at other directions [17]. Therefore, the coupling area on the sea surface can be truncated according to the specular reflection. According to geometrical optical principle, the truncated coupling area is shown as shown in Fig. 2. The unknowns  $M_2$  in the coupling area will be only a small fraction of  $M_1$ . The complexities of multiplication will be decreased to  $O((M_2 + N)^2)$ .

The coupling area of the sea surface corresponding to each target is

$$\begin{aligned} x'_{\min} - h \tan \theta_i &\leq x \leq x'_{\max} + h \tan \theta_i, \\ y'_{\min} - h \tan \theta_i &\leq y \leq y'_{\max} + h \tan \theta_i, \end{aligned} \quad (6)$$



**Figure 2.** Illustration of the coupling area on the sea surface.

where  $\theta_i$  is the incident angle,  $x'_{\max}$ ,  $x'_{\min}$ ,  $y'_{\max}$ ,  $y'_{\min}$  are the maximum and the minimum locations in  $x$  and  $y$  directions of the target.

### 3.2.2. Fast Far-field Approximation

To further accelerate the coupling process between the targets and the sea surface, fast far-field approximation (FAFFA) is applied to calculate the two far coupling groups. Suppose that  $\mathbf{r}_t$  is an observation point located in an observation group  $m$ , and  $\mathbf{r}_s$  is a source point that is located in a source group  $n$ .  $\mathbf{r}_m$  and  $\mathbf{r}_n$  represent the centers of the observation and the source groups, respectively. Under the far-field condition, i.e.,  $|\mathbf{r}_{tm} + \mathbf{r}_{ns}| \ll |\mathbf{r}_{mn}|$ , the scalar Green's function is approximated as

$$\frac{e^{-jk|\mathbf{r}_t - \mathbf{r}_s|}}{4\pi|\mathbf{r}_t - \mathbf{r}_s|} = \frac{e^{-jk|\mathbf{r}_{tm} + \mathbf{r}_{mn} + \mathbf{r}_{ns}|}}{4\pi|\mathbf{r}_{tm} + \mathbf{r}_{mn} + \mathbf{r}_{ns}|} \approx \frac{e^{-jk|\mathbf{r}_{mn}|} e^{-jk\hat{\mathbf{r}}_{mn} \cdot (\mathbf{r}_{tm} + \mathbf{r}_{ns})}}{4\pi|\mathbf{r}_{mn}|}, \quad (7)$$

where  $\hat{\mathbf{r}}_{mn} = \mathbf{r}_m - \mathbf{r}_n / |\mathbf{r}_m - \mathbf{r}_n|$ .

For the iterative process, the observation point and the source point are located on different surfaces ( $S_s$  or  $S_t$ ). In terms of FAFFA, the interactions are separated into the following three steps: aggregation, translation, and disaggregation in far regions. The matrix-vector multiplication can be accelerated by using the fast multipole method (FMM) [23]. And the complexity of multiplication greatly is greatly reduced to  $O((M_2 + N) \log(M_2 + N))$ .

### 3.2.3. Successive Iteration Method

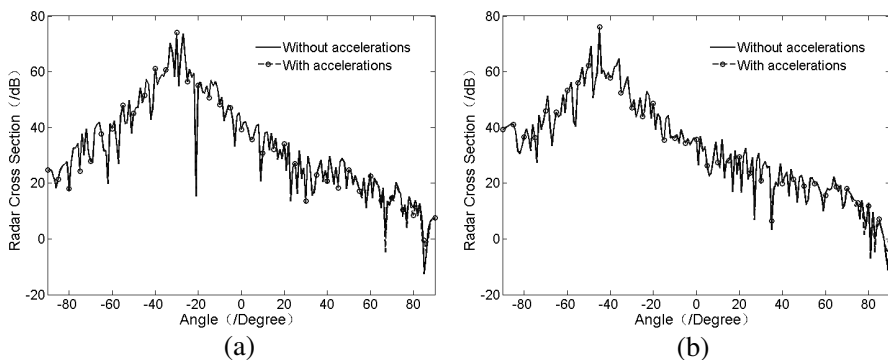
A successive iteration method is firstly proposed in this paper to further expedite the coupling process. There are two iterative processes in computing the composite scattering problem. One is the sea-target iterative process existing between the targets and the sea surface. The other one is the inter-target iteration to solve the matrix equation of the target when uses MLFMA. Successive iteration method is

applied in the inter-target iteration process to reduce the iteration steps of MLFMA while solve the matrix equation. At the first sea-target interaction process, the unknowns on the target surface are initialized as zeros and the excited field is the incident wave and the scattering wave from the sea surface. At the later sea-target interaction processes, the unknowns on the targets are initialized using the results obtained in the former iteration. This successive method can greatly accelerate the convergence of the inter-target iteration at each sea-target interaction process. Because the currents on both the sea surface and the target surface turn stable and do not change much at each sea-target interaction process. The currents on the targets are initialized as

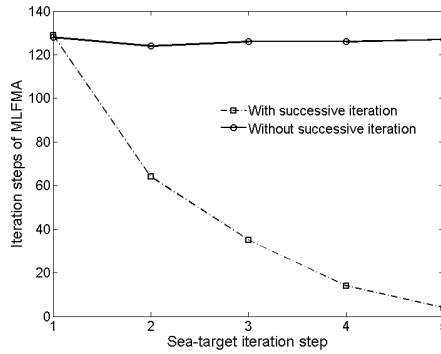
$$\mathbf{J}_t^{(1)}|_{\text{init}} = 0, \quad \mathbf{J}_t^{(n+1)}|_{\text{init}} = \mathbf{J}_t^{(n)}, \quad (8)$$

where  $n$  is the  $n$ -th sea-target interaction process.

An experiment is performed to show the validity and efficiency of this hybrid iteration method. The bistatic radar cross section (RCS) from a target at a altitude of 5 m above a rough sea surface is simulated at the incident angles of  $30^\circ$  and  $45^\circ$ , respectively. The radar working frequency is 300 MHz. The validity of the computational algorithm with accelerations is shown in Fig. 3. Compared with the conventional hybrid method without accelerations, the bistatic scattering radar cross sections are consistent with a relative error of less than 0.32% in a range of  $-90^\circ$  to  $90^\circ$ . The efficiency of this method is demonstrated in Fig. 4 when the incident angle is  $30^\circ$ . The successive iteration method has reduced the convergence steps for the inter-target MLFMA process. For each sea-target interaction process, the MLFMA process need about 130 steps to reach convergence by using the method without successive iteration method. As a contrast, the number of iteration



**Figure 3.** The validity of the acceleration techniques. (a)  $30^\circ$ , (b)  $45^\circ$ .



**Figure 4.** The speed of convergence for the successive iteration method.

**Table 1.** The comparison on time consumptions.

Incident angle	With accelerations ( <i>s</i> )	Without accelerations ( <i>s</i> )
30°	717	13321
45°	1034	16814

steps decreases to about half the number in the former sea-target interaction. Table 1 shows the comparison of the time consumptions with and without acceleration techniques. It demonstrates that when using these acceleration approaches, the time consumption is less than 7% of the time used without these accelerations. These robust and efficient techniques make large scale Doppler simulations from 3-D targets and sea surface possible.

#### 4. DOPPLER SPECTRUM ANALYSIS

Quasi-stationary algorithm is applied to obtain the Doppler spectra of this composite time-evolving model. During a time period  $T$ , the evolving model is separated as a series of time-frozen surfaces with a short time interval  $\tau$ . How to choose this time interval is given in Ref. [20]. Repeat this EM scattering procedure for  $INT(T/\tau)$  times to get a time-varying field from this composite evolving model, then the Doppler spectrum can be obtained by using the standard spectral-estimation technique,

$$S(f) = \frac{1}{T} \left| \int_0^T E_{total}^s(t) e^{-i2\pi ft} dt \right|^2, \quad (9)$$



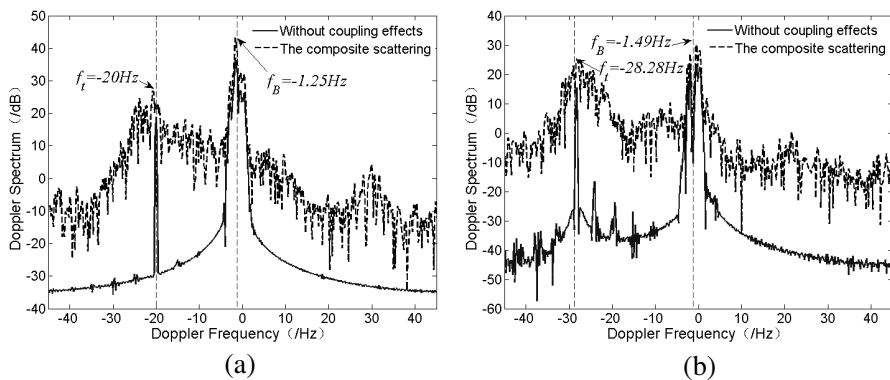
where  $E_{total}^s(t)$  is the vertical (or horizontal) polarization of total scattering field at the time  $t$ .

In order to analyze the EM scattering characteristics of the composite model of the moving targets and the time-evolving sea surface, the EM coupling effects to the Doppler spectra are illustrated first in the following simulation. Then, Doppler spectra of one target moving above the rough sea surface under different conditions are simulated and analyzed. Finally, the Doppler spectra are examined when there are two targets above sea surface.

All the following simulations are performed at the frequency of 300 MHz. The sea surface is seen as dielectric, whereas the targets are regarded as perfectly electric conductor. At moderate incident angles, the vertically polarized (VV) EM backscattering from sea surface is much higher than horizontally polarized (HH) field. The incident and received electric fields are both vertically polarized. Doppler spectra are computed from 500 backscattered field time samples spaced in intervals of 0.01 s. The sea surface is with a length of 200 m and width of 30 m. In the following figures, the vertical dashed lines ( $f_B$ ) are theoretical Bragg frequencies. Whereas, the vertical dashed lines ( $f_t$ ) are Doppler frequencies when the target is treated as a point target.

### 4.1. The Coupling Effects to the Doppler Spectra

To examine the coupling impacts on Doppler spectra, simulations are firstly performed on a target moving above the sea surface. For simplicity the target is moving towards  $x$  axis at a low altitude of 5 m above the sea surface. Fig. 5 shows the Doppler spectra from a moving

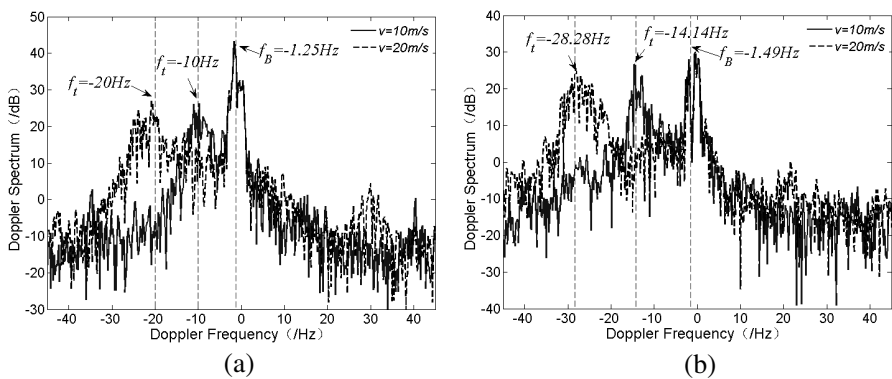


**Figure 5.** The coupling effects between the sea surface and the target. (a)  $30^\circ$ , (b)  $45^\circ$ .

target with a velocity of 20 m/s above the sea surface. During the observation time  $T = 5$  s, the target moves from  $x = 20$  m to  $x = 120$  m. The wind speed  $U_{19.5}$  is 5 m/s. The incident angles are  $30^\circ$  and  $45^\circ$ . When counting the coupling mechanism between the target and the sea surface, a broadening of the Doppler spectra is shown. There are two main peaks centered at the theoretical Doppler frequencies corresponding to the velocity of the target and the evolution of the sea. The coupling effects to the left peak are much more evident than that to the right peak. It can be concluded that the EM scattering from the sea surface greatly affects the backscattering of the target. By contrast, the target has little influence on the sea surface scattering. When taking the mutual coupling between the two parts into account, the transmitting paths of the EM wave are much more complex. Thus more spurious scattering can be found in the Doppler spectra. At certain frequencies some spikes are found in the Doppler spectra. However, compared to the Bragg peak, these spikes are much lower.

#### 4.2. Doppler Spectra from One Moving Target above the Sea

The Doppler spectra of a low moving target with different velocities above the same sea surface is shown in Fig. 6. The incident angles are  $30^\circ$  and  $45^\circ$  corresponding to (a) and (b). These two peaks are corresponding to the sea state and the velocity of the target. The Doppler frequency peaks are centered at  $f_B$  and  $f_t$ . The center Doppler frequency of the target  $f_t$  is directly proportion to its velocity. It can be acquired by  $f_t = 2v \sin \theta_i / \lambda$ , where  $v$  is the velocity of the moving



**Figure 6.** Different target velocities with wind speed of 5 m/s. (a)  $30^\circ$ , (b)  $45^\circ$ .

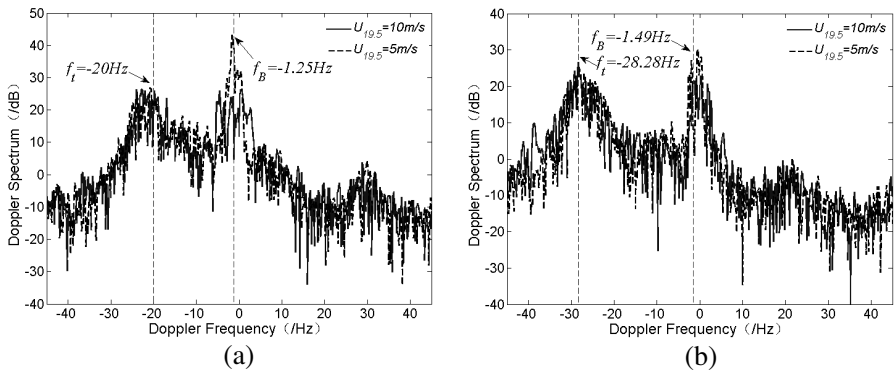
target. When the velocity of the target is 20 m/s,  $f_t$  is  $-20$  Hz, which obeys the Doppler effects.

The center Doppler frequency of the sea surface is Bragg frequency, which indicates that the Bragg scattering is dominate in the EM backscattering from the sea surface. If the deep water gravity waves of length  $\Lambda$  travel at a given phase velocity  $v_p = \sqrt{g\Lambda/2\pi}$ . Then the Bragg frequency can be acquired by

$$f_B = 2v_p \sin \theta_i / \lambda = \sqrt{2gk \sin \theta_i} / 2\pi, \tag{10}$$

When the incident angle is small, the sea echo is much stronger than the backscattering of the target. It is obvious that if the target's velocity is very small or the incident angle is small, the Doppler frequency peak of the target will be submerged in the Bragg peak. Since flying target usually moves much faster than the sea surface, velocity-sensitive radar can eliminate unwanted sea clutter. However, the influence induced by the EM coupling effects can not be neglected.

Comparisons of the Doppler spectra from this target with a velocity of 20 m/s moving under different sea states are shown in Fig. 7. The broadening of the Doppler spectra for rougher sea surface is much more evident. As the incident angle gets larger, the EM backscattering from the sea surface is smaller, the spectral broadening becomes weaker.



**Figure 7.** Different sea states with target velocity of 20 m/s. (a)  $30^\circ$ , (b)  $45^\circ$ .

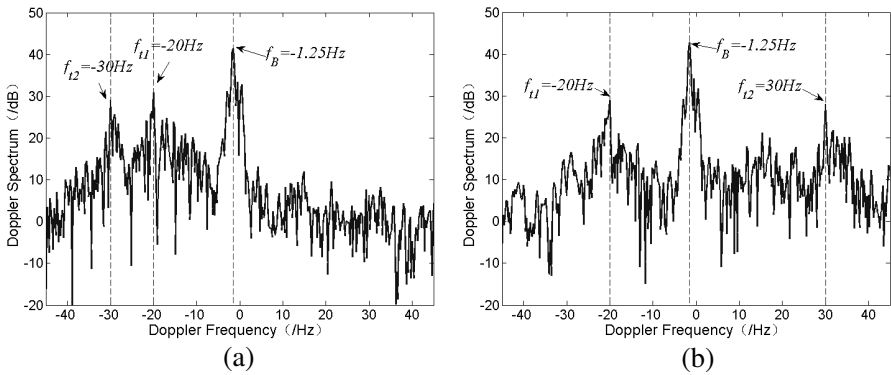
### 4.3. Doppler Spectra from Multiple Moving Targets above the Sea

As another numerical example, Doppler spectra from two moving targets above the time-evolving sea surfaces are simulated. The

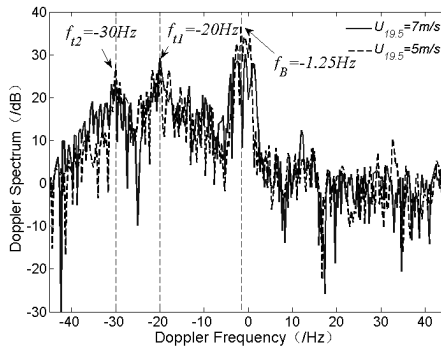
incident angle is  $30^\circ$ . One target is at a height of 3 m with a velocity of 30 m/s. The other one is at a height of 8 m with a velocity of 20 m/s.

Figure 8 shows the Doppler spectra of two targets moving with different velocities above a PM sea surface. The wind speed is 5 m/s. The Doppler spectral peaks locate at the Bragg frequency ( $f_B$ ) which is induced by the evolving of the sea surface. The other two peaks centered at the Doppler frequency when the target is seen as a point target ( $f_{t1}$  and  $f_{t2}$ ). When the targets move at the opposite direction, the Doppler spectral peaks will turn out on the positive frequency axis. In Fig. 8, the two targets can be clearly distinguished from the Doppler spectra.

Comparisons of the Doppler spectra corresponding to different sea states are shown in Fig. 9. It can be seen from the two curves that



**Figure 8.** The Doppler spectra of two targets. (a) In the same direction. (b) In the opposite directions.



**Figure 9.** Comparison of Doppler spectra under different sea states.

when the wind speed gets higher (e.g.,  $U_{19.5} = 7$  m/s), the Doppler spectral broadening is more evident especially when the target signal is weak.

## 5. CONCLUSION

The Doppler simulation of moving targets at low altitude above a 3-D time-evolving sea surface is a challenging problem for the complicated EM scattering mechanism of the composite model as well as the vast computational requirements. In this paper, a hybrid iterative method is applied to solve the composite EM scattering from the targets and the sea surface. Considering the precision and the efficiency, the sea surface is computed by using analytical method while the complex 3-D targets are calculated by using numerical method, then the coupling between sea surface and the targets is obtained by using an iterative process. To reduce the huge computational cost, three acceleration techniques are applied. Examples show that the acceleration techniques can greatly improve the computing efficiency, thus this method makes the Doppler simulation for this 3-D problem possible.

The Doppler spectra from moving targets at low altitude above 3-D time-evolving sea surface under different sea states are simulated. Doppler spectra are greatly influenced by the EM coupling effects between the targets and sea surface. Due to the mutual interaction between the sea surface and the targets, the Doppler spectra of the targets are not a single line. A spectral broadening can be found in the results. Such numerical simulations for the 3-D composite time-evolving model can serve as a useful tool in studying the Doppler spectra for the cases that experimental results are scarce and expensive.

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## REFERENCES

1. Calvo-Gallego, J. and F. Perez-Martinez, "Simple traffic surveillance system based on range-doppler radar images," *Progress In Electromagnetics Research*, Vol. 125, 343–364, 2012.

2. Wang, Y., Y.-M. Zhang, and L.-X. Guo, "Microwave doppler spectra of sea echoes at high incidence angles: Influences of large-scale waves," *Progress In Electromagnetics Research B*, Vol. 48, 99–113, 2013.
3. Valagiannopoulos, C. and N. Uzunoglu, "Simplified model for EM inverse scattering by longitudinal subterranean inhomogeneities exploiting the dawn/dusk ionospheric ridge," *IET Microwaves, Antennas & Propagation*, Vol. 5, 1319–1327, 2011.
4. Sun, R.-Q., M. Zhang, C. Wang, and Y. Chen, "Study of electromagnetic scattering from ship wakes on PEC sea surfaces by the small-slope approximation theory," *Progress In Electromagnetics Research*, Vol. 129, 387–404, 2012.
5. Valagiannopoulos, C. A., "On jamming unfriendly submarine communication by radiating across an island in the vicinity of the opponent's coastline," *Electromagnetics*, Vol. 32, 438–449, 2012.
6. Valagiannopoulos, C. A., "On developing alternating voltage around a rotating circular ring under plane wave excitation in the presence of an eccentrically positioned metallic core," *Progress In Electromagnetics Research M*, Vol. 12, 193–204, 2010.
7. Bi, S. and X. Y. Ren, "Maneuvering target doppler-bearing tracking with signal time delay using interacting multiple model algorithms," *Progress In Electromagnetics Research*, Vol. 87, 15–41, 2008.
8. Valagiannopoulos, C. A., "Study of an electrically anisotropic cylinder excited magnetically by a straight strip line," *Progress In Electromagnetics Research*, Vol. 73, 297–325, 2007.
9. Chen, H., M. Zhang, and H.-C. Yin, "Facet-based treatment on microwave bistatic scattering of three-dimensional sea surface with electrically large ship," *Progress In Electromagnetics Research*, Vol. 123, 385–405, 2012.
10. Wu, Z.-S., J.-J. Zhang, and L. Zhao, "Composite electromagnetic scattering from the plate target above a one-dimensional sea surface: Taking the diffraction into account," *Progress In Electromagnetics Research*, Vol. 92, 317–331, 2009.
11. Liu, Z. and L. Carin, "Efficient evaluation of the half-space Green's function for fast-multipole scattering models," *Microwave and Optical Technology Letters*, Vol. 29, 388–392, 2001.
12. Geng, N., A. Sullivan, and L. Carin, "Fast multipole method for scattering from an arbitrary PEC target above or buried in a lossy half space," *IEEE Transactions on Antennas and Propagation*, Vol. 49, 740–748, 2001.

13. Johnson, J. T., "A numerical study of scattering from an object above a rough surface," *IEEE Transactions on Antennas and Propagation*, Vol. 50, 1361–1367, 2002.
14. Zhang, Y., Y. Yang, H. Braunisch, and J. Kong, "Electromagnetic wave interaction of conducting object with rough surface by hybrid SPM/MOM technique," *Progress In Electromagnetics Research*, Vol. 22, 315–335, 1999.
15. Yang, W., Z. Zhao, C. Qi, and Z. Nie, "Electromagnetic modeling of breaking waves at low grazing angles with adaptive higher order hierarchical legendre basis functions," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 49, 346–352, 2011.
16. Beckmann, P. and A. Spizzichino, *The Scattering of Electromagnetic Waves from Rough Surfaces*, Vol. 1, 511, Norwood, MA, Artech House, Inc., 1987.
17. Tsang, L., J. A. Kong, and K. H. Ding, *Scattering of Electromagnetic Waves, Theories and Applications*, Vol. 27, Wiley-Interscience, 2004.
18. Chew, W. C., T. J. Cui, and J. M. Song, "A FAFMA-MLFMA algorithm for electromagnetic scattering," *IEEE Transactions on Antennas and Propagation*, Vol. 50, 1641–1649, 2002.
19. Chaitin-Chatelin, F. and S. Gratton, "Convergence in finite precision of successive iteration methods under high nonnormality," *BIT Numerical Mathematics*, Vol. 36, 455–469, Sep. 1, 1996.
20. Toporkov, J. V. and G. S. Brown, "Numerical simulations of scattering from time-varying, randomly rough surfaces," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 38, 1616–1625, 2000.
21. Nie, D., M. Zhang, X. Geng, and P. Zhou, "Investigation on doppler spectral characteristics of electromagnetic backscattered echoes from dynamic nonlinear surfaces of finite-depth sea," *Progress In Electromagnetics Research*, Vol. 130, 169–186, 2012.
22. Li, X. and X. Xu, "Scattering and doppler spectral analysis for two-dimensional linear and nonlinear sea surfaces," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 49, 603–611, 2011.
23. Coifman, R., V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: A pedestrian prescription," *IEEE Antennas and Propagation Magazine*, Vol. 35, 7–12, 1993.