PLANE WAVE BEAM PRODUCED BY AN EXCLUSIVE MEDIUM

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Abstract—In this work, a special indefinite medium is used to produce a plane wave beam due to its linear type equifrequency contour resulted by the zero parameter in the permeability tenser. Only one transmitting mode exists in such a medium, composing the plane wave beam propagating along the zero parameter direction. Parameters along the field vectors of the wave are 1, enabling the wave propagation in the medium like in the air, which is totally different from the zero-index medium. A split ring resonator (SRR)-arrays based metamaterial is used to realize such a medium and produce an approximate plane wave beam experimentally. This idea proposes a feasible way to generate a near field plane wave beam with certain beamwidth, which can also be applied in cylindrical coordination or 3D cases.

1. INTRODUCTION

In the family of anisotropic media, indefinite medium is a rising star due to its negative refraction property and the capability of transferring an evanescent wave into a propagation mode [1–8]. The term indefinite implies that the principle items in the permittivity or permeability tensor have different signs, which results in a strong anisotropy and a hyperbolic EFC (equifrequency contour) in an indefinite medium [9]. It is the hyperbolic EFC that enables the all angle negative refraction and existence of the evanescent wave as a propagation mode. Based on

these two properties, a hyperlens can be realized with a subwavelength imaging ability in the far field [10–13]. This tells us an idea that new properties will come out by increasing the anisotropy of a medium. If we decrease the negative permeability (or permittivity) of the indefinite medium to almost zero, the medium owns an extremely strong anisotropy, by which we hope to create surprising properties. Therefore, we do the following work. Considering a uniaxially anisotropic magnetic medium with an isotropic permittivity of $\varepsilon = 1$ and an indefinite permeability tensor:

$$\mu = \mu_0 \begin{pmatrix} \mu_{xx} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

where μ_{xx} is negative. The EFC function of the indefinite medium described by Eq. (2) is hyperbolic [1, 2]:

$$\frac{k_x^2}{\mu_{zz}} + \frac{k_z^2}{\mu_{xx}} = \frac{\omega^2}{c^2}\varepsilon. \tag{2}$$

If we reduce the value of μ_{xx} to zero, the hyperbola is squeezed into a single line. In this case, Eq. (2) degenerates to Eq. (3), as shown
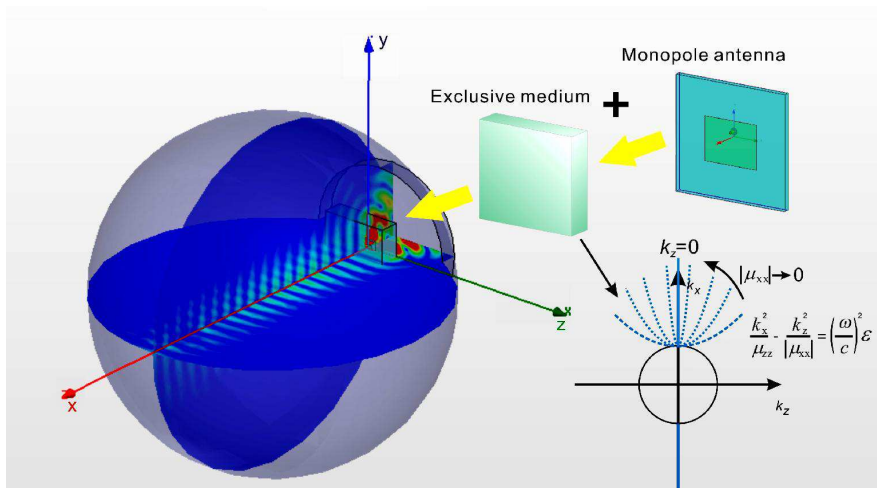

Figure 1. An approximate plane wave beam produced by a monopole antenna covered by an exclusive medium. When the value of μ_{xx} is approaching to zero, the hyperbola is squeezed into a single line, which excludes all transmitting mode except for $k_z = 0$.

below:

$$k_z^2 = \mu_{xx} \left(\frac{\omega^2}{c^2} \varepsilon - \frac{k_x^2}{\mu_{zz}} \right)$$

$$k_z = 0$$

(3)

which means that the exclusive transmitting mode allowed is $k_z = 0$ and the wave can only propagate along the optical axis direction. Thus, we define such a special indefinite medium as an exclusive medium. Accordingly, if a monopole antenna is covered by an exclusive medium, the original spherical wave radiated from the antenna will be forced to propagate along its zero parameter (permittivity or permeability) direction as a plane wave ray, as shown in Fig. 1.

2. THEORY

Let us study the transmission and reflection property of an electromagnetic (EM) wave at the interface between an exclusive medium and the air. As shown in Fig. 2, due to the linear type EFC, any incident EM wave with the mode of $k_z \neq 0$ can not transmit through the medium, and is reflected totally. By contrast, if the source
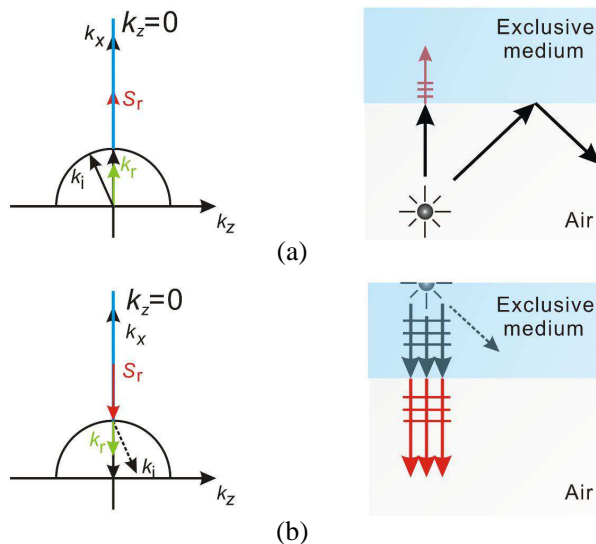

(a)

(b)

Figure 2. The transmission and reflection property of an EM wave at the interface between the exclusive medium and the air. (a) Source in the air. (b) Source in the exclusive medium.

is merged in the exclusive medium, wave produced by any type of the source will be remodeled into the mode of $k_z = 0$ as soon as they radiate from the source, which can be utilized to produce a plane wave beam. Detailed derivation can be found in the supplementary. As a result, the only mode allowed in the exclusive medium is $k_z = 0$, which shows a similar property with the zero-index material. However, wave with $k_z = 0$ propagates in the exclusive medium as if in the air, for both the permittivity ε_{yy}, ε_{zz} and the permeability μ_{yy}, μ_{zz} are equal to 1. The value of the phase velocity is $3 \times 10^8 \, \text{m/s}$, but not infinity. That is why we define it as a new type of medium to distinguish the zero-index material, where waves' phase and magnitude keep constant during the propagation [14–21].

The reflection at the interface is quite low when the wave emits from an exclusive medium to the air. According to the Fresnel reflection theory (Eq. (4)), the reflectance for a normal incidence is almost zero with $\mu_{yy} = \mu_{zz} = 1$, $\varepsilon = 1$. Thus, the waves emitted from any source in the exclusive medium are transformed into a plane wave beam without any reflection at the interface ($r_p = 0$).

$$r_p = \pm \frac{1 - \sqrt{\varepsilon \mu_y}}{1 + \sqrt{\varepsilon \mu_y}} \tag{4}$$

3. EXPERIMENT

Here we implement a 2D exclusive medium for its simpler fabrication and measurement than the 3D one. The exclusive medium is realized by a metamaterial consisted of periodic arrays of Teflon slab and SRRs, etched by lithography techniques on 0.254-mm-thick Rogers 5880 substrate. Fig. 3(a) shows the composition of the sample and the size of the SRR. Because of the magnetic resonance of the SRR generated by the **H** field along the x axis, the x component of the permeability μ_{xx} exhibits a Lorentz dispersion, which is obtained by the retrieval method [22] and plotted in Fig. 3(c). It is shown that 8.9 GHz and 9.85 GHz are the preferred frequencies where μ_{xx} reaches zero while μ_{yy} and μ_{zz} are around 1 for no magnetic resonance occurred [23].

To demonstrate the specification in Fig. 2(b), a near field scan system is used to obtain the **E** field distribution in and outside the exclusive medium metamaterial. As shown in Fig. 3, cylindrical microwave is introduced through an X-band monopole antenna that is attached to the lower plate and encased in the middle of the bulk metamaterial. The lower plate move along two directions in a 2D plane so that the magnitude at the mapping region is sampled by a
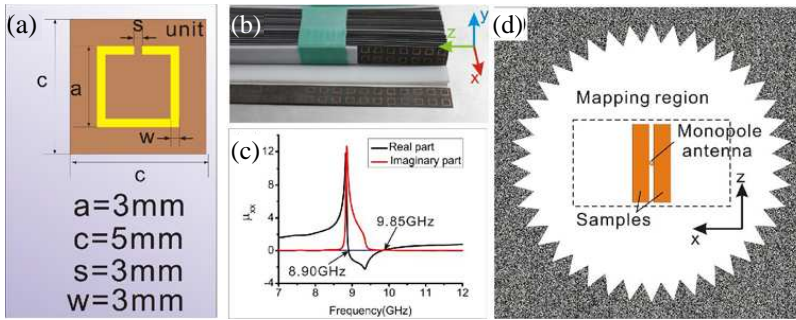
Figure 3. Samples and the scheme of the experiment. (a) The size of a SRR unit. (b) Photograph of the SRRs arrays ($120 \times 27 \times 10$ mm). (c) Permeability of the SRRs array, computed by S-parameter retrieval method. (d) The schematic diagram of the near field scan system. The Monopole antenna which may generate a cylindrical wave is surrounded by the exclusive medium of SRRs arrays.
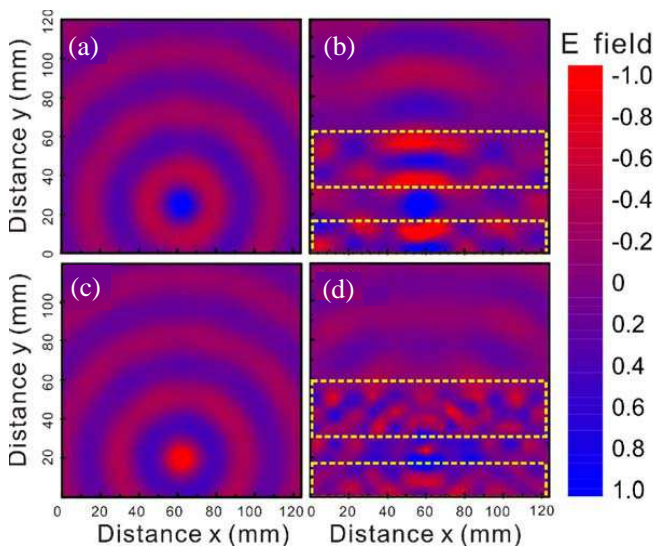

Figure 4. Experiment results. (a) Experimental measurement of the antenna at 8.9 GHz. (b) Antenna covered by the exclusive medium at 8.9 GHz. (c) Antenna at 9.8 GHz. (d) Antenna covered by the exclusive medium at 9.8 GHz.

field-sensing antenna fixed on the upper plate. Microwave over the X band (8–12 GHz) that includes the expected operating frequency of the exclusive medium is generated by the vector net analyzer Agilent ENA 5071C and the complex electric field is acquired at each frequency. After reviewing the field maps at all frequencies, the optimal frequency for the exclusive medium is determined to be 8.9 GHz and 9.8 GHz, in exact agreement with the permeability dispersion spectrum in Fig. 3(c). Measurement results of the cylindrical wave with and without the exclusive medium at the two frequencies are shown in Fig. 4.

4. RESULTS AND ANALYSIS

By contrast with Fig. 4(a), (c) and (b), (d), the cylindrical waves at both the two frequencies are inhibited by the exclusive medium. All omnidirectionally radiated waves with $k_z \neq 0$ are excluded and the only one beam left emits from the metamaterial to the outside along the x direction (i.e., $k_z = 0$). Thus, a plane wave beam is produced by using an exclusive medium covered cylindrical source. The beam width is almost equal to the lateral size of the metamaterial. Comparison of Fig. 4(b) and Fig. 4(d) indicates that the wave front of 9.8 GHz is much more uniform than that of 8.9 GHz. According to Fig. 3(c), $\mu_{xx} = -0.01 + 8.9i$ @ 8.9 GHz and $\mu_{xx} = -0.01 + 0.01i$ @ 9.85 GHz, which shows that the imaginary part of μ_{xx} is much larger than the one at 9.85 GHz due to the strong magnetic resonance. Here we use HFSS 11 to simulate the wave propagation in an exclusive media with high and low loss. The simulation results are shown in Fig. 5. Then we find that the field distributions in Figs. 5(a) & (b) are in good
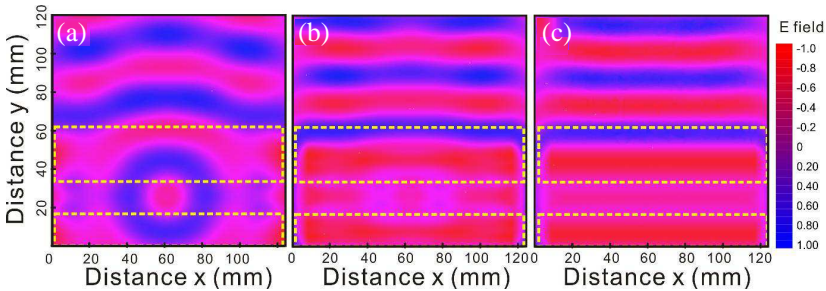

Figure 5. Simulation results by HFSS on the exclusive media, (a) with loss: $\mu_{xx} = -0.01 + 8.9i$ @ 8.9 GHz; (b) low loss: $\mu_{xx} = -0.01 + 0.01i$ @ 9.85 GHz; and (c) no loss: $\mu_{xx} = -0.0001$.

agreement with that in Figs. 4(b) & (d), respectively. Therefore, the better plane wave can be attributed to the low loss at the plasma frequency 9.85 GHz. If the wave emits from an exclusive medium of no loss, as plotted shown in Fig. 5(c), the output wave will be very close to a plane wave mode. Further study shows the decreasing the loss and increasing the large lateral scale of the exclusive medium may greatly improve the quality of the plane wave beam [See the details in the supplementary]. Moreover, the wave propagation in and outside the exclusive medium is continuous without any change on the wavelength in Fig. 5, which reveals that it is quite different from the zero-index material.

4.1. Difference between the Zero-index Medium and the Exclusive Medium

Same performance of transforming a cylindrical wave into a plane wave can also be realized by zero-index medium, which is widely studied in recent years [14–21, 24–26]. People may mix up the two media sometimes. Any EM waves emitted from a zero-index medium keep the same constant phase until they come out of the medium, which results in a uniform wavefront outside the medium. The exclusive medium, however, is quite different from the zero-index material. The zero parameter of the exclusive medium is along the propagation direction of the wave, resulting in a linear type EFC which excludes all the other transmitting modes except the one on the zero parameter orientation. Additionally, the electromagnetic parameters along both the **E** field and the **H** field polarization of the wave are 1 in the exclusive medium, where the magnitude of the wave vibrates with the phase change just like in the air.

Here we use HFSS to simulate the propagation of a plane wave in an exclusive medium, a zero index medium with $\varepsilon = \mu = 0$ and a zero index medium with $\varepsilon = 1$ and $\mu = 0$. Fig. 6 shows the **E** and **H** field distributions in the three types of media. The exclusive medium acts as the air for the wave, as shown in Fig. 6(a). By contrary, vibrated propagations are interrupted in the two zero-index media, in which the phase and magnitude of the wave do not experience any change due to the zero refractive index. Fig. 7 shows the **E** field magnitude distribution along the propagation direction, which also implies the phase change. For the exclusive medium and the zero index medium with $\varepsilon = \mu = 0$, the intensity of the wave does not attenuate due to the matched impedance. Yet in the zero index medium with $\varepsilon = 1$ and $\mu = 0$, the wave decays sharply after it transmitted though the medium, as shown in Fig. 7(c).
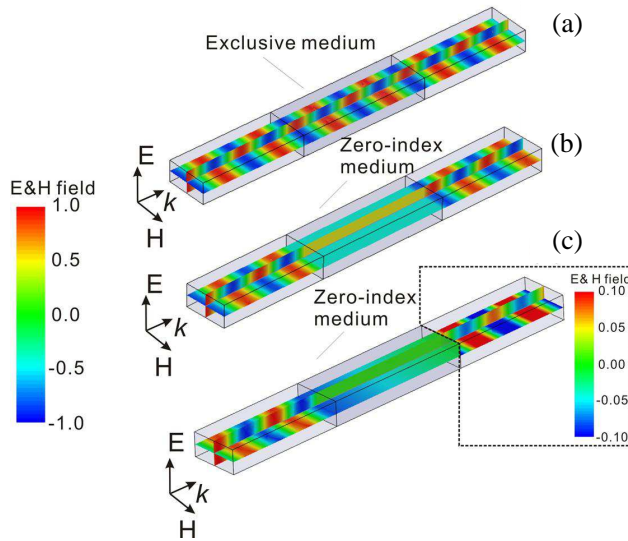
Figure 6. The **E** and **H** field distributions on each polarized plane of a plane wave transmitting through the media of three types. (a) An exclusive medium; (b) a zero index medium with $\varepsilon = \mu = 0$; and (c) a zero index medium with $\varepsilon = 1$ and $\mu = 0$. The inset in (c) shows a reduced scalar to clarify the damped field. The frequency of the plane wave is 11 GHz and the size of the medium is $60 \times 20 \times 10\,\mathrm{mm}^3$. The total simulated space is $160 \times 20 \times 10\,\mathrm{mm}^3$.
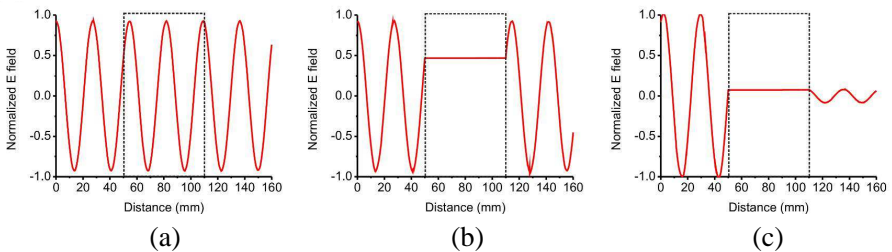

Figure 7. The **E** field magnitude distribution along the propagation direction. (a) An exclusive medium; (b) a zero index medium with $\varepsilon = \mu = 0$; and (c) a zero index medium with $\varepsilon = 1$ and $\mu = 0$.

5. CONCLUSION

In conclusion, an exclusive medium can be used to produce a plane wave beam due to its linear type EFC resulted by the zero parameter in the permeability tenser. By studying the electromagnetic property, it turns out that only one transmitting mode exists in such a medium, which results in a plane wave beam propagating along the zero parameter direction. Parameters along the field vectors of the wave are equal to 1, enabling a perfect match between the medium and the air and making the exclusive medium act as the air to the wave. This is also the largest difference in contrast to the zero-index medium. Experiment results show an approximate plane wave beam can be produced by a cylindrical source covered by an exclusive medium of SRRs arrays. This idea can be further extended to cylindrical coordination or a 3D case, generating a perfect isotropic radiation or a single plane wave beam spatially.

ACKNOWLEDGMENT

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