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# SOME PECULIARITIES OF THE SPATIAL POWER SPECTRUM OF SCATTERED ELECTROMAGNETIC WAVES IN RANDOMLY INHOMOGENEOUS MAGNETIZED PLASMA WITH ELECTRON DENSITY AND EXTERNAL MAGNETIC FIELD FLUCTUATIONS

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**Abstract**—Statistical characteristics of multiply scattered electromagnetic waves in turbulent magnetized plasma with both electron density and external magnetic field fluctuations are considered. Analytical expression for phase fluctuations of scattered radiation is derived using the smooth perturbation method. Correlation and wave structure functions of the phase fluctuations, angle-of-arrivals are obtained for arbitrary correlation functions of fluctuating plasma parameters and external magnetic field taking into account the diffraction effects. The evolution of a double-peaked shape in the spatial power spectrum of scattered radiation is analyzed numerically for both anisotropic Gaussian and power-law spectra of electron density fluctuations using experimental data. Phase portraits of external magnetic field fluctuations have been constructed for different non-dimensional spatial parameters characterizing a given problem.

## 1. INTRODUCTION

The features of light propagation in randomly inhomogeneous statistically isotropic media have been rather well studied [1–3]. However, in many cases turbulent media (biological tissue, atmosphere, ocean, etc.) are anisotropic. For example molecules of thermotropic liquid crystals have a typical length of about 20 angstrom and their longitudinal and transverse sizes are in the ratio $(4 \div 8) : 1$. Structures of greater size are formed in lyotropic systems, where the

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ratio may exceed 15 [4]. Small-angle scattering of UV radiation is typical of algae and plants chloroplast having ellipsoidal shape with diameter from 1 μm to 5 μm and length from 1 μm to 10 μm [4]. At electromagnetic waves scattering in randomly isotropic media cross section of inhomogeneities depends on the direction of the incident wave. In many cases prolate irregularities are oriented along a certain direction. Such an orientation is observed in lyotropic liquid crystals with hexagonal structure [4]. In thermotropic liquid crystals it can be easily realized by an external electric field [5]. Moreover it is well known that polymer macromolecules are oriented along a velocity gradient of a liquid flow [6, 7]. Electron concentration irregularities have anisotropic formations in the ionosphere and are strongly extended along the geomagnetic field [8]. Therefore, investigation of statistical characteristics of scattered radiation in randomly inhomogeneous anisotropic media is of great interest. The authors [9] considered a 2D nonabsorbing medium with random inhomogeneities extended along the direction of propagation of an incident wave. The Fokker-Planck equation for the probability density characterizing propagation of rays was derived. The evolution of the angular distribution of the intensity of an electromagnetic wave at oblique and normal illumination of a thick layer of a random statistically anisotropic medium has been investigated using Monte-Carlo simulations [10, 11]. It was shown that strongly pronounced dip exists in the angular spectrum of a single-scattered radiation along the direction of prolate inhomogeneities caused due to permittivity fluctuations. The same peculiarities have been revealed analytically in the spatial power spectrum (SPS) of a multiple scattered radiation at oblique illumination of random medium with prolate inhomogeneities by mono-directed incident radiation using the smooth perturbation method [12]. The authors have also studied the features of the SPS of scattered radiation in magnetized anisotropic turbulent plasma using complex geometrical optics approximation and perturbation method [13–16].

This paper is devoted to the investigation of statistical characteristics of multiply scattered electromagnetic waves in turbulent magnetized plasma with both electron density and external magnetic field fluctuations. In Section 2 analytical expression for phase fluctuations of scattered radiation in the direction perpendicular to the principle plane is derived using the smooth perturbation method taking into account the diffraction effects. Correlation and wave structure functions of the phase fluctuations for arbitrary correlation functions of fluctuating plasma parameters and external magnetic field are obtained in Section 3. The evolution of a double-peaked shape in the SPS of scattered radiation for both anisotropic Gaussian and power-law spectra of electron density fluctuations using experimental data is analyzed numerically in Section 4. Formation of a gap at different angles of inclination of prolate irregularities with respect to the external magnetic field for the power-law spectrum is considered for the first time. Phase portraits of external magnetic field fluctuations have been constructed for different non-dimensional spatial parameters characterizing given problem.

## 2. FORMULATION OF THE PROBLEM

The features of scattered waves in the magnetized plasma with anisotropic electron density and external magnetic field fluctuations is based on the vector wave equation for the electric field $\mathbf{E}$:

$$\left(\frac{\partial^2}{\partial x_i \partial x_j} - \Delta\delta_{ij} - k_0^2 \varepsilon_{ij}(\mathbf{r})\right)\mathbf{E_j}(\mathbf{r}) = 0. \tag{1}$$

Wave field we introduce as $E_j(\mathbf{r}) = E_{0j}\exp\{\Phi(\mathbf{r})\}$ where $\Phi(\mathbf{r})$ is the complex phase, which is presented as a sum $\Phi(\mathbf{r}) = \varphi_0 + \varphi_1 + \varphi_2 + \ldots$, $\varphi_0 = ik_0x + ik_\perp y$ ($k_\perp \ll k_0$). If electromagnetic wave is propagating along $z$ axis and the vector $\boldsymbol{\tau}$ lies in the $yoz$ plane ($\mathbf{k} \parallel z$, $\langle \mathbf{H_0} \rangle \in yz$-principle plane), the components of the second-rank tensor $\varepsilon_{ij}$ of collisionless magnetized plasma have the following form [17]:

$$\begin{aligned}
&\varepsilon_{xx} = 1 - \frac{v}{1-u}, \quad \varepsilon_{yy} = 1 - \frac{v\left(1 - u\sin^2\alpha\right)}{1-u}, \quad \varepsilon_{zz} = 1 - \frac{v\left(1-u\cos^2\alpha\right)}{1-u},\\
&\varepsilon_{xy} = -\varepsilon_{yx} = i\frac{v\sqrt{u}\cos\alpha}{1-u}, \quad \varepsilon_{yz} = \varepsilon_{zy} = \frac{uv\sin\alpha\cos\alpha}{1-u},\\
&\varepsilon_{xz} = -\varepsilon_{zx} = -i\frac{v\sqrt{u}\sin\alpha}{1-u},
\end{aligned} \tag{2}$$

where $\alpha$ is the angle between the vectors $\mathbf{k}$ and $\mathbf{H_0}$; $\varepsilon_{xy} = i\tilde{\varepsilon}_{xy}$, $\varepsilon_{xz} = -i\tilde{\varepsilon}_{xz}$, $u(\mathbf{r}) = [eH_0(\mathbf{r})/mc\omega]^2$, $v(\mathbf{r}) = \omega_p^2(\mathbf{r})/\omega^2$ are the magneto-ionic parameters, $\omega_p(\mathbf{r}) = [4\pi N(\mathbf{r})e^2/m]^{1/2}$ is the plasma frequency, $\Omega_H(\mathbf{r}) = eH_0(\mathbf{r})/mc$ is the electron gyro-frequency.

If wavelength $\lambda$ is small in comparison with the characteristic linear scales of electron density and external magnetic fluctuations $l_{n,h}$, the scattered waves are concentrated in a narrow solid angle, i.e., scattered waves propagate in the same direction as an incident wave. One of the methods describing multiple scattering in random media is the ray-(optics) approximation, $\sqrt{\lambda L} \ll l_{n,h}$, but it neglects the diffraction effects. If a distance $L$ travelling by wave in magnetized turbulent plasma is substantially big, diffraction effects become essential. In this case, multiple scattering is effectively described by the smooth perturbation method (narrow-angle scattering) for the solution of diffraction effects [1–3]. Dielectric permittivity of turbulent magnetized plasma is a random function of the spatial coordinates $\varepsilon_{ij}(\mathbf{r}) = \varepsilon_{ij}^{(0)} + \varepsilon_{ij}^{(1)}(\mathbf{r})$, $|\varepsilon_{ij}^{(1)}(\mathbf{r})| \ll 1$. The first term is the regular (unperturbed) component of the dielectric permittivity connecting with the ionization distribution in the ionospheric layers at different altitudes; the second term is the fluctuating term of the dielectric permittivity describing both electron density and external magnetic field fluctuations in the ionosphere, which are random functions of the spatial coordinates: $v(\mathbf{r}) = v_0[1 + n_1(\mathbf{r})]$, $u(\mathbf{r}) = u_0[1 + 2h_1(\mathbf{r})]$. We suppose that electron density and external magnetic field fluctuations are statistically independent [3]. Complex phase fluctuations are of the order $\varphi_1 \sim \varepsilon_{ij}^{(1)}$, $\varphi_2 \sim \varepsilon_{ij}^{(1)2}$. Wave field we introduce as $E_j(\mathbf{r}) = E_{0j}\exp(\varphi_0 + \varphi_1 + \varphi_2 + \ldots)$ , $\varphi_0 = ik_\perp y + ik_0 z$ ($k_\perp \ll k_0$). Substituting these expressions in (2) we can easily restore regular and fluctuating components of magnetized plasma permittivity tensor.

In the zero-order approximation the wave equation:

$$\left[\frac{\partial^2\varphi_0}{\partial x_i \partial x_j} + \frac{\partial\varphi_0}{\partial x_i}\frac{\partial\varphi_0}{\partial x_j} + \left(k_\perp^2 + k_0^2\right)\delta_{ij} - k_0^2\varepsilon_{ij}^{(0)}\right]E_{0j} = 0, \tag{3}$$

contains the set of three algebraic equations for $E_{0j}$ ($j = x, y, z$) regular field components:

$$\begin{aligned}
\left(\mu^2 + 1 - \varepsilon_{xx}^{(0)}\right)E_{0x} - i\tilde{\varepsilon}_{xy}^{(0)}E_{0y} + i\tilde{\varepsilon}_{xz}^{(0)}E_{0z} &= 0,\\
i\tilde{\varepsilon}_{xy}^{(0)}E_{0x} + \left(1 - \varepsilon_{yy}^{(0)}\right)E_{0y} - \left(\mu + \varepsilon_{yz}^{(0)}\right)E_{0z} &= 0,\\
i\tilde{\varepsilon}_{xz}^{(0)}E_{0x} + \left(\mu + \varepsilon_{yz}^{(0)}\right)E_{0y} - \left(\mu^2 - \varepsilon_{zz}^{(0)}\right)E_{0z} &= 0,
\end{aligned} \tag{4}$$

where $\mu = k_\perp/k_0$.

Taking into account inequalities characterizing the smooth perturbation method:

$$\left|\frac{\partial\varphi_1}{\partial z}\right| \ll k_0|\varphi_1|, \quad \left|\frac{\partial^2\varphi_1}{\partial z^2}\right| \ll k_0\left|\frac{\partial\varphi_1}{\partial z}\right|, \quad \left|\frac{\partial\varphi_2}{\partial z}\right| \ll k_0|\varphi_2|, \quad \left|\frac{\partial^2\varphi_2}{\partial z^2}\right| \ll k_0\left|\frac{\partial\varphi_2}{\partial z}\right|,$$

in the first approximation we obtain:

$$\begin{aligned}
&\left[\frac{\partial^2\varphi_1}{\partial x_i \partial x_j} + \frac{\partial\varphi_0}{\partial x_i}\frac{\partial\varphi_1}{\partial x_j} + \frac{\partial\varphi_1}{\partial x_i}\frac{\partial\varphi_0}{\partial x_j}\right.\\
&\left.-\delta_{ij}\left(\Delta_\perp + 2ik_\perp\frac{\partial\varphi_1}{\partial y} + 2ik_0\frac{\partial\varphi_1}{\partial z}\right) - k_0^2\varepsilon_{ij}^{(0)}\right]E_{0j} = 0.
\end{aligned} \tag{5}$$

where $\Delta_\perp = (\partial^2\varphi_1/\partial x^2) + (\partial^2\varphi_1/\partial y^2)$ is the transversal Laplasian.

Using two-dimensional Fourier transformation

$$\varphi_1(x,y,z)=\int\limits_{-\infty}^{\infty}dk_x\int\limits_{-\infty}^{\infty}dk_y\psi(k_x,k_y,z)\exp(ik_xx+ik_yy),$$

from (5) we obtain stochastic differential equation for $j=x$ component of the phase fluctuation [16]:

$$\frac{\partial\psi}{\partial z}+\frac{i}{ik_x\Gamma_j-2k_0}\left[ik_x(k_y+k_\perp)P_j+ik_xk_0\Gamma_j-k_y(k_y+2k_\perp)\right]\psi$$
$$=-i\frac{k_0^2}{ik_x\Gamma_j-2k_0}\cdot\left(\varepsilon_{xx}^{(1)}-\tilde{\varepsilon}_{xy}^{(1)}P_j+i\tilde{\varepsilon}_{xz}^{(1)}\Gamma_j\right),\tag{6}$$

Mean electric field components are determined by well-known formulae: $(E_{0y}/E_{0x})=iP_j$, $(E_{0z}/E_{0x})=i\Gamma_j$, where polarization coefficients are [17]:

$$P_j=\frac{2\sqrt{u}(1-v)\cos\alpha}{u\sin^2\alpha\pm\sqrt{u^2\sin^4\alpha+4u(1-v)^2\cos^2\alpha}},$$
$$\Gamma_j=-\frac{v\sqrt{u}\sin\alpha+P_juv\sin\alpha\cos\alpha}{1-u-v+uv\cos^2\alpha}.\tag{7}$$

The upper sign (index $j=1$) corresponds to the extraordinary wave and the lower sign (index $j=2$) corresponds to the ordinary wave. Ordinary and extraordinary waves in magnetized plasma generally are elliptically polarized. External magnetic field radically changes electromagnetic properties of plasma making it magnetized (gyrotropic and anisotropic) medium. Gyrotropy of plasma is revealed in elliptic polarization of normal waves; anisotropy is revealed in dependence of their characteristics (polarization, refractive index and absorption) on the direction of propagation. Wave structure functions [1–3]:

$$D_1(\mathbf{r_1},\mathbf{r_2})=\left\langle\left(\varphi_1(\mathbf{r_1})-\varphi_1(\mathbf{r_2})\right)\left(\varphi_1^*(\mathbf{r_1})-\varphi_1^*(\mathbf{r_2})\right)\right\rangle,$$
$$D_2(\mathbf{r_1},\mathbf{r_2})=\left\langle\left(\varphi_1(\mathbf{r_1})-\varphi_1(\mathbf{r_2})\right)^2\right\rangle,\tag{8}$$

of the amplitude, phase and mutual correlation functions are determined by the expressions [1–3]:

$$D_\chi(\mathbf{r}_1,\mathbf{r_2})=\frac{1}{2}(D_1+\mathrm{Re}D_2),\quad D_S(\mathbf{r}_1,\mathbf{r_2})=\frac{1}{2}(D_1-\mathrm{Re}D_2),$$
$$D_{\chi S}=\frac{1}{2}\mathrm{Im}D_2,\tag{9}$$

Phase variations estimate angle-of-arrivals (AOA) of radio waves. Angle-of-arrivals of scattered electromagnetic waves are calculated using wave structure function [1–3]:

$$\left\langle\theta_x^2\right\rangle = \lim_{\xi\to 0}\frac{D_1(\xi,0,L)}{\xi^2}, \quad \left\langle\theta_y^2\right\rangle = \lim_{\eta\to 0}\frac{D_1(0,\eta,L)}{\eta^2}. \tag{10}$$

where: $\xi = k_0\rho_x$, $\eta = k_0\rho_y$, $\rho_y$ and $\rho_x$ are distances between observation points spaced apart in the principle and perpendicular planes, respectively.

Transverse correlation function of a scattered field $W_{EE^*}(\boldsymbol{\rho}) = \langle E(\mathbf{r})E^*(\mathbf{r}+\boldsymbol{\rho})\rangle$ taking into account that the observation points are spaced apart at a small distance $\boldsymbol{\rho} = \{\rho_x,\, \rho_y\}$ is given in the following form [14]:

$$W_{EE^*}(\boldsymbol{\rho}, k_\perp) = E_0^2 \exp\left\{\mathrm{Re}\left[\frac{1}{2}\left(\left\langle\varphi_1^2(\mathbf{r})\right\rangle + \left\langle\varphi_1^{2*}(\mathbf{r}+\boldsymbol{\rho})\right\rangle\right) + \left\langle\varphi_1(\mathbf{r})\varphi_1^*(\mathbf{r}+\boldsymbol{\rho})\right\rangle + 2\langle\varphi_2\rangle\right]\right\} \cdot \exp(-i\rho_y k_\perp) \tag{11}$$

where $E_0^2$ is the intensity of incident radiation.

Spatial power spectrum of scattered field in case of incident plane wave $W(k',\, k_\perp)$ is easily calculated by Fourier transform of the transversal correlation function of scattered field [2]:

$$W(k, k_\perp) = \int_{-\infty}^{\infty} d\rho_y W_{EE^*}(\rho_y, k_\perp)\exp(ik\rho_y). \tag{12}$$

when the angular spectrum of an incident wave has a finite width and its maximum coincide with $z$ axis, APS of scattered radiation is given:

$$I(k) = \int_{-\infty}^{\infty} dk_\perp W(k, k_\perp)\exp\left(-k_\perp^2\beta^2\right), \tag{13}$$

where $\beta$ characterizes the dispersal of an incident radiation (disorder of an incident radiation), and $k$ is a transverse component of the wave vector of scattered field [1–3].

## 3. SECOND ORDER STATISTICAL MOMENTS OF SCATTERED RADIATION

Two-dimensional spectral component of the phase fluctuation of scattered electromagnetic field (6) along the direction perpendicular to the principle plane in the first approximation can be rewritten in the compact form:

$$\frac{\partial\psi}{dz} + \frac{id_1 - d_2}{\Gamma_j k_x + 2ik_0}\psi(k_x, k_y, z)$$

$$=-\frac{k_0^2}{\Gamma_j k_x+2ik_0}\left\{\varepsilon_{xx}^{(1)}(k_x,k_y,z)-\left[P_j\tilde{\varepsilon}_{xy}^{(1)}(k_x,k_y,z)-\Gamma_j\tilde{\varepsilon}_{xz}^{(1)}(k_x,k_y,z)\right]\right\}, \quad (14)$$

where $d_1 = k_x(k_y + k_\perp)P_j + k_0 k_x \Gamma_j$, $d_2 = k_y(k_y + 2k_\perp)$. The solution of (14) with the boundary condition $\psi(k_x, k_y, z = 0) = 0$ is:

$$\psi(k_x,k_y,z)=-\frac{k_0^2}{\Gamma_j k_x+2ik_0}\int_{-\infty}^{\infty}dz'\left\{\varepsilon_{xx}^{(1)}(k_x,k_y,z')-\left[P_j\tilde{\varepsilon}_{xy}^{(1)}(k_x,k_y,z')\right.\right.$$
$$\left.\left.-\Gamma_j\tilde{\varepsilon}_{xz}^{(1)}(k_x,k_y,z')\right]\right\}\cdot\exp\left[-\frac{d_2-id_1}{\Gamma_j k_x+2ik_0}(L-z')\right], \quad (15)$$

Using (14) and taking into account well-known expressions [1–3]: $\langle\varepsilon_{\alpha\beta}(\boldsymbol{\kappa},z')\varepsilon_{\gamma\delta}(\boldsymbol{\kappa}',z'')\rangle=V_{\alpha\beta,\gamma\delta}(\boldsymbol{\kappa},z'-z'')\delta(\boldsymbol{\kappa}+\boldsymbol{\kappa}')$, $\langle\varepsilon_{\alpha\beta}(\boldsymbol{\kappa},z')\varepsilon_{\gamma\delta}^*(\boldsymbol{\kappa}',z'')\rangle = V_{\alpha\beta,\gamma\delta}(\boldsymbol{\kappa},z'-z'')\delta(\boldsymbol{\kappa}-\boldsymbol{\kappa}')$, changing the variables $z'-z''=\rho_z$, $z'+z''=2\eta$ and following [16] we obtain second order statistical moments of scattered electromagnetic waves for arbitrary correlation functions of electron density and external magnetic field fluctuations (they are statistically independent [3]):

$$\langle\varphi_1(\mathbf{r})\varphi_1(\mathbf{r}+\boldsymbol{\rho})\rangle=\frac{\pi}{2}k_0^2\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\frac{G_1+iG_2}{G_1^2+G_2^2}$$
$$[\Omega_1 V_n(k_x,k_y,iG_3-G_4)+\Omega_2 V_h(k_x,k_y,iG_3-G_4)]$$
$$\cdot\{1-\exp[(G_1-iG_2)L]\}\exp(-ik_x\rho_x-ik_y\rho_y), \quad (16)$$

$$\left\langle\varphi_1^{*2}(\mathbf{r}+\boldsymbol{\rho})\right\rangle=\frac{\pi}{2}k_0^2\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\frac{G_1-iG_2}{G_1^2+G_2^2}[\Omega_1 V_n(k_x,k_y,-iG_3-G_4)$$
$$+\Omega_2 V_h(k_x,k_y,-iG_3-G_4)]\cdot\{1-\exp[(G_1+iG_2)L]\}, \quad (17)$$

$$\left\langle\varphi_1(\mathbf{r})\varphi_1^*(\mathbf{r}+\boldsymbol{\rho})\right\rangle$$
$$=-\pi k_0^4\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\frac{2k_0^2}{d_2\Gamma_j k_x-2d_1k_0}\left\{1-\exp\left[\frac{d_2\Gamma_j k_x-2d_1k_0}{2k_0^2}\right]\right\}$$
$$\cdot\left[\Omega_1 V_n\left(k_x,k_y,-\frac{d_1\Gamma_j k_x+2d_2k_0}{4k_0^2}\right)+\Omega_2 V_h\left(k_x,k_y,-\frac{d_1\Gamma_j k_x+2d_2k_0}{4k_0^2}\right)\right]$$
$$\exp(-ik_x\rho_x-ik_y\rho_y), \quad (18)$$

where

$$\Omega_1=\frac{v_0^2}{(1-u_0)^2}\left[1+u_0-2\sqrt{u_0}(\sin\alpha-\cos\alpha+\sqrt{u_0}\sin\alpha\cos\alpha)\right],$$

$$\Omega_2 = \frac{v_0^2 u_0}{(1-u_0)^4}\left\{4u_0 + (1+u_0)^2 - 2\left[2\sqrt{u_0}(\sin\alpha - \cos\alpha) + (1+u_0)^2 \sin\alpha\cos\alpha\right]\right\},$$

$$G_1 = \frac{1}{k_0^2}(\Gamma_j k_\perp - P_j k_0) k_x k_y,\quad G_2 = \frac{1}{2k_0^2}\left[\Gamma_j (P_j k_\perp + \Gamma_j k_0) k_x^2 + 2k_0 k_y^2\right],$$

$$G_3 = \frac{1}{4k_0^2}\left(2\Gamma_j k_0^2 + 2P_j k_0 k_\perp - \Gamma_j k_y^2\right) k_x,\quad G_4 = \frac{1}{4k_0^2}\left(P_j \Gamma_j k_x^2 + 4k_0 k_\perp\right) k_y,$$

2D spatial spectrum $V_n(\boldsymbol{\kappa},\, z' - z'')$ and $V_h(\boldsymbol{\kappa},\, z' - z'')$ include both electron density and external magnetic field fluctuations; indices denote the product of fluctuating components of second rank tensor (2); $\delta$ is Dirac delta function, the brackets $\langle\rangle$ means ensemble average, $k_x$, $k_y$ and $k_z$ are the spatial wave-numbers in the $x$, $y$ and $z$ directions respectively, the asterisk represents the complex conjugate. Phase fluctuations in a second order approximation for $i = x$ component satisfies stochastic differential equations:

$$iP_j \frac{\partial^2 \varphi_2}{\partial x \partial y} + i\Gamma_j \frac{\partial^2 \varphi_2}{\partial x \partial z} - (k_\perp P_j + k_0 \Gamma_j)\frac{\partial \varphi_2}{\partial x} - \frac{\partial^2 \varphi_2}{\partial y^2} - 2ik_\perp \frac{\partial \varphi_2}{\partial y} - 2ik_0 \frac{\partial \varphi_2}{\partial z} = -iP_j \frac{\partial \varphi_1}{\partial x}\frac{\partial \varphi_1}{\partial y} + \left(\frac{\partial \varphi_1}{\partial y}\right)^2. \tag{19}$$

Using Fourier transformation to (19) we have:

$$\langle \varphi_2(x,y,L)\rangle = i\frac{\pi}{4}k_0 \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \left(iP_j k_x k_y + k_y^2\right)\frac{G_1 + iG_2}{G_1^2 + G_2^2} \left[\Omega_1 V_n(k_x, k_y, iG_3 - G_4) + \Omega_2 V_h(k_x, k_y, iG_3 - G_4)\right] \left\{L + \frac{G_1 + iG_2}{G_1^2 + G_2^2}\left(1 - \exp\left[(G_1 + iG_2)L\right]\right)\right\}. \tag{20}$$

Contrary to the ray-(optics) approximation, the correlation functions contain parameter $k_y^2/k_0^2$ taking into account the diffraction effects. These effects become essential if the condition $L \gg (l_{n,h}^2/\lambda)$ is fulfilled. In the absence of an external magnetic field ($H_0 = 0,\ u_0 = 0$) from (16)–(18) and (20) we obtain [12].

## 4. NUMERICAL RESULTS AND DISCUSSIONS

Irregularities that are responsible for fluctuations of radiation from discrete sources and satellites are mainly located in $F$-region of the ionosphere at a height of $250 \div 400$ km. Data obtained from spaced receiver measurements made at Kingston, Jamaica (during the periods August 1967–January 1969 and June 1970–September 1970) show that the irregularities between heights of 153 and 617 km causing the scintillation of signals from the moving earth satellites (BE-B and BE-C) are closely aligned along the magnetic field lines in the $F$-region [18]. Orientation of the irregularities in the ionosphere has been measured with respect to the geographic north observing a diffraction pattern of the satellite signals (41 MHz) on the ground. The dip angle of the irregularities with respect to the field lines was within 16°. The anisotropic spectral features in the $F$-region are defined for Gaussian and Power-law spectra.

For investigation of the influence of electron density and external magnetic field fluctuations on the deformation of SPS of scattered radiation, we use anisotropic Gaussian correlation function of electron density fluctuation having in the principle $yoz$ plane following form [13]:

$$\tilde{V}_n(k_x, k_y, k_z) = \sigma_n^2 \frac{l_\perp^2 l_\parallel}{8\pi^{3/2}} \exp\left(-\frac{k_x^2 l_\perp^2}{4} - p_1 \frac{k_y^2 l_\parallel^2}{4} - p_2 \frac{k_z^2 l_\parallel^2}{4} - p_3 k_y k_z l_\parallel^2\right). \quad (21)$$

This function is characterized by anisotropy factor of irregularities $\chi = l_\parallel / l_\perp$ (ratio of longitudinal and transverse linear scales of plasma irregularities with respect to the external magnetic field) and the inclination angle of prolate irregularities with respect to the external magnetic field $\gamma_0$. $p_1 = 1 + (1 - \chi^2)^2 \sin^2\gamma_0 \cos^2\gamma_0 / \chi^2$, $p_2 = (\sin^2\gamma_0 + \chi^2 \cos^2\gamma_0)/\chi^2$, $p_3 = (1 - \chi^2)\sin\gamma_0 \cos\gamma_0 / 2\chi^2$, $\bar{l} = l_\parallel (\sin^2\gamma_0 + \chi^2 \cos^2\gamma_0)^{-1/2}$. In isotropic case ($\chi = 1$): $p_1 = p_2 = 1$, $p_3 = 0$; at $\gamma_0 = 0°$: $p_1 = 1/\chi^2$, $p_2 = 1$, $p_3 = 0$. Plasma irregularities are extremely elongated if $\chi \gg 1$.

Measurements of satellite's signal parameters passing through ionospheric layer and measurements aboard of satellite show that in $F$-region of the ionosphere irregularities have power-law spectrum with different spatial scales. Observations suggest that the power-law spectrum is believed to be the most suitable model of ionospheric irregularities. We utilize 3D anisotropic power-law spectrum of electron density irregularities. Generalized correlation function for power-law spectrum of electron density irregularities with a power-law index $p$ has been proposed in [19]. The corresponding spectral function has the form:

$$W_n(\mathbf{k}) = \frac{\sigma_N^2}{(2\pi)^{3/2}} \frac{r_0^3 (k_0 r_0)^{(p-3)/2}}{\left(r_0\sqrt{k^2 + k_0^2}\right)^{p/2}} \frac{K_{p/2}\left(r_0\sqrt{k^2 + k_0^2}\right)}{K_{(p-3)/2}(k_0 r_0)}, \quad (22)$$

where $K_\nu(x)$ is McDonald function, $r_0$ the inner scale of turbulence, and $L_0 = 2\pi/k_0$ the outer scale and $k_0 r_0 \ll 1$. In the interval of wavenumber $k_0 r_0 \ll k r_0 \ll 1$ we introduce two spatial correlation lengths of electron density irregularities $\ell_{||}$ and $\ell_\perp$. For $p > 3$ spatial power-law spectrum could be rewritten as:

$$W_n(\mathbf{k}) = \frac{\sigma_N^2}{2\pi^2} \frac{\Gamma\left(\frac{p}{2}\right)\Gamma\left(\frac{5-p}{2}\right)}{\Gamma\left(\frac{3}{2}\right)} \sin\left[\frac{(p-3)\pi}{2}\right] \frac{\ell_\perp^2 \ell_{||}}{\left[1+\ell_\perp^2\left(k_\perp^2+\chi^2 k_{||}^2\right)\right]^{p/2}}, \quad (23)$$

Experimental investigations of Doppler frequency shift of ionospheric signal show that index of the power-law spectrum of electron density fluctuations is in the range of $3.8 \le p \le 4.6$. Experimental value of the power-law spectrum of the ionosphere ($\langle p \rangle \approx 4$), measured by translucence of satellite signals [20], is within the limits of $p$. Experimental observations of backscattering signals from the artificially disturbed region of the ionosphere by the powerful HF radio emission shows that a lot of artificial ionospheric irregularities of the electron density are stretched along the geomagnetic field. Power-law spectral index was within the limits $p = 1.4 \div 4.8$ for different heating sessions using "Sura" heating facility in the frequency range of $4.7 \div 9$ MHz (ordinary mode) with the effective radiated power $50 \div 70$ mW beamed vertically upwards [21].

Correlation function describing fluctuations of an external magnetic field has anisotropic Gaussian form [14]:

$$V_h(\mathbf{k}) = \frac{\langle h_1^2 \rangle}{8\pi^{3/2}} l_{0x} l_{0y} l_{0z} \exp\left(-\frac{k_x^2 l_{0x}^2}{4} - \frac{k_y^2 l_{0y}^2}{4} - \frac{k_z^2 l_{0z}^2}{4}\right), \quad (24)$$

characteristic linear scales $l_{0x}$, $l_{0y}$, $l_{0z}$ and variance $\langle h_1^2 \rangle$.

Numerical calculations are carried out for: 3 MHz ($\lambda = 100$ m, $k_0 = 6.28 \cdot 10^{-2}\,\mathrm{m}^{-1}$) and 40 MHz ($\lambda = 7.5$ m, $k_0 = 0.84\,\mathrm{m}^{-1}$) incident electromagnetic waves propagating in $F$ region of the ionosphere. At 3 MHz non-dimensional plasma parameters are: $u_0 = 0.22$, $v_0 = 0.28$; at 40 MHz we have: $u_0 = 0.0012$, $v_0 = 0.0133$. If electron density in a slab varies as $0 < v_0 < 1$ there is not an area for ordinary wave's reflection. Under the same condition for the extraordinary waves $0 < v_0 < 1 - \sqrt{u_0}$. The analysis shows that in this case the amplitude and phase fluctuations of the field components of the ordinary wave are less than those of the extraordinary wave. Thus, "inclusion" of an external magnetic field (at $u_0 < 1$) increases the fluctuations of parameters of extraordinary wave and decreases them at ordinary wave's propagation. However, situation can be changed at $u_0 > 1$. In this case, the imposed magnetic field can lower the level of fluctuations of parameters for both waves in comparison to those observed in the absence of an external magnetic field.

Substituting (21) into (16), for anisotropic Gaussian correlation functions of scattered electromagnetic waves by electron density fluctuations we obtain:

$$\langle\varphi_1(\mathbf{r})\varphi_1(\mathbf{r}+\boldsymbol{\rho})\rangle = -2\tilde{\Omega}_1 T\int\limits_{-\infty}^{\infty} ds\int\limits_{-\infty}^{\infty} dx\exp\left[-T^2\left(\frac{1}{4}p_2 m_4^2 x^4+b_1x^2+b_2\right)\right.$$
$$\left.+iT^2\left(b_4x^3+b_5x\right)\right]\cdot\exp(-i\xi x-i\eta s), \qquad (25)$$

where:

$$b_1 = -\frac{1}{64}p_2\Gamma_j^2 s^4+\frac{1}{16}p_2\Gamma_j(\Gamma_j+P_j\mu)s^2+\frac{1}{4}P_j\Gamma_j\left(\frac{1}{2}p_2\mu+p_3\right)s$$
$$+\left[\frac{1}{4\chi^2}-\frac{1}{16}p_2(\Gamma_j+P_j\mu)^2\right],$$
$$b_2 = \frac{1}{4}\left(\frac{1}{\chi^2}+2p_2m_3m_4+4p_3m_3s\right),\quad b_4=\frac{1}{2}p_2m_3m_4,$$
$$b_5 = m_3\left(\frac{1}{2}p_2\mu+p_3\right)s,\quad T=k_0l_{\parallel},$$
$$x=\frac{k_x}{k_0},\quad s=\frac{k_y}{k_0},\quad m_3=\frac{1}{4}\left[(s+\mu)P_j+\Gamma_j\right]\Gamma_j,\quad m_4=\frac{1}{2}(s^2+2s\mu).$$

In isotropic case ($\chi = 1$), in the absence of diffraction effects ($\mu = 0$), for variance of the phase fluctuations in non-magnetized plasma we obtain well-known formula [8]: $\langle\varphi_1^2\rangle = \sqrt{\pi}\sigma_n^2 v_0^2 k_0^2 L l_n/4$. For the correlation function (18) we have:

$$W_\varphi(\xi,\eta,L)$$
$$=\tilde{\Omega}_1 T\int\limits_{-\infty}^{\infty} ds\int\limits_{-\infty}^{\infty} dx\exp\left[-\frac{T^2}{4}\left(p_2m_5^2x^4+b_5x^2+b_7\right)\right]\exp(-i\xi x-i\eta s), \quad (26)$$

where $m_5 = \frac{1}{4}[(s+\mu)P_j+\Gamma_j]\Gamma_j$, $b_7 = \frac{1}{4}p_2s^4+(p_2\mu+2p_3)s^3+(p_1+p_2\mu^2+4p_3\mu)s^2$. Knowledge of these functions allows us to calculate wave structure functions and angle-of-arrivals in the principle and perpendicular planes using (8):

$$D_1(\xi,\eta,L) = \frac{2}{\sqrt{\pi}}\tilde{\Omega}_1 T\int\limits_{-\infty}^{\infty} ds\int\limits_{-\infty}^{\infty} dx\left[1-\cos(\xi x+\eta s)\right]$$
$$\exp\left[-\frac{T^2}{4}\left(p_2m_5^2x^4+b_6x^2+b_7\right)\right], \qquad (27)$$

$$\langle\theta_x^2\rangle = \lim_{\xi\to 0}\frac{D_1(\xi,0,L)}{\xi^2}, \quad \langle\theta_y^2\rangle = \lim_{\xi\to 0}\frac{D_1(0,\eta,L)}{\eta^2}, \tag{28}$$

where $b_6 = \frac{1}{\chi^2} + 2p_2m_5m_6 + 4p_3m_5s$, $m_6 = \frac{1}{2}(s^2 + 2s\mu)$, $\tilde{\Omega}_1 = \frac{B_0}{4\sqrt{\pi}}\frac{T\Omega_1}{\chi}$, $B_0 = \sigma_n^2\frac{\sqrt{\pi}}{4}\frac{Tk_0L}{\chi}$. At $\eta \to \infty$ wave structure functions tend to saturation, which is in agreement with [1–3]. The expression $\mathrm{Re}\langle\varphi_2\rangle$ for anisotropic correlation function has quite bulky form and therefore is omitted in the paper. In isotropic ($\chi = 1$) non-magnetized plasma ($H_0 = 0$) at: $\alpha = 0°$, neglecting diffraction effects ($\mu = 0$) we obtain the well-known formula [8]: $\langle\theta_x^2\rangle = \langle\theta_y^2\rangle = \sigma_n^2\sqrt{\pi}v_0^2L/2l_n$.

Figure 1 illustrates the dependence of wave structure function $D_1$ of scattered ordinary wave ($j = 1$) versus distance between observation points in the principle plane $\eta = k_0\rho_y$ for different angle of inclination $\gamma_0$ of prolate irregularities of electron density fluctuations with respect to the external magnetic field. For anisotropic Gaussian correlation function of electron density fluctuations wave structure function oscillates at small distances tending to saturation with increasing distance between the observation points. We consider the case when the angles of inclination of prolate irregularities are $\gamma_0 = 0°, 10°$, anisotropy factor $\chi = 15$ and scattered ordinary electromagnetic wave propagates in magnetized ionospheric plasma in the direction $\alpha = 20°$ with respect to the external magnetic field. If original electromagnetic wave at 3 MHz, $\mu = 0.06$ are incident on chaotically oriented plasma inhomogeneities, first maxima of the structure function of a scattered ordinary wave appear at $\eta = 28, 33$ and tend to saturation at $\eta = 500, 800$. If an incident electromagnetic

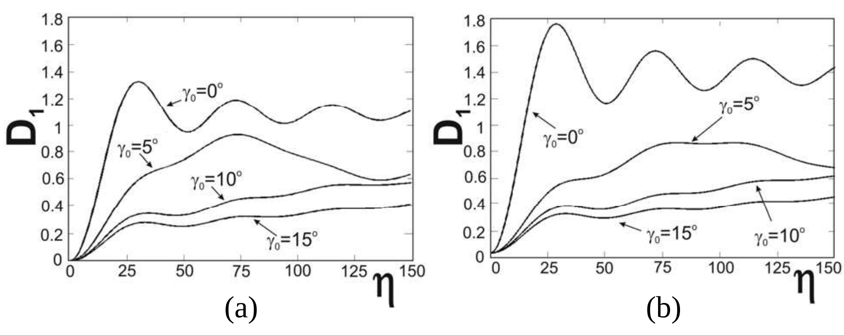


**Figure 1.** Wave structure function $D_1$ versus distance between observation points $\eta$ for different angle of inclination of prolate irregularities $\gamma_0$ with respect to the external magnetic field for an incident electromagnetic wave at (a) 3 MHz; and at (b) 40 MHz.

wave has frequency 40 MHz, $\mu = 0.114$ wave structure function $D_1$ has first maxima at $\eta = 30$, 35 and saturation at $\eta = 400$, 550. Numerical calculations show that increasing the angle of inclination $\gamma_0$, AOA $\langle\theta_y^2\rangle$ in the principle plane decreases. Varying angle $\gamma_0 = 0^\circ \div 10^\circ$ the AOAs are in the intervals $0.2'' \div 3.3'$ and $0.4'' \div 5'$ for frequencies 3 MHz and 40 MHz, respectively.

In magnetized plasma if $T \gg 1$ we use the saddle-point method [22]. Substituting second order statistical moments of scattered electromagnetic waves in (11)–(13), for the SPS of anisotropic Gaussian correlation functions of electron density fluctuations in turbulent magnetized plasma we have:

$$\frac{W_{EE^*}(\eta, L)}{E_0^2} = \exp(-i\eta\mu)\exp\left\{-\sqrt{\frac{\pi}{2}}\frac{\Omega_1 T B_0}{\chi}\int\limits_{-\infty}^{\infty} ds \frac{1}{\sqrt{b_1}}\exp\left(-T^2 b_2\right)\right.$$
$$+\frac{1}{2\sqrt{\pi}}\frac{\Omega_1 T B_0}{\chi}\cdot\int\limits_{-\infty}^{\infty} ds\frac{1}{\sqrt{b_6}}\exp\left(-\frac{T^2}{2}\left[\frac{1}{4}p_2 s^4 + (p_2\mu + 2p_3)\,s^3\right.\right.$$
$$\left.\left.\left.+\left(p_1 + p_2\mu^2 + 4p_3\mu\right)s^2\right]\right)\exp(-i\eta s)\right\}. \qquad (29)$$

Figure 2 illustrates the dependence of normalized correlation function of scattered field in magnetized plasma with prolate irregularities having characteristic linear scale $l_\parallel = 5$ km. Calculations show that a gap is formed when the ordinary waves are passing $L = 25 \cdot 10^2$ km

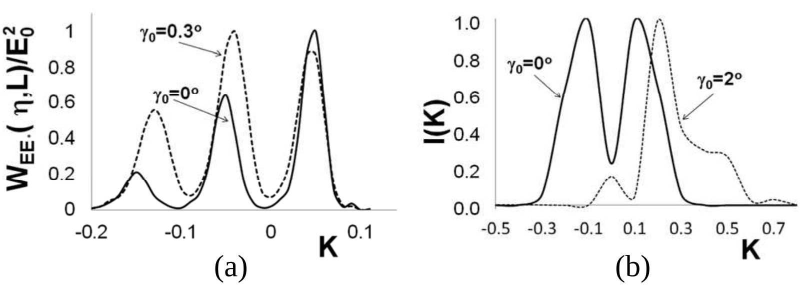


**Figure 2.** Dependence of normalized correlation function of scattered ordinary wave versus $k$ for different angle of inclination $\gamma_0 = 1^\circ, 0.3^\circ, 2^\circ$ at: (a) $\gamma = 190$, $T = 4000$, $\mu = 0.05$, $B_0 = 2200$. (b) Depicts the dependence of SPS of scattered ordinary wave versus $k$ for a finite width of an original mode $\beta = 10$ at $T = 8000$, $\chi = 490$, $B_0 = 2000$ and angle of inclination of prolate irregularities: $\gamma_0 = 0^\circ$, $2^\circ$.

in turbulent plasma with the variance of electron density fluctuations $\sigma_D^2 = 10^{-4}$. Normalized correlation function has the Gaussian shape at the angle of inclination of prolate irregularities with respect to the external magnetic field $\gamma_0 = 2°$. Decreasing $\gamma_0$ this function starts oscillation leading to the formation of a gap in the SPS of scattered ordinary electromagnetic wave.

Figure 3 demonstrates the formation of a gap in the intensity of power spectrum of scattered extraordinary wave for a finite width of an original mode $\beta = 10$ (left figure) at $l_{\|} = 7\,\text{km}$, $T = 6000$, $\chi = 290$, $B_0 = 2000$. Numerical calculations show that SPS has a Gaussian form at angle of inclination of prolate irregularities $\gamma_0 = 1°, 5°$. The gap is formed only at $k = 0.11$ and $\gamma_0 = 0°$, i.e., if directions of stretched irregularities and external magnetic field coincide. Right figure illustrates the evaluation of a gap travelling by extraordinary electromagnetic wave in turbulent magnetized plasma with prolate irregularities having characteristic linear scale $l_{\|} = 10\,\text{km}$. The curves have symmetrical forms with respect to $k = 0$. The gap of the SPS increases 14 times when the wave is passing a distance 3000 km.

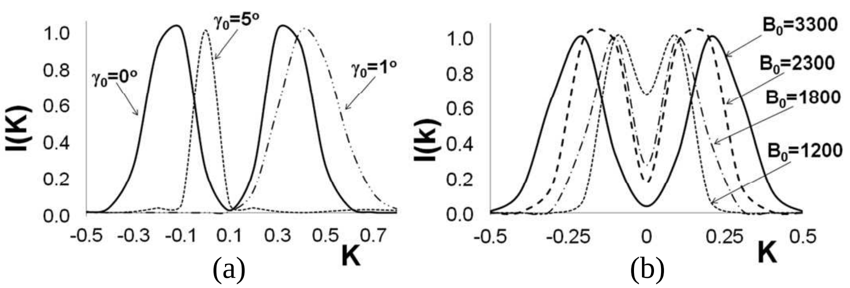


**Figure 3.** Dependence of the SPS of scattered extraordinary wave versus $k$ for a finite width of an original mode $\beta = 10$: (a) at fixed parameter $B_0 = 2000$ and different angle of inclination $\gamma_0 = 0°, 1°, 5°$; (b) for different parameter $B_0$ at fixed $\gamma_0 = 0°$.

If $T \gg 1$ SPS of anisotropic power-law correlation functions of electron density fluctuations in turbulent non-magnetized plasma has the form:

$$\frac{W_{EE^*}(\xi,\eta,L)}{E_0^2} = \exp(-i\eta\mu)\cdot\exp\left(\pi^{3/2}\frac{T^2 v_0^2 Q_0}{\chi\Gamma\left(\frac{p}{2}\right)}k_0 L\left\{-\frac{1}{2}\Gamma\left(\frac{p-1}{2}\right)\right.\right.$$
$$\int\limits_{-\infty}^{\infty} ds\frac{1}{(1+C_1 s^2)^{p-1}} + \left(\frac{\chi\xi}{2T}\right)^{(p-1)/2}$$

$$\int_{-\infty}^{\infty} ds \frac{\exp(-i\eta s)}{[1+\Phi(s)]^{(p-1)/2}} K_{\frac{p-1}{2}}\left(\frac{\chi\xi}{T}[1+\Phi(s)]\right)\Bigg\}\Bigg), \tag{30}$$

where: $Q_0 = \frac{\sigma_n^2}{\pi^{5/2}}\Gamma(\frac{p}{2})\Gamma(\frac{5-p}{2})\sin[\frac{(p-3)\pi}{2}]$, $\Phi(s) = \frac{T^2}{4}[s^4 + 4\mu s^3 + 4(\frac{1}{\chi^2} + \mu^2)s^2]$, $C_1 = \frac{T^2}{\chi^2}(1+\chi^2\mu^2)$, $\Gamma(x)$ is the gamma function, $K_\nu(x)$ is the McDonald function.

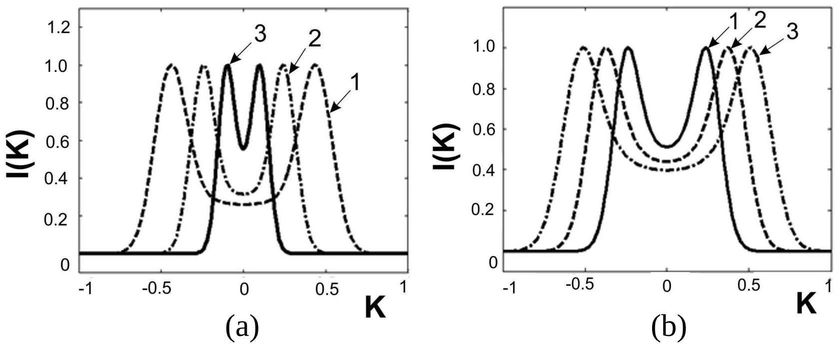


**Figure 4.** SPS of the power-law correlation function of electron density fluctuations ($p = 4$) of scattered radiation in non-magnetized plasma for different parameter of anisotropy of electron density fluctuations $\chi = 100$, 130, 160; (a) $B_0 = 50$ and (b) $B_0 = 50$, 75, 100 at $\chi = 100$.

Figure 4 depicts SPS of scattered electromagnetic waves in non-magnetized plasma with power-law electron density fluctuations (index $p = 4$) at different anisotropy factor of prolate irregularities $\chi = 100$, 130, 160. Parameters of an incident wave 3 MHz, $\alpha = 20°$, $\beta = 10$, $\mu = 0.06$. Numerical calculations were carried out for $T = 15 \cdot 10^3$, which is equivalent to the characteristic longitudinal scale of irregularities $l_{\|} \approx 240$ km. Analyses show that at fixed parameter $B_0 = 50$ by increasing an anisotropy factor $\chi = 100 \div 160$, SPS is broadening 3.4 times and a gap increases (a). At fixed parameter of anisotropy $\chi = 100$ SPS is broadening 1.4 times and a dip of the gap increases. Formation of a gap in the SPS and evaluation of its double-humped shape for power-law spectrum of electron density fluctuations in non-magnetized plasma at different angle of inclination of prolate irregularities with respect to the external magnetic field is considered for the first time.

Correlation function of the phase fluctuations (18) caused by external magnetic field fluctuations and describing by anisotropic correlation function (24) has the following form:

$$W_h(\xi,\eta,L)=\tilde{\Omega}_2\int\limits_{-\infty}^{\infty}dx\int\limits_{-\infty}^{\infty}ds\exp\left[-T_h^2\left(m_6^2+Y_0^2s^2\right)\right]$$
$$\exp\left\{-T_h^2\left[m_5^2x^4+\left(2m_5m_6+X_0^2\right)x^2\right]-i\xi x-i\eta s\right\}\quad(31)$$

where: $\tilde{\Omega}_2=\frac{\langle h_1^2\rangle\Omega_2}{2\sqrt{\pi}}k_0LT_h^2X_0Y_0$, $T_h=\frac{k_0l_{0z}}{2}$, $X_0=\frac{l_{0x}}{l_{0z}}$, $Y_0=\frac{l_{0y}}{l_{0z}}$. In polar coordinate system can be written as:

$$W_h(X_h,Y_h,L)=\tilde{\Omega}_2\int\limits_{0}^{\infty}dx x\int\limits_{0}^{2\pi}d\varphi\exp\left[-T_h^2\left(X_0^2\cos^2\varphi+Y_0^2\sin^2\varphi\right)\right]$$
$$\exp\left[-2iT_h(X_h\cos\varphi-Y_h\sin\varphi)\right]$$
$$\cdot\exp\left[-T_h^2\left(D_0x^2+D_1x^3+D_2x^4+D_3x^5+D_4x^6\right)\right],\quad(32)$$

where: $X_h=\frac{\rho_x}{l_{0z}}$, $Y_h=\frac{\rho_y}{l_{0z}}$, $D_0=\mu\sin\varphi$, $D_1=\mu\sin\varphi[\sin^2\varphi+\frac{1}{2}(\Gamma_j+P_j\mu)\Gamma_j\cos^2\varphi]$,

$$\tilde{D}_2=\frac{1}{4}\left[\frac{1}{4}(P_j\mu+\Gamma_j)^2\Gamma_j^2\cos^4\varphi+\sin^4\varphi+(3P_j\mu+\Gamma_j)\Gamma_j\sin^2\varphi\cos^2\varphi\right],$$
$$\tilde{D}_3=\frac{1}{8}P_j\Gamma_j\sin\varphi\cos^2\varphi\left[(P_j\mu+\Gamma_j)\Gamma_j\cos^2\varphi+2\sin^2\varphi\right],$$
$$\tilde{D}_4=\frac{1}{16}P_j^2\Gamma_j^2\sin^2\varphi\cos^4\varphi.$$

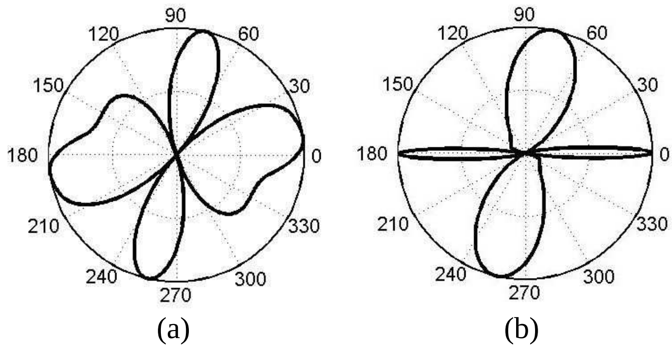


**Figure 5.** Phase portraits of the correlation function of phase fluctuations for different non-dimensional linear parameters characterizing the task at: (a) $X_h=4.5\times10^{-2}$, $Y_h=10^{-2}$; (b) $X_h=9\times10^{-3}$, $Y_h=2\times10^{-3}$.

Figure 5 demonstrates the phase portraits of the phase correlation function of scattered electromagnetic wave at 40 MHz when $\mu = 0.114$, $\alpha = 20^\circ$ and for different non-dimensional parameters $Y_h$ and $X_h$characterizing a ratio of distance between observation points in the principle and perpendicular planes ($\rho_y$ and $\rho_x$ respectively) to the longitudinal characteristic spatial scale of an external magnetic field fluctuations $l_{0z}$. Numerical calculations show that the behavior of phase portraits substantially depends on these parameters.

## 5. CONCLUSION

Statistical characteristics of multiply scattered electromagnetic waves in turbulent magnetized and non-magnetized plasma with both electron density and external magnetic field fluctuations are investigated analytically and numerically using experimental data of the $F$-region of the ionosphere. Analytical expression for phase fluctuations of scattered radiation in the direction perpendicular to the principle plane is derived using the smooth perturbation method taking into account diffraction effects. Correlation and wave structure functions of the phase fluctuations are obtained for arbitrary correlation functions of fluctuating plasma parameters and external magnetic field. Numerical calculations are carried out for 3 MHz and 40 MHz original electromagnetic wave. Investigation has shown that for anisotropic Gaussian correlation function of electron density fluctuations wave structure function oscillates at small distances tending to saturation with increasing distance between observation points. Strongly pronounced dip appears in the spatial power spectrum of scattered ordinary electromagnetic wave along prolate irregularities at oblique illumination of magnetized plasma by mono-directed incident radiation. Double-humped shape having symmetric form becomes deeper by increasing distance traveling by ordinary wave in magnetized ionospheric plasma. Upsizing anisotropy factor and a distance traveling by electromagnetic wave in non-magnetized plasma with power-law spectrum of electron density fluctuations, spatial power spectrum is broadening and a gap is increasing. Formation of a gap in the spatial power spectrum and evaluation of its double-humped shape for power-law spectrum of electron density fluctuations in plasma at different angle of inclination of prolate irregularities with respect to the external magnetic field is considered for the first time. The obtained results have wide practical applications at observations of electromagnetic waves propagation in the ionosphere and remote sensing.

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