

## AN IMPEDANCE-PERMEABILITY SELF-RESONANCE OF INDUCTANCE COIL WITH METAMATERIALS

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**Abstract**—An impedance-permeability ( $Z$ - $\mu_r$ ) resonance phenomenon is firstly found and numerically demonstrated when electromagnetic metamaterials with negative permeability are firstly introduced into inductance coil. Numerical results reveal that the impedance-permeability relationship exhibits an extraordinary self-resonant phenomenon at a certain negative value of relative permeability, which is related to the dimensions of the core but nearly independent of the coil size. Such a mechanism is proposed to increase the sensitivity of eddy current (EC) sensors up to about 270 times, offering a new method to greatly improve the sensitivity of EC sensors and the spatial resolution with micrometer scale.

### 1. INTRODUCTION

In semiconductor industry, it is an important task to accurately detect the thickness distribution of metallic layers of  $\sim 1\ \mu\text{m}$  thickness on wafer. Different methods have been discussed and the eddy current (EC) method has been proved to be one of the most proper methods due to the limitations of technologies and working conditions [1]. The EC testing method, which is a non-destructive and non-contact detection method for microscopy [2] and for measuring displacement, film thickness and defects in metals, can achieve a high resolution and a wide measurement range [3–10]. In recent years, EC technology has been developed and a measurement range of  $0.1$ – $1\ \mu\text{m}$  can be achieved by a kind of high-frequency technique [1]. Hence, the problem can be solved adequately by these developments. However, this technique works in microwave range which is complex and not easy

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for practical instrumentation. Besides, as the semiconductor process develops, higher accuracy is needed and nanometer-scale thickness measurement becomes an urgent requirement [11]. However, the increase of the resolution of EC sensors runs into a bottle neck by using the common design and common materials. Fortunately, electromagnetic metamaterials [12–18], attracting much attention recently, offer a possibility to arbitrarily control the electromagnetic parameters and electromagnetic wave propagation [19, 20], such as the negative permittivity, negative permeability, negative refractive index [21–23], and invisible cloaking [24–27], etc.. And these metamaterials have many potential applications [28–30]. In the design of eddy current sensors, the magnetic properties of sensor materials play a very important role. Utilization of advantages of the extraordinary properties of these new materials has theoretical and practical significance for improving the performance of EC sensors. In this study, electromagnetic metamaterials are firstly introduced into the design of EC sensors as the core of the coil and numerical results reveal that the impedance-permeability relationship exhibits an extraordinary self-resonant phenomenon. Then, the feasibility to increase the sensitivity and spatial resolution by the mechanism is numerically demonstrated.

## 2. NUMERICAL MODEL

To analyze the influence of the cores with different permeability on the coil impedance, an EC coil model with a cylinder core is calculated with the finite element method (FEM) software COMSOL Multiphysics. The simulation model is axisymmetric (as shown in Fig. 1) and basic parameters of the model are as follows.

Here  $N$  is the number of turns of the coil,  $R_i$  the inner radius of the coil,  $w$  the cross section width of the coil,  $h$  the coil height,  $l$  the liftoff height (the distance between the bottom of the coil and the top of the conductor below),  $R_c$  the radius of the cylinder core,  $h_c$  the height of the core,  $t$  the thickness of the conductor,  $f$  the simulation frequency, and  $\mu_r$  the relative permeability of the core. Since electromagnetic parameters of metamaterials can be arbitrarily controlled, the  $\mu_r$  values ranging from positive to negative are used in the simulation. Copper films are used in the simulation.

## 3. RESULTS AND DISCUSSION

As it is known, the magnetic field of the coil will be enhanced when a soft iron core is put into the coil. Hence the resolution of the coil

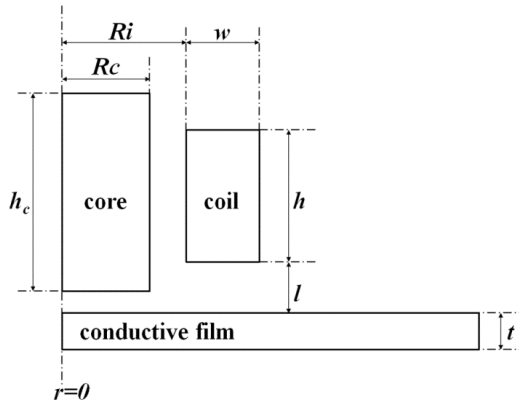


Figure 1. Schematic of the simulation model.

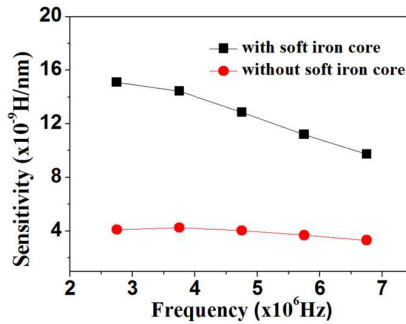
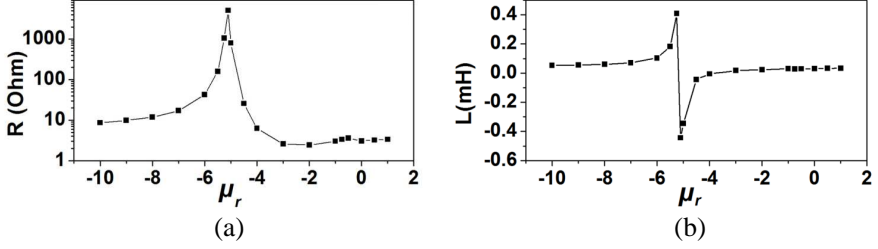


Figure 2. Influence of the soft iron core on thickness sensitivity.

will be increased as well. Thickness sensitivities of EC sensors with and without soft iron core are calculated by changing the metal film thickness from 500 nm to 1000 nm and shown in Fig. 2 ( $N = 110$ ,  $R_i = 1.5 \times 10^{-3}$  m,  $w = 1 \times 10^{-3}$  m,  $h = 2 \times 10^{-3}$  m;  $l = 0.5 \times 10^{-3}$  m,  $R_c = 0.75 \times 10^{-3}$  m,  $h_c = 2.5 \times 10^{-3}$  m). Here, the slope of inductance-thickness relationship between the cases of 500 nm thickness and 1000 nm thickness is used to state the thickness sensitivity in Fig. 2. It can be seen that the thickness sensitivity of the sensor is nearly tripled by inserting a soft iron core.

As to metamaterials with negative relative permeability, the impedance change of the coil while  $\mu_r$  changes from positive to negative is shown in Fig. 3 ( $t = 0.5 \times 10^{-3}$  m,  $f = 10^6$  Hz, and other parameters are the same as that of Fig. 2). In some simple cases, the impedance of the coil is positively correlated with the relative permeability (positive



**Figure 3.** Impedance change while  $\mu_r$  changes from positive to negative values.

value) of the core. For example, for a coil with a closed magnetic core, the inductance of the coil is linear with the relative permeability of the core. When  $\mu_r$  is negative, however, the relationship is complex. According to Fig. 3, in a negative  $\mu_r$  range, the inductance of the coil turns out to be negative. And combining the change of the resistance, the impedance of the coil seems to be self-resonant with the negative relative permeability. The symbol  $\mu_{r0}$  is used to denote the self-resonant relative permeability and  $\mu_{r0}$  is approximately  $-5.1$  in Fig. 3. And it can be seen that the impedance change is vast around the self-resonant point.

The MnZn ferrites exhibit magnetic resonance and negative relative permeability at about megahertz frequencies, which can also be adjusted by controlling the size of the magnetic domain, crystal grain, sinter curve and the element components [31, 32]. Thus, these materials could provide the proper  $\mu_r$  value right at the resonant point.

The self-resonant curve of impedance  $Z$  and permeability  $\mu_r$  seems to accord with the Drude-Lorentz model [21]. According to the expression (1) in Ref. [21], the curve of  $Z$  and  $\mu_r$  can be fitted using the data in Fig. 3. And the impedance  $Z$  can be rewritten as:

$$\frac{iZ}{a} = 1 - \frac{\mu_{r0}^2 - \mu_{rp}^2}{\mu_r^2 - \mu_{r0}^2 + i\gamma\mu_r} \quad (1)$$

The  $\mu_{r0}$  in the expression is the self-resonant permeability. The  $\mu_{rp}$  is the  $\mu_r$  value when inductance equals zero and  $\mu_{rp} > \mu_{r0}$ . Symbols  $a$  and  $\gamma$  are parameters, and  $i = \sqrt{-1}$ . The fitting result shows a great agreement to the expression with correlation coefficient  $R = 0.999969$  and  $\mu_{r0} = -5.12$ . This reveals that the  $Z$ - $\mu_r$  self-resonant phenomenon accords with Drude-Lorentz model indeed.

To study the influence of the geometry size on the self-resonant point, the inner radius  $Ri$  and the core radius  $Rc$  are changed respectively while other parameters of the model remain constant. The

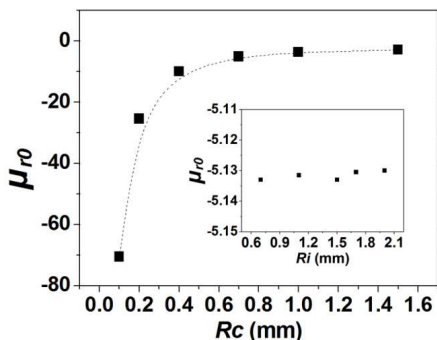


Figure 4. Shift of  $\mu_{r0}$  with  $Ri$  and  $Rc$ .

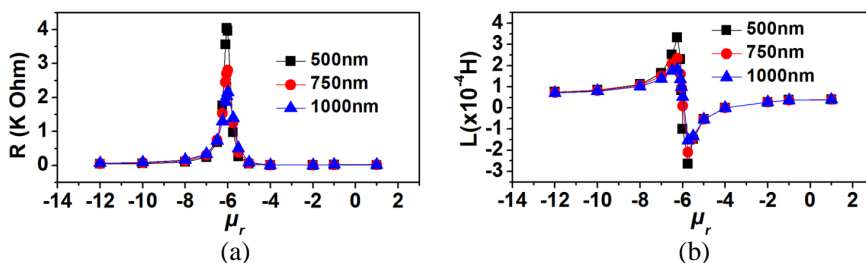


Figure 5. Impedance curves of different film thickness.

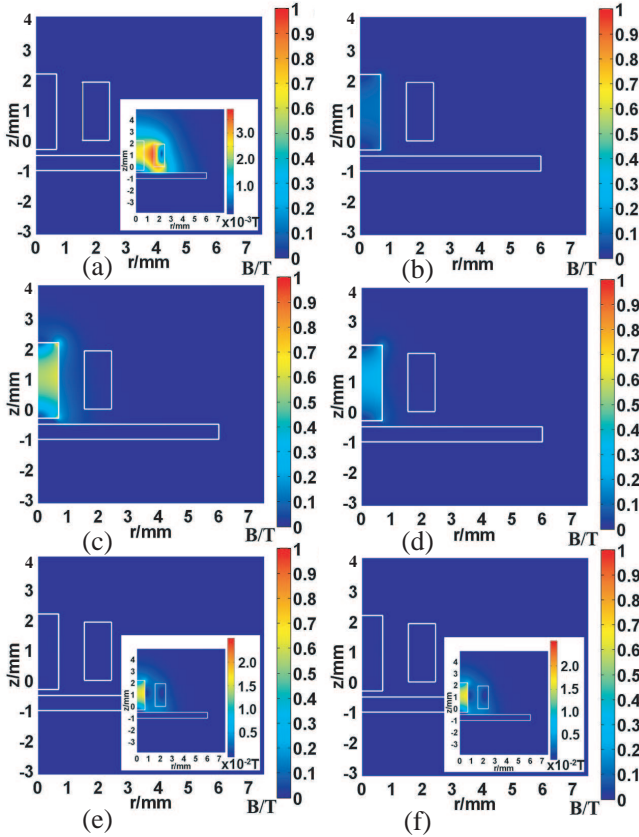
dependences of  $\mu_{r0}$  on the  $Ri$  and  $Rc$  are shown in Fig. 4. It shows that  $\mu_{r0}$  varies significantly with  $Rc$  while it is almost independent on  $Ri$ . Besides, the  $\mu_{r0}$  value varies fast when  $Rc$  is smaller than 0.7 mm, and then it becomes flat after 0.7 mm, showing an approximate inverse relation between  $\mu_{r0}$  and  $Rc$ .

Comparing with the ‘normal’ situation of  $\mu_r > 1$ , the self-resonant phenomenon with negative  $\mu_r$  is quite extraordinary. Since the impedance change is vast around the self-resonant point, the sensitivity of the coil sensor is expected to be greatly enhanced. This property may be applied to EC measurement.

The impedances of EC sensors with different film thickness varying from 500 nm to 1000 nm are calculated and shown in Fig. 5. The value  $\mu_{r0}$  equals about  $-6$  in this thickness range. It shows that the impedance change around the self-resonant point is much larger than that in other regions. When  $\mu_{r0} = -6$ , the thickness sensitivity by resistance is about 3.61 Ohm/nm, while the value is  $-0.013$  Ohm/nm when  $\mu_r = 1$ . So the thickness sensitivity of the coil increases about

277 times by inserting the metamaterials core, which is much bigger than the increase by inserting a soft iron core. Besides, by using this mechanism, even sensors with small coils could have enough sensitivity, so the spatial resolution of the coil can be increased as well in this way.

The question remains that why the impedance-permeability relationship can exhibit such a self-resonant phenomenon as relative permeability changes. A preliminary analysis about the distributions of magnetic field (magnetic flux density) is shown in Fig. 6. The magnetic field of all simulation results are analyzed but only the results with relative permeability  $\mu_r = 1, -4.5, -5, -5.5, 1000, -1000$  are



**Figure 6.** Magnetic field distribution. (a)  $\mu_r = 1$ . (b)  $\mu_r = -4.5$ . (c)  $\mu_r = -5$ . (d)  $\mu_r = -5.5$ . (e)  $\mu_r = 1000$ . (f)  $\mu_r = -1000$ , the color bar scale of the main figures have been unified to 0 ~ 1 T for direct compare. And the inserted figures are auto scale results of each situation.

shown in Fig. 6 for convenient plotting. (All parameters are the same as that of Fig. 3).

It can be seen from the result (not shown in Fig. 6) that when relative permeability becomes approximately zero, there is no magnetic flux in the core, being in agreement with the magnetic insulation property of  $\mu_r = 0$ . And according to Fig. 6, when  $\mu_r < 0$ , the magnetic field can transmit through the core, but the magnetic flux density mostly concentrates in some parts and the value is much greater than the value of  $\mu_r = 1$ . Besides, the magnetic flux density increases with  $\mu_r$  for  $\mu_r < \mu_{r0}$  and decreases with  $\mu_r$  for  $\mu_r > \mu_{r0}$ . This means the magnetic flux density reaches its peak at  $\mu_{r0}$ , being in agreement with the self-resonant phenomenon near the  $\mu_{r0}$ . Overall, some great magnetic flux density appeared after inserting negative relative permeability cores into the coil and a self-resonant phenomenon is caused in the field.

However, even though the magnetic field of the coil changed a lot while  $\mu_r$  decreases from positive to negative, when  $\mu_r$  is much smaller than zero, the magnetic field will tend to be same with the situation of  $\mu_r$  is much larger than zero, as shown in Fig. 6 with the results of (e)  $\mu_r = 1000$  and (f)  $\mu_r = -1000$ . And impedances of these two situations are almost the same as well. This reveals the influence of the core on the coil is equivalent while absolute value of its permeability is much larger than zero. However, how to explain this extraordinary resonance phenomenon still needs more analysis and some experimental verification as well.

#### 4. CONCLUSION

In summary, the impact of introducing metamaterials into eddy current sensors is studied. It reveals that the impedance-permeability relationship exhibits an extraordinary self-resonant phenomenon at a certain negative relative permeability, which is related to the dimensions of the core but nearly independent to the coil size. It is thought that a capacitance-like factor or effective capacitance excited in the coil impedance with the insert of a negative permeability core may be the origin of the phenomenon. Such a mechanism can be proposed to increase the sensitivity of EC sensors to about 270 times, offering a new approach to increase the sensitivity of EC sensors and the spatial resolution of the sensors with micrometer scale size. However, the cause of this phenomenon still remains to be further explored and analyzed, which could provide more reference and guide for the design of EC sensors and the research of related magnetic problems.

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