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TE AND TM MODES OF A VANE-LOADED CIRCULAR CYLINDRICAL WAVEGUIDE FOR GYRO-TWT APPLICATIONS

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Abstract—The dispersion equation governing the guided propagation of *TE* and *TM* fast wave modes of a circular cylindrical waveguide loaded by metal vanes positioned symmetrically around the wave-guide axis is derived from the exact solution of a homogeneous boundary value problem for Maxwell's equations. The dispersion equation takes the form of the solvability condition for an infinite system of linear homogeneous algebraic equations. The approximate dispersion equation corresponding to a truncation of the infinite-order coefficient matrix of the infinite system of equations to the coefficient matrix of a finite system of equations of sufficiently high order is solved numerically to obtain the cut-off wave numbers of the various propagating modes. Each cut-off wave number gives rise to a unique dispersion curve in the shape of a hyperbola in the ω-β plane.

1. INTRODUCTION

The propagation characteristics of *TE* and *TM* modes of a vane-loaded circular cylindrical waveguides have been attracting the attention of researchers in microwave engineering in view of their potential application in broadband gyro-TWT amplifiers. The earliest work on the modal analysis of a vane-loaded circular cylindrical waveguide appears to be by Singh et al. [1] in 1999. A method of controlling the gain-frequency response by the beam and the magnetic-field parameters was proposed in [2] by the same authors based on the results of [1]. A two-stage vane-loading of gyro-TWT for high gains and bandwidths was proposed by Agarwal et al. in [3] based again

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on the results of [1]. The dispersion relations 'derived' in [1] were modified by the same authors for tapered vanes in [4]. In a paper [5] published in 2002, Singh et al. have used the dispersion relations of [1] for finding the dispersion characteristics of 'TE_{01}' and 'TE_{21}' modes for a typical set of vane-parameter values. In the papers by Singh et al. [6–10], various methods of analyzing/improving the frequency-response characteristics of gyro-TWT amplifiers have been proposed based again on the results from [1]. The papers [11] by Singh and Basu and [12] by Singh were devoted to analytical studies of the interaction structure of vane-loaded gyro-TWT amplifiers starting again from the cold-wave dispersion relations appearing in [1].

Notwithstanding the impressive list of applications of the vane-loaded structure cited in the previous paragraph, the 'derivation' of the dispersion equations for this structure attempted in [1] is seriously flawed rendering the results and conclusions of [2–12] to be of questionable validity for the following reason: The azimuthal dependence of the assumed form of the solution for the field components in the annular region containing the vanes does not permit the boundary condition on the radial component of the electric field, viz., the radial electric field component should vanish on the lateral boundaries (located on the radial planes passing through the waveguide axis) of the perfectly conducting vanes, to be satisfied; more fundamentally the assumed form of solution is not capable of ensuring a null electromagnetic field everywhere inside the vane region.

In this paper, the dispersion relation for the TE and TM modes supported by a vane-loaded circular cylindrical waveguide are derived from an exact solution of a homogeneous boundary value problem for Maxwell's equations, i.e., a solution of Maxwell's equations satisfying all the boundary conditions of the problem. Rigorous approaches to solving the boundary value problem for coaxial waveguides and cavities with azimuthally slotted inserts have also been presented(though without full details of analysis) in [13, 14].

2. PROBLEM FORMULATION

We consider an infinitely long circular cylindrical hollow waveguide of inner radius b with L wedge-shaped metal projections (vanes), which are periodically disposed with an angular period $2\pi/L$ around the waveguide axis, running along the entire length of the waveguide (see Fig. 1 for a cross-sectional view). As seen from the cross-sectional view in Fig. 1, the vanes extend from the waveguide wall up to radius a.

We take the axis of the vane-loaded waveguide along the z-coordinate of a cylindrical coordinate system (r, θ, z). The slots

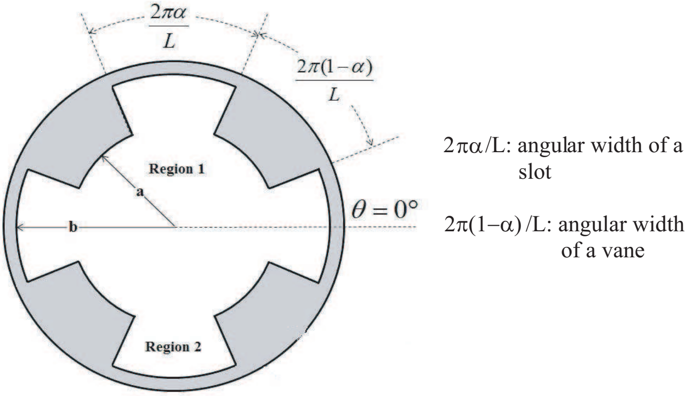

Figure 1. Cross-sectional view of a vane-loaded circular cylindrical waveguide with $L = 4$.

between adjacent vanes are numbered anticlockwise from 0 to $L-1$ with the ℓth slot occupying the two-dimensional region $\{(r,\theta): a < r < b,\ \pi(2\ell-\alpha)/L < \theta < \pi(2\ell+\alpha)/L\}$ in the cross-sectional plane of the waveguide where $0 < \alpha < 1$. Thus the angular width of each slot is $2\pi\alpha/L$ and that of each vane is $2\pi(1-\alpha)/L$. The vanes occupy the region $\bigcup_{\ell=0}^{L-1} \{(r,\theta)\colon a \leq r \leq b,\ \pi(2\ell+\alpha)/L \leq \theta \leq \pi(2(\ell+1)-\alpha)/L\}$ in the cross-sectional plane.

In order to identify the appropriate form of the solution of the Helmholtz equation for the axial electric field component $E_z\,(r,\,\theta,\,z)$ (in the case of TM modes) and the axial magnetic field component $H_z\,(r,\,\theta,\,z)$ (in the case of TE modes), it is expedient to divide the domain $\{(r,\theta): 0 \leq r < b,\ 0 \leq \theta < 2\pi\}$ in the cross-sectional plane into Region 1: $\{(r,\theta): 0 \leq r \leq a,\ 0 \leq \theta < 2\pi\}$ and Region 2: $\{(r,\theta): a < r < b,\ 0 \leq \theta < 2\pi\}$. Modeling the waveguide walls and the vanes to be perfectly conducting, the z-dependence of the non-zero field components may be assumed to be according to the propagation factor $e^{-j\beta z}$ with β (propagation phase constant) corresponding to single-frequency wave propagation in the positive z-direction. Thus the field components (phasors) may be expressed as

$$E_i(r,\theta,z) = e_i(r,\theta)e^{-j\beta z}, \quad i = r,\theta,z$$

$$H_i(r,\theta,z) = h_i(r,\theta)e^{-j\beta z}, \quad i = r,\theta,z$$

In the following, we make use of the standard notation $\mathbf{Z}$, $\mathbf{N}$ and $\mathbf{N_0}(\underline{\Delta}\mathbf{N} \cup \{0\})$ for the ring of integers, the set of natural numbers (positive integers) and the set of non-negative integers respectively. Moreover, the indicator function I_X of a set X is defined as

$$I_X(x) = \begin{cases} 1 & \text{if} \quad x \in X \\ 0 & \text{if} \quad x \notin X \end{cases}$$

and for a real number x, $\lfloor x \rfloor$ and $\lceil x \rceil$ denote respectively the largest integer less than or equal to x and the smallest integer greater than or equal to x.

3. DERIVATION OF DISPERSION EQUATIONS

The steps involved in the solution of the homogeneous boundary value problem for Maxwell's equations arising in the guided propagation of *TE* and *TM* waves for the model of a vane-loaded circular cylindrical waveguide introduced in the previous section are

(i) Identify the infinite-series expansion for the solution of the Helmholtz equation for the axial field component appropriate to regions 1 and 2.

(ii) Find the expansions for the transverse field components using that for the axial field components in both regions.

(iii) Obtain the linear algebraic relations among the expansion coefficients by enforcing the electromagnetic boundary conditions across the interface separating region 1 and region 2.

(iv) Reduce the infinite set of linear algebraic relations to an infinite system of homogeneous linear equations for the coefficients of the field expansions in region 2.

(v) Deduce the sought after dispersion equation as the solvability condition for a nontrivial solution of the above infinite system of homogeneous linear equations.

We now carry out the above steps for the *TE* and *TM* modes of a vane loaded circular cylindrical waveguide.

3.1. Case 1: TE Modes

The two tangential field components $h_{mz}(r, \theta)$ and $e_{m\theta}(r, \theta)$, $m = 0, 1, 2, \ldots, \lfloor L/2 \rfloor$ involved in the boundary conditions at $r = a$ for region 1 ($0 \le r < a$) admit the infinite series representation

$$h_{mz}(r, \theta) = \sum_{p \in \mathbf{Z}} A_{mp} J_{m+pL}(k_c r) e^{-j(m+pL)\theta} \tag{1a}$$

$$\begin{aligned} e_{m\theta}(r,\theta) &= \left(j\omega\mu_0/k_c^2\right)\partial h_{mz}/\partial r \\ &= (j\omega\mu_0/k_c)\sum_{p\in\mathbf{Z}} A_{mp}J'_{m+pL}(k_c r)e^{-j(m+pL)\theta} \end{aligned} \tag{1b}$$

where $k_c^2 = k_0^2 - \beta^2 > 0$ since $k_0/|\beta| = \omega/c|\beta| = v_p/c > 1$ for fast waves and A_{mp}, $p \in \mathbf{Z}$, are the (complex) expansion coefficients which are arbitrary at this stage and where ω is the operating angular frequency, $k_0 = \omega/c$ is the free-space wave number, $c = (\mu_0\varepsilon_0)^{-1/2}$ is the speed of light in vacuum and v_p is the phase speed of the TE waves. In (1) and in the following, J_υ and Y_υ denote υth order Bessel functions of first and the second kind respectively and a prime superscript denotes differentiation with respect to the argument. A vane-loaded circular cylindrical waveguide is seen to support at most $\lfloor L/2\rfloor$ sets of non-axisymmetric modes ($m > 0$) in addition to the single set of axisymmetric modes ($m = 0$). This characteristic may be contrasted with that of a circular cylindrical hollow waveguide for which the azimuthal mode number $m \in \mathbf{N_0}$. The expressions for the z-component of the magnetic field and θ-component of the electric field (apart from the factor $e^{-j\beta z}$) appropriate to region 2 ($a < r < b$) are

$$\begin{aligned} h_{mz}(r,\theta) &= \sum_{\ell=0}^{L-1} e^{-j2\pi m\ell/L} I_{(-\pi\alpha/L,\pi\alpha/L)}(\theta - 2\pi\ell/L) \\ &\quad \sum_{s=0}^{\infty} B_{ms} g_s(k_c r)\cos\left(sL\left(\theta - 2\pi\ell/L\right)/\alpha\right) \end{aligned} \tag{2a}$$

$$\begin{aligned} e_{m\theta}(r,\theta) &= \left(\frac{j\omega\mu_0}{k_c}\right)\sum_{\ell=0}^{L-1} e^{-j2\pi m\ell/L} I_{(-\pi\alpha/L,\pi\alpha/L)}(\theta - 2\pi\ell/L) \\ &\quad \sum_{s=0}^{\infty} B_{ms} g'_s(k_c r)\cos\left(sL\left(\theta - 2\pi\ell/L\right)/\alpha\right) \end{aligned} \tag{2b}$$

where B_{ms}, $s \in \mathbf{N_0}$, are arbitrary expansion coefficients and

$$g_s(k_c r) = J_{sL/\alpha}(k_c r)Y'_{sL/\alpha}(k_c b) - J'_{sL/\alpha}(k_c b)Y_{sL/\alpha}(k_c r), \quad a<r<b \tag{3}$$

The summation over the indicator functions in (2) ensures that there is no electromagnetic field inside the region of the vanes viz., for $-\pi\alpha/L < \theta - 2\pi\ell/L < \pi\alpha/L$, $\ell = 0, 1, 2, \ldots, L-1$, $a < r < b$. It may be observed from (2) and (3) that the boundary conditions on the bottom and the side walls of slots are automatically satisfied by the form of solution assumed for the field components in region 2, viz.

(i) $e_{m\theta}(r,\theta)$ vanishes at $r = b$ for $-\pi\alpha/L < \theta - 2\pi\ell/L < \pi\alpha/L$, $\ell = 0, 1, \ldots, L-1$

and

(ii) $e_{mr}(r,\theta) = -(j\omega\mu_0/k_c^2 r)\partial h_{mz}/\partial\theta$ vanishes at $\theta = \pi(2\ell \pm \alpha)/L$, $\ell = 0, 1, \ldots, L-1$ for $a < r < b$.

Now it remains only to enforce the continuity of $e_{m\theta}(r,\,\theta)$ across the interface $r = a$ between regions 1 and 2, and the continuity of $h_{mz}(r,\,\theta)$ across the slot openings located at $r = a$, for $-\pi\alpha/L < \theta - 2\pi\ell/L < \pi\alpha/L$, $\ell = 0, 1, \ldots, L-1$. The continuity of $e_{m\theta}(r,\,\theta)$ across $r = a$ requires

$$\sum_{q\in\mathbf{Z}} A_{mq} J'_{m+qL}(k_c a) e^{-jqL\theta} = \sum_{\ell=0}^{L-1} e^{jm(\theta-2\pi\ell/L)} I_{(-\pi\alpha/L,\pi\alpha/L)}(\theta-2\pi\ell/L) \sum_{s=0}^{\infty} B_{ms} g'_s(k_c a)\cos\left(sL\left(\theta-2\pi\ell/L\right)/\alpha\right) \quad (4)$$

Since both sides of (4) are periodic in the variable θ with period $2\pi/L$, it is sufficient to satisfy the above continuity condition over the span of one period, say over the interval $(-\pi/L,\,\pi/L)$. Thus (4) is equivalent to

$$\sum_{q\in\mathbf{Z}} A_{mq} J'_{m+qL}(k_c a) e^{-jqL\theta} = e^{jm\theta} I_{(-\pi\alpha/L,\pi\alpha/L)}(\theta) \sum_{s=0}^{\infty} B_{ms} g'_s(k_c a)\cos(sL\theta/\alpha) \quad \text{for } -\pi/L \le \theta < \pi/L \qquad (5)$$

The infinite series on the left side of (5) may be considered to be the complex- exponential Fourier series representation of the function defined by the right side over the interval $(-\pi/L, \pi/L)$. Multiplying both sides of (5) by $e^{jpL\theta}$ and integrating with respect to θ from $-\pi/L$ to π/L, we have

$$A_{00} = \alpha g'_0(k_c a) B_{00}/J'_0(k_c a) \qquad (6a)$$

and for $m + |p| \neq 0$

$$A_{mp} = \begin{cases} \big((p+m/L)\sin((p+m/L)\pi\alpha)/\pi J'_{m+pL}(k_c a)\big) \displaystyle\sum_{s=0}^{\infty}(-1)^s g'_s(k_c a)\frac{B_{ms}}{((p+m/L)^2 - s^2/\alpha^2)} & \text{if } \alpha\,|p+m/L| \notin \mathbf{N} \\ \big((p+m/L)\sin((p+m/L)\pi\alpha)/\pi J'_{m+pL}(k_c a)\big) \displaystyle\sum_{\substack{s=0\\ s\neq\alpha|m+p/L|}}^{\infty}(-1)^s g'_s(k_c a)\frac{B_{ms}}{((p+m/L) - s^2/\alpha^2)} \\ \quad +\alpha g'_{\alpha|p+m/L|}(k_c a) B_{m(\alpha|p+m/L|)}/2J'_{m+p/L}(k_c a) & \text{if } \alpha\,|p+m/L| \in \mathbf{N} \end{cases} \qquad (6b)$$

The continuity of $h_{mz}(r,\theta)$ across the slot openings on the interface $r=a$ may be expressed as

$$e^{-jm(\theta-2\pi l/L)}\sum_{p\in\mathbf{Z}}A_{mp}J_{m+pL}(k_ca)e^{-jpL\theta}=I_{(-\pi\alpha/L,\pi\alpha/L)}(\theta-2\pi\ell/L)$$

$$\sum_{k=0}^{\infty}B_{mk}g_k(k_ca)\cos\left(kL(\theta-2\pi\ell/L)/\alpha\right)\quad\text{for}\quad\ell=0,1,\ldots,L-1 \tag{7}$$

Since both sides of (7) are periodic in θ with period $2\pi/L$, it is sufficient to enforce the continuity of $h_{mz}(r,\theta)$ across $r=a$ only for $\theta\in(-\pi\alpha/L,\pi\alpha/L)$. Thus we need to ensure

$$\sum_{p\in\mathbf{Z}}A_{mp}J_{m+pL}(k_ca)e^{-j(m+pL)\theta}=\sum_{k=0}^{\infty}B_{mk}g_k(k_ca)\cos\left(kL\theta/\alpha\right) \tag{8}$$

only for $-\pi\alpha/L\leq\theta<\pi\alpha/L$. The right side of (8) may be considered to be the Fourier cosine-series representation of the function on the left side of (8) restricted to the interval $(-\pi\alpha/L,\,\pi\alpha/L)$. Multiplying both sides of (8) by $\cos(sL\theta/\alpha)$ and integrating with respect to θ from $-\pi\alpha/L$ to $\pi\alpha/L$, we have

$$B_{m0}g_0(k_ca)=\sum_{p\in\mathbf{Z}}A_{mp}J_{m+pL}(k_ca)\operatorname{sinc}((p+m/L)\theta) \tag{9a}$$

and for $s\geq 1$

$$B_{ms}g_s(k_ca)=(2(-1)^s/\pi\alpha)\sum_{p\in\mathbf{Z}}A_{mp}(p+m/L)J_{m+pL}(k_ca)$$

$$\frac{\sin(p+m/L)\pi\alpha}{((p+m/L)^2-s^2/\alpha^2)}\ \text{ if }\ \mathbf{Z}\cap\{-m/L+s/\alpha,-m/L-s/\alpha\}\text{ is empty}$$

$$(2(-1)^s/\pi\alpha)\sum_{\substack{p\in\mathbf{Z}\\ p\neq -m/L\pm s/\alpha}}A_{mp}(p+m/L)J_{m+pL}(k_ca)\frac{\sin(p+m/L)\pi\alpha}{((p+m/L)^2-s^2/\alpha^2)}$$

$$+J_{(sL/\alpha)}(k_ca)A_{m(-m/L+s/\alpha)}I_{\mathbf{Z}}(-m/L+s/\alpha)+J_{(-sL/\alpha)}(k_ca)$$

$$A_{m(-m/L-s/\alpha)}I_{\mathbf{Z}}(-m/L-s/\alpha)$$

$$\text{if }\mathbf{Z}\cap\{-m/L\pm s/\alpha\}\text{ is nonempty} \tag{9b}$$

In (9a)

$$\operatorname{sinc}X\underline{\underline{\Delta}}\sin\pi X/\pi X\quad\text{if}\quad X\neq 0$$
$$1\quad\text{if}\quad X=0$$

where the symbol '$\underline{\underline{\Delta}}$' denotes equality by definition. Substituting for A_{mp}, $p\in\mathbf{Z}$, from (6) in terms of B_{ms}, $s\in\mathbf{N_0}$, into (9), we end up with the infinite system of homogeneous linear algebraic equations for B_{ms}, $s \in \mathbf{N_0}$:

$$\left[g_0(k_ca)-\alpha g_0'(k_ca)\sum_{p\in\mathbf{Z}}(J_{m+pL}(k_ca)/J'_{m+pL}(k_ca))\mathrm{sinc}^2(p+m/L)\alpha\right]B_{m0}=$$
$$\sum_{k=1}^{\infty}\Big((-1)^k g_k'(k_ca)/\pi\Big)\left(\sum_{\substack{p\in\mathbf{Z}\\ p\neq -m/L\pm k/\alpha}}\frac{J_{m+pL}(k_ca)}{J'_{m+pL}(k_ca)}\frac{\sin^2(p+m/L)\pi\alpha}{((p+m/L)^2-k^2/\alpha^2)\pi\alpha}\right)$$
$$B_{mk} \tag{10a}$$

and for $s \geq 1$

$$B_{ms}g_s(k_ca) = \frac{2}{\pi^2\alpha}\sum_{k=0}^{\infty}(-1)^{s+k}g_k'(k_ca)\left(\sum_{\substack{p\in\mathbf{Z}\\ p\neq -m/L\pm k/\alpha}}\frac{J_{m+pL}(k_ca)}{J'_{m+pL}(k_ca)}\right.$$
$$\left.\frac{(p+m/L)^2\sin^2(p+m/L)\pi\alpha}{\left(\left(p+\frac{m}{L}\right)^2-\frac{s^2}{\alpha^2}\right)\left(\left(p+\frac{m}{L}\right)^2-\frac{k^2}{\alpha^2}\right)}\right)B_{mk} \text{ if } \mathbf{Z}\cap\{-m/L\pm s/\alpha\} \text{ is empty}$$

$$\frac{2}{\pi^2\alpha}\sum_{k=0}^{\infty}(-1)^{s+k}\,g_k'(k_ca)\left(\sum_{\substack{p\in\mathbf{Z}\\ p\neq -m/L\pm s/\alpha\\ p\neq -m/L\pm k/\alpha}}\frac{J_{m+pL}(k_ca)}{J'_{m+pL}(k_ca)}\right.$$
$$\left.\frac{(p+m/L)^2\sin^2(p+m/L)\pi\alpha}{\left(\left(p+\frac{m}{L}\right)^2-\frac{s^2}{\alpha^2}\right)\left(\left(p+\frac{m}{L}\right)^2-\frac{k^2}{\alpha^2}\right)}\right)B_{mk}+\frac{\alpha}{2}\frac{J_{sL/\alpha}(k_ca)}{J'_{SL/\alpha}(k_ca)}$$
$$(I_{\mathbf{Z}}(-m/L+s/\alpha)+I_{\mathbf{Z}}(-m/L-s/\alpha))\,g_s'(k_ca)B_{ms}$$
$$\text{if } \mathbf{Z}\cap\{-m/L\pm s/\alpha\} \text{ is nonempty} \tag{10b}$$

Defining

$$b^{TE}_{m0k}\triangleq\delta_{0k}\left(1-\alpha\frac{g_0'(k_ca)}{g_0(k_ca)}\sum_{p\in\mathbf{Z}}\frac{J_{m+pL}(k_ca)}{J'_{m+pL}(k_ca)}\mathrm{sinc}^2((p+m/L)\alpha)\right)$$

$$-(1-\delta_{0k})\frac{(-1)^k}{\pi}\frac{g'_k(k_c a)}{g_0(k_c a)}\sum_{\substack{p\in\mathbf{Z}\\ p\neq -\frac{m}{L}\pm\frac{k}{\alpha}}}\frac{J_{m+pL}(k_c a)}{J'_{m+pL}(k_c a)}$$

$$\frac{\sin^2(p+m/L)\pi\alpha}{((p+m/L)^2-k^2/\alpha^2)\pi\alpha} \tag{11a}$$

and for $s \geq 1$

$$b^{TE}_{msk} \triangleq \delta_{sk}-\frac{2(-1)^{s+k}}{\pi^2\alpha}\frac{g'_k(k_c a)}{g_s(k_c a)}\sum_{\substack{p\in\mathbf{Z}\\ p\neq -m/L\pm k/\alpha}}\frac{J_{m+pL}(k_c a)}{J'_{m+pL}(k_c a)}$$

$$\frac{(p+m/L)^2\sin^2(p+m/L)\pi\alpha}{\left((p+m/L)^2-\frac{s^2}{\alpha^2}\right)\left((p+m/L)^2-\frac{k^2}{\alpha^2}\right)} \quad \text{if } \mathbf{Z}\cap\{-m/L\pm s/\alpha\} \text{ is empty}$$

$$\delta_{sk}\left(1-\frac{\alpha}{2}\frac{g'_s(k_c a)}{g_s(k_c a)}\frac{J_{sL/\alpha}(k_c a)}{J'_{sL/\alpha}(k_c a)}\left(I_{\mathbf{Z}}(-m/L+s/\alpha)+I_{\mathbf{Z}}(-m/L-s/\alpha)\right)\right)$$

$$-\frac{2(-1)^{s+k}}{\pi^2\alpha}\frac{g'_k(k_c a)}{g_s(k_c a)}\sum_{\substack{p\in\mathbf{Z}\\ p\neq -m/L\pm s/\alpha\\ p\neq -m/L\pm k/\alpha}}$$

$$\frac{J_{m+pL}(k_c a)(p+m/L)\sin^2(p+m/L)\pi\alpha}{J'_{m+pL}(k_c a)\left((p+m/L)^2-\frac{s^2}{\alpha^2}\right)\left((p+m/L)^2-\frac{k^2}{\alpha^2}\right)}$$

$$\text{if } \mathbf{Z}\cap\{-m/L\pm s/\alpha\} \text{ is nonempty} \tag{11b}$$

we may rewrite (10) as

$$\sum_{k=0}^{\infty} b^{TE}_{msk} B_{mk} = 0 \quad \text{for} \quad s\in\mathbf{N_0} \tag{12}$$

which is an infinite system of linear homogeneous algebraic equations for determining the expansion coefficients B_{ms}, $s\in\mathbf{N_0}$. In (11) and in the following, the symbol δ_{pq}, p, $q\in\mathbf{Z}$, stands for the Kronecker delta defined by

$$\delta_{pq} = \begin{cases} 1 & \text{if} \quad p=q \\ 0 & \text{if} \quad p\neq q \end{cases}$$

Rewriting the two-sided series appearing in the expressions (11) for b^{TE}_{msk} s, $k\in\mathbf{N_0}$, as a one-sided series, it may be observed that the pth term of the resulting one-sided series is of order p^{-3} as $p\to\infty$. This asymptotic decay rate is sufficient to guarantee the rapid convergence of the series representing b_{msk}^{TE}, $\forall s,\ k \in \mathbf{N_0}$. For a nontrivial solution of (12) for the expansion coefficients B_{ms}, $s \in \mathbf{N_0}$, it is necessary that the determinant of the infinite order coefficient matrix $\mathbf{A}_m^{TE} \triangleq \left[b_{msk}^{TE}\right]_{s,k\in N_0}$ is zero, that is

$$\left|\mathbf{A}_m^{TE}\right| = 0 \tag{13}$$

The solvability condition (13) gives, in principle, the eigenvalue equation for determining the normalized cut-off wave numbers $k_{ca,mn}^{(h)}$, $n \in \mathbf{N}$, of the mth ($m = 0, 1, 2, \ldots, \lfloor L/2 \rfloor$) TE mode supported by the vane-loaded circular cylindrical waveguide where $k_{ca} \triangleq k_c a$. The dispersion curves of the TE mode with azimuthal mode number m are then given by the family of rectangular hyperbolas $k_{oa}^2 - \beta_a^2 = (k_{ca,mn}^{(h)})^2, n \in \mathbf{N}$, in the $\beta_a - k_{oa}$ plane where $k_{0a} \triangleq k_0 a$ and $\beta_a \triangleq \beta a$. For a specified root $k_{ca,mn_0}^{(h)}$ of the eigenvalue Equation (13), the expansion coefficients $B_{ms}^{(n_0)}$, $s \in \mathbf{N_0}$, can be determined in terms of any one of them, say $B_{mo}^{(n_0)}$ from (12), and the expansion coefficients $A_{mp}^{(n_0)}$, $p \in \mathbf{Z}$, are then given in terms of $B_{ms}^{(n_0)}$, $s \in \mathbf{N_0}$, by the relations (6).

3.2. Case 2: TM Modes

The two tangential field components $e_{mz}(r,\theta)$ and $h_{m\theta}(r,\theta)$, $m = 0, 1, 2, \ldots, \lfloor L/2 \rfloor$ for region 1 appearing in the boundary conditions at $r = a (0 \le r < a)$ admit the infinite series representation

$$e_{mz}(r,\theta) = \sum_{p\in\mathbf{Z}} A_{mp} J_{m+pL}(k_c r) e^{-j(m+pL)\theta} \tag{14a}$$

$$\begin{aligned} h_{m\theta}(r,\theta) &= -\left(j\omega\varepsilon/k_c^2\right)\partial e_{mz}/\partial r \\ &= -(j\omega\varepsilon/k_c)\sum_{p\in\mathbf{Z}} A_{mp} J'_{m+pL}(k_c r) e^{-j(m+pL)\theta} \end{aligned} \tag{14b}$$

where $k_c^2 = k_0^2 - \beta^2 > 0$ and the (complex) expansion coefficients A_{mp}, $p \in \mathbf{Z}$, are arbitrary at this stage. The expressions for $e_{mz}(r,\theta)$ and $h_{m\theta}(r,\theta)$ appropriate to region 2 ($a < r < b$) are

$$\begin{aligned} e_{mz}(r,\theta) = &\sum_{\ell=0}^{L-1} e^{-j2\pi m\ell/L} I_{(-\pi\alpha/L,\pi\alpha/L)}(\theta - 2\pi\ell/L) \\ &\sum_{s=1}^{\infty} B_{ms} h_s(k_c r) \sin\left(sL(\theta - 2\pi\ell/L)/\alpha\right) \end{aligned} \tag{15a}$$

$$h_{m\theta}(r,\theta) = (-j\omega\varepsilon/k_c)\sum_{\ell=0}^{L-1} e^{-j2\pi m\ell/L} I_{(-\pi\alpha/L,\pi\alpha/L)}(\theta - 2\pi\ell/L)$$

$$\sum_{s=1}^{\infty} B_{ms} h_s'(k_c r)\sin\left(sL(\theta - 2\pi\ell/L)/\alpha\right) \quad (15b)$$

where B_{ms}, $s \in \mathbf{N}$, are arbitrary expansion coefficients and

$$h_s(k_c r) = Y_{(sL/\alpha)}(k_c b)J_{(sL/\alpha)}(k_c r) - J_{(sL/\alpha)}(k_c b)Y_{(sL/\alpha)}(k_c r), \quad a<r<b$$

It may be observed from (14) and (15) that the boundary conditions on the bottom and the side walls of the slots are automatically satisfied by the form of solution assumed for the field components in region 2, viz., (i) $e_{mz}(r,\theta)$ vanishes at $r = b$ for $-\pi\alpha/L < \theta - 2\pi\ell/L < \pi\alpha/L$, $\ell = 0, 1, \ldots, L-1$ and (ii) $e_{mr}(r,\theta) = -(j\beta/k_c^2)\partial e_{mz}/\partial r$ vanishes at $\theta = \pi(2\ell \pm \alpha)/L$, $\ell = 0, 1, \ldots, L-1$ for $a < r < b$. Thus we need only to ensure the continuity of $e_{mz}(r,\theta)$ across the interface $r = a$ between regions 1 and 2, and the continuity of $h_{m\theta}(r,\theta)$ across the slot openings located at $r = a$, for $-\pi\alpha/L < \theta - 2\pi\ell/L < \pi\alpha/L$, $\ell = 0, 1, \ldots, L-1$. The continuity of $e_{mz}(r, \theta)$ across $r = a$ requires

$$\sum_{q\in\mathbf{Z}} A_{mq} J_{qL}(k_c a)e^{-jqL\theta} = \sum_{\ell=0}^{L-1} e^{jm(\theta-2\pi\ell/L)} I_{(-\pi\alpha/L,\pi\alpha/L)}(\theta - 2\pi\ell/L)$$

$$\sum_{s=1}^{\infty} B_{ms} h_s(k_c a)\sin\left(sL(\theta - 2\pi\ell/L)/\alpha\right) \quad (16)$$

As for the TE modes, it is sufficient to enforce the continuity of $e_{mz}(r, \theta)$ over the interval $(-\pi/L, \pi/L)$ in view of the $2\pi/L$ periodicity of both sides of (16). Thus (16) is equivalent to

$$\sum_{q\in\mathbf{Z}} A_{mq} J_{m+qL}(k_c a)e^{-jqL\theta} = e^{jm\theta} I_{(-\pi\alpha/L,\pi\alpha/L)}(\theta)$$

$$\sum_{s=1}^{\infty} B_{ms} h_s(k_c a)\sin(sL\theta/\alpha) \text{ for } -\pi/L \le \theta < \pi/L \quad (17)$$

The infinite series on the left side of (17) is nothing but the complex exponential Fourier series of the function defined by the right side over the interval $(-\pi/L, \pi/L)$. Multiplying both sides of (17) by $e^{jpL\theta}$ and integrating with respect to θ from $-\pi/L$ to π/L, we have for the Fourier coefficients

$$A_{mp} J_{m+pL}(k_c a) = (j/\pi\alpha)\sin(p + m/L)\pi\alpha \sum_{s=1}^{\infty} (-1)^s s h_s(k_c a)$$

$$B_{ms}/\big((p+m/L)^2 - s^2/\alpha^2\big) \qquad \text{if } \alpha\,|p+m/L| \notin \mathbf{N}$$

$$(j/\pi\alpha)\sin(p+m/L)\pi\alpha \sum_{\substack{s=1 \\ s\neq\alpha|p+m/L|}}^{\infty} (-1)^s s h_s(k_c a) B_{ms}/\big((p+m/L)^2 - s^2/\alpha^2\big)$$

$$+j(\alpha/2)h_{\alpha|p+m/L|}(k_c a)(\delta_{p(-m/L+s/\alpha)} - \delta_{p(-m/L-s/\alpha)})B_{m(\alpha|p+m/L|)}$$

$$\text{if } \alpha|p+m/L| \in \mathbf{N} \quad (18)$$

The continuity of $h_{m\theta}(r,\,\theta)$ across the slot openings on the interface $r = a$ may be expressed as

$$e^{-jm(\theta-2\pi\ell/L)} \sum_{p\in\mathbf{Z}} A_{mp} J'_{m+pL}(k_c a) e^{-jpL\theta}$$

$$= I_{(-\pi\alpha/L,\pi\alpha/L)}(\theta - 2\pi\ell/L) \sum_{k=1}^{\infty} B_{mk} h'_k(k_c a) \sin(k\ell(\theta - 2\pi\ell/L)/\alpha) \quad (19)$$

Once again the continuity of $h_{m\theta}(r,\theta)$ over the first slot opening located at $r = a$ for $-\pi\alpha/L < \theta < \pi\alpha/L$ need only be enforced in view of the $2\pi/L$ periodicity of both sides of (19). Thus (19) is equivalent to

$$\sum_{p\in\mathbf{Z}} A_{mp} J'_{m+pL}(k_c a) e^{-j(m+pL)\theta} = \sum_{k=1}^{\infty} B_{mk} h'_k(k_c a) \sin(qL\theta/\alpha)$$

$$\text{for } -\pi\alpha/L \le \theta < \pi\alpha/L \quad (20)$$

The right side of (20) may be considered to be the Fourier sine-series representation of the function on the left side restricted to the interval $(-\pi\alpha/L, \pi\alpha/L)$. Multiplying both sides of (20) by $\sin(sL\theta/\alpha)$ and integrating with respect to θ from $-\pi\alpha/L$ to $\pi\alpha/L$, we have

$$h'_s(k_c a) B_{ms} = \big(2j(-1)^s s/\pi\alpha^2\big) \sum_{p\in\mathbf{Z}} A_{mp} J'_{m+pL}(k_c a) \frac{\sin((p+m/L)\pi\alpha)}{(s^2/\alpha^2 - (p+m/L)^2)}$$

$$\text{if } \mathbf{Z} \cap \{-m/L \pm s/\alpha\} \text{ is empty}$$

$$\big(2j(-1)^s s/\pi\alpha^2\big) \sum_{\substack{p\in\mathbf{Z} \\ p\neq -m/L\pm s/\alpha}} A_{mp} J'_{m+pL}(k_c a) \frac{\sin((p+m/L)\pi\alpha)}{(s^2/\alpha^2 - (p+m/L)^2)}$$

$$-jI_{\mathbf{Z}}(-m/L + s/\alpha) J'_{sL/\alpha}(k_c a) A_{m(-m/L+s/\alpha)}$$

$$+jI_{\mathbf{Z}}(-m/L - s/\alpha) J'_{-sL/\alpha}(k_c a) A_{m(-m/L-s/\alpha)}$$

$$\text{if } \mathbf{Z} \cap \{-m/L \pm s/\alpha\} \text{ is nonempty} \quad (21)$$

Substituting for A_{mp}, $p \in \mathbf{Z}$, from (18) in to (21) we have

$$B_{ms} = \frac{2(-1)^s s}{\pi^2\alpha^3} \sum_{k=1}^{\infty} (-1)^k \frac{k h_k(k_c a)}{h'_s(k_c a)}$$

$$\left(\sum_{\substack{p\in\mathbf{Z} \\ p\neq -\frac{m}{L}\pm\frac{k}{\alpha}}} \frac{J'_{m+pL}(k_c a)}{J_{m+pL}(k_c a)} \frac{\sin^2(p+m/L)\pi\alpha}{\left((p+m/L)^2 - \frac{s^2}{\alpha^2}\right)\left((p+m/L)^2 - \frac{k^2}{\alpha^2}\right)} \right) B_{mk}$$

$$\text{if } \mathbf{Z} \cap \{-m/L \pm s/\alpha\} \text{ is empty}$$

$$\frac{2(-1)^s s}{\pi^2\alpha^3} \sum_{k=1}^{\infty} (-1)^k \frac{k h_k(k_c a)}{h'_s(k_c a)} \left(\sum_{\substack{p\in\mathbf{Z} \\ p\neq -\frac{m}{L}\pm\frac{s}{\alpha} \\ p\neq -\frac{m}{L}\pm\frac{k}{\alpha}}} \frac{J'_{m+pL}(k_c a)}{J_{m+pL}(k_c a)} \right.$$

$$\left. \frac{\sin^2(p+m/L)\pi\alpha}{\left((p+m/L)^2 - \frac{s^2}{\alpha^2}\right)\left((p+m/L)^2 - \frac{k^2}{\alpha^2}\right)} \right) B_{mk}$$

$$+\frac{\alpha}{2}\left(\frac{h_s(k_c a) J'_{sL/\alpha}(k_c a)}{h'_s(k_c a) J_{sL/\alpha}(k_c a)} (I_{\mathbf{Z}}(-m/L+s/\alpha) + I_{\mathbf{Z}}(-m/L-s/\alpha))\right) B_{ms}$$

$$\text{if } \mathbf{Z} \cap \{-m/L \pm s/\alpha\} \text{ is nonempty} \quad (22)$$

Defining for s, $k \in \mathbf{N}$

$$b^{TM}_{msk} \triangleq \delta_{sk} - \frac{2(-1)^s sk}{\pi^2\alpha^3} \frac{h_k(k_c a)}{h'_s(k_c a)} \left(\sum_{\substack{p\in\mathbf{Z} \\ p\neq -\frac{m}{L}\pm\frac{k}{\alpha}}} \frac{J'_{m+pL}(k_c a)}{J_{m+pL}(k_c a)} \right.$$

$$\left. \frac{\sin^2(p+m/L)\pi\alpha}{((p+m/L)^2 - s^2/\alpha^2)((p+m/L)^2 - k^2/\alpha^2)} \right)$$

$$\text{if } \mathbf{Z} \cap \{-m/L \pm s/\alpha\} \text{ is empty}$$

$$\delta_{sk}\left(1-\frac{\alpha}{2}\frac{h_s(k_ca)}{h_s'(k_ca)}\frac{J'_{sL/\alpha}(k_ca)}{J_{sL/\alpha}(k_ca)}\left(I_{\mathbf{Z}}(-m/L+s/\alpha)+I_{\mathbf{Z}}(-m/L-s/\alpha)\right)\right)$$

$$-\frac{2(-1)^s sk}{\pi^2\alpha^3}\frac{h_k(k_ca)}{h_s'(k_ca)}\left(\sum_{\substack{p\in\mathbf{Z}\\ p\neq -m/L\pm s/\alpha\\ p\neq -m/L\pm k/\alpha}}\frac{J'_{m+pL}(k_ca)}{J_{m+pL}(k_ca)}\right.$$

$$\left.\frac{\sin^2(p+m/L)\pi\alpha}{\left((p+m/L)^2-s^2/\alpha^2\right)\left((p+m/L)^2-k^2/\alpha^2\right)}\right)$$

$$\text{if } \mathbf{Z}\cap\{-m/L\pm s/\alpha\} \text{ is nonempty} \qquad (23)$$

we may recast (22) as an infinite system of homogeneous linear algebraic equations for B_{ms}, $s\in\mathbf{N}$:

$$\sum_{k=1}^{\infty} b_{msk}^{TM}B_{mk}=0 \quad \text{for} \quad s\in\mathbf{N} \qquad (24)$$

In order that a nontrivial solution for B_{ms}, $s\in\mathbf{N}$, exists, the determinant of the infinite order coefficient matrix $\mathbf{A}_m^{TM}\triangleq[b_{msk}^{TM}]_{s,k\in\mathbf{N}}$ must be zero, that is

$$\left|\mathbf{A}_m^{TM}\right|=0 \qquad (25)$$

The solvability condition (25) delivers, in principle, the eigenvalue equation for determining the normalized cut-off wave numbers $k_{ca,mn}^{(e)}$, $n\in\mathbf{N}$, of the mth ($m=0,1,\ldots,\lfloor L/2\rfloor$) TM mode supported by the vane-loaded circular cylindrical waveguide. The dispersion curves of the TM mode with azimuthal mode number m are then given by the family of rectangular hyperbolas

$$k_{0a}^2-\beta_a^2=(k_{ca,mn}^{(e)})^2, \quad n\in\mathbf{N}$$

in the β_a-k_{oa} plane. For a specified root $k_{ca,mn_0}^{(e)}$ of the eigenvalue Equation (25), the TM mode expansion coefficients $B_{ms}^{(n_0)}$, $s\in\mathbf{N}$, can be determined in terms of any one of them, say $B_{m1}^{(n_0)}$ from (24), and the expansion coefficients $A_{mp}^{(n_0)}$, $p\in\mathbf{Z}$, are then given in terms of $B_{ms}^{(n_0)}$, $s\in\mathbf{N}$, by the relations (18).

4. NUMERICAL SOLUTION OF DISPERSION EQUATIONS

Before attempting to solve the dispersion Equations (13) and (25) numerically to obtain the cut-off wave numbers of the TE_m- and TM_m-modes for specified values of waveguide parameters, α, L and b/a, it is of course necessary to truncate the infinite-order coefficient matrices $\mathbf{A}_m^{TE}$ and $\mathbf{A}_m^{TM}$ to finite-order matrices $\hat{\mathbf{A}}_m^{TE}$ and, $\hat{\mathbf{A}}_m^{TM}$, say of orders $(N+1)\times(N+1)$ and $N\times N$ respectively, and estimate the entries of the truncated matrices by symmetrically truncating the rapidly converging infinite series for them from the $-M$th term to the Mth term for a sufficiently large value of M.

In this paper, numerical computation of the cut-off wave numbers is carried out only for the axisymmetric TE_0- and TM_0-modes (i.e., corresponding $m = 0$) as only these modes appear to be ideally suited for interaction with an axisymmetric electron beam employed in gyro-TWT amplifiers. The following values of the waveguide parameters are chosen for the numerical computation: $\alpha = 1/2$, $b/a = 6/5$, $8/5$, $L = 3, 4$. The entries of the truncated coefficient matrices $\hat{\mathbf{A}}_0^{TE}$ and $\hat{\mathbf{A}}_0^{TM}$ may now be read directly from (11) and (23) by setting $m = 0$ and $\alpha = 1/2$:

$$\begin{aligned} b_{0k}^{TE} &= \delta_{0k}\left(1-\frac{g_0'(k_c a)}{2g_0(k_c a)}\sum_{p=-M}^{M}(J_{pL}(k_c a)/J_{pL}'(k_c a))\operatorname{sinc}^2(p/2)\right) \\ &\quad -(1-\delta_{0k})(-1)^k\frac{g_k'(k_c a)}{\pi g_0(k_c a)}\sum_{\substack{p=-M\\ p\neq\pm 2k}}^{M}\left(p^2 J_{pL}(k_c a)\sin\left(p\pi/2\right)\right. \\ &\quad \left.\operatorname{sinc}^2(p/2)/\left(p^2-4k^2\right)J_{pL}'(k_c a)\right) \end{aligned} \tag{26a}$$

and for $s \geq 1$

$$\begin{aligned} b_{sk}^{TE} &= \delta_{sk}(1-g_s'(k_c a)J_{2sL}(k_c a)/2g_s(k_c a)J_{2sL}'(k_c a)) \\ &\quad -4(-1)^{s+k}\left(g_k'(k_c a)/\pi^2 g_s(k_c a)\right) \\ &\quad \sum_{\substack{p=-M\\ p\neq\pm 2s,\pm 2k}}^{M} p^2\frac{J_{pL}(k_c a)}{J_{pL}'(k_c a)}\frac{\sin^2(p\pi/2)}{\left(p^2-4s^2\right)\left(p^2-4k^2\right)} \\ &\qquad \text{for}\quad k=0,1,\ldots,N \end{aligned} \tag{26b}$$

$$b_{sk}^{TM} = \delta_{sk}(1-h_s(k_c a)J_{2sL}'(k_c a)/2h_s'(k_c a)J_{2sL}(k_c a))$$

$$-16(-1)^s sk(h_k(k_ca)/\pi^2 h_s'(k_ca))$$
$$\sum_{\substack{p=-M\\p\neq\pm 2s,\pm 2k}}^{M} \frac{J_{pL}'(k_ca)}{J_{pL}(k_ca)} \frac{\sin^2(p\pi/2)}{\left(p^2-4s^2\right)\left(p^2-4k^2\right)}$$
$$\text{for} \quad k=,1,\ldots,N \tag{27}$$

In (26) and (27) we have omitted the first '0' subscript on the matrix entries for simplicity. All the non-zero entries of the matrices $\hat{\mathbf{A}}_0^{TE}$ and $\hat{\mathbf{A}}_0^{TM}$ are estimated by partially summing the rapidly converging infinite series for them from the −30th term to the 30th term; in other words we are taking M to be 30. A matrix truncation order N as low as 2 for $L=4$ in the case of TE_0 modes and for $L=3$ and 4 in the

Table 1. Normalized cut-off wave numbers of axisymmetric TE_{0n} and TM_{0n} modes for $\alpha = 1/2$.

	$k_{ca,0n}^{(h)}$				
b/a \ n	1	6/5		8/5	
		$L=3$	$L=4$	$L=3$	$L=4$
1	3.831	1.99	1.997	1.502	1.503
2	7.015	2.399	3.425	2.359	2.377
3	10.173	3.392	3.831	2.377	2.39
4	13.323	3.831	4.522	2.399	3.45
5	16.47	6.284	6.046	3.447	3.831
6	19.615	6.312	7.015	3.831	4.193
7	22.76	6.889	7.199	4.141	5.409
8	25.903	7.015	7.566	5.359	6.549
	$k_{ca,0n}^{(e)}$				
b/a \ n	1	6/5		8/5	
		$L=3$	$L=4$	$L=3$	$L=4$
1	2.404	8.144	10.193	6.198	10.026
2	5.52	8.265	10.199	6.209	10.03
3	8.653	9.762	13.25	6.379	12.206
4	11.791	9.795	13.272	6.381	12.208
5	14.93	9.937	16.037	8.4	14.249
6	18.071	10.033	16.128	8.49	14.319
7	21.211	11.65	17.432	9.762	15.885
8	24.352	11.832	17.441	9.793	15.887

case of the TM_0 modes is found to be adequate to stabilize the first 8 roots of the dispersion equations. However, the truncation order has to be increased to 10 in order to stabilize the 8th root for $L = 3$ in the case of TE_0 modes. The estimated values of the TE_0 and TM_0-mode cut-off wave numbers $k_{ca,0n}^{(e)}$ and $k_{ca,0n}^{(h)}$ for $n = 1, 2, \ldots, 8$, are listed in Table 1. The TE_0- and TM_0-mode cut-off wave numbers of a circular cylindrical waveguide of radius $a(b/a = 1)$ are also displayed in Table 1 for comparison.

5. DISCUSSION AND CONCLUSIONS

It may be seen from the form of the field expansions (1) and (2) for the TE modes and (14) and (15) for the TM modes that replacing the azimuthal mode number m by $L-m$ $(m = 0, 1, 2, \ldots, \lceil L/2-1 \rceil)$ has the effect of only switching from an azimuthal variation of the field components according to the factor $e^{-jm\theta}$ to an azimuthal variation according to the factor $e^{jm\theta}$. However, the matrix entries b_{msk}^{TE}, $s,\ k \in \mathbf{N_0}$, and b_{msk}^{TM}, $s,\ k \in \mathbf{N}$, are seen to remain invariant under such a change of sign in the azimuthal mode number m. Thus each of the $\lceil L/2 \rceil - 1$ non-axisymmetric TE_{mn} and TM_{mn} modes for $(m = 1, 2, \ldots, \lceil L/2-1 \rceil)$ may be considered to be two-fold degenerate. Identifying two TE_{mn} or TM_{mn} with the azimuthal mode number m for $(|m| = 1, \ldots, \lceil L/2-1 \rceil)$ differing by a sign, the variation of the TE- and TM-mode field components with respect to the angle variable θ can occur only in $\lfloor L/2 \rfloor + 1$ essentially distinct ways. This behavior is in marked contrast to the behaviour of the TE and the TM modes of a circular cylindrical waveguide for which the azimuthal mode number magnitude $|m|$ can assume any value in $\mathbf{N}_0$. It may also be inferred from the form of the dispersion equations for the TE-and TM-modes of a vane-loaded circular cylindrical waveguide $(b/a > 1)$ that none of the TE_{mn} mode is degenerate with any of the TM_{mn} mode for $m = 0, 1, \ldots, \lfloor L/2 \rfloor$, $n \in \mathbf{N}$ unlike the TE_{0n} and TM_{1n} modes $(n \in \mathbf{N})$ of a circular cylindrical waveguide $(b/a = 1)$. Thus the TE_{mn} and the TM_{mn} modes, $m = 0, 1, \ldots, \lfloor L/2 \rfloor$, $n \in \mathbf{N}$, of a vane-loaded circular cylindrical waveguide $(b/a > 1)$ taken together form a complete system of orthogonal modes in terms of which any 'arbitrary' propagating wave supported by the vane-loaded structure may be expanded [15]. The orthogonality and the completeness properties of these waveguide modes make an analysis of small-signal or large-signal amplification in a gyro-TWT using a modal expansion very attractive. An orthogonal expansion using the TE_{0n} and TM_{0n} modes alone will of course suffice for analyzing the amplification characteristics of an axisymmetric electromagnetic field configuration.

The effect of varying the values of the parameters b/a and L on the TE_{0n} and the TM_{0n} cut-off wave numbers may be seen from Table 1. The cut-off frequency of the TE_{0n} mode is seen to be always smaller than that of TM_{0n} mode, that is, irrespective of the values of parameters b/a and L. No attempt is made here to compare the numerical results for the TE_{0n} modes with those of [1] since the 'method' used in [1] arrive at the k_{ca} values is seriously flawed as explained in the introductory section of this paper. As a matter of fact, estimates of the cut-off wave numbers of the TE_{0n} modes have to be extracted from the 'dispersion curves' as no table of cut-off wave numbers is provided in [1].

We conclude with the remark that the analysis of the *TE*- and the *TM*-modes of a vane-loaded circular cylindrical waveguide presented in this paper parallels the analysis of the solenoidal and the irrotational modes of a cylindrical magnetron cavity presented in [16].

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