Progress In Electromagnetics Research M, Vol. 29, 79–93, 2013

DETERMINING THE EFFECTIVE ELECTROMAGNETIC PARAMETERS OF BIANISOTROPIC METAMATERIALS WITH PERIODIC STRUCTURES

Lei Chen*, **Zhenya Lei, Rui Yang, Xiaowei Shi, and Jiawei Zhang**

National Key Laboratory of Antennas and Microwave Technology, Xidian University, Xi'an 710071, P. R. China

Abstract—A straightforward approach is proposed to retrieve the effective electromagnetic parameters of a slab of bianisotropic material from the scattering parameters. We first obtain the values of the impedance and refractive index of a slab of metamaterial, followed by the deduction of the expressions for determining these electromagnetic parameters including permittivity, permeability and magnetoelectric coupling coefficient. Then, comparisons between the results coming respectively from retrieval technique and analytical method are made. Finally, we demonstrate the properties of split-ring resonator materials in other two orientations with respect to the incident plane wave and apply the proposed method to anisotropic materials to reveal its generality.

1. INTRODUCTION

Metamaterials or artificial materials have received considerable attention over the past several years due to their attractive properties. The materials consisting of split-ring resonantors (SRRs) and wires can present simultaneously negative permeability and permittivity, which leads to the negative refractive index within a certain range of frequency [1]. Since artificial materials present bulk properties when the spacing of the components is much smaller than the wavelength, hence the determination of the effective electromagnetic parameters becomes necessary.

Received 2 January 2013, Accepted 4 February 2013, Scheduled 7 February 2013
* Corresponding author: Lei Chen (1225chenlei@163.com).

The description of metamaterials using effective parameters provides a convenient means to understand the behavior of the artificial materials without considering the local field distribution. These parameters can be calculated by numerical calculation of the electromagnetic fields [2] or equivalent circuit models [3], but these two methods may show deficiency for practical applications. A well-known approach that can be applied to both numerical simulations and experiments is the retrieval technique derived from the scattering (S) parameters. The retrieval technique, first proposed by Nicolson and Ross [4], can be applied to both traditional materials and the artificial materials. In recent years, a variety of retrieval techniques have been proposed for the purpose of obtaining the effective parameters associated with a metamaterial. Smith et al. first applies the technique to materials composed of SRRs and finite rods [5] and then discusses the retrieval of inhomogeneous materials [6]. Some improved retrieval schemes based on the initial approach are also presented in recent years [7–11]. The parameters of a material can also be obtained by experiments based on the measurement of the S parameters using waveguide method [12]. However, most techniques reported can only be applied to isotropic or anisotropic materials.

It is known that artificial materials are intrinsically anisotropic because of the orientations of the rings and rods in space and some may be bianisotropic due to their special structures, such as the SRR, which may exhibit bianisotropy in some cases. There are also some special designs that make use of the bianisotropy [13]. When calculating the effective parameters of bianisotropic metamaterials, we have to take the magnetoelectric coupling into consideration, which plays an important role in the propagation of electromagnetic waves. Only then, the true effective electromagnetic parameters can be achieved.

Actually, there have been several published papers about the calculation of bianisotropic metamaterials. Marques et al. provides an equivalent circuit model for a SRR structure with the magnetoelectric coupling considered in the analysis of quasi-static fields [14]. Chen et al. presents a retrieval technique from S parameters in three incident directions and an optimization approach for the determination of a bianisotropic material [15, 16]. Then, Li et al. develops a relatively simple retrieval method from the transmission and reflection coefficients [17]. Some other investigations of the bianisotropic materials also appear in recent years [18–21]. In this paper, we propose a straightforward approach to retrieve the effective electromagnetic parameters of a bianisotropic material from S parameters using the transfer-matrix method. In the determination of the permittivity ε_z, permeability μ_y and magnetoelectric coupling coefficient ξ_0 of the

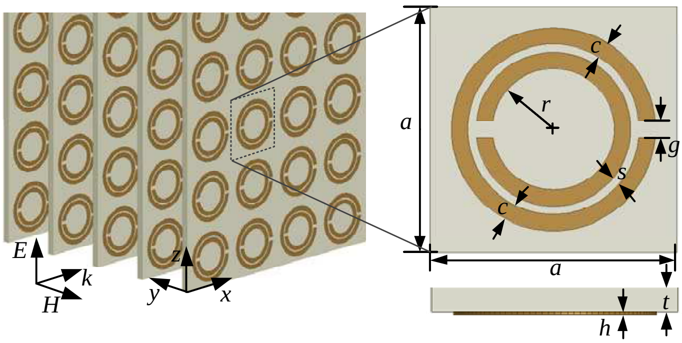


Figure 1. The SRRs printed on the substrates with the metallization in yellow. A plane wave is incident in the x direction polarized in the z direction. For numerical calculation, we choose the dimensions as $a = 3$ mm, $r = 0.74$ mm, $c = 0.2$ mm, $s = 0.1$ mm, $g = 0.2$ mm, $t = 0.3$ mm, $h = 35\,\mu$m and assume the permittivity of the substrate is ε_0 for convenience.

medium shown in Fig. 1, only one direction of the incident plane wave is considered and values can be readily determined knowing the impedance and refractive index of a material. Although the proposed technique is applicable to any structures with bianisotropy, we derive it from the SRR composed of two rings each interrupted with a small gap as shown in Fig. 1.

2. RETRIEVAL FOR THE BIANISOTROPIC METAMATERIALS

It has been pointed out that the SRR presents bianisotropy in the case shown in Fig. 1. The electric fields in the z direction will induce magnetic dipoles and the magnetic fields in the y direction will induce electric dipoles due to the asymmetry of the structure. It means that electric dipoles cannot only be excited by the electric fields but also by the magnetic fields. Similarly, magnetic dipoles cannot only be excited by the magnetic fields but also by the electric fields. We assuming that the material is reciprocal and that the harmonic time dependence is$e^{-j\omega t}$, thus, the constitutive relationship can be written as

$$\vec{D} = \bar{\bar{\varepsilon}} \cdot \vec{E} + \bar{\bar{\xi}} \cdot \vec{H}, \qquad \vec{B} = \bar{\bar{\mu}} \cdot \vec{H} + \bar{\bar{\zeta}} \cdot \vec{E}, \tag{1}$$

where

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}, \quad \bar{\bar{\mu}} = \mu_0 \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix},$$
$$\bar{\bar{\xi}} = \frac{1}{c} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -i\xi_0 & 0 \end{pmatrix}, \quad \bar{\bar{\zeta}} = \frac{1}{c} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i\xi_0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2}$$

with the ε_0 and μ_0 being the permittivity and permeability of free space respectively, and c being the speed of light in vacuum. When a plane wave polarized in the z direction is incident in the x direction, only two components of electromagnetic fields (E_z and H_y) and three electromagnetic parameters (ε_z, μ_y and ξ_0) are included.

Most of the metamaterials presented over the past several years are formed by periodic structures. It is convenient to extract the parameters from a unit cell of periodic structures through numerical approaches. It has been demonstrated that under the condition of long wavelength the materials with periodic structures can be viewed as a homogeneous medium and their properties can be described by the effective medium parameters [1]. The transfer-matrix method [22] has been used to retrieve the effective electromagnetic parameters of a homogeneous or inhomogeneous material [5, 6]. We here try to present a general approach for the retrieval of the electromagnetic parameters of bianisotropic metamaterials making use of the transfer-matrix method. Fig. 2 shows a slab of bianisotropic material placed in air.

For a material consisting of periodic structures, there exists a phase advance per unit cell, which can always be defined according

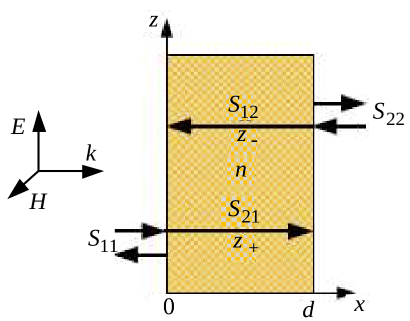


Figure 2. A plane wave incident in the x direction into a slab of bianisotropic material with a thickness of d and the corresponding S parameters of the material are illustrated.

to the periodicity. The properties of a periodic structure can be determined from the transfer-matrix which associated with the fields of a unit cell. We express the fields in the form of a vector $\mathbf{F} = (E, H')^T$ [6], where E and H' are the complex amplitudes of the electric and magnetic fields located on the left-hand and right-hand faces of a slab. The magnetic field is a reduced field with the normalization form $H' = (+i\omega\mu_0)H$. The relations of fields on two sides of a slab can be expressed as

$$\mathbf{F}(x + d) = e^{i\alpha d}\mathbf{F}(x) = \mathbf{TF}(x), \tag{3}$$

where α is the phase advance per unit cell which relates the fields on the two sides of a unit cell. $\mathbf{T}$ is the one dimensional transfer matrix of a slab

$$\mathbf{T} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} \cos(nk_0 d) & -\frac{z}{k_0}\sin(nk_0 d) \\ \frac{z}{k_0}\sin(nk_0 d) & \cos(nk_0 d) \end{pmatrix}, \tag{4}$$

with k_0 being the wave number of light in free space, z being the wave impedance of a slab and n being the refractive index. Eq. (3) allows us to find the dispersion relation by solving

$$T_{11}T_{22} - \chi(T_{11} + T_{22}) + \chi^2 - T_{12}T_{21} = 0, \tag{5}$$

where $\chi = \exp(i\alpha d)$. The elements of the transfer matrix can be expressed in terms of S parameters [22],

$$\begin{aligned} T_{11} &= \frac{(1+S_{11})(1-S_{22})+S_{21}S_{12}}{2S_{21}}, \quad T_{12} = \frac{(1+S_{11})(1+S_{22})-S_{21}S_{12}}{2S_{21}}, \\ T_{21} &= \frac{(1-S_{11})(1-S_{22})-S_{21}S_{12}}{2S_{21}}, \quad T_{22} = \frac{(1-S_{11})(1+S_{22})+S_{21}S_{12}}{2S_{21}}. \end{aligned} \tag{6}$$

Using the condition of $\det(\mathbf{T}) = 1$ in Eq. (5), and substituting Eq. (6) into Eq. (5), we have,

$$\cos(nk_0 d) = \cos(\alpha d) = \frac{1 - S_{11}S_{22} + S_{21}^2}{2S_{21}}. \tag{7}$$

Thus, the refractive index n can be obtained from the implicit expression (7), which has many solutions for n to the different branches of the inverse cosine. To choose the correct one, we deal with it as the method described in Ref. [7], that we will not detail here. For a passive medium, the solved n needs to satisfy the condition $n'' \geq 0$, where the $(\cdot)''$ represents the imaginary part operator.

For a homogeneous or symmetric material, the value of z will be unique, while it will result in two different values for an inhomogeneous or asymmetric material. The impedance z is an intrinsic parameter which relates the electric field to the magnetic field as $z = E/H'$.

From Eq. (3), two equivalent expressions for the impedance of a slab can be obtained by

$$z = \frac{T_{12}}{\chi - T_{11}} = \frac{\chi - T_{22}}{T_{21}}. \tag{8}$$

Solving Eq. (5) and then substituting the χ into the Eq. (8), we can find two roots for the impedance of a metamaterial,

$$z_{\pm} = \frac{(T_{11} - T_{22}) \pm \sqrt{(T_{22} - T_{11})^2 + 4T_{12}T_{21}}}{2T_{21}}. \tag{9}$$

The two roots of Eq. (9) correspond to the two propagation directions of a plane wave. Substitution of Eq. (6) into Eq. (8) leads to

$$z_{\pm} = \frac{(S_{11} - S_{22}) \pm \sqrt{(1 - S_{11}S_{22} + S_{12}S_{21})^2 - 4S_{12}S_{21}}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}. \tag{10}$$

Up to now, we have obtained the characteristic impedance which associates with a metamaterial with bianisotropic structures in terms of S parameters. What we should pay attention to is that for a passive medium the impedance needs to satisfy the conditions $z^{+\prime} \geq 0$ and $(-z^{-})' \geq 0$, where the $(\cdot)'$ denotes the real part operator. For a unit cell of SRR shown in Fig. 1, it can be seen from Fig. 3 that S_{12} is equal to S_{21} and the magnitudes of S_{11} and S_{22} are almost the same, while the phase of S_{11} is different with that of S_{22}. In this case, the application of the standard retrieval method [5] will lead to an inaccurate result.

Assuming that there is a plane wave propagating in the direction of $+x$ polarized in the $+z$ direction. The constitutive relationship

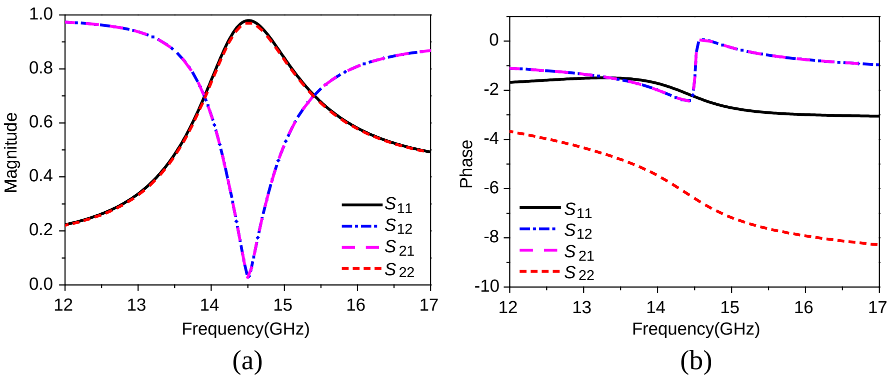


Figure 3. (a) Magnitude and (b) phase of the simulated S parameters for a unit cell of the SRR-based material in Fig. 1.

between electric field and magnetic field can be described as

$$\begin{aligned} D_z &= \varepsilon_0 \varepsilon_z E_z - ic^{-1} \xi_0 H_y \\ B_y &= \mu_0 \mu_y H_y + ic^{-1} \xi_0 E_z \end{aligned}, \tag{11}$$

With the substitution of Eq. (11) into Maxwell's equations, we can derive the wave equation

$$\nabla^2 E_z + \omega^2 \varepsilon_0 \mu_0 (\varepsilon_z \mu_y - \xi_0^2) E_z = 0. \tag{12}$$

From the Eq. (12), the refractive index can be found,

$$n^2 = \varepsilon_z \mu_y - \xi_0^2. \tag{13}$$

For a passive material, the roots have to be chosen properly to guarantee the condition $\mathrm{Im}(n) \geq 0$. Otherwise, the exponentially growing solutions will occur, which violates the energy conservation.

The impedances of a bianisotropic material for the wave propagation in the right-hand or left-hand directions are $Z_\pm$, the values of which are given by the equations $Z_+ = E^+/H^+$ and $Z_- = E^-/H^-$. From Maxwell's equations, the formulas of the impedances can be obtained,

$$z_\pm = \frac{Z_\pm}{Z_0} = \frac{\mu_y}{\pm n - i\xi_0}, \tag{14}$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the vacuum impedance. We note from Eq. (14) that the impedance of the right-hand direction is not equal to that of the left-hand direction.

Using the Eqs. (12) and (13) we can obtain the expressions for ε_z, μ_y and ξ_0 as follows,

$$\varepsilon_z = \frac{n + i\xi_0}{z_+}, \tag{15}$$

$$\mu_y = (n - i\xi_0) z_+, \text{ and} \tag{16}$$

$$\xi_0 = in \frac{z_- + z_+}{z_- - z_+}. \tag{17}$$

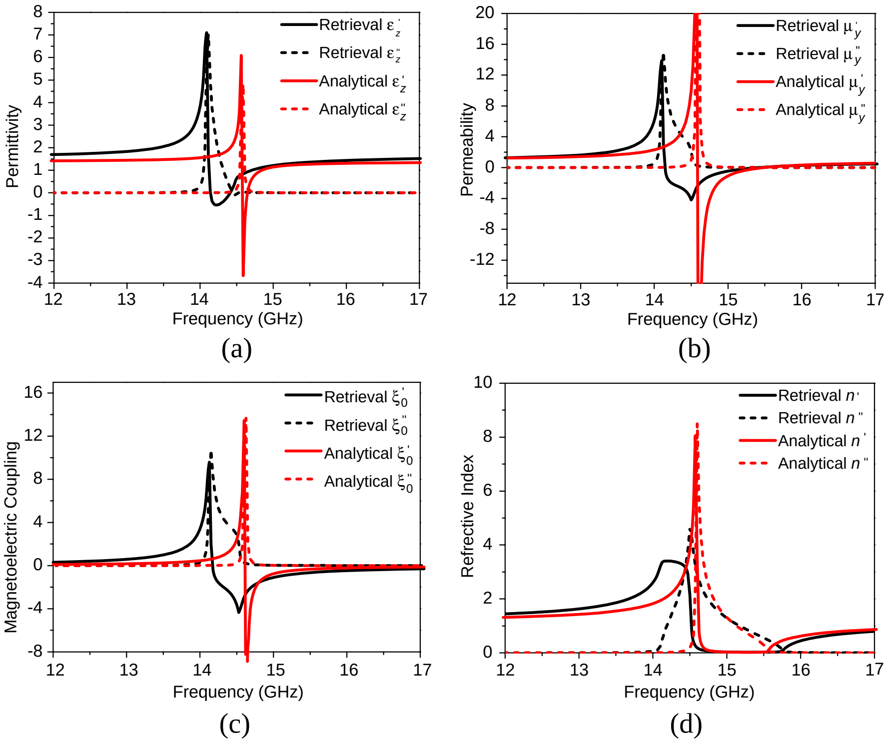
The refractive index of and impedances for a bianisotropic medium can be obtained from Eqs. (7) and (10). So far, the expressions for the effective electromagnetic parameters (ε_z, μ_y and ξ_0) that associate with the bianisotropy have been obtained. We find that the effective parameters can be readily determined from Eqs. (15)–(17) knowing the refractive index and the impedance of a material. The calculation results using the proposed retrieval technique are shown in Fig. 4.

3. COMPARISONS BETWEEN THE RESULTS RESPECTIVELY FROM RETRIEVAL TECHNIQUE AND ANALYTICAL METHOD

In this section, the theoretical formulations of the SRR will be given, which are deduced from equivalent circuit models by Ricardo Marques [14, 23]. The analytical model gives insight into the relationship between the physical properties and geometrical parameters of an SRR-based metamaterial shown in Fig. 1. The frequency of resonance and the polarizabilities are calculated by the analytical method, and electromagnetic parameters of bulk SRR media can be derived from the poparizabilities caused by the exerted fields [14]. Thus, we have

$$\varepsilon_z = 1 + \frac{1}{\varepsilon_0 a^3}\left[\varepsilon_0 \frac{16}{3} r_{ext}^3 + 4d_{eff}^2 r_0^2 C_{pul}^2 L \left(\frac{\omega_0^2}{\omega}\right)\left(\frac{\omega^2}{\omega_0^2 - \omega^2 - j\omega\gamma}\right)\right], \quad (18)$$

$$\mu_y = 1 + \frac{\mu_0}{a^3}\left[\frac{\pi^2 r_0}{L}\left(\frac{\omega^2}{\omega_0^2 - \omega^2 - j\omega\gamma}\right)\right], \text{ and} \quad (19)$$



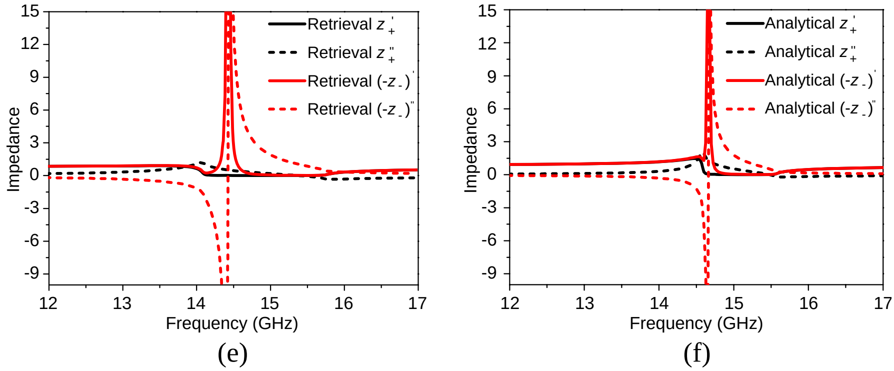


Figure 4. Comparisons of the retrieval and analytical results from the equivalent circuit model for SRR structure shown in Fig. 1. The electromagnetic parameters: permittivity, permeability and magnetoelectric coupling coefficient are presented respectively in (a), (b) and (c) with black lines denoting the retrieval results and red lines the analytical results. (d) The refractive index as well as (e) the retrieval and (f) the analytical impedances from two directions are also provided.

$$\xi_0 = -j\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{a^3}\left[-2jr_0^3 d_{eff} C_{pul}\frac{\omega_0^2}{\omega}\left(\frac{\omega^2}{\omega_0^2-\omega^2-j\omega\gamma}\right)\right], \tag{20}$$

where $d_{eff} = c+s$, $\gamma = R/L$, $r_0 = r+c+s/2$, and $r_{ext} = r+2c+s$. L and C_{pul} are the total inductance of the SRR and the p.u.l. capacitance between the rings respectively. Here, ohmic losses involving heat loss and radiation loss have been taken into consideration, which play an important role in the characterization of metamaterials. From the equivalent circuit model of the SRR, we find that the SRR behaves as a resonant LC circuit, and that the frequency of resonance is given by

$$\omega_0^2 = \frac{1}{\left(\frac{1}{2}\pi r_0 C_{pul} + C_{gap}\right) L}, \tag{21}$$

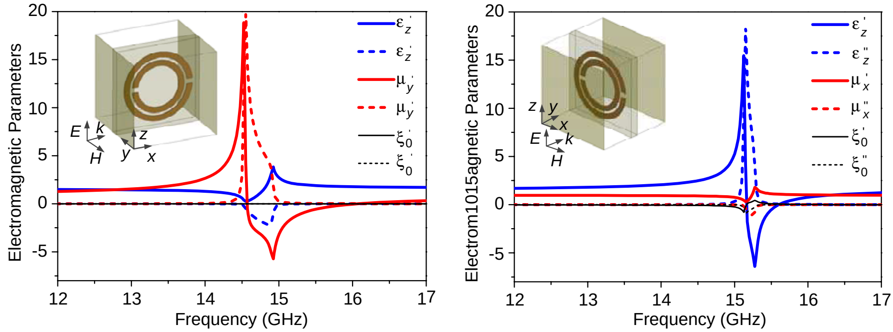
where the values of C_{pul} and L can be calculated from the approximate analytical equations [23]. C_{gap} is the capacitance of the gap, which does not be considered in the analysis of the circuit model proposed by Ricardo Marques. We find that it will result in a great offset of resonant frequency without consideration of the capacitance of the gaps. If the gap is narrow, its capacitance can be written as $C_{gap} = \varepsilon_0 ch/g + C_0$, with $C_0 = \varepsilon_0(h+c+g)$ [24]. According to the formulations given in Ref. [23], we have $L = 3.9287 \times 10^{-9}\,\mathrm{H}$, $C_{pul} = 1.6806 \times 10^{-11}\,\mathrm{F}$, $C_{gap} = 4.1557 \times 10^{-15}\,\mathrm{F}$ and $R = 0.4901\,\Omega$ at the frequency of resonance.

The curves of the electromagnetic parameters ε_z, μ_y and ξ_0 from both retrieval technique and analytical method are similar, which can be seen from Figs. 4(a)–4(c). Especially, the magnetoelectric coupling is such strong that it cannot be neglected. Fig. 4(d) gives the refractive index through analytical and retrieval method. The frequency regimes of different models can be estimated from the curves obtained from retrieval technique in Fig. 4(d). Below the frequency of 14.1 GHz and above 15.7 GHz, the wave is propagating. From 14.1 to 14.5 GHz, the wave is in the resonant crystal band-gap. From 14.5 to 15.7 GHz, modes are evanescent. The resonant frequency achieved from the retrieval technique is 14.5 GHz, while that occurs at 14.6 GHz using the analytical method. The frequency 14.1 GHz acts as the interface of propagation modes and resonant crystal modes. From Figs. 4(e) and 4(f), we found two different impedances for two opposite propagation directions, which obtained respectively from the retrieval and analytical results. The poor accuracy of the analytical method for parameter calculation and the deficiency for complex models limits its applications. On the comparison of the values of the electromagnetic parameters achieved from retrieval technique and analytical method, we see that they do not quite agree with each other in the range of resonant frequency. This is because the fact that the metamaterial particles are not infinitely small compared to the wavelength, which results in the effect of space dispersion. Detailed descriptions of the space dispersion for isotropic materials have been presented in Refs. [25, 26].

4. RETRIEVAL OF THE METAMATERIALS COMPOSED OF DIFFERENT SRR STRUCTURES

It has been pointed that the matematerials based on SRR structures present bianisotropy or anisotropy, which is related to the incidence of the electromagnetic wave [16]. We can see from the Eqs. (1) and (2) that only the $\vec{y}$ component of $\vec{H}$ can contribute to $\vec{D}$, and similarly only the $\vec{z}$ component of $\vec{E}$ can contribute to $\vec{B}$. Hence, negative permeability appears if the magnetic field is perpendicular to the plane of the SRR, and similarly the negative permittivity appears when the electric field is parallel to the plane of the SRR. It means that only three of all six orientations [16], can we see the resonance and that in the other cases the SRR presents isotropic properties. Of the three cases, one orientation of SRR with strong magnetoelectric coupling has been demonstrated in the previous section, while the properties of material in other two cases are shown in Figs. 5(a) and 5(b).

Observing Fig. 5(a), we know that the edge-coupled SRR (EC-



(a) (b)

Figure 5. Values of the electromagnetic parameters of the EC-SRR and its two orientations with respect to the incident plane wave are shown in (a) and (b).

SRR) presents primarily a magnetic response and a small inverse electric response, and that the magnetoelectric coupling coefficient is equal to zero, which can be explained by the symmetry of the SRR in the direction of the wave propagation. Yet, a strong electric response appears in the Fig. 5(b) with weak inverse magnetic response and small electromagnetic coefficient, which can be explained by the fact that there being an electric filed parallel to the plane of the SRR, but no magnetic field perpendicular to it. In rough calculation, the magnetoelectric coupling of the SRR in Fig. 5(b) can be neglected and it behaves as an anisotropic media.

As a general method for the retrieval of bianisotropic media, we expect it can also be applied to anisotropic media. Since the existence of bianisotropy for a medium is due to the asymmetry of the unit structure, symmetric units can be constructed to avoid the bianisotropy. The nonbianisotropic SRR (BN-SRR) was initially proposed in Ref. [27] in order to avoid the bianisotropy of EC-SRR while keeping a coplanar design. It can be seen that the NB-SRR has an inverse symmetry with regard to its center. The broadside-coupled SRR (BC-SRR) proposed in Ref. [14] has the same inverse symmetry as the NB-SRR, with the advantage of potentially much smaller electric size because of the capacitance of the broadside-coupled strips [28]. The NB-SRR and BC-SRR have the same dimensions as the EC-SRR and the permittivity of the substrate is assumed to be ε_0. For these two structures, we expect the bianisotropic terms tend to zero using the proposed retrieval method.

From the Figs. 6(a) and 6(b), it can be seen that the retrieved

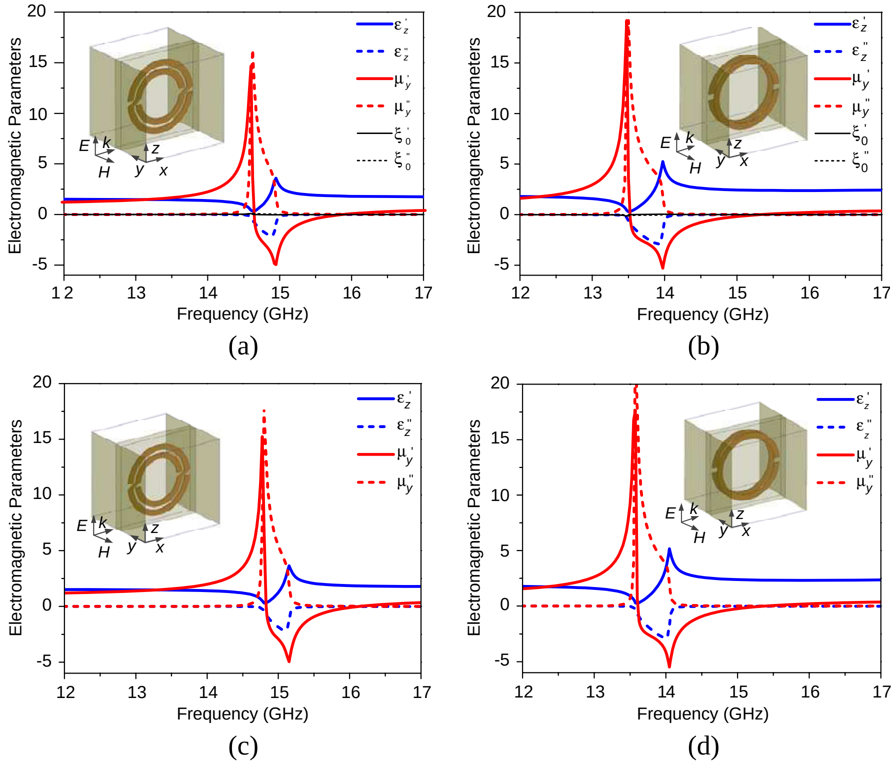


Figure 6. Retrieval results of the NB-SRR and BC-SRR using the proposed technique are shown in (a) and (b). As a comparison, the results obtained from the standard retrieval method [5] are given in (c) and (d).

ξ_0 is close to zero in the whole frequency range. This means these two structures do not present bianisotropy. Moreover, the retrieved results using the proposed method are identical with that from the standard retrieval method [5] which are presented in Figs. 6(c) and 6(d). The successful retrieval of the electromagnetic parameters reveals that although the proposed retrieval method is deduced for the determination of bianisotropic media, it can also be applied to the anisotropic media.

5. CONCLUSIONS

In this paper, the expressions for calculating the effective electromagnetic parameters of bianisotropic materials are deduced in terms of the S parameters followed by the comparisons between the retrieval technique and analytical method. Then, the properties of the SRR in different incidence of an electromagnetic wave are demonstrated. Finally, the application of the retrieval technique to anisotropic materials is verified to be valid. It indicates that the retrieval method is a general method which can be used to retrieve both bianisotropic and anisotropic materials.

ACKNOWLEDGMENT

This work is supported by the Fundamental Research Funds for the Central Universities (K5051202039) from China, and Innovation Funds for Excellent Returned Overseas Chinese Talents from Xidian University.

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