Progress In Electromagnetics Research B, Vol. 48, 375–394, 2013

AXIALLY SYMMETRIC TRANSIENT ELECTROMAGNETIC FIELDS IN A RADIALLY INHOMOGENEOUS BICONICAL TRANSMISSION LINE

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Abstract—In the present paper a novel mathematical model of physical processes of transient electromagnetic waves excitation and propagation in a biconical transmission line with radially inhomogeneous magneto-dielectric filling is proposed. The model is based on time domain mode expansions over spherical waves. The basis functions of the mode expansions are calculated analytically. The mode expansion coefficients are governed by Klein-Gordon-Fock equation with coefficients depending on a radial spatial coordinate. The explicit finite difference time domain computational scheme is derived to calculate the mode expansion coefficients. Dependences of cutoff frequencies of higher modes of TE and TM waves on the line geometry and dielectric filling are studied. In order to calculate electromagnetic field in the line with higher accuracy, just finite number of terms in the mode expansions is required. Electromagnetic field excited by the transient electric ring current is calculated in both homogeneous and radially inhomogeneous biconical transmission line. It is shown that there is a possibility to increase the bandwidth of the line via introduction of partial dielectric filling without changing the line geometrical size.

1. INTRODUCTION

Various conical and biconical metallic conducting structures are key bodies for radar applications. A surface of such objects contains

Received 13 January 2013, Accepted 12 February 2013, Scheduled 13 February 2013
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the typical singularity and can be simply described analytically. In fact, many problems of electromagnetic scattering on conical-like bodies have been solved. Some types of antennas based on biconical lines are proposed for transient electromagnetic waves radiating and receiving [1–6]. In order to improve or develop some biconical antenna it is necessary to understand the evolution of electromagnetic fields inside it. The paper [7] is one of the first works where the general treatment of the biconical antenna theory is given. Further, the electromagnetic properties and characteristics of the biconical antenna have been investigated in [5, 6, 8]. In papers [9, 10] a problem of transient TEM wave excitation and diffraction in the biconical antenna has been solved in the time domain using an approach based on mode matching incorporated into the finite difference time stepping scheme for the mode channels. The problems of electromagnetic waves excitation in complex structures of slotted cones have been solved rigorously in [11, 12].

It is known that waveguides with partial dielectric filling have richer electromagnetic properties than hollow ones. One can see that introducing a partial dielectric filling in the biconical line can lead to improving electromagnetic parameters of the line. In the paper [13] the eigenmodes in a layered biconical transmission line are found. The radiation of elementary electric dipole with harmonic time-dependences isolated by a dielectric sphere is calculated with using the critical sections of inhomogeneous horn conception in [14]. On the basis of previous results [13, 14] a new type of compact UWB antennas is designed in [15].

In the studies mentioned above the authors consider the excitation of the fundamental TEM wave which has no dispersion in the biconical line. Nevertheless higher TE and TM modes can be also excited in a biconical transmission line. So, goal of the present paper consists in the study of processes of excitation and propagation of the transient TE, TM and TEM electromagnetic waves in a biconical transmission line with some radial-inhomogeneous magneto-dielectric filling. Generally, there are two suitable approaches in the transient electromagnetic theory: Finite Difference Time Domain method (FDTD) [16] and methods based on Mode Expansions in Time Domain (METD) [17–44]. FDTD is most general and universal approach used in the time domain electromagnetics. It can be applied to solve problems with arbitrary geometry. METD is more special method, which is appropriate to solve problems of electromagnetic waves excitation and propagation in waveguides [17–28], cavities [29–33], free space, some other media and structures [34–39]. Also METD is developed for studying the transient electromagnetic processes in conical transmission lines [40–

44]. In order to use METD for solving electromagnetic pulse diffraction problems on a junction of two regular transmission lines, Mode Matching Time Domain Method was developed [9, 10].

In the present paper a new mathematical model based on METD is proposed to study the processes of transient electromagnetic waves excitation and propagation in an asymmetrical biconical transmission line with radial-inhomogeneous magneto-dielectric filling [42, 43]. In the framework of the proposed model the process of transient TE wave excitation by the ring electric current in both homogeneous and radially inhomogeneous biconical line is analyzed. Also transformations of signal spectrum in the line are considered. It is shown that introduction of a partial dielectric filling in the biconical line increases its excitation efficiency. Some results of the present work were previously reported in [44].

2. PROBLEM STATEMENT

A regular asymmetrical biconical transmission line is considered in Figure 1. Two circular cones with the common vertex and axis form this line. In the spherical coordinate system the surfaces of the upper and lower cones are defined via the equations $\theta = \theta_1$ and $\theta = \theta_2$, respectively. On the surface of the cones the boundary conditions are imposed that correspond to perfect electric conductor (PEC). The space between cones is filled with a radially inhomogeneous magneto-dielectric medium. It means that the relative permittivity and permeability depend on the radial coordinate only $\varepsilon = \varepsilon(r)$, $\mu = \mu(r)$. The constraints for permittivity and permeability are given as: $\varepsilon \geq 1$, $\mu \geq 1$. The electromagnetic field in the line is excited

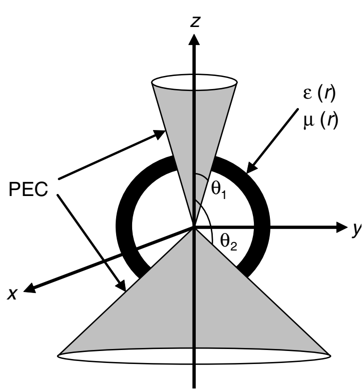

Figure 1. An asymmetrical radially inhomogeneous biconical line.

with certain transient electric $\vec{\mathcal{J}}(\vec{r},t)$ and magnetic $\vec{\hat{\mathcal{J}}}(\vec{r},t)$ currents and electric $\rho(\vec{r},t)$ and magnetic $\hat{\rho}(\vec{r},t)$ charges. The electric $\vec{\mathcal{E}}(\vec{r},t)$ and magnetic $\vec{\mathcal{H}}(\vec{r},t)$ field strengths in the line are to be found.

The electromagnetic fields are governed by Maxwell equations:

$$\nabla\times\vec{\mathcal{H}} = \frac{\partial\vec{\mathcal{D}}}{\partial t} + \vec{\mathcal{J}}, \quad \nabla\times\vec{\mathcal{E}} = -\frac{\partial\vec{\mathcal{B}}}{\partial t} - \vec{\hat{\mathcal{J}}}, \quad \nabla\cdot\vec{\mathcal{D}} = \rho, \quad \nabla\cdot\vec{\mathcal{B}} = \hat{\rho}, \tag{1}$$

with taking into account the constitutive equations:

$$\vec{\mathcal{D}}\left(\vec{r},t\right) = \varepsilon_0\varepsilon\left(r\right)\vec{\mathcal{E}}\left(\vec{r},t\right), \quad \vec{\mathcal{B}}\left(\vec{r},t\right) = \mu_0\mu\left(r\right)\vec{\mathcal{H}}\left(\vec{r},t\right). \tag{2}$$

The boundary conditions on the cone surfaces are:

$$\vec{\varphi}_0\cdot\vec{\mathcal{E}}\Big|_L = 0, \quad \vec{\theta}_0\cdot\vec{\mathcal{H}}\Big|_L = 0, \quad \vec{r}_0\cdot\vec{\mathcal{E}}\Big|_L = 0. \tag{3}$$

Here ε_0 and μ_0 are permittivity and permeability of free space, respectively; $\vec{r}_0$, $\vec{\varphi}_0$ and $\vec{\theta}_0$ are orts of the spherical coordinate system.

3. PROBLEM SOLUTION

In order to find the electromagnetic field in the line Mode Basis Method (O. A. Tretyakov method) for conical transmission lines [42–44] is applied here. In the framework of the method, the electric $\vec{\mathcal{E}}(\vec{r},t)$ and magnetic $\vec{\mathcal{H}}(\vec{r},t)$ field strengths are presented as the field expansions over spherical TE, TM and TEM waves:

$$\begin{aligned}
\vec{\mathcal{E}}\left(\vec{r},t\right) &= \vec{E}\left(\vec{r},t\right) + \vec{r}_0 E_r\left(\vec{r},t\right), \\
\vec{\mathcal{H}}\left(\vec{r},t\right) &= \vec{H}\left(\vec{r},t\right) + \vec{r}_0 H_r\left(\vec{r},t\right), \\
\sqrt{\varepsilon_0}\vec{E}\left(\vec{r},t\right) &= \frac{1}{r}\left(\sum_m e_m^H\left(r,t\right)\vec{E}_m^H\left(\theta,\varphi\right) + \sum_n e_n^E\left(r,t\right)\vec{E}_n^E\left(\theta,\varphi\right)\right. \\
&\quad \left. + \sum_k e_k^T\left(r,t\right)\vec{E}_k^T\left(\theta,\varphi\right)\right), \\
\sqrt{\mu_0}\vec{H}\left(\vec{r},t\right) &= \frac{1}{r}\left(\sum_m h_m^H\left(r,t\right)\vec{H}_m^H\left(\theta,\varphi\right) + \sum_n h_n^E\left(r,t\right)\vec{H}_n^E\left(\theta,\varphi\right)\right. \\
&\quad \left. + \sum_k h_k^T\left(r,t\right)\vec{H}_k^T\left(\theta,\varphi\right)\right), \\
\sqrt{\varepsilon_0}E_r\left(\vec{r},t\right) &= \frac{1}{r^2}\sum_n e_n^r\left(r,t\right)q_n\Phi_n^E\left(\theta,\varphi\right), \\
\sqrt{\mu_0}H_r\left(\vec{r},t\right) &= \frac{1}{r^2}\sum_m h_m^r\left(r,t\right)p_m\Phi_m^H\left(\theta,\varphi\right).
\end{aligned} \tag{4}$$

Vector and scalar functions that depend on angular coordinates θ, φ are the basis functions. Scalar functions that depend on the radial coordinate r and time t are coefficients of the expansions. These coefficients are often called as mode amplitudes. The superscripts indicate a type of wave. The index H shows that the basis functions and mode amplitudes correspond to the transverse electric (TE) waves. Similarly, the index E indicates the transverse magnetic (TM) waves and index T corresponds to the transverse electromagnetic (TEM) waves. The quantities p_m and q_n are spectral parameters of eigenvalue boundary problems [43]. For simplicity we consider symmetrical electromagnetic fields and sources that do not depend on the angular coordinate φ.

TE-waves. The basis functions for TE waves have the form:

$$\begin{aligned}
\Phi_m^H(\theta) &= C_m^H\left[P_{\nu_m}(\cos\theta) + A_m P_{\nu_m}(-\cos\theta)\right],\\
\vec{E}_m^H(\theta) &= -\vec{\varphi}_0 C_m^H \sqrt{\frac{\nu_m+1}{\nu_m}}\frac{1}{\sin\theta}\left[P_{\nu_m+1}(\cos\theta) - \cos\theta P_{\nu_m}(\cos\theta)\right.\\
&\quad \left.-A_m\left(P_{\nu_m+1}(-\cos\theta) + \cos\theta P_{\nu_m}(-\cos\theta)\right)\right],\\
\vec{H}_m^H(\theta) &= \vec{\theta}_0 C_m^H \sqrt{\frac{\nu_m+1}{\nu_m}}\frac{1}{\sin\theta}\left[P_{\nu_m+1}(\cos\theta) - \cos\theta P_{\nu_m}(\cos\theta)\right.\\
&\quad \left.-A_m\left(P_{\nu_m+1}(-\cos\theta) + \cos\theta P_{\nu_m}(-\cos\theta)\right)\right],
\end{aligned} \tag{5}$$

where spectral parameters ν_m are the roots of the equation:

$$\begin{aligned}
&\left.\frac{dP_{\nu_m}(\cos\theta)}{d\theta}\right|_{\theta=\theta_1}\left.\frac{dQ_{\nu_m}(\cos\theta)}{d\theta}\right|_{\theta=\theta_2}\\
&-\left.\frac{dP_{\nu_m}(\cos\theta)}{d\theta}\right|_{\theta=\theta_2}\left.\frac{dQ_{\nu_m}(\cos\theta)}{d\theta}\right|_{\theta=\theta_1} = 0
\end{aligned} \tag{6}$$

and the normalized constants are defined as follow:

$$A_m = -\frac{\left.\frac{dP_{\nu_m}(\cos\theta)}{d\theta}\right|_{\theta=\theta_1}}{\left.\frac{dP_{\nu_m}(-\cos\theta)}{d\theta}\right|_{\theta=\theta_1}},$$

$$C_m^H = \sqrt{\frac{2}{\int\limits_{\theta_1}^{\theta_2}\left[P_{\nu_m}(\cos\theta) + A_m P_{\nu_m}(-\cos\theta)\right]^2\sin\theta d\theta}}.$$

Here $m = 1, 2, 3, \ldots$ are numbers of TE-waves; $P_\nu(\cdot)$ and $Q_\nu(\cdot)$ are Legendre functions of the first and second kinds respectively and $p_m = \sqrt{\nu_m(\nu_m+1)}$. The basis functions satisfy the orthogonality conditions [43]:

$$\frac{1}{2}\int\limits_{\theta_1}^{\theta_2} \Phi_m^H(\theta)\,\Phi_{m'}^H(\theta)\sin\theta d\theta = \delta_{mm'},$$
$$\frac{1}{2}\int\limits_{\theta_1}^{\theta_2} \vec{E}_m^H(\theta)\cdot\vec{E}_{m'}^H(\theta)\sin\theta d\theta = \delta_{mm'}, \tag{7}$$
$$\frac{1}{2}\int\limits_{\theta_1}^{\theta_2} \vec{H}_m^H(\theta)\cdot\vec{H}_{m'}^H(\theta)\sin\theta d\theta = \delta_{mm'}.$$

The mode amplitudes are governed by the following system of evolutionary equations:

$$\begin{aligned} &\varepsilon(r)\mu(r)\frac{\partial^2 e_m^H}{\partial\tau^2} - \frac{\partial^2 e_m^H}{\partial r^2} + \frac{d\mu}{dr}\frac{1}{\mu(r)}\frac{\partial e_m^H}{\partial r} + \frac{p_m^2 e_m^H}{r^2} = J_1^H(r,\tau), \\ &\frac{\partial h_m^H}{\partial\tau} = -\frac{1}{\mu(r)}\frac{\partial e_m^H}{\partial r} + J_2^H(r,\tau), \\ &\frac{\partial h_m^r}{\partial\tau} = -\frac{1}{\mu(r)}e_m^H + J_3^H(r,\tau). \end{aligned} \tag{8}$$

Known sources of the electromagnetic field have form:

$$\begin{aligned} J_1^H(r,\tau) &= -\mu(r)\frac{\partial}{\partial r}\left[\frac{r}{\mu(r)}\frac{\sqrt{\varepsilon_0}}{p_m}\frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_{\theta_1}^{\theta_2}\left(\nabla_\perp\cdot\hat{\vec{J}}\right)\Phi_m^H(\theta)\sin\theta d\theta d\varphi\right] \\ &\quad + \frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_{\theta_1}^{\theta_2}\left\{r\frac{\partial}{\partial\tau}\left(\mu(r)\sqrt{\mu_0}\left[\vec{J}\times\vec{r}_0\right]\right) - \sqrt{\varepsilon_0}\nabla_\perp\hat{J}_r\right\} \\ &\quad \cdot\vec{H}_m^H(\theta)\sin\theta d\theta d\varphi, \\ J_2^H(r,\tau) &= \frac{r}{\mu(r)}\frac{\sqrt{\varepsilon_0}}{p_m}\frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_{\theta_1}^{\theta_2}\left(\nabla_\perp\cdot\hat{\vec{J}}\right)\Phi_m^H(\theta)\sin\theta d\theta d\varphi, \\ J_3^H(r,\tau) &= -\frac{r^2}{\mu(r)}\frac{\sqrt{\varepsilon_0}}{p_m}\frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_{\theta_1}^{\theta_2}\hat{J}_r\Phi_m^H(\theta)\sin\theta d\theta d\varphi. \end{aligned}$$

Here $\tau = ct$, c is the light velocity in vacuum, the dot and the cross indicate the dot product and cross product respectively, $\nabla_\perp =$

$\vec{\theta}_0\partial_\theta + \vec{\varphi}_0(\sin\theta)^{-1}\partial_\varphi$ is a part of Hamilton operator "nabla" in the spherical coordinate system.

TM-waves. The basis functions have the form:

$$\begin{aligned}
\Phi_n^E(\theta) &= C_n^E\left[P_{\chi_n}(\cos\theta) + B_n P_{\chi_n}(-\cos\theta)\right], \\
\vec{E}_n^E(\theta) &= \vec{\theta}_0 C_n^E\sqrt{\frac{\chi_n+1}{\chi_n}}\frac{1}{\sin\theta}\left[P_{\chi_n+1}(\cos\theta) - \cos\theta P_{\chi_n}(\cos\theta)\right. \\
&\qquad \left.- B_n\left(P_{\chi_n+1}(-\cos\theta) + \cos\theta P_{\chi_n}(-\cos\theta)\right)\right], \\
\vec{H}_n^E(\theta) &= \vec{\varphi}_0 C_n^E\sqrt{\frac{\chi_n+1}{\chi_n}}\frac{1}{\sin\theta}\left[P_{\chi_n+1}(\cos\theta) - \cos\theta P_{\chi_n}(\cos\theta)\right. \\
&\qquad \left.- B_n\left(P_{\chi_n+1}(-\cos\theta) + \cos\theta P_{\chi_n}(-\cos\theta)\right)\right],
\end{aligned} \tag{9}$$

where the spectral parameters χ_n are the roots of the equation:

$$P_{\chi_n}(\cos\theta_1)\,Q_{\chi_n}(\cos\theta_2) - P_{\chi_n}(\cos\theta_2)\,Q_{\chi_n}(\cos\theta_1) = 0 \tag{10}$$

and the normalized constants are defined as follow:

$$\begin{aligned}
B_n &= -\frac{P_{\chi_n}(\cos\theta_1)}{P_{\chi_n}(-\cos\theta_1)}, \\
C_n^E &= \sqrt{\frac{2}{\int_{\theta_1}^{\theta_2}\left[P_{\chi_n}(\cos\theta) + B_n P_{\chi_n}(-\cos\theta)\right]^2\sin\theta d\theta}}.
\end{aligned}$$

Here $n = 1, 2, 3, \ldots$ are numbers of TM-waves and $q_n = \sqrt{\chi_n(\chi_n+1)}$. The basis functions satisfy the orthogonality conditions:

$$\begin{aligned}
\frac{1}{2}\int_{\theta_1}^{\theta_2}\Phi_n^E(\theta)\,\Phi_{n'}^E(\theta)\sin\theta d\theta &= \delta_{nn'}, \\
\frac{1}{2}\int_{\theta_1}^{\theta_2}\vec{E}_n^E(\theta)\cdot\vec{E}_{n'}^E(\theta)\sin\theta d\theta &= \delta_{nn'}, \\
\frac{1}{2}\int_{\theta_1}^{\theta_2}\vec{H}_n^E(\theta)\cdot\vec{H}_{n'}^E(\theta)\sin\theta d\theta &= \delta_{nn'}.
\end{aligned} \tag{11}$$

The mode amplitudes are governed by the following system of evolutionary equations:

$$\begin{aligned}
&\varepsilon(r)\mu(r)\frac{\partial^2 h_n^E}{\partial\tau^2} - \frac{\partial^2 h_n^E}{\partial r^2} + \frac{d\varepsilon}{dr}\frac{1}{\varepsilon(r)}\frac{\partial h_n^E}{\partial r} + \frac{q_n^2 h_n^E}{r^2} = J_1^E(r,\tau), \\
&\frac{\partial e_n^E}{\partial\tau} = -\frac{1}{\varepsilon(r)}\frac{\partial h_n^E}{\partial r} + J_2^E(r,\tau),
\end{aligned}$$

$$\frac{\partial e_n^r}{\partial \tau} = -\frac{1}{\varepsilon(r)} h_n^E + J_3^E(r,\tau). \qquad (12)$$

Known sources of the TM electromagnetic waves have form:

$$\begin{aligned} J_1^E(r,\tau) &= -\varepsilon(r)\frac{\partial}{\partial r}\left[\frac{r}{\varepsilon(r)}\frac{\sqrt{\mu_0}}{q_n}\frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_{\theta_1}^{\theta_2}\left(\nabla_\perp\cdot\vec{J}\right)\Phi_n^E(\theta)\sin\theta\, d\theta d\varphi\right] \\ &\quad +\frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_{\theta_1}^{\theta_1}\left\{r\frac{\partial}{\partial\tau}\left(\varepsilon(r)\sqrt{\varepsilon_0}\left[\vec{r}_0\times\hat{\vec{J}}\right]\right)-\sqrt{\mu_0}\nabla_\perp J_r\right\} \\ &\quad \cdot\vec{E}_n^E(\theta)\sin\theta d\theta d\varphi, \\ J_2^E(r,\tau) &= \frac{r}{\varepsilon(r)}\frac{\sqrt{\mu_0}}{q_n}\frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_{\theta_1}^{\theta_2}\left(\nabla_\perp\cdot\vec{J}\right)\Phi_n^E(\theta)\sin\theta d\theta d\varphi, \\ J_3^E(r,\tau) &= -\frac{r^2}{\varepsilon(r)}\frac{\sqrt{\mu_0}}{q_n}\frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_{\theta_1}^{\theta_2} J_r\Phi_n^E(\theta)\sin\theta d\theta d\varphi. \end{aligned}$$

<u>TEM-waves.</u> The basis functions have form:

$$\vec{E}^T(\theta) = \sqrt{\frac{2}{\ln\left(\frac{1-\cos\theta_2}{1-\cos\theta_1}\frac{\sin\theta_1}{\sin\theta_2}\right)}}\frac{\vec{\theta}_0}{\sin\theta},\ \vec{H}^T(\theta) = \sqrt{\frac{2}{\ln\left(\frac{1-\cos\theta_2}{1-\cos\theta_1}\frac{\sin\theta_1}{\sin\theta_2}\right)}}\frac{\vec{\varphi}_0}{\sin\theta}. \qquad (13)$$

The basis functions satisfy the orthogonality conditions:

$$\frac{1}{2}\int\limits_{\theta_1}^{\theta_2}\vec{E}^T(\theta)\cdot\vec{E}^T(\theta)\sin\theta d\theta = 1,\ \frac{1}{2}\int\limits_{\theta_1}^{\theta_2}\vec{H}^T(\theta)\cdot\vec{H}^T(\theta)\sin\theta d\theta = 1. \qquad (14)$$

The mode amplitudes are governed by the following system of evolutionary equations:

$$\begin{aligned} &\varepsilon(r)\mu(r)\frac{\partial^2 e_k^T}{\partial\tau^2}-\frac{\partial^2 e_k^T}{\partial r^2}+\frac{d\mu}{dr}\frac{1}{\mu(r)}\frac{\partial e_k^T}{\partial r} = \frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_{\theta_1}^{\theta_2}\left\{\frac{\vec{f}_1(r,\theta,\varphi,t)}{r^2}\right. \\ &\left.-\frac{\vec{f}_2(r,\theta,\varphi,t)}{r^2\varepsilon(r)}-\mu(r)\frac{\partial}{\partial r}\left(\frac{1}{r^2\varepsilon(r)\mu(r)}\right)\int\limits_0^r\vec{f}_2(r',\theta,\varphi,t)\,dr'\right\} \\ &\cdot\vec{H}_k^T\sin(\theta)\,d\theta d\varphi, \end{aligned}$$

$$\frac{\partial h_k^T}{\partial \tau}=-\frac{1}{\mu(r)}\frac{\partial e_k^T}{\partial r}+\frac{1}{r^2\varepsilon(r)\mu(r)}\frac{1}{4\pi}\int_0^{2\pi}\int_{\theta_1}^{\theta_2}\left\{\int_0^r \vec{f}_2\left(r',\theta,\varphi,t\right)dr'\right\} \tag{15}$$

$$\cdot\,\vec{H}_k^T\sin\left(\theta\right)d\theta d\varphi.$$

where $\vec{f}_1(r,\theta,\varphi,t)=r^3\partial_\tau(\mu_{\|}[\mu_0^{1/2}\vec{J}\times\vec{r}_0])-r^2\nabla_\perp\varepsilon_0^{1/2}\hat{J}_r$, $\vec{f}_2(r,\theta,\varphi,t)=r^2[\vec{r}_0\times\nabla_\perp\varepsilon_0^{-1/2}\rho]-\partial_r(r^3\varepsilon_{\|}\varepsilon_0^{1/2}\hat{\vec{J}})$.

4. NUMERICAL RESULTS

In the current section the proposed mathematical model is applied for studying a process of transient TE waves excitation in the biconical line with a radial-inhomogeneous dielectric filling. In order to excite the electromagnetic waves in the line a transient ring electric current is used. The current has the form:

$$\vec{J}\left(\vec{r},t\right)=\vec{\varphi}_0\delta\left(r-R\right)\delta\left(\theta-\frac{\pi}{2}\right)f\left(t\right). \tag{16}$$

Here $\delta(\cdot)$ is the Dirac delta function, $f(t)$ an arbitrary time-depending function, and R the ring radius. From right hand sides of Equations (8), (12) and (15) one can see that the current (16) really excites TE waves only. In subsequent calculations the time-depending function in a form of Laguerre pulse is chosen:

$$f\left(t\right)=\left(\frac{t}{T}\right)^2\left(1-\frac{t}{3T}\right)\exp\left(-\frac{t}{T}\right)\eta\left(t\right). \tag{17}$$

where $\eta(t)$ is the Heaviside step function, T is a scaling parameter. The form of the Laguerre pulse for $T=33.36$ ps is presented in Figure 2. Also the spectrum of this signal is shown in Figure 3.

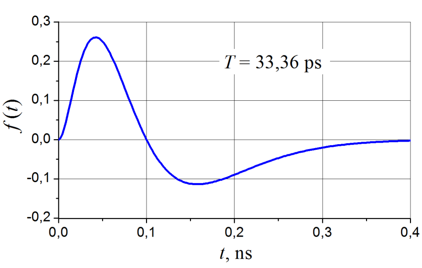

Figure 2. The Laguerre pulse.

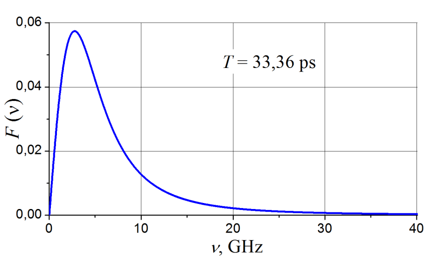

Figure 3. The spectrum of Laguerre pulse.

The authors of work [6] recommend using the biconical antenna with flare angle of 60° as a reference antenna for ultrawideband 2–12 GHz applications. The antenna length should be approximately equal to one wavelength of the central frequency. For this reason the parameters of the line and the excitation current are chosen as follow: $\theta_1 = 60°$, $\theta_2 = 120°$, $T = 33.36$ ps and $R = 0.5$ cm.

The discussion about convergence of expansions over spherical waves (4) is given in the paper [45] in detail. It was found that spherical expansions have a fast convergence and number of main terms in the series (4) is defined by upper frequency in the signal spectrum and the geometrical sizes of electromagnetic field sources.

In the framework of the proposed model, the electromagnetic field in the biconical line is calculated (see Equations (4)–(5)) via the following relations:

$$\vec{E}\left(r,\theta,t\right) = \frac{-\vec{\varphi}_0}{\sqrt{\varepsilon_0} r \sin\theta} \sum_{m=1}^{\infty} e_m^H\left(r,t\right) C_m^H \sqrt{\frac{\nu_m+1}{\nu_m}} \left[P_{\nu_m+1}\left(\cos\theta\right) \right.$$
$$\left. -\cos\theta P_{\nu_m}\left(\cos\theta\right) - A_m\left(P_{\nu_m+1}\left(-\cos\theta\right) + \cos\theta P_{\nu_m}\left(-\cos\theta\right)\right)\right], \quad (18)$$

$$\vec{H}\left(r,\theta,t\right) = \frac{\vec{\theta}_0}{\sqrt{\mu_0} r \sin\theta} \sum_{m=1}^{\infty} h_m^H\left(r,t\right) C_m^H \sqrt{\frac{\nu_m+1}{\nu_m}} \left[P_{\nu_m+1}\left(\cos\theta\right) \right.$$
$$\left. -\cos\theta P_{\nu_m}\left(\cos\theta\right) - A_m\left(P_{\nu_m+1}\left(-\cos\theta\right) + \cos\theta P_{\nu_m}\left(-\cos\theta\right)\right)\right], \quad (19)$$

$$H_r\left(r,\theta,t\right) = \frac{1}{\sqrt{\mu_0} r^2} \sum_{m=1}^{\infty} h_m^r\left(r,t\right) C_m^H \sqrt{\nu_m\left(\nu_m+1\right)} \left[P_{\nu_m}\left(\cos\theta\right) \right.$$
$$\left. + A_m P_{\nu_m}\left(-\cos\theta\right)\right]. \quad (20)$$

The mode amplitudes $e_m^H(r,t)$, $h_m^H(r,t)$ and $h_m^r(r,t)$ satisfy to Equation (8). Equation (8) is written in the convenient form to derive an explicit finite-difference scheme. A uniform grid $u(r,t) \to u|_i^j := u(i\Delta r, j\Delta t)$ are used. Here Δr and Δt are steps in radial spatial coordinate and time, respectively. The derivatives are calculated approximately via central finite differences:

$$\begin{aligned} \frac{\partial\, u|_i^j}{\partial r} &\approx \frac{u|_{i+1}^j - u|_{i-1}^j}{2\Delta r}, \qquad \frac{\partial^2\, u|_i^j}{\partial r^2} \approx \frac{u|_{i+1}^j - 2\, u|_i^j + u|_{i-1}^j}{\Delta r^2}, \\ \frac{\partial\, u|_i^j}{\partial t} &\approx \frac{u|_i^{j+1} - u|_i^{j-1}}{2\Delta t}, \qquad \frac{\partial^2\, u|_i^j}{\partial t^2} \approx \frac{u|_i^{j+1} - 2\, u|_i^j + u|_i^{j-1}}{\Delta t^2}. \end{aligned} \quad (21)$$

Taking into account the current (16), Equation (21) and condition $\mu(r) = 1$, we arrive to the following an explicit finite-difference scheme for system of differential Equations (8):

$$\begin{aligned} e_m^H\big|_i^{j+1} &= \alpha_i^m\, e_m^H\big|_i^j + \beta_i\left(e_m^H\big|_{i+1}^j + e_m^H\big|_{i-1}^j\right) \\ &\quad - e_m^H\big|_i^{j-1} + F_m\left(i\Delta r, j\Delta\tau\right), \\ h_m^H\big|_i^{j+1} &= h_m^H\big|_i^{j-1} - \frac{c\Delta t}{\Delta r}\left(e_m^H\big|_{i+1}^j - e_m^H\big|_{i-1}^j\right), \\ h_m^r\big|_i^{j+1} &= h_m^r\big|_i^{j-1} - 2c\Delta t\, e_m^H\big|_i^j. \end{aligned} \tag{22}$$

where $F_m(r,t) = \frac{c\sqrt{\mu_0}\Delta t^2}{2\varepsilon(r)} r\delta(r-R)\frac{df(t)}{dt} C_m^H \sqrt{\frac{\nu_m+1}{\nu_m}} P_{\nu_m+1}(0)(1-A_m)$, $\alpha_i^m = 2 - \frac{c^2\Delta t^2}{\Delta r^2\, \varepsilon|_i}\left(2 + \frac{\nu_m(\nu_m+1)}{i^2}\right)$, $\beta_i = \frac{c^2\Delta t^2}{\Delta r^2\, \varepsilon|_i}$.

In order to verify the developed computational scheme we solve a simple problem about radiation of the ring current (16) in a vacuum. In this case $\varepsilon(r) = 1$, $\nu_m = m$, $C_m^H = \sqrt{2m+1}$, $A_m = 0$ and from (18) the solution for electric field has form:

$$\vec{E}\left(r,\theta,t\right) = \frac{\vec{\varphi}_0}{\sqrt{\varepsilon_0} r \sin\left(\theta\right)} \sum_{m=1}^{\infty} \sqrt{\frac{m\left(m+1\right)}{2m+1}} e_m^H\left(r,t\right)\left[P_{m-1}\left(\cos\left(\theta\right)\right)\right. \\ \left. - P_{m+1}\left(\cos\left(\theta\right)\right)\right]. \tag{23}$$

The mode amplitudes $e_m^H(r,t)$ are calculated numerically using the finite-difference scheme (22). This problem also has the analytical solution:

$$\vec{E}\left(r,\theta,t\right) = \vec{\varphi}_0 \frac{\mu_0 R^2}{4\pi} \int_0^{2\pi} \frac{f'\left(t - \frac{\sqrt{r^2+R^2+2rR\sin(\theta)\cos(\xi)}}{c}\right)}{\sqrt{r^2+R^2+2rR\sin\left(\theta\right)\cos\left(\xi\right)}} \cos(\xi) d\xi, \tag{24}$$

where the prime in Equation (24) means differentiation on the argument.

In Figure 4 the time-dependences of electric field magnitude in the free space are presented. They are calculated at two different space points by means of both mode basis method (23) and Equation (24). In Figure 4 as well as in the figures below an axis of abscissa indicates a real time multiplied by the light velocity in vacuum. In order to calculate the time-dependences, the first ten nonzero terms are taken into account in series (23) only. One can see that keeping the first ten spherical waves provides a high accuracy in the calculations of electromagnetic fields in the free space (bi-conical line is absent). As mentioned in [44, 45], mode expansions (18)–(20) of the field in the biconical line converge if they converge for same sources of the field in the free space. Thus the first ten nonzero spherical waves are more

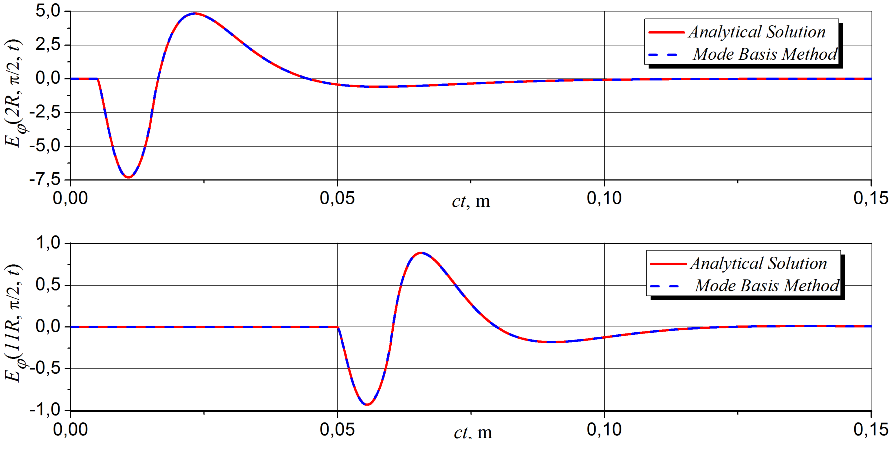

Figure 4. Time-dependences of electric field magnitude in vacuum.

Table 1. Spectral parameter.

m	ν_m	m	ν_m
1	2.627061	11	32.51252
3	8.545464	13	38.510596
5	14.527461	15	44.509185
7	20.519652	17	50.508105
9	26.515296	19	56.507252

than enough to calculate electromagnetic fields in the biconical line with high accuracy and we are assured that the computational scheme on the basis of (18)–(20), (22) is correct.

In order to use the solution for electromagnetic field in the biconical line (18)–(20) it is necessary to calculate spectral parameter of the line ν_m for desired angles via Equation (6). For example, in Table 1 the spectral parameter ν_m for the first ten odd spherical waves (the even spherical waves are not excited by the current (16)) is presented for biconical line with angles $\theta_1 = 60^\circ$ and $\theta_2 = 120^\circ$.

In Figure 5 the time-dependences of electric field magnitude in the hollow biconical line with angles $\theta_1 = 60^\circ$ and $\theta_2 = 120^\circ$ are presented at different distances from the current (16). The dependences are calculated directly at $\theta = 90^\circ$. Figure 5 shows the pulse waveform transformation with increasing of the radial spatial coordinate. The waveform transformation appears due to two factors. The first one is a spherical attenuation of the wave as $1/r$. The second one consists in the existence of TE wave's dispersion. The spherical attenuation follows from the energy conservation law and exists at any space point. But action of the dispersion attenuation appears near the sources of waves. In the upper panel of Figure 5 one can see that wave attenuation occurs faster than $1/r$, as well as the qualitative transformation of the pulse waveform takes place. As can be seen in the lower panel of Figure 5 after distance $r \approx 4R$ the pulse waveform is transformed

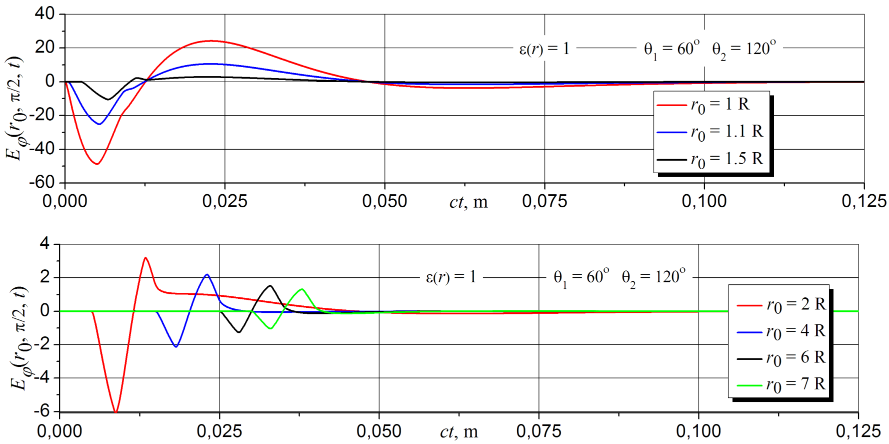

Figure 5. Time-dependences of electric field magnitude in biconical line without filling.

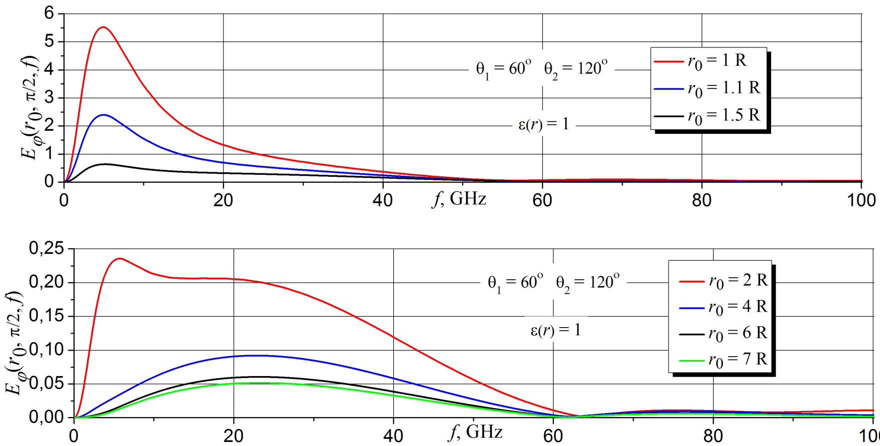

Figure 6. Transformation of the electric field spectrum in the biconical line without filling.

due to spherical attenuation only. It means that in space $r > 4R$ all spherical modes excited by current (16) become propagating waves. The transformation of the pulse waveform leads to transformation of pulse spectrum also. In Figure 6 the spectra of the electric field magnitudes in the biconical line are shown. The calculated spectra correspond to TE pulse waves which are presented in Figure 5. As can be seen in Figure 6 the transformation of the spectrum also confirms the previous result where essential influence of dispersion TE waves appears only in the bounded space domain near the field source. As seen in the lower panel of Figure 6 the spectrum magnitude near frequency 60 GHz goes to zero. It means that directivity pattern of the biconical line have zero at direct $\theta = 90°$ at this frequency. Zeros of the directivity pattern depend on the angles θ_1, θ_2 and the line filling.

It is obviously that the line filling with a dielectric leads to increasing electrical sizes without changing the geometrical ones. Thus the dielectric introducing in the line is a way to improve its radiation characteristics. Below we consider the biconical line with radial-inhomogeneous dielectric filling. The permittivity has a piecewise constant profile:

$$\varepsilon(r) = \begin{cases} 3, & r \leq 6R \\ 1, & r > 6R \end{cases} . \qquad (25)$$

The time-dependence of the electric field in the radial-inhomogeneous biconical line with the dielectric filling (25) and same parameters of the line and excited current as in previous calculations is shown in Figure 7. The field is calculated at $r = 11R$ and $\theta = 90°$. The radiated field consists of series of separated decreasing pulses. They appear due to reflections at the dielectric interface and in the vicinity of vertex of the cones.

The first radiated pulse from dielectric ball is shown in the upper panel of Figure 8 in the details. In the lower panel of Figure 8 its

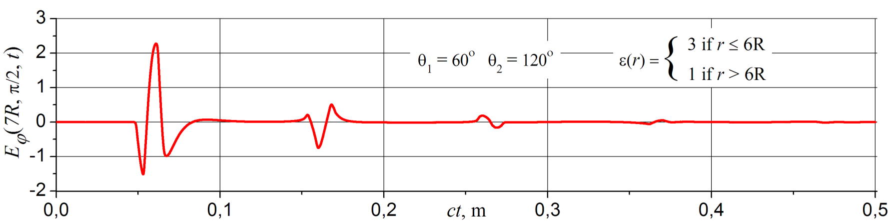

Figure 7. Time-dependence of electric field magnitude in the radial-inhomogeneous biconical line.

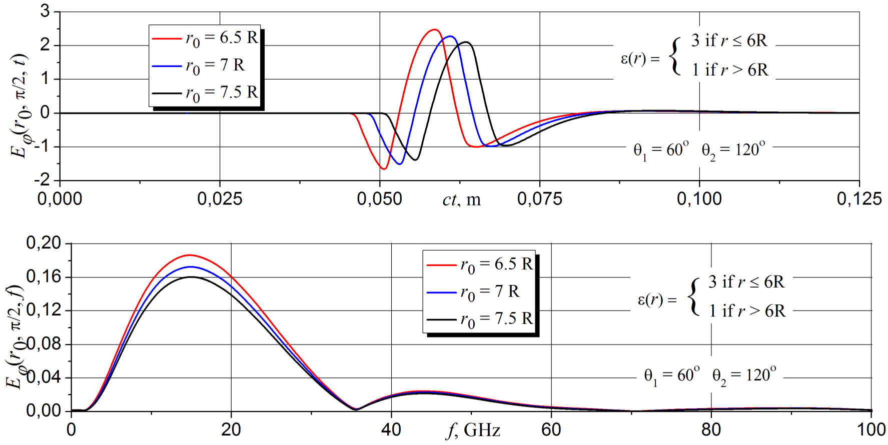

Figure 8. The first radiated pulse from dielectric ball in biconical line and its spectrum.

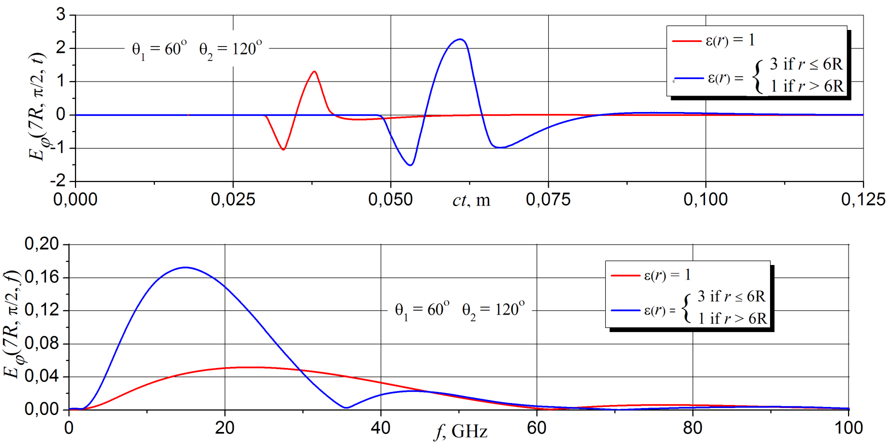

Figure 9. Comparison of radiated pulses in biconical line with different dielectric filling and its spectra.

spectrum is presented. As can be seen in Figure 8 the first pulse has only uniform spherical attenuation as $1/r$.

In the upper panel of Figure 9 the radiated pulse in the biconical line without filling $\varepsilon(r) = 1$ is compared with the first radiated pulse in the line with radial-inhomogeneous filling having form (25). The time-dependences of electric field magnitudes are calculated at same space point ($r = 7R$, $\theta = 90°$). In the lower panel of Figure 9 the spectra of such pulses are shown. The first radiated pulse (some part of total radiated field) from dielectric ball has higher amplitude than that radiated one in the hollow biconical line. It is due to in the dielectric ball excited pulse has amplitude up to $\sqrt{\varepsilon} = \sqrt{3}$ times higher than in the vacuum. Also the first pulse has expected time delay $5R(\sqrt{\varepsilon} - 1) = 0.018\,\text{m}$. In Figure 9 the curves show a possibility to increase a radiation efficiency of the biconical line by means of partial dielectric filling.

5. CONCLUSIONS

In the present paper a new mathematical model of processes of excitation and propagation of axially symmetrical transient electromagnetic fields in the biconical line with radial-inhomogeneous magneto-dielectric filling is proposed. On the basis of the model an efficient computational scheme is obtained. The validity of the scheme is verified for the limit case of free space. In the framework of the scheme the transient TE electromagnetic wave is calculated both in the hollow biconical transmission line and in the line with piecewise constant dielectric filling along radial direction. A possibility to increase the radiation efficiency of compact electromagnetic field sources in the biconical line by means of partial dielectric filling is shown. The developed computational scheme can be used to find an optimal magneto-dielectric filling for asymmetrical biconical lines and antennas based on such lines.

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