

ROBUST SUPERDIRECTIVE BEAMFORMING FOR HF CIRCULAR RECEIVE ANTENNA ARRAYS

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Abstract—Superdirective beamforming can highly reduce the aperture size of high-frequency receive array. At the same time, the closely spaced elements of a small aperture array can make it low efficiency and sensitivity to the array uncertainty, which limit its application in practice. Using a parameter called sensitivity factor, we found that array efficiency and robustness against array error could be considered simultaneously. On that basis, we derive a novel superdirective beamforming criterion based on a constrained sensitivity factor for the HF circular receive array. New method is analytical and computationally inexpensive. Through making the directive gain with a given sensitivity factor maximum, we calculate the optimal weights of the array elements. To illustrate the proposed method can increase the acceptance of HF superdirective receive arrays in practice, several numerical results are provided.

1. INTRODUCTION

Array signal processing has been widely used in sensing and data-acquisition systems ranging from radar [1], mobile communications [2–5], cognitive radio [6] and medical imaging [7]. A versatile approach of array signal processing is beamforming [1–19], which is used to detect and enhance a desired signal while suppressing interference and noise at the output of an array of sensors. It is well known that the benefits brought by beamforming such as signal-to-noise ratio enhancement and spatial resolution are restricted by the ratio of array aperture size relative to the working wavelength. HF (high frequency) systems [20] can extend the communication range beyond the horizon limits of

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UHF and higher-frequency signals. However, to achieve high gain and angular resolution, its receive array aperture size are obliged to be very huge as its working wavelength is at least 10 meters. How to reduce the array aperture size without sacrificing gain are the insistent demands of the HF systems.

Among those well-established theories and technologies, superdirectivity [20–24] is a huge attraction to the researchers. This concept claims that a small size array can perform as high directivity as a large-size array, while the directivity of the large-size array is achieved by conventional beamforming. In the other words, superdirective beamforming is helpful for us to reduce the vast aperture size of HF receive array. As the electric size of the array antennas is too small and the array antenna elements are closely spaced, the small aperture array using superdirective beamforming exists many flaws, such as narrow bandwidth, low efficiency and extremely sensitive to array perturbation. For small aperture HF receive arrays using superdirective beamforming, efficiency and sensitive to array uncertainty are two major limitations.

To increase the acceptance of superdirective beamforming in practice, a great amount of work has been done. According to the discoveries of Newman [24] and Barrick [20], the minimum efficiency of HF receive arrays must guarantee the attenuated external noise is greater than the internal receiver noise, or the low efficiency can make the weight sum of array signals close to zero.

Robustness against array uncertainty is another crucial challenge. Although the optimum performance of superdirective beamforming is very intriguing, the array uncertainty can make this theoretical performance even worse than that of conventional beamforming. To increase its robustness against array uncertainty, many methods [22–28] were presented from different perspectives. Most of these methods could be essentially classified as diagonal loading methods, and they have a common defect that how to choose an optimal loading value to calculate weight are not clearly illustrated.

Aiming to break through the limitations of superdirective beamforming applied in practical HF receive arrays, we present a new method. Taking a parameter called sensitivity factor as constraints on array efficiency and robustness against array uncertainty, the proposed method could obtain a good tradeoff between directive gain, array efficiency and robustness against array uncertainty. Compared with the existing methods, the solution of new constrained optimum directive gain method is more analytical and easier to implement.

The rest of this paper is organized as follows. The problem of interest is introduced, and several related algorithms are reviewed in

Section 2. Section 3 presents the proposed method. To verify the validity of the proposed algorithm, numerical examples are presented in Section 4. Finally, we make conclusions in Section 5.

2. PROBLEM AND RELATE METHODS

Consider a circular array constituting of M idealized short vertical dipole elements. Supposing the plane-wave signal arriving from the direction (θ, φ) , the propagation range difference between the i th ($i = 0, 1, 2, \dots, M - 1$) element and the original point will be $\tau_i = r \sin(\theta) \cos(i\beta - \varphi)$, where $\beta = 2\pi/M$ and r is the radius. Letting k as the radio wave number, the steering vector can be expressed as:

$$\mathbf{a}(\theta, \varphi) = [\sin \theta e^{jk\tau_0} \quad \sin \theta e^{jk\tau_1} \quad \dots \quad \sin \theta e^{jk\tau_{M-1}}]^T \quad (1)$$

Assuming \mathbf{w} as the weight vector for beamforming, the radiation pattern in the preset direction (θ_0, φ_0) is

$$F(\theta, \varphi) = \mathbf{w}^H(\theta_0, \varphi_0)\mathbf{a}(\theta, \varphi) \quad (2)$$

where H denotes complex conjugate transpose.

The array directive gain is defined as

$$G(\theta_0, \varphi_0) = \frac{4\pi|F(\theta_0, \varphi_0)|^2}{\int_0^{2\pi} \int_0^\pi \sin \theta |F(\theta, \varphi)|^2 d\theta d\varphi} \quad (3)$$

The above product can represent as follows:

$$G(\theta_0, \varphi_0) = \frac{\mathbf{w}^H N \mathbf{w}}{\mathbf{w}^H R \mathbf{w}} \quad (4)$$

where

$$N = a(\theta_0, \varphi_0)a^H(\theta_0, \varphi_0) \quad (5)$$

and

$$R = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi \sin \theta \mathbf{a}(\theta, \varphi)\mathbf{a}^H(\theta, \varphi) d\theta d\varphi \quad (6)$$

Considering the antenna element is short vertical dipole, the integral result of matrix R can be expressed as:

$$R_{ij} = \begin{cases} \frac{2}{3}, & i = j \\ \frac{kd_{ij}}{\sin kd_{ij}} - \frac{1}{(kd_{ij})^2} \left[\frac{kd_{ij}}{\sin kd_{ij}} - \cos kd_{ij} \right], & i \neq j. \end{cases} \quad (7)$$

where d_{ij} is the distance between the i th element and j th element.

The solution of the maximum directive gain is an optimization problem. As N and D are both Hermitian matrices, the optimization

problem can be viewed as an eigenvalue problem. According to the matrix theory, the optimum \mathbf{w} is referred to as the eigenvector corresponding to the non-zero eigenvalue G . The optimal \mathbf{w} and the gain itself can be written in the form [20]:

$$\mathbf{w}_{opt} = R^{-1}\mathbf{a}(\theta_0, \varphi_0), \quad G_{opt} = \mathbf{a}^H(\theta_0, \varphi_0)R^{-1}\mathbf{a}(\theta_0, \varphi_0) \quad (8)$$

Optimum directive gain method could provide the theoretical maximum gain. Meanwhile, it brings about low efficiency and high sensitivity to array uncertainty, both of which are unacceptable. To avoid the negative effects brought by low efficiency, Ref. [20] conducts a research on array efficiency. The array efficiency could be described as:

$$\eta = \frac{\mathbf{w}^H N \mathbf{w}}{M \mathbf{w}^H \mathbf{w}} \quad (9)$$

For high-frequency radio/ radar receivers, the minimum efficiency must guarantee the attenuated external noise is greater than internal receiver noise, or low efficiency can make the weight sum of array signals close to zero. At 10 MHz, external receiver noise is typically 55 dB larger than internal receiver noise. Suppose the dipole elements are each connect to a high-impedance preamplifier with a noise figure of 10 dB and a 10 dB “cushion” to make sure that external noise dominates. Then the array at 10 MHz must have a minimum efficiency of -35 dB, based on these example numbers. To ensure that array efficiency must be higher than this calculated efficiency, the method in Ref. [20] based on optimum directive gain method tests the number of elements, spacings between elements so that a small array using superdirective beamforming is designed.

Sensitivity factor K is introduced in [21] to describe the performance of the array to array uncertainty. In general, the larger K is, the array is more sensitive to array uncertainty. K is defined as:

$$K = \frac{\mathbf{w}^H \mathbf{w}}{\mathbf{w}^H N \mathbf{w}} \quad (10)$$

To increase the acceptance of superdirective beamforming applied in high-frequency array, [24] proposed a constrained gain optimization method with a given tolerance sensitivity. By finding a tradeoff between increased directive gain and increased sensitivity, constrained gain optimization method makes directive gain, efficiency and sensitivity acceptable and realizable. The problem of maximizing the array gain subject to a constraint on the sensitivity factor could be expressed by a function:

$$Q = \frac{\mathbf{w}^H N \mathbf{w}}{\mathbf{w}^H D \mathbf{w}} + v \frac{\mathbf{w}^H \mathbf{w}}{\mathbf{w}^H D \mathbf{w}} \quad (11)$$

in which v is a Lagrange multiplier. The detailed solution is given in Ref. [27]. The constrain optimum weight \mathbf{w} could be written as:

$$\hat{\mathbf{w}}_{opt} = q(R + \delta I)^{-1} \mathbf{a}(\theta_0, \varphi_0) \tag{12}$$

where δ is scalar constant and I is the identity matrix. Although q is a rather complicate quantity, which relates to \mathbf{w} , it has no effect on the normalization of $\hat{\mathbf{w}}_{opt}$. Thus,

$$\hat{\mathbf{w}}_{opt} = (R + \delta I)^{-1} \mathbf{a}(\theta_0, \varphi_0), \tag{13}$$

$$\hat{\mathbf{G}}_{opt} = \mathbf{a}^H(\theta_0, \varphi_0)(R + \delta I)^{-1} \mathbf{a}(\theta_0, \varphi_0) \tag{14}$$

It is seen that optimum directive gain method is a special case of constrained optimum directive gain method when $\delta = 0$. In fact, δ provides a continuous monotonic parameterization between the optimum directive gain method ($\delta = 0$) and the conventional beamforming ($\delta = \infty$). It seems very simple to get a tradeoff between directive gain and sensitivity to the array uncertainty by loading a proper value δ . Nevertheless, as the value δ and sensitivity factor K is not directly linked with the solution, it needs to take a multi-test to obtain a proper value δ in practice. Thus, a more effective method to obtain an optimal weight must be developed.

3. PROPOSED ALGORITHM

Aiming to increase the acceptance of superdirective beamforming applied in HF array, we herein provide a new method in which the optimal weight is directly linked with the given constrained value. Compared with constrained gain optimization method, proposed method can be more efficiently.

It is easy to find array efficiency and sensitivity factor are reciprocal relationship. So array efficiency and sensitivity factor could be controlled simultaneously by the same constrained value. Combined with the latter discussion, we found that decreasing sensitivity to array uncertainty is harder than maintaining array efficiency and sensitivity factor is more convenient for our formulation. Thus, we use sensitivity factor as the constrained value in the following.

Assume the minimum array efficiency we need is η_0 , the maximum sensitivity factor must be smaller than $\frac{1}{M\eta_0}$. Considering a sensitivity factor is always greater than or equal to $\frac{1}{M}$. So the bound of the given sensitivity factor K_0 could be

$$\frac{1}{M} \leq K_0 \leq \frac{1}{M\eta_0} \tag{15}$$

For a certain small aperture array, optimum directive gain method is the most sensitive to array uncertainty, which has the largest sensitivity factor. Here,

$$K_{\max} = \frac{\mathbf{a}^H(\theta_0, \varphi_0)R^{-2}\mathbf{a}(\theta_0, \varphi_0)}{[\mathbf{a}^H(\theta_0, \varphi_0)R^{-1}\mathbf{a}(\theta_0, \varphi_0)]^2} \quad (16)$$

To decrease sensitive to array uncertainty and maintain array efficiency higher than the calculated efficiency, the given sensitivity factor should be

$$\frac{1}{M} < K_0 < \min\left(\frac{1}{M\eta_0}, \frac{\mathbf{a}^H(\theta_0, \varphi_0)R^{-2}\mathbf{a}(\theta_0, \varphi_0)}{[\mathbf{a}^H(\theta_0, \varphi_0)R^{-1}\mathbf{a}(\theta_0, \varphi_0)]^2}\right) \quad (17)$$

Under the condition of (17), our proposed method is illustrated as follows. Using the distortionless constraint $\mathbf{w}^H \mathbf{a}(\theta_0, \varphi_0) = 1$, sensitivity factor becomes $K_0 = \mathbf{w}^H \mathbf{w}$ and the maximization of G becomes $\min_{\mathbf{w}} \mathbf{w}^H R \mathbf{w}$. Therefore, the proposed method becomes:

$$\min_{\mathbf{w}} \mathbf{w}^H R \mathbf{w} \text{ s.t. } \mathbf{w}^H \mathbf{a}(\theta_0, \varphi_0) = 1, \quad \mathbf{w}^H \mathbf{w} \leq K_0 \quad (18)$$

The solution to (18) can be found by minimizing the function

$$F(\mathbf{w}, \lambda, \mu) = \mathbf{w}^H R \mathbf{w} + \lambda(\mathbf{w}^H \mathbf{w} - K_0) + \mu(\mathbf{w}^H \mathbf{a}(\theta_0, \varphi_0) - 1) \quad (19)$$

where λ, μ are real-valued Lagrange multipliers with μ being arbitrary and $\lambda > 0$ satisfying $R + \lambda I > 0$. Taking the gradient of $F(\mathbf{w}, \lambda, \mu)$ and equating it to zeros gives

$$\bar{\mathbf{w}} = \mu(R + \lambda I)^{-1} \mathbf{a}(\theta_0, \varphi_0) \quad (20)$$

Using the equation $\bar{\mathbf{w}}^H \mathbf{a}(\theta_0, \varphi_0) = 1$, we obtain

$$\mu = - \left[\mathbf{a}^H(\theta_0, \varphi_0)(R + \lambda I)^{-1} \mathbf{a}(\theta_0, \varphi_0) \right]^{-1} \quad (21)$$

then,

$$\bar{\mathbf{w}} = \frac{(R + \lambda I)^{-1} \mathbf{a}(\theta_0, \varphi_0)}{\mathbf{a}^H(\theta_0, \varphi_0)(R + \lambda I)^{-1} \mathbf{a}(\theta_0, \varphi_0)} \quad (22)$$

Similar to constrained gain optimization method, the key point of method is finding a proper loading value λ . The solution of λ is linked with K_0 by the following equation:

$$\bar{\mathbf{w}}^H \bar{\mathbf{w}} = \frac{\mathbf{a}^H(\theta_0, \varphi_0) \left(R + \hat{\lambda} I \right)^{-2} \mathbf{a}(\theta_0, \varphi_0)}{\left[\mathbf{a}^H(\theta_0, \varphi_0) \left(R + \hat{\lambda} I \right)^{-1} \mathbf{a}(\theta_0, \varphi_0) \right]^2} = K_0 \quad (23)$$

Under the condition $K_0 < \frac{\mathbf{a}^H(\theta_0, \varphi_0)R^{-2}\mathbf{a}(\theta_0, \varphi_0)}{[\mathbf{a}^H(\theta_0, \varphi_0)R^{-1}\mathbf{a}(\theta_0, \varphi_0)]^2}$, we have a unique solution $\hat{\lambda}$ for the left side of (23) is a monotonically decreasing function of $\hat{\lambda}$, which has been proven in Ref. [19]. Hence, $\hat{\lambda}$ can be obtained by numerical methods, such as bisection method, Newton's method. Next, we will discuss how to define an upper bound on $\hat{\lambda}$. The eigendecomposition of R can be expressed as

$$R = U\Lambda U^H \tag{24}$$

where U is comprised by the eigenvectors of R , and $\Lambda = \text{diag}(\gamma_1, \gamma_2, \dots, \gamma_M)$ is a diagonal matrix comprised by eigenvalues, satisfying $\gamma_1 \geq \gamma_2 \geq \dots \geq \gamma_M$. Using (24),

$$(R + \hat{\lambda}I)^{-1} = U(\Lambda + \hat{\lambda}I)^{-1}U^H \tag{25}$$

Let $z = U^H\mathbf{a}(\theta_0, \varphi_0)$, (23) could be written as

$$\sum_{i=1}^M \frac{|z_i|^2}{(\gamma_i + \hat{\lambda})^2} \bigg/ \left[\sum_{i=1}^M \frac{|z_i|^2}{\gamma_i + \hat{\lambda}} \right]^2 = K_0 \tag{26}$$

Further, we derive

$$K_0 \leq \frac{\|\mathbf{a}(\theta_0, \varphi_0)\|^2}{(\gamma_M + \hat{\lambda})^2} \bigg/ \frac{\|\mathbf{a}(\theta_0, \varphi_0)\|^4}{(\gamma_1 + \hat{\lambda})^2} = \frac{(\gamma_1 + \hat{\lambda})^2}{M(\gamma_M + \hat{\lambda})^2} \tag{27}$$

which gives the following upper bound on $\hat{\lambda}$

$$\hat{\lambda} \leq \frac{\gamma_1 - (MK_0)^{1/2}\gamma_M}{(MK_0)^{1/2} - 1} \tag{28}$$

Considering $\hat{\lambda} \geq 0$, the bound of $\hat{\lambda}$ is

$$0 \leq \hat{\lambda} \leq \frac{\gamma_1 - (MK_0)^{1/2}\gamma_M}{(MK_0)^{1/2} - 1} \tag{29}$$

Using (25), $\bar{\mathbf{w}}$ could be rewritten as

$$\bar{\mathbf{w}} = \frac{U(\Lambda + \hat{\lambda}I)^{-1}U^H\mathbf{a}(\theta_0, \varphi_0)}{\mathbf{a}^H(\theta_0, \varphi_0)U(\Lambda + \hat{\lambda}I)^{-1}U^H\mathbf{a}(\theta_0, \varphi_0)} \tag{30}$$

To sum up, we implement the proposed method as follows.

Step 1) Compute the eigendecomposition of R .

Step 2) According to the practical knowledge, choose a K_0 satisfying (17) and solve (26) for $\hat{\lambda}$ via, for example, bisection method.

Step 3) Use the $\hat{\lambda}$ obtained in Step 2 and (30) to calculate the optimal weight vector $\bar{\mathbf{w}}$.

4. NUMERICAL RESULTS

Combined with the illustration in Section 2, we assume the minimal array efficiency of -35 dB must be achieved. Numerical examples in

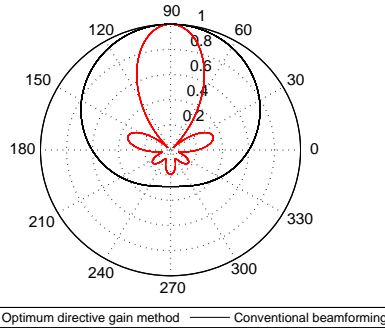


Figure 1. The radiation patterns in ideal circumstance.

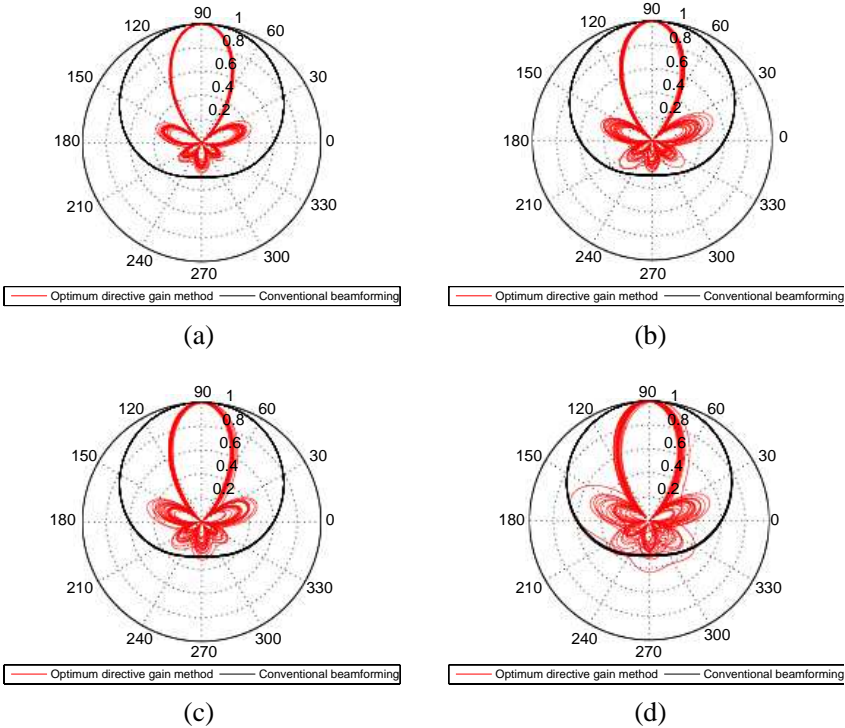


Figure 2. The radiation patterns when sensors exist amplitude error. (a) -40 dB amplitude error. (b) -37 dB amplitude error. (c) -34 dB amplitude error. (d) -31 dB amplitude error.

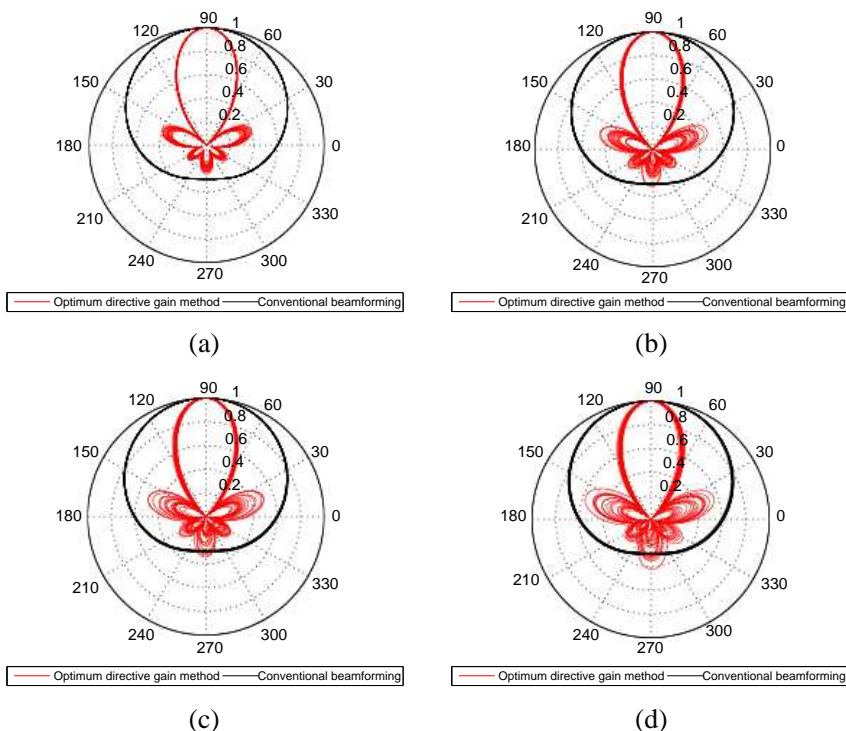


Figure 3. The radiation patterns when sensors exist phase error. (a) 0.3° phase error. (b) 0.6° phase error. (c) 0.9° phase error. (d) 1.2° phase error.

Ref. [20] demonstrated that optimum directive gain method could make array efficiency higher than the minimum array efficiency only if the number of elements, spacings are properly selected. However, it hasn't been considered its robustness. In order to make a better comparison of those mentioned methods, we design an array used in all the examples bellowed with its lowest efficiency of -23.4 dB. We consider a planar circular array of $M = 7$ short vertical dipole elements uniformly spaced on a circumference with a radius of 4.5 m. The antenna array works at 10 MHz. Besides, as to array error, only amplitude error and phase error are considered, which meet independent and identically Gaussian distribution in assumption.

The examples are arranged as follows. Firstly, based on results shown in Figure 2 and Figure 3, we could see the robustness of optimum directive gain method is rather poor, as its radiation patterns are severely distorted by tiny array error. Secondly, in Figure 5, we could

see proposed method show certain robustness to array uncertainty, which is equivalent to constrained gain optimization method under a same sensitivity factor. Lastly, to verify novel method is more convenient for practical use, the rough relationships between sensitivity factor K_0 and its corresponding maximum affordable array uncertainty and directive gain are presented in Figure 5 and Figure 6, one can easily obtain a tradeoff between directive gain and robustness to array uncertainty by choosing a proper sensitivity factor.

4.1. The Robustness of Optimum Directive Gain Method

The maximum affordable array error can be considered as a measure of array robustness. We obtain the maximum affordable array error through an approximation test. In the test, as the array error increases, the previous value is assumed as the maximum affordable array error while the distortions of radiation patterns become unacceptable. The detailed proceeded as follows:

Step 1) Assume there is no amplitude and phase error, we calculate the radiation patterns of optimum directive gain method and conventional beamforming in the preset direction ($90^\circ, 90^\circ$), the results of which are shown in Figure 1.

Step 2) Assume the array sensors only exist amplitude error. We test the radiation patterns while the amplitude error increases from -40 dB to -31 dB in a step of 3 dB. Compare the radiation patterns in Figure 2 with the radiation patterns in ideal circumstance, we find the maximum affordable amplitude error.

Step 3) Assume the array sensors only exist phase error. We test the radiation patterns while the phase error increases from 0.3° to 1.2° in a step of 0.3° . Compare the radiation patterns in Figure 3 with the radiation patterns in ideal circumstance, we find the maximum affordable phase error.

Observe the radiation patterns in Figure 2 and Figure 3, we find that the distortions of the radiation patterns are unacceptable when the amplitude error reaches -31 dB or the phase error reaches 1.2° . Thus, we regard -34 dB and 0.9° as the maximum affordable amplitude error and phase error, respectively. It can be perceived that the robustness of optimum directive gain method is rather weak, and its implement has a high demanding of actual engineering environment.

4.2. The Robustness of the Proposed Method

By loading a proper value, constrained gain optimization method could lessen the sensitivity to array uncertainty and make array efficiency

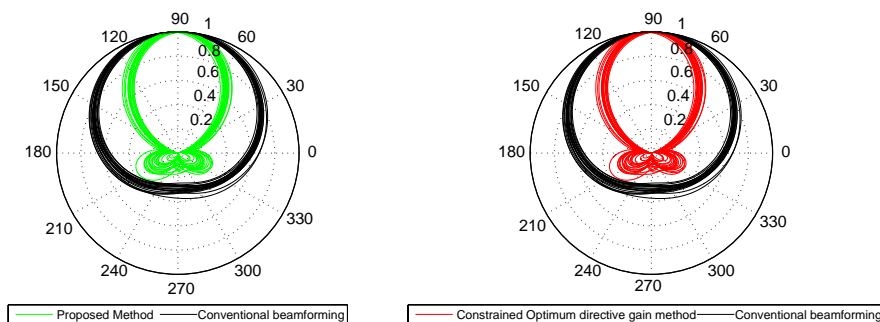


Figure 4. The robustness performances of (a) proposed method and (b) constrained optimum directive gain method.

higher than the minimum array efficiency. Nevertheless, it hasn't provided a clear method to calculate the optimal loading value.

Figure 4 presents the performances of proposed method and constrained optimum directive gain method while the random amplitude error and phase error are fixed at -25 dB and 5° . It can be seen that the radiation patterns of two methods are favorable and nearly identical. That is because the two methods are calculated under a same sensitivity factor $K_0 = 0.8$ and the robustness to array error can be highly improved only if the sensitivity factor is properly chosen in design. It's worth noting that the loading value $\delta = 0.34$ of constrained gain optimization method to reach the given sensitivity factor is by a multi-test while the weights of proposed method are calculated directly in the given sensitivity factor. Apparently, the novel method is more convenient in practical use than constrained gain optimization method.

4.3. The Performances of Proposed Method with Different Given Sensitivity Factor

In Figure 5 and Figure 6, we could see the performance of proposed method changes as the given sensitivity factor K_0 varies. In fact, this two figures not only show the performance of proposed method but also the performances of conventional beamforming and optimum directive gain method. With respect to the assumed array, $K_0 = 31$ means proposed method is close to optimum directive gain method, and $K_0 = 0.15$ means proposed method is close to conventional beamforming. As the distortions of radiation patterns are hard to ameliorate when the random array error reaches a certain extent, the curves in Figure 5 stop at $K_0 = 0.5$. Thus, the maximum affordable amplitude error and phase errors of the proposed method could be

regarded as -15 dB and 11.4° .

As it is seen from Figure 5 and Figure 6, the proposed method can provide an accurate tradeoff between directive gain and robustness to array uncertainty by changing sensitivity factor. Although the proposed method cannot replace array calibration, it highly reduces the requirements on the accuracy of calibration. In practical implement, the actual array error (including the residual error after calibration) is unknown, the proposed method can provide a favorable radiation pattern only if array error is less than the maximum affordable error corresponding to the given sensitivity factor. Meanwhile, as directive gain decreases monotonically with the increase of K_0 , one can increase K_0 to obtain the best directive gain provided that the radiation pattern is acceptable.

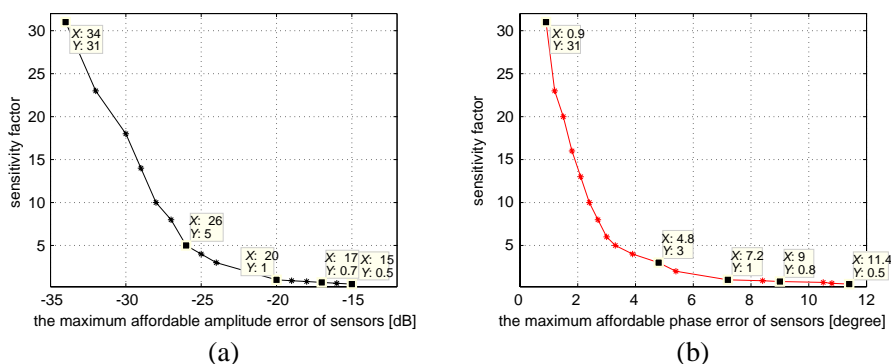


Figure 5. The maximum affordable array error corresponding to a given sensitivity factor. (a) The maximum affordable amplitude error. (b) The maximum affordable phase error.

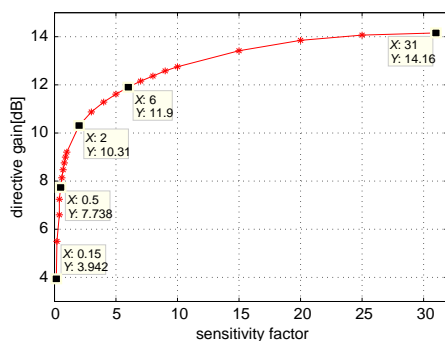


Figure 6. The directive gain corresponding to a given sensitivity factor.

5. CONCLUSION

In this paper, we propose a robust method to increase the acceptance of superdirective beamforming applied in practical HF receive arrays. Compared with the existing methods, the proposed method not only presents a good tradeoff between directive gain, array efficiency and robustness against array error, but also provides an analytical solution to the given sensitivity factor. With numerical examples, the effectiveness and convenience of the proposed method are demonstrated.

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