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MODELING OF THE DIRECT LIGHTNING STRIKE ON A TOWERS CASCADE EQUIPPED WITH ITS PROTECTIONS

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Abstract—In this paper, a direct time domain approach based on the corresponding transmission lines equations and Finite Difference Time Domain (FDTD) method is proposed to analyze a direct lightning strike to a cascade of transmission line towers. The proposed model deals with a real case of towers being connected by ground wires and equipped with grounding systems with different topologies, as well (vertical or horizontal conductor buried in the ground, crow's feet in the ground ...). In particular, this work realistically represents the tower geometry and accounts for the propagation phenomena along the tower and between the towers. The proposed direct time domain approach deals with rather complex electrical devices (towers, ground wires and grounding systems), but at the same time requires very low computational cost and also provides relatively simple implementation. Some illustrative computational examples related to some engineering applications are given in the paper.

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1. INTRODUCTION

Lightning phenomenon has always been a cause of disturbances in the use of electricity. On one hand, there are demanding requirements on the quality of electrical systems (reliability, availability, continuity of service ...) and on the other hand, there is also permanent concern to minimize the production costs.

Thus, the modeling of lightning phenomena has become a rather hot-issue and it is now addressed as one of the major concerns of the distributors, material manufacturers, designers, or installers.

A study of the lightning effects involves two steps:

- Predict what may happen on a given installation and recommend solutions for improvement. This is possible by using specialized software that simulates the behavior of installations, validated by experience.
- A technical and economic study of insulation coordination taking into account the cost of installations, maintenance, service disruptions.

To ensure the protection of the power network against the natural phenomenon of lightning which has a random character, it is important to know the level of overvoltages. Performing the measurements of overvoltages in realistic conditions is rather expensive, very difficult and sometimes even impossible. The proposed work aims to provide a simulation tool easy to use with very low computational time which will complement the measurements and undertake parametric studies.

Full wave modeling of an electrical device consisting of towers, ground wires and grounding systems by taking into account realistic geometry is extremely difficult task. As an alternative, a time domain modeling approach based on the transmission line (TL) approximation and the finite difference time domain (FDTD) method is proposed. The towers equipped with related ground wires and grounding systems are considered to be an equivalent radial network of straight conductors excited directly by an equivalent current source due to lightning strike. All straight wires are assumed to have open ends, while only the lower end in contact with the ground is connected to ground via vertical or horizontal electrode, respectively.

The validation of the work is carried out by comparing the calculated results with those published in the literature pertaining to some simple applications arising from the use of the ATP commercial software [1] (for three vertical conductors) and vector fitting method [2] (for single towers). Our approach allows a more complete modeling of the electrical tower because in the literature it is usually represented simply by a single vertical conductor or else the analysis is performed in frequency domain with the use of the inverse Fourier transform for the transition in the time.

2. MODELING OF THE INTERACTION BETWEEN A LIGHTNING DISCHARGE AND A TOWERS CASCADE

A geometry of the towers cascade exposed to direct lightning strike is shown in Figure 1. Note that the electrical towers are equipped with ground wire and terminated with grounding system with different topologies.

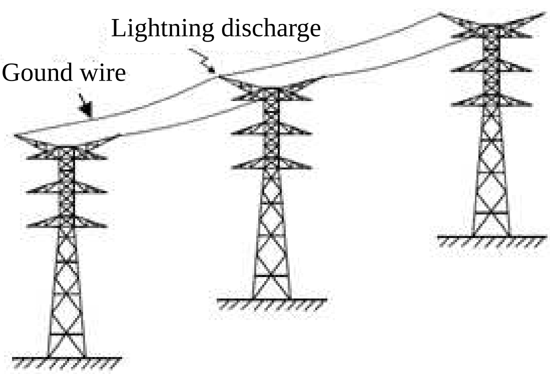


Figure 1. Towers connected with ground wires.

In general, the matrix system in time domain for the analysis of the considered problem can be written, as follows:

$$[A]\,[X] = [B] \tag{1}$$

where:

$$[A] = \begin{bmatrix} [A_1] \\ [A_2] \end{bmatrix}$$

stands for the matrix of the network topology.

Furthermore, sub matrix $[A_1]$ represents the propagation effects to appear on the electrical device consisting of towers, ground wires and grounding systems, while sub matrix $[A_2]$ represents the relationship between electric currents and voltages at each node (deduced by the application of Kirchhoff's laws). Vector $[X]$ contains the unknown currents and voltages at each node, while $[B]$ is the excitation vector composed of two sub vectors $[B_1]$ and $[B_2]$ representing an equivalent current source or plane wave, respectively due to a lightning strike.

2.1. Tower Modeling

Within the present work, based on the transmission line theory [3] a model representing the tower with a number of interconnected segments including the arms, with each column containing four conductors (Figure 2(a)) is used. The problem can be reduced to a single conductor using conductor bundling formulas [4]. The representation of the tower from Figure 2(a) by a number of interconnected straight wires is shown in Figure 2(b) [3].

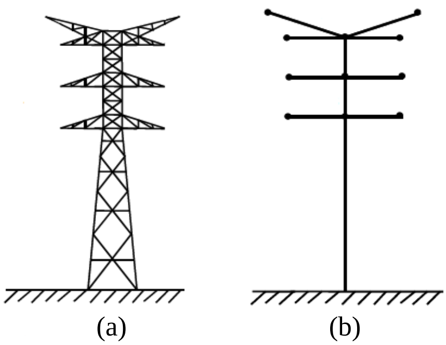


Figure 2. Electric tower and its equivalent representation by interconnected conductors.

Considering the tower as a radial network of lines (vertical and horizontal) interconnected by nodes, the entire tower cascade connected by overhead lines is treated as a complex network topology.

In this study, the direct lightning strike is modeled as a complex network of lines charged and excited by a generator located at the injection point of energy arising from the lightning discharge (Figure 3).

In order to simulate behavior of different tower grounding systems during lightning transient, a vertical or horizontal electrode, respectively, is used at each leg of the transmission line tower as shown in Figure 3.

The response of these towers for direct lightning waves can be modeled by the telegrapher's equations of transmission lines (TL):

$$\begin{cases} \frac{\partial v(x,t)}{\partial x} + Ri(x,t) + L\,\frac{\partial i(x,t)}{\partial t} = 0 \\ \frac{\partial i(x,t)}{\partial x} + C\,\frac{\partial v(x,t)}{\partial t} = 0 \end{cases} \tag{2}$$

L, R and C are the inductance, the resistance and the capacitance of the line segments respectively.

These parameters can be obtained by expressions from [5] for the horizontal and inclined ones. On the other hand, the vertical segments parameters are calculated by using A. Ametani [4] or J. A. Gutierrez [3] expressions. For the horizontal and vertical buried conductors we use the expressions [6–8].

The voltages and currents shown in Figure 4 denote the currents and voltages at each node in the equivalent tower network.

2.2. Matrix System Construction for Evaluating Voltages and Currents

To study the problem of transient wave propagation along an electric tower excited by a current generator, the following matrix equation is to be solved [9]:

$$[A]\ [X] = [B] \tag{3}$$

The proposed approach leads to the solution of the partial differential equations set (propagation over line segments) with current-voltage relations on the nodes.

2.2.1. Time Domain Representation of the Line by a Quadruple

Modeling the direct lightning strike on a metallic wire structure can be analyzed in the time domain using the transmission line approach featuring the simplified Telegraphers equations [10] represented by system (2). A direct solution in the time domain is carried out via the FDTD. Thus, the partial derivatives are replaced by the finite

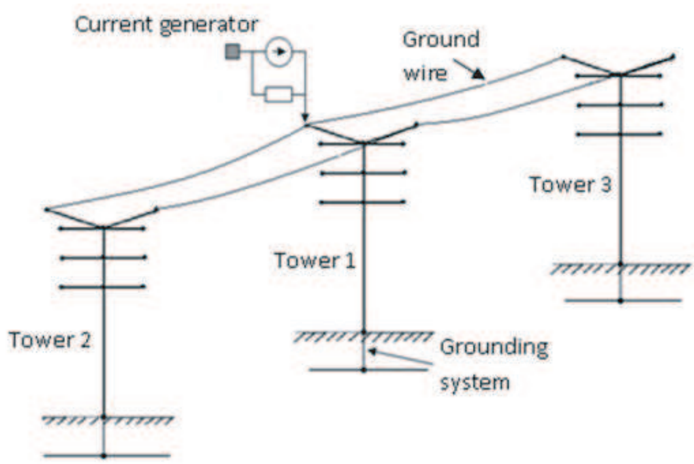


Figure 3. Segments network equivalent towers cascade form.

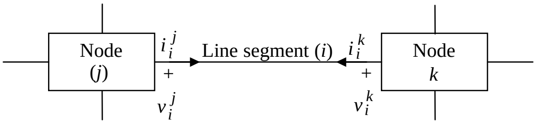


Figure 4. Definition of voltages and currents at the ends of a line segment.

differences using the following discrete notation:

$$v_k^n \equiv v\left[(k-1)\,\Delta x, n\Delta t\right] \tag{4}$$

$$i_k^n \equiv i\left[\left(k-\frac{1}{2}\right)\Delta x, (n+\frac{1}{2})\Delta t\right] \tag{5}$$

Note that the spatio-temporal discretization alternating voltage node and current node separated by $\Delta x/2$ in space and $\Delta t/2$ in time. The nodes at the ends of the line are voltages, and they are given by:

$$\left(\frac{C}{\Delta t}\right) v_1^n = \left(\frac{C}{\Delta t}\right) v_1^{n-1} - \frac{i_1^{n-1/2} - i_0^{n-1/2}}{\Delta x/2} \tag{6}$$

$$\left(\frac{C}{\Delta t}\right) v_{k_{\max}+1}^n = \left(\frac{C}{\Delta t}\right) v_{k_{\max}+1}^{n-1} - \frac{i_{k_{\max}+1}^{n-1/2} - i_{k_{\max}}^{n-1/2}}{\Delta x/2} \tag{7}$$

It is worth noting that:

$$v_1^n = (v\,(0))^n\,, \quad v_{k_{\max}+1}^n = (v\,(L))^n$$

In order to represent the line by a quadruple we create two fictitious current nodes at the end of line (for $x = 0$ and $x = L$).

Thus, it follows:

$$i_0^{n-1/2} = (i\,(0))^{n-1/2} \quad \text{for } x = 0$$

$$i_{k_{\max}+1}^{n-1/2} = (i\,(L))^{n-1/2} \quad \text{for } x = L$$

Assuming the approximations:

$$(i\,(0))^{n-1/2} = \frac{(i\,(0))^n + (i\,(0))^{n-1}}{2} \tag{8}$$

$$(i\,(L))^{n-1/2} = \frac{(i\,(L))^n + (i\,(L))^{n-1}}{2} \tag{9}$$

one obtains the equation to the first end of the line $k = 1\,(x = 0)$:

$$\left(\frac{C}{\Delta t}\right)(v\,(0))^n - \frac{1}{\Delta x}(i\,(0))^n$$
$$= \left(\frac{C}{\Delta t}\right)(v\,(0))^{n-1} + \frac{1}{\Delta x}(i\,(0))^{n-1} - \frac{2\cdot i_1^{n-1/2}}{\Delta x} \tag{10}$$

At the other end $k = k_{\max} + 1\,(x = L)$:

$$\left(\frac{C}{\Delta t}\right)(v\,(L))^n + \frac{1}{\Delta x}\,(i\,(L))^n$$
$$= \left(\frac{C}{\Delta t}\right)(v\,(L))^{n-1} - \frac{1}{\Delta x}\,(i\,(L))^{n-1} + \frac{2 \cdot i_{k_{\max}}^{n-1/2}}{\Delta x} \tag{11}$$

And the form represented by a quadruple is derived:

$$\begin{bmatrix} \left(\frac{C}{\Delta t}\right) & -\frac{1}{\Delta x} & 0 & 0 \\ 0 & 0 & \left(\frac{C}{\Delta t}\right) & \frac{1}{\Delta x} \end{bmatrix} \begin{bmatrix} (v\,(0))^n \\ (i\,(0))^n \\ (v\,(L))^n \\ (i\,(L))^n \end{bmatrix}$$
$$= \begin{bmatrix} \left(\frac{C}{\Delta t}\right)(v\,(0))^{n-1} + \frac{1}{\Delta x}\,(i\,(0))^{n-1} - \frac{2 \cdot i_1^{n-1/2}}{\Delta x} \\ \left(\frac{C}{\Delta t}\right)(v\,(L))^{n-1} - \frac{1}{\Delta x}\,(i\,(L))^{n-1} + \frac{2 \cdot i_{k_{\max}}^{n-1/2}}{\Delta x} \end{bmatrix} \tag{12}$$

2.2.2. Construction of the Sub Matrices $[A_1]$ and $[A_2]$

The sub matrix $[A_1]$ is obtained from the relationship (12). The contribution of the line with index i appears as follows:

$$[A_1] = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & \left(\frac{C_i}{\Delta t}\right) & -\frac{1}{\Delta_x} & 0 & 0 & \dots \\ \dots & [0] & [0] & \left(\frac{C_i}{\Delta t}\right) & \frac{1}{\Delta x_i} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \tag{13}$$

The sub matrix $[A_2]$ is derived using the laws of Kirchhoff in voltage and current (KCL and KVL) at each node m of the tower [9, 10]:

$$\sum_{k=1}^{N} \left([Y_k^m]\;[v_k^m] + [Z_k^m]\;[i_k^m]\right) = [P^m] \tag{14}$$

where $[Z_k^m]$ and $[Y_k^m]$ are impedances or admittances matrices resulting from the application of Kirchhoff's laws (KVL and KCL) at node m. $[P^m]$ is the vector of current or voltage localized sources.

2.2.3. Construction of the Sub Vectors $[B_1]$ and $[B_2]$

Sub vector $[B_1]$ is constructed from the second member of the matrix system (12). For the conductor of index i at instant $t = n \cdot \Delta t$ it follows:

$$[B_1] = \begin{bmatrix} \left(\frac{C}{\Delta t}\right)(v_i(0))^{n-1} + \frac{1}{\Delta x_i}(i_i(0))^{n-1} - \frac{2 \cdot i_{i1}^{n-1/2}}{\Delta x_i} \\ \left(\frac{C}{\Delta t}\right)(v_i(L))^{n-1} + \frac{1}{\Delta x_i}(i_i(L))^{n-1} - \frac{2 \cdot i_{ik_{\max}}^{n-1/2}}{\Delta x_i} \end{bmatrix} \quad (15)$$

The currents $i_{i1}^{n-1/2}$ and $i_{ik_{\max}}^{n-1/2}$ are calculated from the following recurrence equations:

$$i_k^{n+1/2} = \left(\frac{L}{\Delta t} + \frac{R}{2}\right)^{-1} \left(\left(\frac{L}{\Delta t} - \frac{R}{2}\right) i_k^{n-1/2} - \frac{v_{k+1}^n - v_k^n}{\Delta x}\right) \quad (16)$$

$$v_k^n = \left(\frac{C}{\Delta t}\right)^{-1} \left(\left(\frac{C}{\Delta t}\right) v_k^{n-1} - \frac{v_k^{n-1/2} - v_{k-1}^{n-1/2}}{\Delta x}\right) \quad (17)$$

Sub vector $[B_2]$ contains zeros, except the node in which the lightning strike occurs. In the case of modeling of lightning injection by using the bi-exponential generator one has:

$$[B_2] = \begin{bmatrix} 0 \\ \vdots \\ I_0\left(\exp\left(-\alpha \cdot n\left(\Delta t\right)\right) - \exp\left(-\beta \cdot n\left(\Delta t\right)\right)\right) \\ \vdots \\ 0 \end{bmatrix} \quad (18)$$

2.2.4. Construction of the Unknown Vector $[X]$

Vector $[X]$ includes the unknown currents and voltages at each node in the network. For the conductor segment index i at instant $t = n \cdot \Delta t$ one has:

$$[X] = \left[\dots \quad (v_i(0))^n \quad (i_i(0))^n \quad (v_i(L))^n \quad (i_i(L))^n \quad \dots\right]^{transpose} \quad (19)$$

3. VALIDATION OF THE PROPOSED MODEL

To compare the results calculated by using transmission line theory and FDTD method with the results published [1, 2] (computed by means of ATP software), the configuration of the instrumented tower installed at Morro do Cachimbo Station [11], illustrated in Figure 5(a) [1] is considered.

The tower is excited at the top by a step current with two different front times as indicated in Figure 6. In proposed tower model, three vertical parallel conductors connected to grounding resistances of $10\,\Omega$

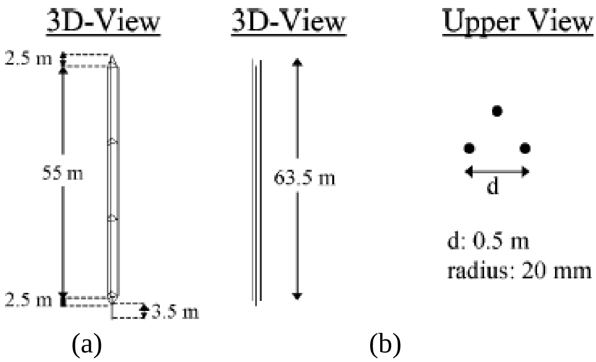


Figure 5. Morro do Cachimbo Tower [1]. (a) Full representations. (b) Simplified representations.

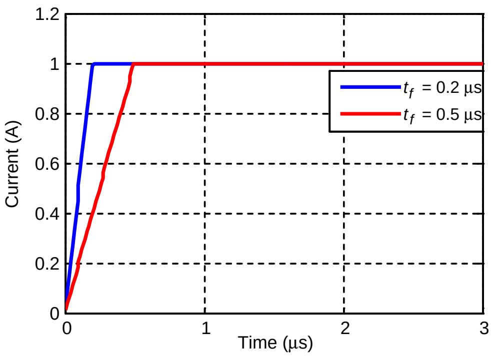


Figure 6. Shape of the injected current at the top.

at their bottom and to surge impedance of $1500\,\Omega$ at their tops have been considered, as depicted in Figure 5.

Note that the expressions of the parameters per unit length, proposed by Gutierrez et al. [3], are used. It is worth noting that these parameters arise from the antenna theory formalism.

Figures 7 and 8 show the transient current induced at the top of the tower at instant $t_f = 0.2\,\mu\text{s}$ and $t_f = 0.5\,\mu\text{s}$, respectively.

It should be emphasized that the calculated results are in satisfactory agreement with the results available in [1], thus proving the robustness of the proposed transmission line (TL) approach and the FDTD solution method.

The next configuration of interest is related to UHV tower and is shown in Figure 9(a). The UHV tower is struck by a direct lightning

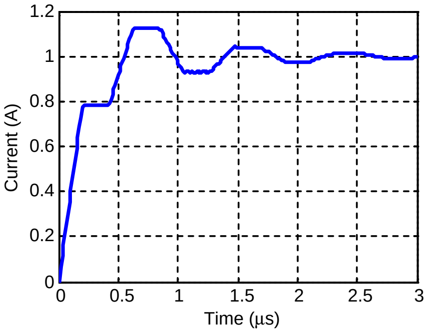


Figure 7. Current at the top of the tower ($t_f = 0.2\,\mu\text{s}$).

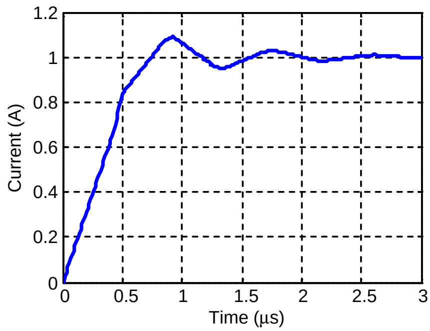


Figure 8. Current at the top of the tower ($t_f = 0.5\,\mu\text{s}$).

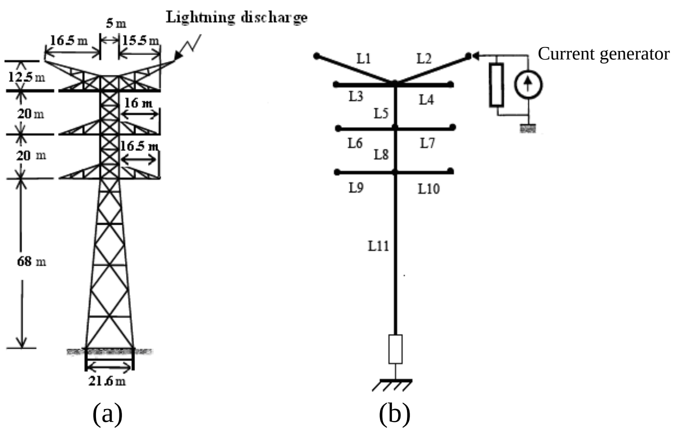


Figure 9. Configuration to study the direct lightning strike.

strike and modeled as a vertical column with horizontal and inclined arms as shown in Figure 9(b).

The direct strike is represented via the double-exponential current generator:

$$i\,(t) = I_0 \left(e^{-\alpha t} - e^{-\beta t}\right) \tag{20}$$

with $I_0 = 1.06537\,\text{kA}$, $\alpha = 1.88 \times 10^4\,\text{s}^{-1}$, and $\beta = 1.6 \times 10^6\,\text{s}^{-1}$.

The typical current waveform is shown in Figure 10.

The tower is connected at its lower base by a square grounding ($21.6\,\text{m} \times 21.6\,\text{m}$). The proposed model neglects the effect of the tower on the current at the channel base. The per unit length longitudinal and transverse parameters of the horizontal and inclined segments are

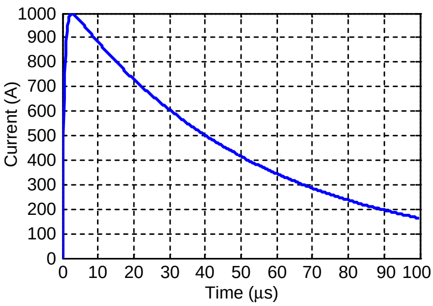


Figure 10. Current injected at the top of tower.

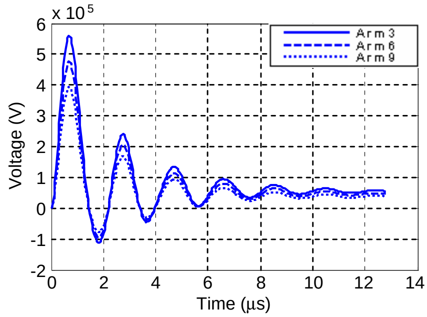


Figure 11. Transient voltage in different arms.

calculated using the formalism described in [9]. For vertical segments, the per unit length parameters are calculated by using the formalism developed by Ametani et al. [4].

In Table 1, the geometrical data associated with the tower are given.

Table 1. Different distances associated with the studied tower.

Distance between two phase arms	20 m
Distance between arm and ground wire	5.2 m
Vertical column diameter	0.3 m
Vertical column height	108 m
Arms diameter	0.2 m
Phase arm length (horizontal)	15.5 m
Ground wire arm length (inclined)	18.5 m

The transient voltage induced in different arms of the tower structure, presented in Figure 9, is shown in Figure 11.

The obtained results are in a satisfactory agreement with the results computed by using the vector fitting method [2].

4. ANALYSIS OF DIRECT LIGHTNING STRIKE IMPACT ON A TOWERS CASCADE

The principal disadvantage of the configuration considered in the previous case is the fact that the ground wire connected to the tower, which the main objective is to capture the direct lightning strikes and consequently protect the power lines, was not taken into account.

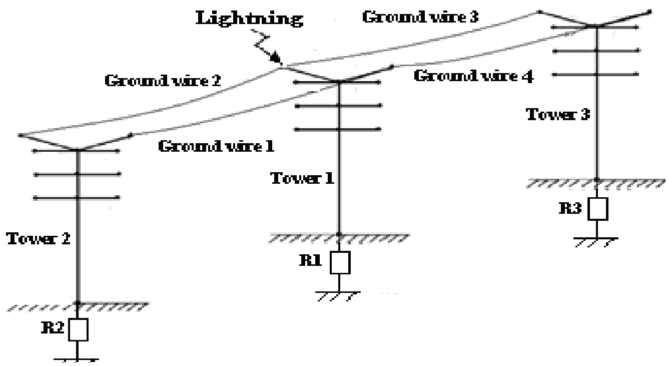


Figure 12. Equivalent geometry of three towers.

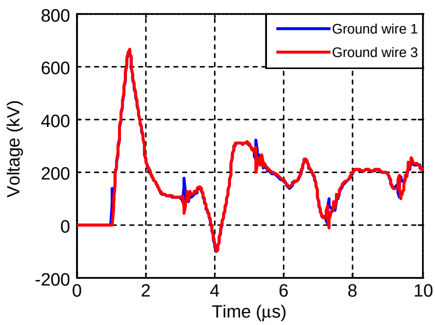


Figure 13. Voltage at the ends of ground wires 1 and 3.

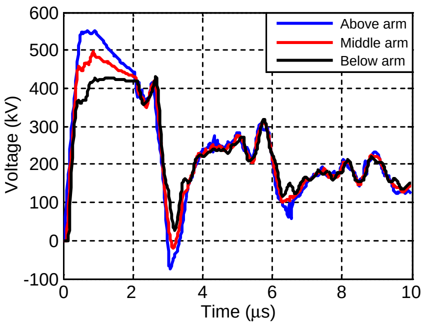


Figure 14. Voltage at the free arms of Tower 1.

Thus, further analysis includes three towers connected with a 300 m pair of ground wires. Figure 12 shows the equivalent tower configuration being considered.

The parameters of the double-exponential lightning current are: $I_0 = 10\,\text{kA}$, $\alpha = 1.88 \times 10^4\,\text{s}^{-1}$, and $\beta = 1.6 \times 10^6\,\text{s}^{-1}$.

The distances and dimensions are kept the same for each tower, as in previous case, with related grounding resistances $R_1 = R_2 = R_3 = 60\,\Omega$. Note that the horizontal and inclined segments, respectively are treated as in [9], while the vertical segments are treated as in [7].

Figure 13 and Figure 14 clearly show very high values of overvoltages on the ground wires with a succession of peaks which decrease gradually. This fact justifies their use for primary protection of power against lightning. Moreover, the role of the ground wires is very important in areas of very high Keraunic rate.

Figure 15 and Figure 16 show the transient currents induced at

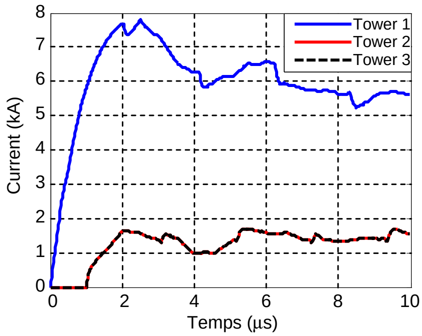


Figure 15. Current at the top of towers.

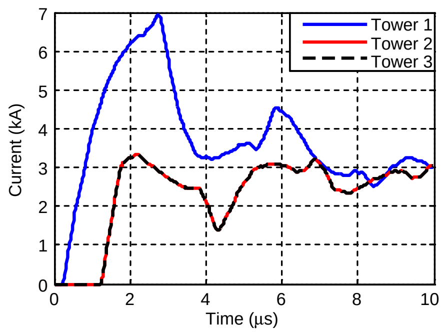


Figure 16. Current at the base of towers.

the top and the base of the three towers.

The current induced at the top of the central tower is the most important (Figure 15), as this is the point of the direct lightning strike. Given the perfect symmetry of the device and the central injection strike point, the transient currents induced on the two others towers (2 and 3) are identical. The results presented in Figure 16 show that the currents at the base of the three towers tend to overlap at time instants $t > 9\,\mu s$. Thus, in the absence of the propagation along the ground wire and the vertical column of the tower only a lumped-constant circuit exists and the sum of three currents leads to the injected current (Kirchhoff's law). Namely, the total sum of currents is around 8.5 kA (approximatively the value of the injected current at the top). These observations confirm the proposed approach to be valid.

It is worth emphasizing that the computational time required for the calculation is relatively low.

In the previous examples the grounding system of the tower was considered as a simple load resistor. However, in the disturbance regime, the grounding system of the tower plays a very important role. Consequently, to demonstrate the role of a grounding system, a more realistic device is analyzed by taking into account the propagation along the grounding (ground rod or horizontal buried conductors).

4.1. Towers Cascade with Vertical Grounding Electrode

First scenario is related to the case that each tower terminates at its lower part by a picket (conductor of 0.014 m radius) with 5 m height buried vertically. The grounding electrode is considered as a transmission line and the parameters per unit length are calculated from the expressions given in [5–8]. The lightning strike is assumed to occur at the point of interconnection, as depicted in Figure 17. The entire cascade of towers — ground wires — grounding system is treated as a network of lines excited by a lightning strike.

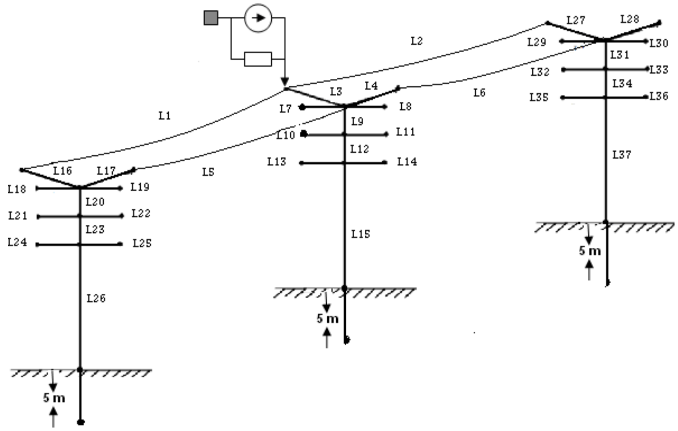


Figure 17. Towers cascade with vertical electrodes.

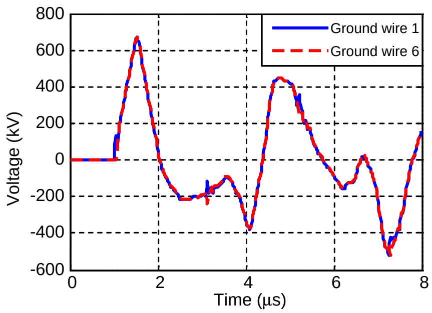


Figure 18. Voltage at the ends of ground wires 1 and 6.

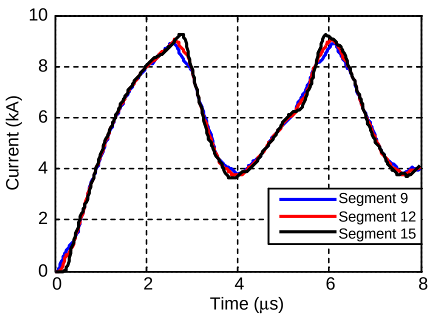


Figure 19. Current along column of middle tower.

The parameters of the double-exponential lightning current are: $I_0 = 10\,\text{kA}$, $\alpha = 1.88 \times 10^4\,\text{s}^{-1}$, and $\beta = 1.6 \times 10^6\,\text{s}^{-1}$. The soil resistivity is $\rho = 100\,\Omega \cdot \text{m}$ and its permittivity $\varepsilon_r = 4$.

The electric parameters of horizontal and inclined segments are obtained as it is proposed in [9], while vertical segments are treated as in [3]. The transient voltages induced along the ground wires, tower and electrodes are shown in Figures 18 to 20.

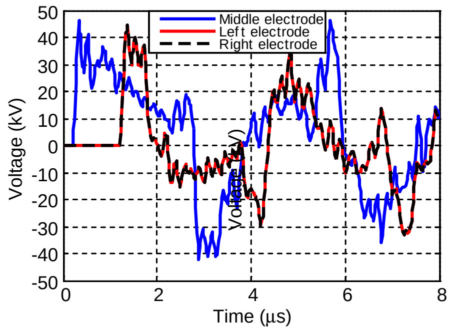


Figure 20. Voltage at the first extremities of electrodes.

4.2. Towers Cascade with Horizontal Grounding Electrode

Now, the case where the grounding system is a 5 m length conductor (2.5 m on each side) buried horizontally at a depth of 0.8 m is of interest. The towers cascade above the ground is of the same geometry and the segments notations are like the one shown in Figure 17, only the horizontal conductor representing the grounding is different.

For both types of electrodes the overvoltages induced on cable guards have almost the same waveform with certain differences in amplitudes (Figure 18 and Figure 21). The currents induced on the central vertical column of the middle tower keep the same shape with a slight amplitude decrease (Figure 19 and Figure 22). The voltages at horizontal electrodes are nearly of half value comparing to the vertical ones, due to the fact that the electrode has the same length but is divided into equal two parts.

It can be noted that the grounding systems of towers for horizontal

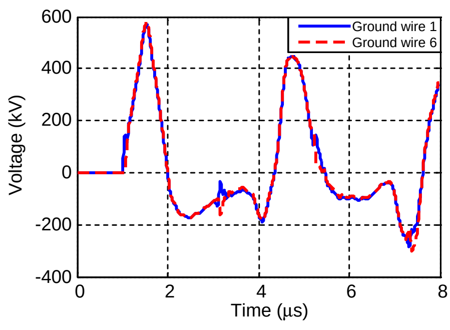
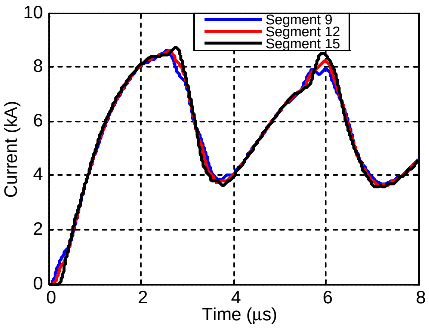


Figure 21. Voltage at the ends of ground wires 1 and 6.



captionCurrent along column of middle tower.

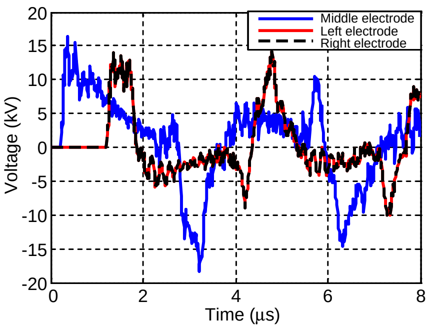


Figure 22. Voltage at the middle of the horizontal electrodes.

and vertical conductor (Figure 20 and Figure 23) lead almost to the same result when the length (grounding electrode) in direct contact with the ground is the same.

5. CONCLUSION

In this work, a direct lightning strike on a cascade of electrical towers, fitted or not with ground wires and connected at their bottom by grounding system with vertical and horizontal topologies, is analyzed.

The formulation is posed directly in the time domain and based on the Transmission Line (TL) theory as TL seems to be appropriate enough to the filamentary nature of the body and arms of the tower. To properly analyze the interaction between lightning discharge on the tower, TL formalism also includes an excitation in the form of an equivalent current generator. The numerical solution is carried out by means of FDTD method. The proposed approach is validated through the comparison with the results reported in relevant publications.

The illustrative computational examples are related to the towers cascade connected with ground wires with both sides, terminated by horizontal/vertical electrodes as its grounding system and subjected to direct injection of the lightning strike. The results obtained via proposed TL approach confirm the crucial role of the ground wire as a protection of power lines, given the high values of induced transient currents and voltages along the ground wire tower, respectively. Basically, these transients can cause serious disturbances in the power distribution system.

This proposed model can be used for insulation coordination in power systems as it allows one to quantify lightning induced transient voltages that may arise; between a ground wire and phase conductor, between the shield wire and finitely conducting ground and the increasing of the GPR (ground potential rise). It is worth underlining that all the calculations have been carried out with very low CPU time.

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