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MODEL SELECTION FOR INVESTIGATION OF THE FIELD DISTRIBUTION IN A REVERBERATION CHAMBER

Xiaoming Chen*

Chalmers University of Technology, Gothenburg 412 96, Sweden

Abstract—In this work two model selection criteria, i.e., Akaike's information criterion (AIC) and minimum description length (MDL), are applied to measurements in a RC with Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions as the distribution candidate set. In spite of small differences of the AIC and MDL tests (due to their different penalty terms on distribution parameters), both criteria result in similar conclusions. Results show that the Rayleigh distribution provides the overall good fit to the Cartesian field amplitude, especially for an overmoded RC, and that the Weibull distribution provides good fit to the Cartesian field amplitude in an undermoded or loaded RC. In addition, it is found that both the Rician and Weibull distributions provide improved approximations of the Cartesian field amplitude in a loaded RC with non-negligible unstirred components and that the transition from undermoded RC to overmoded RC depends not only on the operating frequency and mode-stirrer efficiency (as it is commonly believed) but also on source stirring and RC loading.

1. INTRODUCTION

The reverberation chamber (RC) has been used for electromagnetic compatibility (EMC) tests as well as over-the-air (OTA) measurements of wireless devices [1–13]. Due to the complicated and time-varying test conditions, various RC measurements are ubiquitously analyzed from the statistical point of view. Thus it is of fundamental importance to have a good statistic model of the RC field. Since the (overmoded) RC represents a rich scattering environment, it is usually assumed that the Cartesian field is complex Gaussian, i.e., the real and imaginary parts of the Cartesian field components are Gaussian distributed. Namely

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* Corresponding author: Xiaoming Chen (xiaoming.chen@chalmers.se).

the amplitude of the Cartesian field in a RC is Rayleigh distributed [1]. On the other hand, there are studies showing that the Cartesian field in an undermoded RC follows the Weibull or Bessel K distribution [3, 4].

Goodness-of-fit (GOF) tests [14] were popularly used in studying the distribution of the random field in a reverberation chamber (RC) [5, 6]. Such studies assume that the GOF test in use is equally powerful for all distributions, which does not hold in general [15–17]. Unlike GOF, the Akaike's information criterion (AIC) [15] approach is a special model selection method [17] that allows fair comparisons of different distribution candidates. Another advantage of the model selection method over GOF is that the latter suffers from overfitting (i.e., the model with more parameters tends to offer more flexibility in fitting specific data and the nice fitting tends to break down for another independent data) problem whereas the former does not due to its penalty term on the number of parameters. Therefore AIC has been employed in selecting the best approximating distribution in an earlier work. Nevertheless, there are different selection criteria (or model selection methods) with different properties. The success of each of these criteria in model selection has been mixed. These criteria should only be used as indicators of the fitness of the distribution candidates. Thus it might be necessary to examine the same distribution candidate set with different selection criteria. For this reason, we, in this work, use both AIC and minimum description length (MDL) [16], which is another model selection criterion, to test the field distribution in a RC.

It is well known that the Cartesian field in a well stirred (overmoded) RC follows complex Gaussian distribution [1]. Any distribution deviation represents a RC imperfection (i.e., an undermoded RC [3]). Then it is reasonable to claim that the RC is undermoded when the Cartesian field is otherwise distributed. In this paper, based on the measurements in a RC and with Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions as the candidate set, it is found that the Weibull distribution offers good fit for the undermoded RC, and that the Rayleigh distribution provides the overall good fit to RC measurements (except for the low frequencies), and all the other distributions in the candidate set give relatively poor fitness (except for the Rician distribution for the measurements in a loaded RC with non-negligible unstirred components). In addition, it is shown that the transition from undermoded RC to overmoded RC depends not only on the operating frequency and mode-stirrer efficiency (as it is commonly believed) but also on source stirring (or platform stirring) [7, 8] and RC loading (or quality factor Q). Furthermore, by comparing the AIC and MDL results, it is shown that the MDL prefers the Rayleigh distribution to the Weibull one, although the general trends of both selections are similar. This phenomenon can be explained by the fact that MDL puts more penalties on the distribution with more parameters (cf. Section 2).

2. AIC AND MDL

The selection of a suitable distribution out of a given candidate set J involves the calculation of the discrepancy between the true cumulative distribution function (CDF or distribution) F and each candidate distribution $G_{j|\boldsymbol{\theta}} j = 1, \ldots, |J|$, where $|J|$ denotes the cardinality of J and $\boldsymbol{\theta}$ represents the $p\times$ 1 parameter vector (with p being a positive integer). The detailed derivation of the AIC and the MDL can be found in [15, 16], respectively. For the sake of conciseness, the AIC and the MDL are directly given here as

$$\mathrm{AIC}_j = -2\sum_{n=1}^{N} \ln g_{j|\hat{\boldsymbol{\theta}}}(x_n) + 2p, \tag{1}$$

$$\mathrm{MDL}_j = -\sum_{n=1}^{N} \ln g_{j|\hat{\boldsymbol{\theta}}}(x_n) + \frac{p}{2}\ln N \tag{2}$$

respectively, where ln denotes the natural logarithm, $g_{j|\hat{\boldsymbol{\theta}}}$ the corresponding probability density functions (PDF) of $G_{j|\hat{\boldsymbol{\theta}}}$, x_n the nth sample of the field amplitude, and N the sample number. The corresponding maximum likelihood (ML) parameter estimator is

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \frac{1}{N}\sum_{n=1}^{N} \ln g_{j|\boldsymbol{\theta}}(x_n). \tag{3}$$

As can be seen from (1) and (2), the AIC and the MDL are rather similar; the major difference is the penalty terms (i.e., the MDL contains a penalty term that increases with the sample number, while AIC does not). The successes of both criteria in model selection have been mixed. Hence this work uses both AIC and MDL for approximating the field distribution in a RC. Note that the AIC and MDL values are difficult to interpret directly in that the AIC and MDL values of different (reasonably assumed) candidate distributions are usually in the same order of magnitude. Similarly, comparisons of the empirical CDF of the measured data and those of the distribution candidates do not provide interpretable distinctions (the corresponding results are therefore omitted). Therefore, one has to resort to the AIC and MDL weights [15] for better distinctions. The AIC and MDL weights are defined as

$$w_j = \frac{\exp\left(\phi_j/2\right)}{\sum\limits_{l=1}^{J} \exp\left(\phi_l/2\right)} \tag{4}$$

where $\phi_j = \mathrm{AIC}_j - \min\limits_{l}\{\mathrm{AIC}_l\}$ for the AIC and $\phi_j = \mathrm{MDL}_j - \min\limits_{l}\{\mathrm{MDL}_l\}$ for the MDL. The weights (4) represent relative feasibilities of different candidates, ranging from 0 (the worst fit) to 1 (the best fit). In other words, a larger weight means a better fit.

3. CANDIDATE DISTRIBUTION SET

The Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions are believed to be the most relevant models for the distribution of the Cartesian field amplitude in the RC. Their PDFs and corresponding free parameter ML estimators are given in the following subsections.

The Rayleigh distribution is probably the most common statistical model for an overmoded RC [1]. Its PDF can be expressed as (for notational convenience, the subscript $_{j|\theta}$ is dropped hereafter)

$$g(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{5}$$

where the free parameter is $\theta = \sigma$. Thus the Rayleigh distribution has only one scalar parameter, i.e., $p = 1$. The ML estimator of σ is [18]

$$\hat{\sigma} = \sqrt{\frac{1}{2N} \sum_{n=1}^{N} x_n^2}. \tag{6}$$

The PDF of the Rician distribution is

$$g(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2 + \upsilon^2}{2\sigma^2}\right) I_0\left(\frac{x\upsilon}{\sigma^2}\right) \tag{7}$$

where I_0 is the modified Bessel function of the first kind with order zero, and the free parameter vector is $\boldsymbol{\theta} = [\upsilon \ \sigma]^T$ (the superscript ^{T} denotes transpose). Thus the Rician distribution has two scalar parameters, i.e., $p = 2$. Unfortunately, the ML estimator, in this case, does not exist. Therefore, we have to resort to the numerical ML estimation [18], which utilizes the *fminsearch* function in MATLAB (based on the Nelder-Mead algorithm). Note that, without further specification, all the numerical ML estimations used in this paper resort to this function.

The Nakagami distribution (that includes Rayleigh and Rician distributions as special cases) is a popular statistical model. The PDF of the Nakagami distribution is

$$g(x) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{2m-1} \exp\left(-\frac{mx^2}{\Omega}\right) \tag{8}$$

where Γ is the gamma function, and the free parameter vector is $\boldsymbol{\theta} = [m\Omega]^T$. Hence the Nakagami distribution has two scalar parameters, i.e., $p = 2$. For $m = 1$, the Nakagami distribution reduces to the Rayleigh distribution. The Nakagami distribution can well approximate the Rician distribution by letting $K = m - 1 + \sqrt{m^2 - m}$. The ML estimators of Ω and m are

$$\hat{\Omega} = \frac{1}{N} \sum_{n=1}^{N} x_n^2, \tag{9}$$

$$\hat{m} = \left(2 \ln \hat{\Omega} - \frac{2}{N} \sum_{n=1}^{N} \ln x_n^2\right)^{-1}, \tag{10}$$

respectively.

The PDF of the Bessel K distribution is

$$g(x) = \frac{t^{M+1}}{2^{M-1}\Gamma(M)} x^M K_{M-1}(tx) \tag{11}$$

where the free parameter vector is $\boldsymbol{\theta} = [M\ t]^T$, and K_{M-1} denotes the modified Bessel function of the second kind with the order of $M - 1$. The closed form ML parameter estimator does not exist for the Bessel K distribution. Hence, we resort to the numerical ML estimation $\hat{\boldsymbol{\theta}}$ [18].

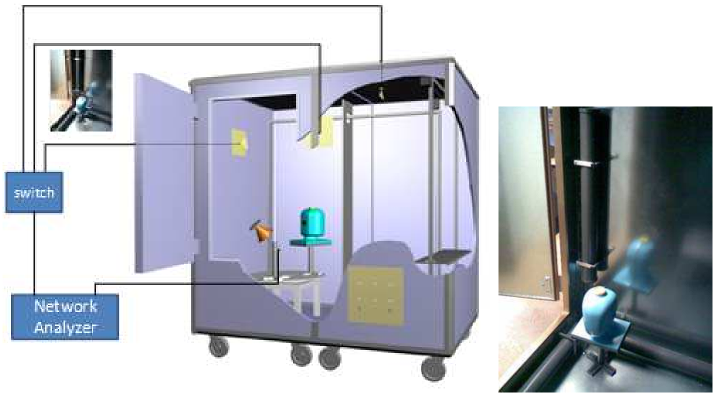
The PDF of the Weibull distribution is

$$g(x) = ba^{-b} x^{b-1} \exp\left(-(x/a)^b\right) \tag{12}$$

where the free parameter vector is $\boldsymbol{\theta} = [a\ b]^T$. For $b = 2$, the Weibull distribution reduces to the Rayleigh distribution. A closed-formed ML estimator for the Weibull distribution does not exist. Thus one has to resort to the numerical ML estimation. In this case, $\hat{\boldsymbol{\theta}}$ can be obtained by calling the available function *wblfit*($\mathbf{x}$) in MATLAB, where $\mathbf{x}$ is a vector of the measured field amplitudes.

4. MEASUREMENTS AND RESULTS

Measurements were performed from 500 to 2000 MHz in a RC with a size of $1.80 \times 1.75 \times 1.25\,\mathrm{m}^3$ (a drawing of which is shown in Fig. 1).



(a) (b)

Figure 1. Drawing of (a) Bluetest RC with two mechanical plate stirrers, one platform and three wall antennas and (b) a photo showing the head phantom and the location of the three absorber-filled PVC cylinders of *load*2 configuration.

Its fundamental mode resonance frequency is $f_0 = 119\,\text{MHz}$, giving a lowest usable frequency (LUF) of about $6f_0 = 717\,\text{MHz}$ (see [6] and reference therein). Note that this LUF corresponds to a well-stirred and unloaded RC. Provided that the stirrers are less effective and/or the RC is loaded, the actual LUF should be larger than 717 MHz. It has two plate mode-stirrers, a turn-table platform (on which a wideband discone antenna is mounted), and three antennas mounted on three orthogonal walls (referred to as wall antennas hereafter). The wall antennas are actually wideband half-bow-tie antennas. The measurement setup (or stirring sequence) of the RC is chosen such that: The turn-table platform was step-wisely moved to 20 platform positions evenly distributed over one complete platform rotation; at each platform position the two plates were simultaneously and step-wisely moved to 50 positions (equally spanned on the total distances that they can move). At each stirrer position and for each wall antenna a full frequency sweep was performed by the VNA with a frequency step of 1 MHz, during which the S-parameters are sampled as a function of frequency and stirrer position. Thus there are $20 \times 50 = 1000$ stirrer positions per frequency point. The same measurement procedure were repeated for three loading conditions: *load*0 (unloaded RC), *load*1 (head phantom that is equivalent to a human head in terms of microwave absorption), and *load*2 (the head phantom plus three Polyvinyl Chloride (PVC) cylinders filled with microwave absorbers cut in small pieces). Hereafter measured data from these different loading configurations are simply referred to as *load*0, *load*1, or *load*2 data.

In the post processing, only the S-parameter samples corresponding to one of the wall antennas are used (the statistics of the samples corresponding to the other two wall antennas are quite similar). As mentioned in Section 3, the random field amplitude is denoted as x and the measured ($N = 1000$) amplitude samples are stacked into one column vector denoted as $\mathbf{x}$.

Although GOF has been rather popular in studying the field distribution in the RC, it should be noted that the comparison is only valid when the GOF test is equally powerful in both distributions,

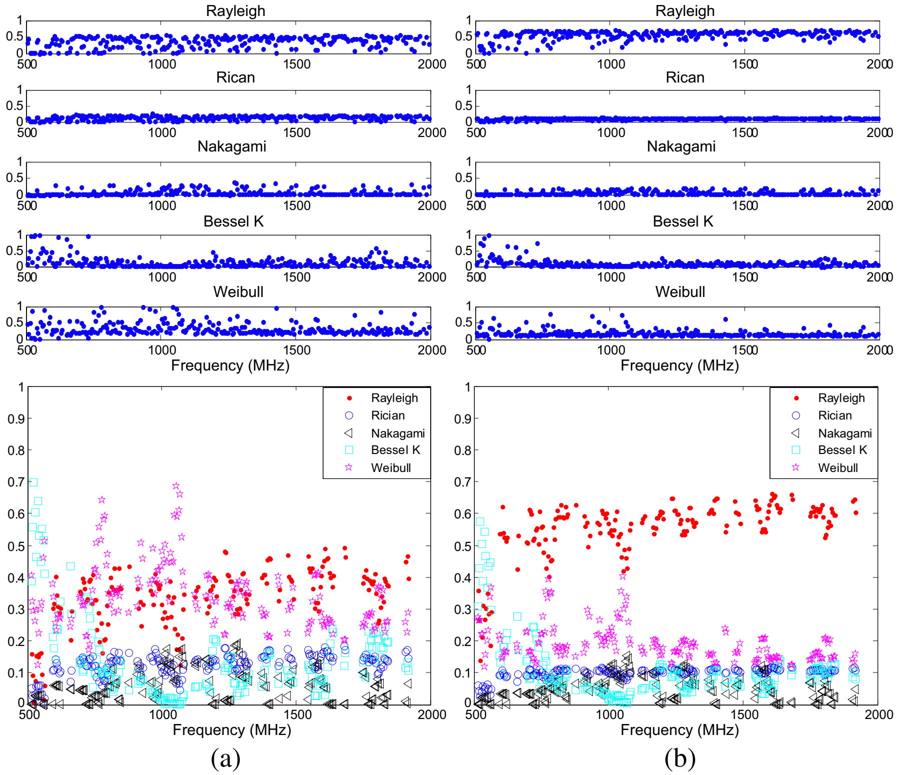


Figure 2. Comparison of (a) AIC weights and (b) MDL weights for Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions based on the *load*0 data. The upper graphs correspond to the weights without frequency stirring. The lower graphs correspond to the weights with 5-MHz frequency stirring.

which does not hold in general. In addition, the GOF test also suffers from the overfitting problem, which prevents fair comparison of different distributions with different parameter numbers (i.e., the Weibull distribution has two scalar parameters and the Rayleigh distribution has only one scalar parameter). Therefore, one has to resort to the AIC and/or the MDL for fair comparisons of different candidate distributions.

Figure 2 shows the comparison of the AIC and MDL weights for the candidate distribution based on the *load*0 data. The upper graphs correspond to the case without frequency stirring of the calculated weights. In order to compare different distributions more clearer, a 5-

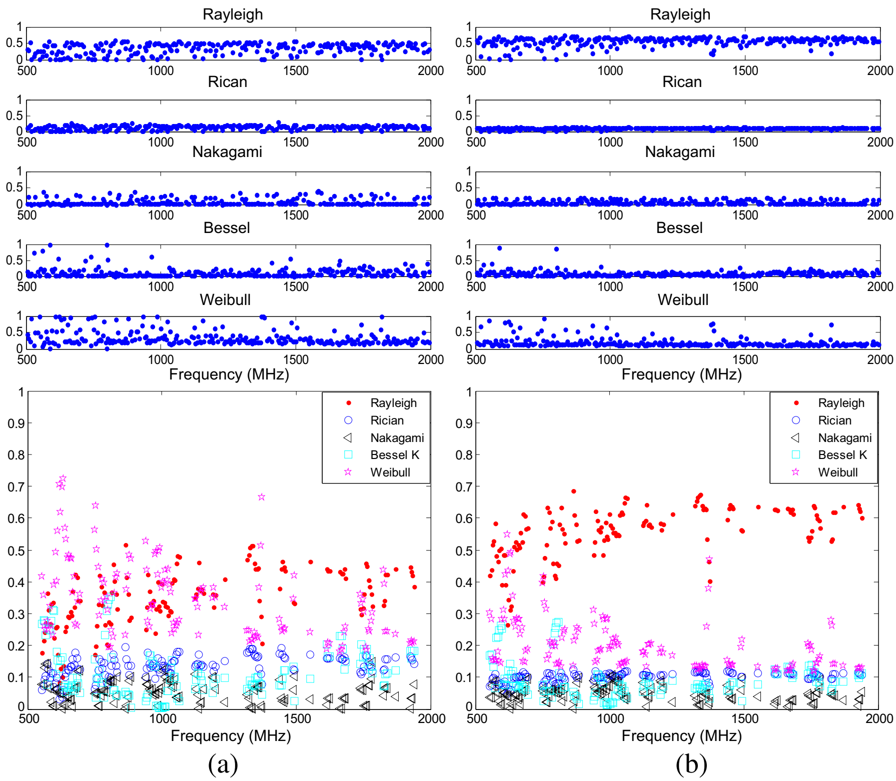


Figure 3. Comparison of (a) AIC weights and (b) MDL weights for Rayleigh, Rician, Nakagami, Bessel K and Weibull distributions based on the *load*1 data. The upper graphs correspond to the weights without frequency stirring. The lower graphs correspond to the weights with 5-MHz frequency stirring.

MHz frequency stirring is applied to the weights, and the corresponding results are shown in the lower graphs. Note that because of the frequency stirring, the peaky points at certain frequencies are averaged out. It can be seen that in the higher frequencies the Rayleigh distribution provides the best fit and that in the lower frequencies both Bessel K and Weibull distributions provide better fit. Note that, for an AIC or MDL, the best candidate may not necessarily have a weight of unity and that the best fit simply corresponds to the largest weight since the AIC test provides relative fitness. This implies that, for the unloaded RC, the Cartesian field in an undermoded RC (at lower frequencies) is more likely to be Weibull or Bessel K distributed and

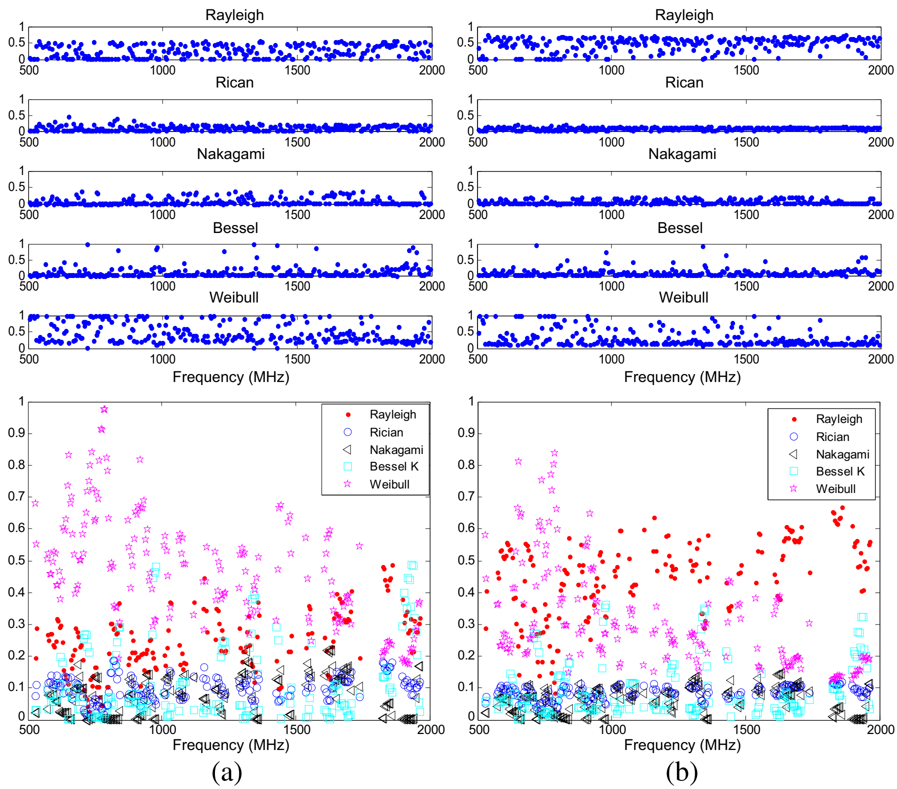


Figure 4. Comparison of (a) AIC weights and (b) MDL weights for Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions based on the *load*2 data. The upper graphs correspond to the weights without frequency stirring. The lower graphs correspond to the weights with 5-MHz frequency stirring.

that the Cartesian field in an overmoded RC (at higher frequencies) is more probable to be Rayleigh distributed.

It is shown that by loading the RC it is possible to create Rician distributed Cartesian field. According to [14], the unstirred multipath component (UMC) has the same effect as the line-of-sight (LOS) component. By locating the loads in the corners of the RC, they reduce only the scattered power not the LOS or UMC power. Hence the K-factor can be increased by loading. In order to study, the feasibility of the Rician distribution in the RC, the AIC and the MDL are applied to the *load*1 and *load*2 data. The corresponding weights are shown in Figs. 3 and 4, respectively. It is seen that even with increasing loading,

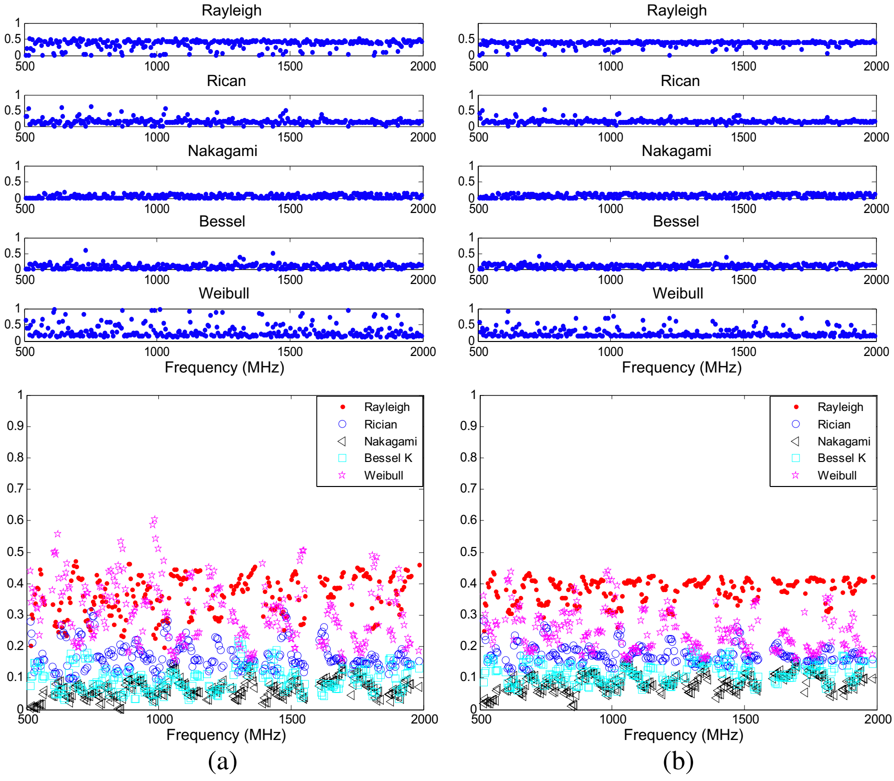


Figure 5. Comparison of (a) AICC weights and (b) MDL weights for Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions based on a platform subset of the *load*0 data. The upper graphs correspond to the weights without frequency stirring. The lower graphs correspond to the weights with 5-MHz frequency stirring.

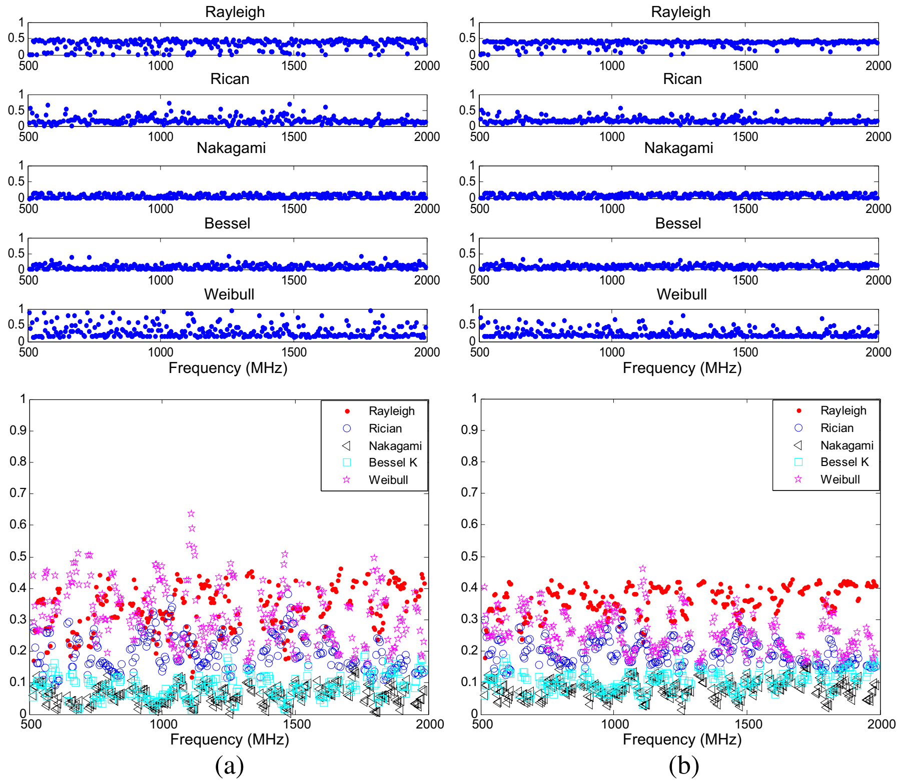


Figure 6. Comparison of (a) AICC weights and (b) MDL weights for Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions based on a platform subset of the *load*1 data. The upper graphs correspond to the weights without frequency stirring. The lower graphs correspond to the weights with 5-MHz frequency stirring.

the Rician distribution is not feasible for the RC Cartesian field. The reasons why the Rician distribution is inferior to the Rayleigh distribution are that both transmit and receive antennas are non-directive and that the platform stirring effectively reduces the potential unstirred components even with increasing loading [19]. On the other hand, the Weibull distribution shows better fit to measurements in the loaded RC.

In order to be able to observe the Rician distributed Cartesian field, one has to restrict the platform position to one and with one wall antenna, because the LOS components are different with different platform position and wall antennas [19]. Instead of doing another set of measurements with one platform position, a subset of the measured data corresponding to one platform position is selected. The corresponding reduced data is referred hereafter as a platform subset of the data (and different platform subsets have very similar statistics). Figs. 5–7 show the corresponding weights of a platform subset of *load*0, *load*1 and *load*2 data, respectively. It can be seen that the weights of the Rician distribution increases with increasing loading and that it almost becomes comparable to that of the Rayleigh distribution, which is reasonable because the corresponding K-factor, in this case, is around 1 (see Fig. 3(a) in [19]).

From Figs. 2–7, it can be seen that the feasibility frequency

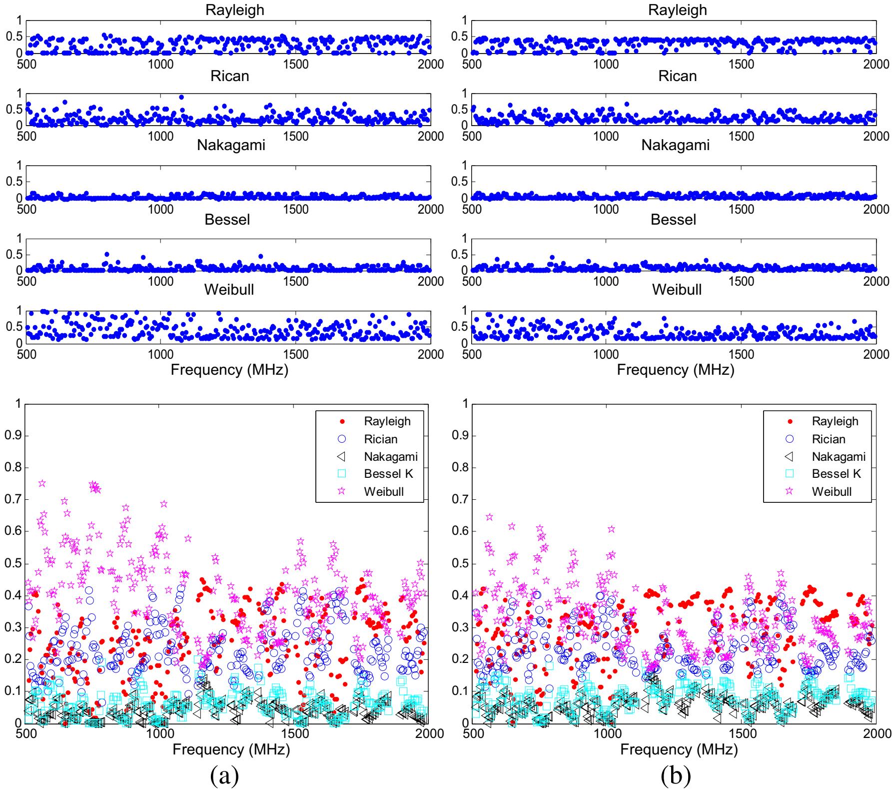


Figure 7. Comparison of (a) AICC weights and (b) MDL weights for Rayleigh, Rician, Nakagami, Bessel K, and Weibull distributions based on a platform subset of the *load*2 data. The upper graphs correspond to the weights without frequency stirring. The lower graphs correspond to the weights with 5-MHz frequency stirring.

of the Weibull distribution (i.e., the frequency above which the Weibull distribution reasonably fits the measured data) increases with increasing loading. This implies that the RC's actual LUF is affected by the loading. Comparing with the MDL weight, it can be seen, in general, that the AIC weight is slightly lower for the Rayleigh distribution and higher for the Weibull distribution. This is due to the fact that the AIC and the MDL use different penalty terms (cf. Section 2). Nevertheless, both weights indicate the same conclusions that the Cartesian field is more probably Rayleigh distributed for overmoded RC and Weibull distributed for undermoded or heavily loaded RC. Provided that platform stirring is not employed and the K-factor is equal or larger than unity, the Rician distribution can provide good fit to the measured data. The Bessel K distribution results in good fit only for unloaded and undermoded RC; once the RC is loaded it becomes inferior to the Weibull distribution. The Nakagami distribution, however, gives the worst overall performance in almost all cases.

5. CONCLUSION

In this paper, AIC and MDL are introduced to select the best approximating distribution for the field in a RC. Unlike GOF tests, AIC and MDL provides fair comparisons between different distribution candidates (with possibly different scalar parameter numbers). With the Rayleigh, Rician, Nakagami, Bessel K and Weibull distributions as the candidate set, both AIC and MDL are applied to the measured data in a RC. It is found that the Cartesian field in an undermoded RC is most fitted by the Weibull distribution and that the Rayleigh distribution approximated the Cartesian field in an overmoded RC the best. By restricting the platform position to one, it is shown that the Rician distribution provides better approximation with increasing loading. It is also found that the Weibull distribution provides better fits to the measured data with larger loading. The intuitive explanation for this is that with increasing loading the LUF (i.e., the transition from undermoded RC to overmoded RC) increases.

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