ABSOLUTE ADAPTIVE CS MODEL AND MODIFIED STRONG TRACKING UNSCENTED FILTER FOR HIGH MANEUVERING TARGET TRACKING

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Abstract—Absolute adaptive *current* statistical (AACS) model and modified strong tracking unscented filter (MSTUF) are proposed for maneuvering target tracking (MTT) under nonlinear measurement in this paper. The key point of the AACS model is to associate the *instantaneous acceleration variance* with some elements of state covariance matrix by constructing acceleration increment models of the acceleration limit and acceleration mean in the CS model, while the maneuvering frequency can adjust itself according to the change of the measurement residual. MSTUF is proposed for high maneuver tracking under nonlinear measurement by incorporating the modified strong tracking filter (STF) into the unscented filter (UF). Since the state covariance, process noise covariance and maneuvering frequency can adjust themselves jointly according to the residual, the proposed algorithm, called the AACS-MSTUF, has a good performance on both maneuver and non-maneuver. Simulation results indicate that the overall performance of the proposed algorithm is better than the interacting multiple-model unscented filter (IMM-UF), UF and original strong tracking unscented filter (STUF) based on the CS model (CS-STUF) when considering tracking accuracy, stability, convergence and computational complexity.

1. INTRODUCTION

Maneuvering target tracking (MTT) has been an important and challenging problem for many years in the field of signal processing [1]. The main problem of MTT is how to deal with the unknown fast change in the maneuvering acceleration. Many techniques and

methods [2–5] have been suggested to solve the problem during last years. Acceleration modeling techniques [6], input estimation (IE) techniques [7] and multiple-model (MM) methods [8, 9] are three main approaches.

Singer suggested a zero-mean and time-correlated acceleration model, which has been one of the foundations in the problem of state estimation for maneuvering targets. Zhou and Kumar suggested a mean-adaptive acceleration model, called the *current* statistical (CS) model [10], which is recognized as an effective model for tracking maneuvering targets. However, the performance of these models often depends on the prior parameters of maneuvering targets, such as the maneuvering frequency and acceleration limits, etc.. The tracking performance will be seriously affected by the inappropriate value of the prior parameters [11]. IE techniques, which are not reliable on the prior information about maneuvering acceleration, consider the maneuvering acceleration as an unknown input and estimate it with least square method, but they need additional effort for the detection of acceleration. Khaloozadeh and Karsaz suggested the modified input estimation (MIE) [12], which considers the unknown acceleration as a new augmented component of the target state and estimate it with Kalman filter. New filters were incorporated into the MIE to improve its performance on high maneuver in [13–15]. However, different tracking algorithms based on MIE are actually different filters based on constant-acceleration (CA) model without acceleration noise. It's impossible to attain better tracking performance on high maneuver than tracking algorithms based on the CA model. Interacting multiple-model (IMM) algorithm is considered as a good compromise between the tracking performance and the computational complexity [16], but the tracking accuracy still depends on the match degree of pre-designed models with the actual situation of a maneuvering target [17]. In addition, IMM algorithm usually suffers from the competitions among different models and large computational load imposed by using multiple sub-filters [18].

To solve the problems mentioned above, this paper converts the CS model into an absolute adaptive CS (AACS) model by constructing the acceleration increment models of the acceleration limits and acceleration mean, while the adaptation method of maneuvering frequency is introduced. Considering the nonlinear relation between radar measurements and target states, modified strong tracking unscented filter (MSTUF) is put forward to improve the state estimation performance of UF on MTT.

The main contribution of this paper is to propose the AACS model and MSTUF algorithm. In the AACS model, the process

noise of the CS model is associated with the state covariance and the maneuvering frequency of the CS model can adjust itself adaptively by comparing the measurement residual with the measurement prediction error. In the MSTUF algorithm, the state covariance consists with the residuals in different measurement channels better. Finally, the model parameter, process noise and state covariance can match the measurement residual rapidly, which leads to the superiority of the proposed algorithm.

The paper is organized as follows: in Section 2, the CS model is reviewed; in Section 3, the AACS model and MSTUF algorithm are presented in detail; in Section 4, the proposed algorithm, CS-UF, CS-STUF and IMM-UF are compared; in Section 5, conclusion remarks are made.

2. THE REVIEW OF CS MODEL

The state equation of the CS model and the measurement equation in two-dimensional cases are described as follows, respectively:

$$\mathbf{X}_{k+1} = \mathbf{F}\mathbf{X}_k + \mathbf{U}\bar{\mathbf{a}}_k + \mathbf{W}_k \tag{1}$$

$$\mathbf{Z}_{k+1} = \begin{bmatrix} \sqrt{x_{k+1}^2 + y_{k+1}^2} \\ \arctan\left(\frac{y_{k+1}}{x_{k+1}}\right) \end{bmatrix} + \mathbf{V}_{k+1} \tag{2}$$

where $\mathbf{X}_k = [x_k, \dot{x}_k, \ddot{x}_k, y_k, \dot{y}_k, \ddot{y}_k]^T$ is target state vector, $\bar{\mathbf{a}}_k = [\bar{a}_k^x, \bar{a}_k^y]^T$ the acceleration mean vector at time kT (k is the time index and T the sample interval), $\bar{\mathbf{a}}_k$ the expectation of $\mathbf{a}_k$; the variables (x_k, y_k), $(\dot{x}_k, \dot{y}_k)$ and $(\ddot{x}_k, \ddot{y}_k)$ represent the target position, velocity, acceleration in the x and y coordinate, respectively; $\mathbf{F} = \text{blkdiag}(\mathbf{F}_x, \mathbf{F}_y)$ is the state transition matrix, $\mathbf{U} = \text{blkdiag}(\mathbf{U}_x, \mathbf{U}_y)$ the acceleration mean input matrix, $\text{blkdiag}(\cdot)$ the construction of a block diagonal matrix from input arguments, $\mathbf{W}_k$ the process noise with covariance matrix $\mathbf{Q}_k$, $\mathbf{Z}_{k+1} = [r_{k+1}, \theta_{k+1}]^T$ the radar measurement vector comprised of range r and angle θ, and $\mathbf{V}_{k+1}$ the Gaussian measurement noise with covariance matrix $\mathbf{R}$. The expression for $\mathbf{F}_x$, $\mathbf{F}_y$, $\mathbf{U}_x$, $\mathbf{U}_y$ and $\mathbf{Q}_k$ are

$$\mathbf{F}_x = \mathbf{F}_y = \begin{bmatrix} 1 & T & (\alpha T - 1 + e^{-\alpha T})/\alpha^2 \\ 0 & 1 & (1 - e^{-\alpha T})/\alpha \\ 0 & 0 & e^{-\alpha T} \end{bmatrix},$$

$$\mathbf{U}_x = \mathbf{U}_y = \begin{bmatrix} T^2/2 - (\alpha T - 1 + e^{-\alpha T})/\alpha^2 \\ T - (1 - e^{-\alpha T})/\alpha \\ 1 - e^{-\alpha T} \end{bmatrix},$$

$\mathbf{Q}_k = 2\alpha\sigma_k^2 q_{cs}$, where α is the maneuvering frequency. q_{cs} can be referred to [10], and instantaneous acceleration variance σ_k^2 can be expressed as

$$\sigma_k^2 = \begin{cases} \frac{4-\pi}{\pi}\left(a_{\max} - \bar{a}_{k+1}\right)^2 & \bar{a}_k \geq 0 \\ \frac{4-\pi}{\pi}\left(a_{-\max} + \bar{a}_{k+1}\right)^2 & \bar{a}_k < 0 \end{cases} \tag{3}$$

where $a_{-\max}$ is the preset negative acceleration limit, not necessarily equal to positive acceleration limit $a_{\max}$. A key underlying assumption of the CS model is that $\bar{a}_{k+1} \triangleq E[a_{k+1}|\mathbf{Z}^k] \approx \hat{\ddot{x}}_k$, which is stated explicitly in [6]. As can be seen from Equation (3), when tracking the non-maneuvering or low maneuvering targets, too large process noise will degrade the tracking accuracy; when the actual maneuvering acceleration exceeds the preset acceleration limits, the tracking performance will deteriorate seriously [19]. In addition, the preset maneuvering frequency cannot adjust to the change of maneuvering types.

3. THE PROPOSED AACS-MSTUF ALGORITHM

3.1. The Proposed AACS Model

In order to eliminate the shortcomings of the CS model, the AACS model is proposed to make a better match between the process noise and the actual maneuver by constructing the acceleration increment models of the acceleration limits and acceleration mean in the CS model.

Taking the acceleration in the x coordinate as an instance, according to the Taylor series expansion of the target acceleration $a(t)$:

$$\begin{aligned} a(t) = {} & a(t_0) + \frac{1}{1!}a^{(1)}(t_0)(t - t_0) \\ & + \frac{1}{2!}a^{(2)}(t_0)(t - t_0)^2 + \ldots + \frac{1}{n!}a^{(n)}(t_0)(t - t_0)^n \end{aligned} \tag{4}$$

and its discrete-time equivalent expression (let $t_0 = kT$, $t = (k+1)T$),

$$a_{k+1} = a_k + Ta_k^{(1)} + \frac{T^2}{2!}a_k^{(2)} + \frac{T^3}{3!}a_k^{(3)} + \ldots + \frac{T^n}{n!}a_k^{(n)} \tag{5}$$

where $a_k^{(n)}$ is nth-order derivative of $a(t)$ at kT time instant, and it can be concluded that the possible acceleration limits can be predicted from the acceleration and its higher-order derivative at the previous time instant. Thus, based on Equation (5), the prediction model of the acceleration limits can be expressed as

$$a_{\pm\max} = \ddot{x}_k + \Delta\ddot{x}_k \tag{6}$$

where $\Delta \ddot{x}_k = T a_k^{(1)} + \frac{T^2}{2!} a_k^{(2)} + \ldots + \frac{T^n}{n!} a_k^{(n)}$ is defined as *acceleration-increment* in this paper. The assumption $\bar{a}_{k+1} \triangleq E[a_{k+1}|\mathbf{Z}^k] \approx \hat{\ddot{x}}_k$ in the CS model can be replaced by a more accurate version based on Equation (5):

$$\bar{a}_{k+1} \overset{\Delta}{=} E[a_{k+1}|\mathbf{Z}^k] = \hat{\ddot{x}}_k + \Delta \hat{\ddot{x}}_k \tag{7}$$

Equation (3) can be also modified as

$$\sigma_k^2 = \frac{4-\pi}{\pi} \left(\tilde{\ddot{x}}_k + \Delta \tilde{\ddot{x}}_k \right)^2 \tag{8}$$

where $\tilde{\ddot{x}}_k$ and $\Delta \tilde{\ddot{x}}_k$ are the estimation errors of acceleration and acceleration increment. Because $a_k^{(1)}$, $a_k^{(2)}$, ..., $a_k^{(n)}$ cannot be estimated directly due to the state dimension limitation in the CS model, $\Delta \tilde{\ddot{x}}_k$ is approximated with the state estimation errors which can be obtained from the filter output based on some assumptions in this paper. Ignoring the higher-order derivative than $a_k^{(1)}$, $\Delta \tilde{\ddot{x}}_k = T \tilde{a}_k^{(1)}$ can be derived. Assuming $a^{(1)}(t_0)$ can be approximated as the function of $\mathbf{x}$, where $\mathbf{x} = [\dot{x}(t_0), \ddot{x}(t_0)]^T$:

$$a^{(1)}(t_0) \approx f(\mathbf{x}) = \frac{2}{(t-t_0)^2} [\dot{x}(t_0) - \ddot{x}(t_0)(t-t_0)] \tag{9}$$

According to the idea of the optimal stochastic linearization [20], when the variable $\mathbf{x}$ of the function $f(\mathbf{x})$ is random, it's better to express $f(\mathbf{x})$ as a function that is close to $f(\mathbf{x})$ in some probabilistic sense. Since $\dot{x}(t_0)$ and $\ddot{x}(t_0)$ are model states which are random variables, $a^{(1)}(t_0)$ and its estimation error $\tilde{a}^{(1)}(t_0)$ should be expressed as

$$a^{(1)}(t_0) \approx \hat{a}^{(1)}(t_0) + \mathbf{B}(\mathbf{x} - \hat{\mathbf{x}}) \quad \text{and} \quad \tilde{a}^{(1)}(t_0) \approx \mathbf{B}\tilde{\mathbf{x}} \tag{10}$$

where $\hat{a}^{(1)}(t_0) = f(\hat{\mathbf{x}})$ is the estimate of $a^{(1)}(t_0)$, $\mathbf{B} = E[(a^{(1)}(t_0) - \hat{a}^{(1)}(t_0))\tilde{\mathbf{x}}^T] E[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^T]^{-1}$ represents the statistical correlation between $a^{(1)}(t_0)$ and $\mathbf{x}$. According to Equation (9), the estimation error of $a^{(1)}(t_0)$ can be approximated by the estimation error of $\mathbf{x}$:

$$a^{(1)}(t_0) - \hat{a}^{(1)}(t_0) \approx \frac{2}{(t-t_0)^2} [\tilde{\dot{x}}(t_0) - \tilde{\ddot{x}}(t_0)(t-t_0)]$$

$$= \frac{2}{(t-t_0)^2} [1 - (t-t_0)] \tilde{\mathbf{x}} \tag{11}$$

Let $t = (k+1)T$, $t_0 = kT$, $\tilde{a}^{(1)}(t_0)$ can be expressed as

$$\tilde{a}_k^{(1)} \approx \mathbf{B}\tilde{\mathbf{x}}_k \tag{12}$$

where $\tilde{\mathbf{x}}_k = [\dot{\tilde{x}}_k, \ddot{\tilde{x}}_k]^T$, $\mathbf{B} = [2/T^2 \quad -2/T]E[\tilde{\mathbf{x}}_k\tilde{\mathbf{x}}_k^T]E[\tilde{\mathbf{x}}_k\tilde{\mathbf{x}}_k^T]^{-1} = [2/T^2 \quad -2/T]$. Equation (8) can be approximated as

$$\sigma_k^2 = \frac{4-\pi}{\pi}\left(\ddot{\tilde{x}}_k + \tilde{a}_k^{(1)}T\right)^2$$

$$\approx \frac{4-\pi}{\pi}\left(E[\ddot{\tilde{x}}_k\ddot{\tilde{x}}_k^T] + \mathbf{B}E[\tilde{\mathbf{x}}_k\tilde{\mathbf{x}}_k^T]\mathbf{B}^T T^2 + \mathbf{B}E[\tilde{\mathbf{x}}_k\ddot{\tilde{x}}_k^T]T + E[\ddot{\tilde{x}}_k\tilde{\mathbf{x}}_k^T]\mathbf{B}^T T\right) (13)$$

where $E[\ddot{\tilde{x}}_k\ddot{\tilde{x}}_k^T] = \mathbf{P}_k(\ddot{x}, \ddot{x})$, $E[\tilde{\mathbf{x}}_k\tilde{\mathbf{x}}_k^T] = [\mathbf{P}_k(\dot{x}, \dot{x}), \mathbf{P}_k(\dot{x}, \ddot{x}); \mathbf{P}_k(\ddot{x}, \dot{x}), \mathbf{P}_k(\ddot{x}, \ddot{x})]$, $E[\tilde{\mathbf{x}}_k\ddot{\tilde{x}}_k^T] = [\mathbf{P}_k(\dot{x}, \ddot{x}), \mathbf{P}_k(\ddot{x}, \ddot{x})]^T$, $E[\ddot{\tilde{x}}_k\tilde{\mathbf{x}}_k^T] = [\mathbf{P}_k(\ddot{x}, \dot{x}), \mathbf{P}_k(\ddot{x}, \ddot{x})]$, $\mathbf{P}_k(\dot{x}, \dot{x})$, $\mathbf{P}_k(\dot{x}, \ddot{x})$, $\mathbf{P}_k(\ddot{x}, \dot{x})$ and $\mathbf{P}_k(\ddot{x}, \ddot{x})$ are the corresponding elements of the state covariance matrix.

As can be seen from Equation (13), the instantaneous acceleration variance is associated absolutely with the elements of state covariance matrix based on some assumptions and approximations in the AACS model. If the filter can evaluate state estimation errors reasonably, the process noise will adjust itself adaptively to the maneuver.

3.2. The Proposed MSTUF Algorithm

Although the unscented filter (UF) has been broadly used to solve the problem of nonlinear state estimation, its performance may be seriously shrunk if the target maneuvers because the prediction covariance and the gains of the UF cannot match the variation of the residuals. A strong tracking filter (STF) was proposed by Zhou [21], which makes the output residuals approximate to Gaussian white noise by selecting appropriate time varying gains online. Compared with some conventional filters, this filter has a stronger robustness for mismatching model parameters, a stronger capability for estimating target states with sudden changes and a moderate computational complexity, etc. Strong Tracking Unscented Filter (STUF) is introduced by incorporating the STF into the UF in literature [22]. However, the residual variance sum, which is used to compute fading factors in the STF and STUF, has not explicit physical meanings under the radar measurement comprised of ranges and angles. Furthermore, the absolute value of the angle residual is generally far less than that of the range residual, which leads to the insensitivity of fading factors to the change of angle residual, indicating the high maneuver.

In order to solve the problem, a new computational method of the fading factors is introduced and the STUF using new fading factors is called *MSTUF*. The main idea of the new method is to select the larger ratio as the suboptimal fading factor after obtaining the ratios of the residual variance to the error variance of measurement prediction

in the range channel and angle channel when excluding the influence of the measurement noise variance. In the following paragraphs, the MSTUF algorithm is described in detail:

Step1: State prediction and sampling:

$$\hat{\mathbf{X}}_{k+1|k} = \mathbf{F}\hat{\mathbf{X}}_k, \quad \mathbf{P}_{k+1|k} = \mu_{k+1}(\mathbf{F}\mathbf{P}_k\mathbf{F}^T + \mathbf{Q}_k) \qquad (14)$$

where μ_{k+1} is the single suboptimal fading factor, used to force the prediction covariance based on the model to accord with the residual, and $\hat{\mathbf{X}}_{k+1|k}$ and $\mathbf{P}_{k+1|k}$ are state prediction and corresponding covariance, respectively. The sigma points and corresponding weights can be obtained:

$$\begin{cases} \mathbf{x}_{k+1|k}^0 = \hat{\mathbf{X}}_{k+1|k} \\ \mathbf{x}_{k+1|k}^i = \hat{\mathbf{X}}_{k+1|k} + \left(\sqrt{(n+\kappa)\mathbf{P}_{k+1|k}}\right)_i \quad i = 1, \ldots, n \\ \mathbf{x}_{k+1|k}^{i+n} = \hat{\mathbf{X}}_{k+1|k} - \left(\sqrt{(n+\kappa)\mathbf{P}_{k+1|k}}\right)_i \quad i = 1, \ldots, n \end{cases}$$

$$\begin{cases} W_{(m)}^0 = \kappa/(n+\kappa) \\ W_{(c)}^0 = \kappa/(n+\kappa) + 1 - \lambda^2 + \beta \\ W_{(m)}^i = W_{(c)}^i = 1/\left[2(n+\kappa)\right] \quad i = 1, \ldots, 2n \end{cases} \qquad (15)$$

where n is the state dimension; $(\cdot)_i$ represents the ith row or column; $W_{(m)}^i$ and $W_{(c)}^i$ are the weights for sample mean and sample covariance; λ and β are the design parameters that can appropriately control the higher order errors; $\kappa = \lambda^2(n+\upsilon) - n$, υ is the parameter that ensures the semi-positive definiteness of $(n+\kappa)\mathbf{P}_{k+1|k}$. Guidelines for the choice of λ, β and κ can be found in literature [23].

Step2: Measurement prediction:

$$\mathbf{z}_{k+1|k}^i = h(\mathbf{x}_{k+1|k}^i), \quad \hat{\mathbf{Z}}_{k+1|k} = \sum_{i=0}^{2n} W_{(m)}^i \mathbf{z}_{k+1|k}^i,$$

$$\mathbf{P}_{zz} = \mu_{k+1} \sum_{i=0}^{2n} W_{(c)}^i (\mathbf{z}_{k+1|k}^i - \hat{\mathbf{Z}}_{k+1|k})(\cdot)^T + \mathbf{R}_{k+1} \qquad (16)$$

Step3: State and state covariance updating

$$\hat{\mathbf{X}}_{k+1} = \hat{\mathbf{X}}_{k+1|k} + \mathbf{K}_{k+1}(\mathbf{Z}_{k+1} - \hat{\mathbf{Z}}_{k+1|k}),$$

$$\mathbf{P}_{k+1} = \mathbf{P}_{k+1|k} - \mathbf{K}_{k+1}\mathbf{P}_{zz}\mathbf{K}_{k+1}^T$$

$$\mathbf{K}_{k+1} = \mathbf{P}_{xz}\mathbf{P}_{zz}^{-1} \quad \mathbf{P}_{xz} = \sum_{i=0}^{2n} W_{(c)}^i (\mathbf{x}_{k+1|k}^i - \hat{\mathbf{X}}_{k+1|k})(\mathbf{z}_{k+1|k}^i - \hat{\mathbf{Z}}_{k+1|k})^T \qquad (17)$$

where $\mathbf{K}_{k+1}$ is the filter gain.

The proposed computational method of μ_{k+1} is

$$\mu_{k+1} = \begin{cases} c_{k+1} & c_{k+1} > 1 \\ 1 & c_{k+1} \leq 1 \end{cases} \tag{18}$$

where

$$c_{k+1} = \max \left\{ \frac{[\text{diag}(\mathbf{N}_{k+1})]_j}{[\text{diag}(\mathbf{M}_{k+1})]_j} \right\} \Bigg|_{j=1,2,\ldots,m}, \quad \mathbf{N}_{k+1} = \mathbf{S}_{k+1} - \gamma \mathbf{R}_{k+1}$$

$$\mathbf{M}_{k+1} = \sum_{i=0}^{2n} W_{(c)}^i (\hat{\mathbf{z}}_{k+1}^i - \hat{\mathbf{Z}}_{k+1|k})(\cdot)^T \quad \mathbf{S}_{k+1} = \begin{cases} \mathbf{v}_1 \mathbf{v}_1^T & k=0 \\ \frac{\rho \mathbf{S}_k + \mathbf{v}_{k+1} \mathbf{v}_{k+1}^T}{\rho+1} & k \geq 1 \end{cases},$$

$\text{diag}(\cdot)$ represents the vector comprised of diagonal elements of the matrix, $[\cdot]_j$ the jth element of the vector, m the measurement dimension, $\mathbf{S}_{k+1}$ the residual second-order moment, $\mathbf{v}_{k+1}$ the measurement residual vector, $0 < \rho \leq 1$ a forgetting factor, and $\gamma \geq 1$ a softening factor which can make the value of state estimation more smooth. Since the new fading factor reflects the higher maneuver between range channel and angle channel, the MSTUF algorithm will be more reliable when coping with the high maneuver under nonlinear radar measurement.

3.3. The Adaptation Method of Maneuvering Frequency

According to the idea used in the computational method of new fading factor, the high maneuver detection function is established as

$$D_k = \max \left\{ \frac{[\text{diag}(\mathbf{v}_k \mathbf{v}_k^T)]_i}{[\text{diag}(\mathbf{P}_{zz})]_i} \right\} \Bigg|_{i=1,2,\ldots,m} \tag{19}$$

The adaptation method of the maneuvering frequency is

$$\alpha = \begin{cases} \alpha_0 D_k & D_k \geq M \\ \alpha_0 & D_k < M \end{cases} \tag{20}$$

where α_0 is the initial maneuvering frequency, M is the high maneuver detection threshold, generally taking 3 [22].

4. SIMULATION RESULTS

To validate the proposed model and tracking algorithm, the IMM-UF (CV-CA-CA) algorithm [17], UF algorithm and STUF algorithm based on the CS model (CS-UF and CS-STUF) are compared with the proposed algorithm in terms of real time tracking accuracy and computational complexity. In the simulation, the sampling interval

$T = 1$ s and sampling 300 and 180 times in the Scenario 1 and Scenario 2, respectively. The measurement accuracy of range and angle are 50 m and $0.1°$, respectively. The parameters used in CS-UF, CS-STUF and IMM-UF are given as follows: maneuvering frequency and acceleration limits in the CS model are $\alpha = 0.06$ and $a_{\pm \max} = \pm 90$ m/s^2, respectively; transition probabilities in IMM-UF are $\pi_{ii} = 0.9$, $\pi_{ij} = 0.05$, $i, j = 1, 2, 3$, $i \neq j$. The root mean squared error (RMSE) of the target position at time k and the mean position estimation error (Error) at all sampling times are defined as in Equations (21) and (22), respectively:

$$\text{RMSE}(k) = \left\{ \frac{1}{M} \sum_{i=1}^{M} \left\| \mathbf{x}^i(k) - \hat{\mathbf{x}}^i(k) \right\|_2^2 \right\}^{1/2} \qquad (21)$$

$$\text{Error} = \frac{1}{N} \sum_{k=1}^{N} \text{RMSE}(k) \qquad (22)$$

where $M = 100$ is the number of times of Monte Carlo simulation, N the total number of samples, and $\mathbf{x}^i(k)$ and $\hat{\mathbf{x}}^i(k)$ are the actual and estimated target positions at time k. The RMSE and Error of the velocity and acceleration are defined in the same way.

The radar position is (0 km, 0 km). In the Scenario 1 and Scenario 2, the target moves from position (50 km, -5 km) with initial velocity (280 m/s, 12 m/s) and position (10 km, 8 km) with initial velocity (426 m/s, 0 m/s). Figure 1 and Figure 3 give the target trajectories in the Scenario 1 and Scenario 2, respectively. The actual
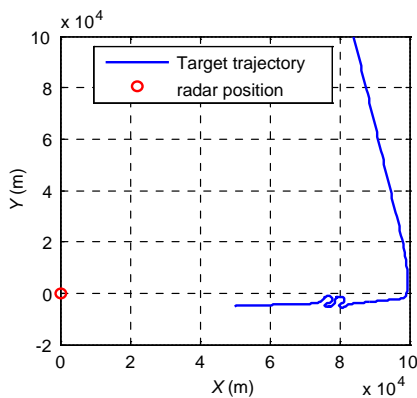

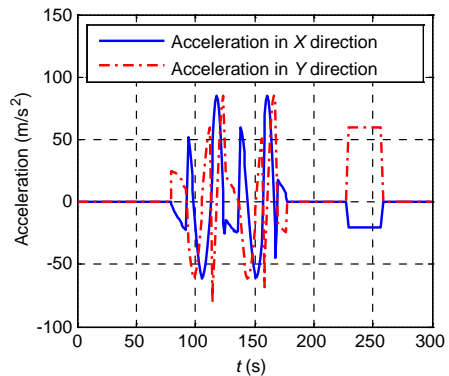
Figure 1. True target trajectory in the Scenario 1.

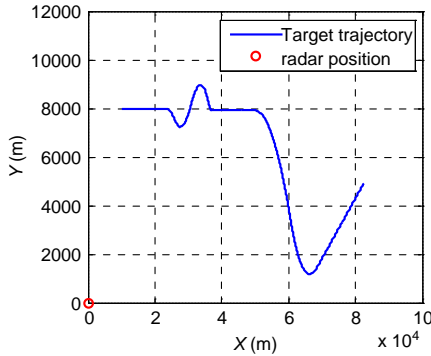**Figure 2.** True target acceleration in the Scenario 1.

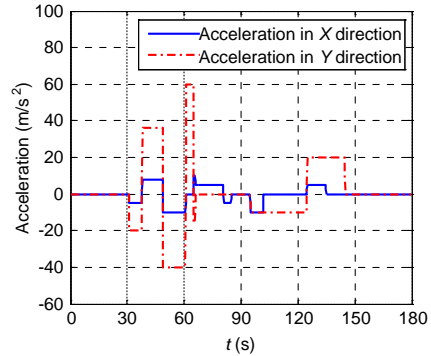**Figure 3.** True target trajectory in the Scenario 2.

Figure 4. True target acceleration in the Scenario 2.
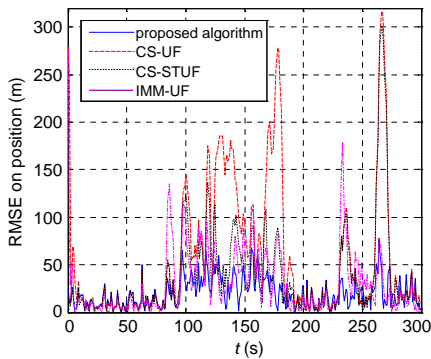

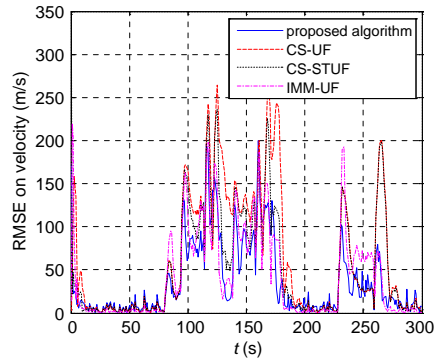**Figure 5.** RMSE on position.

Figure 6. RMSE on velocity.

accelerations that show the maneuvers of the target in the Scenario 1 and Scenario 2 are described in Figure 2 and Figure 4, respectively.

4.1. Target Tracking Performance Comparison in Scenario 1

Figure 5, Figure 6 and Figure 7 show RMSE of estimated position, velocity and acceleration corresponding to the four algorithms in the Scenario 1.

During non-maneuvering (1∼80 s, 181∼230 s and 261∼300 s), the stable tracking accuracy of IMM-UF is a little bit higher than other algorithms. This is because that CV model in IMM-UF can match the non-maneuver better than other algorithms. The stable tracking accuracy of the proposed algorithm is a little bit lower than other algorithms because the process noise is more sensitive to the change of state estimation errors.
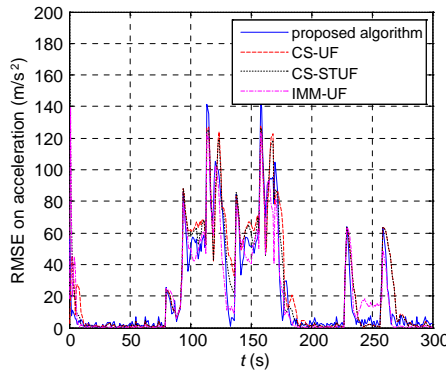
Figure 7. RMSE on acceleration.

Table 1. Tracking performance comparison.

	Position Error/m	Velocity Error/(m/s)	Acceleration Error/(m/s^2)	Consumption Time/s
Proposed algorithm	20.1109	37.2975	22.6252	0.1149
CS-UF	60.0575	64.3813	26.7989	0.0972
CS-STUF	37.7247	53.3776	26.6610	0.1014
IMM-UF	30.5977	42.9384	21.5659	0.2279

Whenever the maneuver starts (81st second and 231st second) or stops (180thsecond and 260th second), it's obvious that the proposed algorithm has the lowest error peak and the fastest convergence. The main reason is that the state covariance and process noise variance can adjust themselves jointly according to the measurement residual. But in the IMM-UF, the switching from non-maneuver model to maneuver model is always behind the maneuver, which results in the highest error peak. CS-STUF does not have better performance on CA turn maneuver than CS-UF due to the fading factor's insensitivity to the change of angle residual.

During the continuous anomalous high maneuver, the tracking accuracy of proposed algorithm is the highest and most stable. During the stable CA turn motion, the proposed algorithm has the similar tracking accuracy as CS-UF and CS-STUF while IMM-UF has the lowest tracking accuracy due to the model competition.

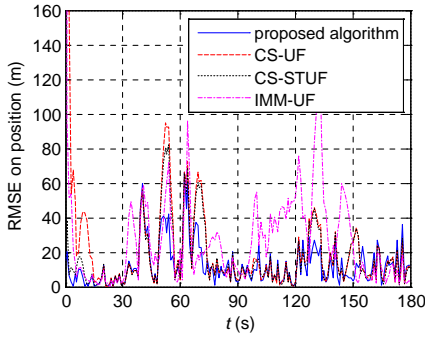Table 1 lists the consumption time and mean estimation errors
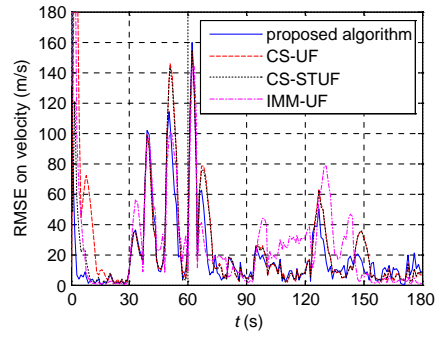
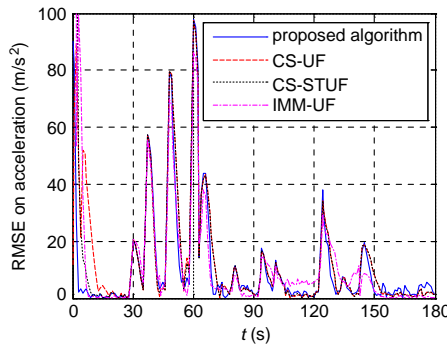**Figure 8.** RMSE on position.   **Figure 9.** RMSE on velocity.

Figure 10. RMSE on acceleration.

(Errors) on position, velocity and acceleration during the observation time for the proposed algorithm, CS-UF, CS-STUF and IMM-UF. When comparing with CS-UF and CS-STUF, the proposed algorithm does much better in tracking stability and accuracy while it doesn't need the prior information on target maneuvering acceleration. When comparing with IMM-UF, the proposed algorithm not only tracks the maneuvering target more stable and more accurate on position and velocity but also reduces about 50% computation time.

4.2. Target Tracking Performance Comparison in Scenario 2

Figure 8, Figure 9 and Figure 10 show RMSE of estimated position, velocity and acceleration corresponding to the four algorithms in the Scenario 2. Table 2 lists the consumption time and the Errors on position, velocity and acceleration.

Table 2. Tracking performance comparison.

	Position Error/m	Velocity Error/(m/s)	Acceleration Error/(m/s^2)	Consumption Time/s
Proposed algorithm	12.7621	20.9085	11.3392	0.0682
CS-UF	23.9062	27.6869	13.5137	0.0578
CS-STUF	16.4795	26.0776	16.2634	0.0602
IMM-UF	28.1577	29.4595	13.5471	0.1343

In this scenario, it's clear that the proposed algorithm still has the best overall performance while IMM-UF has the worst tracking performance, which is mainly due to the model competition during 95 s∼145 s. Of course, the better performance of CS-UF and CS-STUF than that of IMM-UF is based on the reasonable prior acceleration limits. The stable tracking accuracy of the proposed algorithm is a little lower than other algorithms during non-maneuvering, which is the same as Scenario 1.

5. CONCLUSION

An AACS-MSTUF algorithm comprised of AACS model and MSTUF algorithm have been proposed for high maneuvering target tracking in this paper. The AACS model associates the process noise with the state covariance and makes the maneuvering frequency adaptive. The MSTUF algorithm uses a new computational method of fading factor to provide better state estimation when the target maneuvers. Simulation results show the superiority of the proposed algorithm, which can be attributed to the joint adjustment of the state covariance, process noise covariance and maneuvering frequency to the change of residuals in range and angle channels. The idea that associate the process noise with the state covariance can be also used in other nonlinear dynamic models to make the process noise adaptive, such as reentry target models. The further research will focus on it.

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