Progress In Electromagnetics Research M, Vol. 28, 57–71, 2013

A SIGNAL MODEL BASED ON COMBINATION CHAOTIC MAP FOR NOISE RADAR

Qilun Yang[1, 2] *, Yunhua Zhang[2], and Xiang Gu[2]

[1]University of Chinese Academy of Sciences, Beijing 100049, China

[2]The Key Laboratory of Microwave Remote Sensing, Chinese Academy of Sciences, Beijing 100190, China

Abstract—We propose a combination chaotic map (CCM) signal model to resolve the limited-word-length problem in digitally realizing chaotic signals used for noise radar. The proposed CCM has approximated infinite dimension, much more complicated phase space structure as well as better chaotic properties. The radar signal based on CCM presents much lower PSLR of auto-correlation as well as much flatter power spectrum, so it is very suitable for generating wide-band radar signal. Simulation experiments are conducted to show the good performance of the signal.

1. INTRODUCTION

Chaotic signal is one kind of pseudo-noise signals generated from deterministic system. Chaotic signal may vary significantly if initial condition fluctuates, and it is also non-periodic and unpredictable in long time [1]. Chaotic noise signals have shown very good properties in radar applications [2–4], and radar signals generated from chaotic map have shown very good performance of high range resolution, low probability of intercept and interference, and optimum use of frequency spectrum [5–8]. Compared with other noise signals, chaotic signals are much easier to generate and control with low cost. Therefore, chaotic signals are more and more widely paid attention to in radar field recently.

However, the sidelobes of some radar signals generated from chaotic map are not so good, so they are not suitable for high resolution radar imaging [9, 10]. To lower the sidelobes of radar signals based on chaotic map, Reference [11] suggested making use

Received 17 November 2012, Accepted 28 December 2012, Scheduled 3 January 2013
* Corresponding author: Qilun Yang (yangqilun110@163.com).

of the chaotic weak-structure property to produce chaotic map, based on this idea Multi-Segment Bernoulli (MSB) system was proposed; Reference [12] suggested utilizing high dimensional chaotic map with more complicated phase space structure to generate chaotic frequency modulated (FM) radar signal; Reference [13] took advantage of Hyper Chaotic Logistic Phase Coded (HCLPC) signal along with Tikhonov regularization method to lower sidelobes. Nonetheless, all of these methods didn't take limited quantization word length (LQWL) into consideration when digitally realizing chaotic maps. In real digital system, quantization word length is limited when digital device is used to realize chaotic map, and the usual maximum quantization word length is about 14 bits in DDS (Direct Digital Synthesizer) nowadays [14]. As we know, chaotic signals produced from digital device are pseudo-random signals essentially, whose maximum period is restricted by LQWL. For example, when LQWL is M, the maximum period of one-dimensional chaotic signals cannot exceed 2^M, while N-dimensional chaotic signals cannot exceed 2^{NM}.

To resolve the effect of LQWL on chaotic map and get much better noise radar signal, we first propose a combination chaotic map (CCM), and then apply it to generating FM radar signal. Owing to the much more complicated phase space structure, combination map has been applied to communication and image encryption [15–17], but little to radar field at present. The proposed CCM in this paper is made up of Logistic map and Bernoulli map. Here, the Logistic map is used to produce the parameters used for Bernoulli maps, and the Bernoulli mapping sequences with different parameters are combined to finally get the CCM.

The rest of the paper is organized as follows. In Section 2, the generation of CCM, as well as the thus generated FM radar signal is introduced. In Section 3, the characteristics of approximated infinite dimension of the CCM are shown. In Section 4, the good performance of the CCM and the corresponding FM radar signal are tested. Finally Section 5 concludes the paper.

2. FM RADAR SIGNAL BASED ON CCM

2.1. Generation of CCM

The form of one-dimensional chaotic map can be expressed as f: $\phi \rightarrow \phi$, let $\phi\,(n\Delta t)$ be the discrete form of $\phi\,(t)$, then

$$\phi[(n+1)\Delta t] = \phi_{n+1} = f[\phi(n\Delta t)] \tag{1}$$

where Δt represents sampling interval, and $f(\cdot)$ is a nonlinear mapping function, making the sequence $\{\phi_0, \phi_1, \ldots, \phi_n\}$ exhibit fractal behavior [18].

Logistic map [19] is a well-known nonlinear map which can be formulated as

$$x_{n+1} = f(x_n) = u \cdot x_n \cdot (1 - x_n) \qquad 0 < x_n < 1 \tag{2}$$

where u is the control parameter, and Logistic map will exhibit chaotic behavior when $3.569945 < u \leq 4$.

Bernoulli map [20, 21] can be defined as:

$$x_{n+1} = f(x_n) = \begin{cases} Bx_n + \frac{1}{2}, & x_n < 0 \\ Bx_n - \frac{1}{2}, & x_n \geq 0 \end{cases} \qquad -\frac{1}{2} \leq x_n \leq \frac{1}{2} \tag{3}$$

To guarantee Bernoulli map be of chaotic property, B should be chosen between 1.4 and 2.

Both Logistic map and Bernoulli map are one-dimensional chaotic maps. As pseudo-random sequence, the maximum period of both Logistic and Bernoulli sequences will not exceed 2^M when LQWL is M. In order to improve the chaotic property of chaotic sequences and increase their periods when quantization word length is limited, this paper proposes a CCM by using the above Logistic map and Bernoulli map, where the Logistic map is used to produce parameter B needed for Bernoulli map, then Bernoulli sequences with different parameters can be combined to get CCM sequence. The form of CCM sequence $\{\phi\}$ can be expressed by the following equation:

$$\begin{cases} x_{m+1} = u \cdot x_m \cdot (1 - x_m) & 0 < x_m < 1 \\ B_m = 1.4 + 0.6 \cdot x_m & 0 \leq m \leq M \\ y_{m,n+1} = g(y_{m,n}) = \begin{cases} B_m y_{m,n} + 0.5, & y_{m,n} < 0 \\ B_m y_{m,n} - 0.5, & y_{m,n} \geq 0 \end{cases} & 0 \leq n \leq N \\ \Phi = \{\{y_{0,0}, y_{0,1}, \ldots, y_{0,N}\}, \{y_{1,0}, y_{1,1} \ldots, y_{1,N}\}, \\ \qquad \ldots, \{y_{M,0}, y_{M,1}, \ldots, y_{M,N}\}\} \end{cases} \tag{4}$$

where $\{x_n\}$ is the Logistic sequence. In view of the fact that the boundary of Logistic sequence is $0 < x_n < 1$, and the limit of parameter B is $1.4 < B < 2$, one can let $B_m = 1.4 + 0.6 \cdot x_m$. Figure 1 shows the structure of CCM.

2.2. FM Radar Signal Based on Chaotic Map

The form of FM radar signal based on chaotic map can be expressed as follows:

$$s(t) = A \exp[j2\pi K \Phi(t)] = A \exp\left[j2\pi K \int_0^t \phi(u) du\right] \tag{5}$$

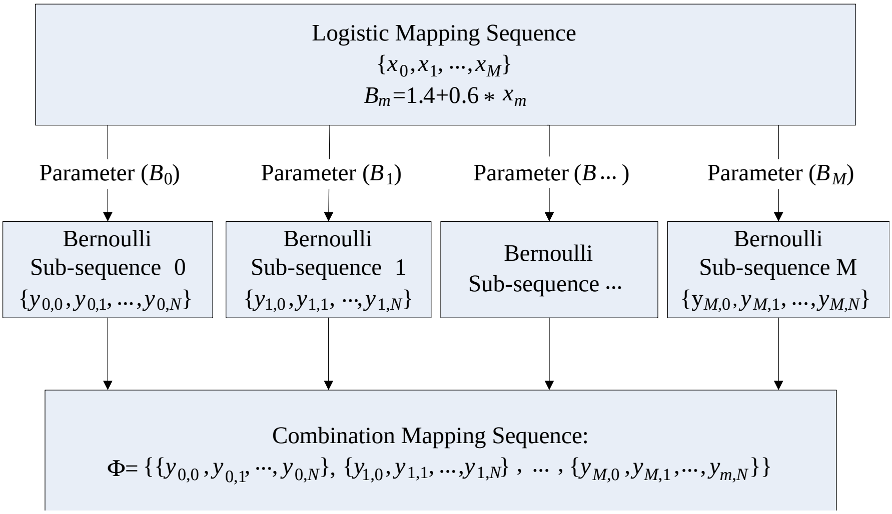

Figure 1. The structure of CCM.

where A is the amplitude of radar signal and K the coefficient of frequency modulation. The $K \cdot \phi(t)$ term indicates the instantaneous frequency of radar signal, and the frequency interval of $s(t)$ is $K \cdot \phi_{\min}(t) \leq f \leq K \cdot \phi_{\max}(t)$. The Nyquist theorem should be guaranteed when sampling the radar signal without distortion, i.e., $f_s \geq K \cdot [\phi_{\max}(t) - \phi_{\min}(t)]$, so the sample interval is $t_s = 1/f_s$.

The discrete form of $s(t)$ can be then expressed as:

$$s(n \cdot t_s) = A \exp\left\{ j2\pi K \int_0^{n \cdot t_s} \phi(u) du \right\} = A \exp\left\{ j2\pi K \sum_{i=0}^{n} \phi_i \cdot t_s \right\} \quad (6)$$

Under the condition of $\phi_{\min}(t) = -\frac{1}{2}$, $\phi_{\max}(t) = \frac{1}{2}$, as well as let $t_s = 1/f_s = 1/K$, (6) can then be transformed into the following form:

$$s(n) = A \exp\left\{ j2\pi \sum_{i=0}^{n} \phi_i \right\} = A \exp\left\{ j2\pi\phi_0 + j2\pi \sum_{i=1}^{n} \phi_i \right\} \quad (7)$$

3. ANALYSIS OF APPROXIMATE INFINITE DIMENSION ON CCM

In this section we will show by simulation that the sequences are uncorrelated for two Bernoulli maps with different parameters of B_1 and B_2, though the difference is very tiny. In the simulation, we

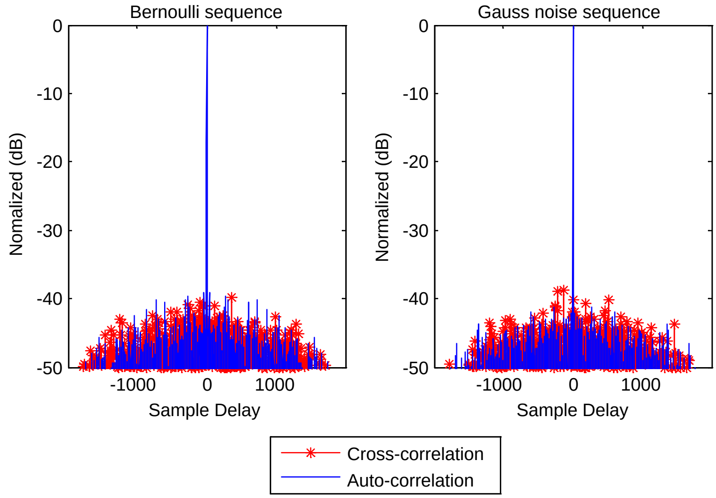

Figure 2. Comparison between normalized auto-correlation and cross-correlation. (a) Bernoulli sequence with different parameters having a difference of $1/10^6$. (b) Gauss noise sequence.

set $B_1 = 1.5, B_2 = 1.500001$, i.e., the difference between B_1 and B_2 is as small as $1/10^6$; the sequence length is set to be 2000. To ensure reliable simulation, we generate the Bernoulli map 50 times with random initial conditions, and then average the obtained 50 auto-correlations and cross-correlations. Figure 2 shows the simulation result, from which one can see that the normalization cross-correlation of Bernoulli sequences is below -40 dB, it is close to cross-correlation of Gauss noise sequences. Therefore, the Bernoulli sequences with different parameters can be regarded as uncorrelated.

The property of Bernoulli sequence is only determined by parameter B, and the Bernoulli sequence corresponding to a certain B has one dimension. If we do not take LQWL into account, then the number of parameter B_m produced from Logistic map could approach infinite because of chaotic property, that is to say the number of Bernoulli sequences corresponding to B_m approaches infinite. Therefore, when we combine Bernoulli sequences with approximately infinite B_m to get CCM sequence, the CCM sequence will have approximated infinite dimension as well [22].

If LQWL is considered, the chaotic map will become pseudo-random sequence with periodicity definitely. When the length of chaotic mapping sequence is longer than the period, the sequence will repeat. For example, if the quantization word length is M, the maximum sequence periods of both Logistic and Bernoulli map will not exceed 2^M. However, if the Logistic sequence's period is 2^{M_1} ($M_1 \leq M$) and the Bernoulli sequence's period is 2^{M_2} ($M_2 \leq M$), then the CCM sequence's period will be at the magnitude of $\underbrace{2^{M_2} \cdot 2^{M_2} \ldots 2^{M_2}}_{2^{M_1}} = 2^{M_1 \cdot M_2}$, so the period of CCM sequence can be much longer than that of Logistic and Bernoulli maps.

4. PERFORMANCE OF CCM AND THE CORRESPONDING FM SIGNAL

4.1. Performance of CCM

Figure 3 shows the phase space structure of CCM and that of Logistic and Bernoulli maps. In the simulation, the initial values are selected randomly within range, and the parameters are selected as $u = 4$ for both Logistic map and CCM, and $B = 2$ for Bernoulli map. The structure of Logistic map's phase space and that of Bernoulli map's phase space are one-to-one correspondence, so the structures are robust and distinct, and at the same time, they can be easily recognized and reconstructed [23, 24]. However, the CCM's phase space structure is one-to-many correspondence, it is to say even we know the CCM's value and phase space structure at present, we can't predict its values in future, so CCM is of very good confidentiality characteristic.

Figure 4 shows the auto-correlation of CCM, as well as comparison with that of Logistic and Bernoulli maps. Figure 4(a) gives the results of auto-correlation of the three chaotic mapping sequences without considering the effect of LQWL, and Figure 4(b) shows the results obtained when 14-bit of LQWL is considered. The sequence length in

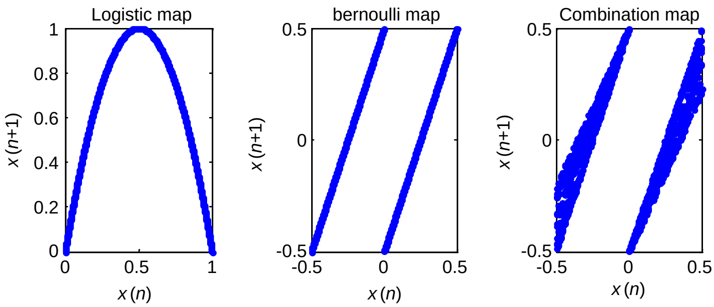

Figure 3. The phase space structures of the Logistic, Bernoulli and Combination chaotic maps.

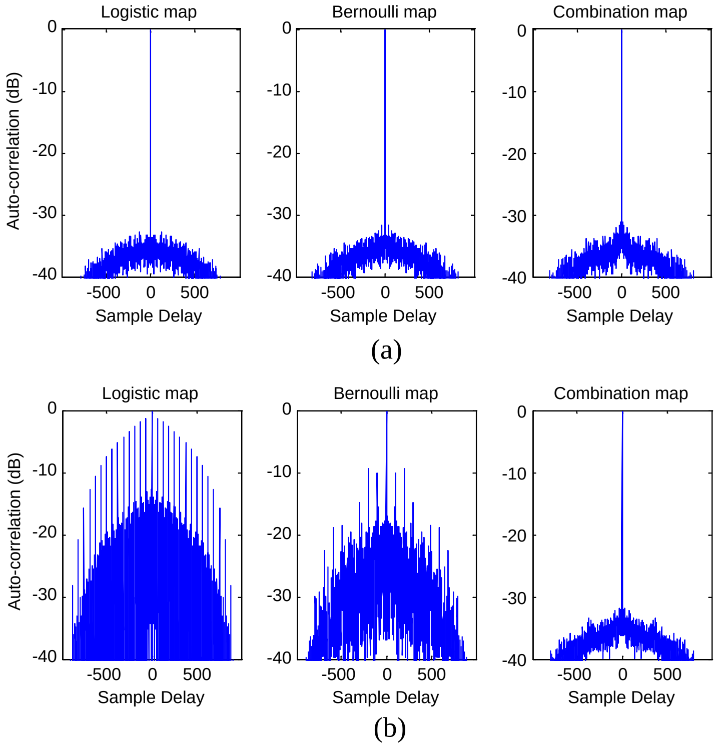

Figure 4. Auto-correlations of the sequences of Logistic, Bernoulli and Combination chaotic maps. (a) Without considering LQWL. (b) 14-bit of LQWL considered.

the simulation is 1000. To guarantee reliability, we conduct sequence simulation 50 times with random initial conditions, and then average the auto-correlations.

From Figure 4(a) one can see that when we do not take LQWL into consideration, the performances of the all auto-correlations are very similar with PSLR around -31.5 dB. However, when 14-bit LQWL is considered, the performance deteriorates remarkably for the Logistic and Bernoulli maps, with PSLRs increasing to -1.5 dB and -10 dB, respectively, but it almost has no change for CCM. It is clearly shown that the LQWL effect destroys the chaotic properties of Logistic and Bernoulli maps, while has little impact on CCM.

It is well known that entropy can be used to measure the randomness of variable, the bigger the entropy, the more random the variable, or the smaller the entropy, the less random the variable. For continuous variable, the entropy is defined as:

$$S = -\int_{-\infty}^{\infty} p(x) \ln[p(x)] dx \tag{8}$$

where x is the corresponding continuous variable and $p(x)$ is its probability density. For discrete variables, the entropy [25] is defined as:

$$S = -\sum_{k=1}^{M} P_k \ln[P_k] \tag{9}$$

where M is the number of segments within range and P_k the probability of variable in the kth segment. In the following, we shall conduct comparison between the entropies of CCM, Logistic map and Bernoulli map. Figure 5 presents the simulation results, from which one can see that when neglecting the LQWL effect, the entropy of CCM is almost the same as that of Bernoulli map, and the entropy of Logistic map is the smallest. It means that the CCM and the Bernoulli map have

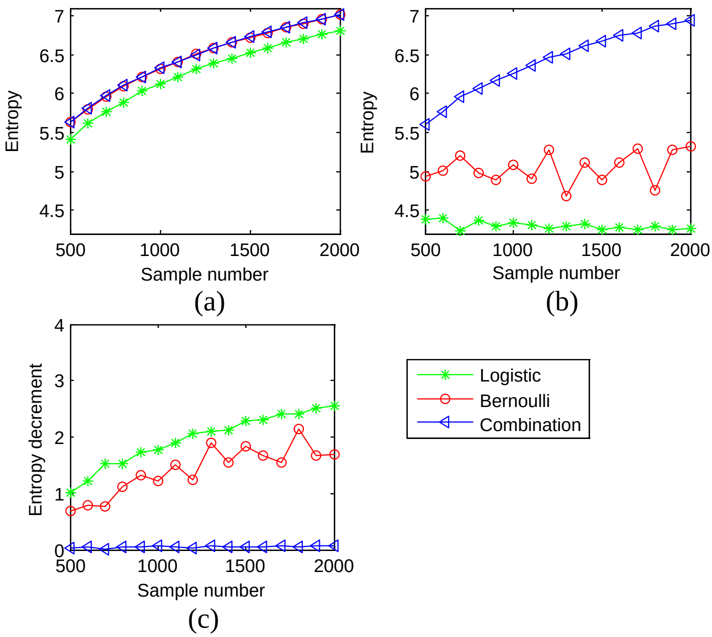

Figure 5. Entropies of CCM, Logistic map and Bernoulli map. (a) Entropies without considering the LQWL effect. (b) Entropies with LQWL of 14-bit. (c) The entropy decrement between (a) and (b).

almost the same randomness, while the randomness of Logistic map is the worst. When the LQWL of 14-bit is used, the entropies of Bernoulli map and Logistic map decrease remarkably as the sequence number increases, while the entropy of CCM changes very little as shown in Figure 5(c). The results indicate that the Logistic and Bernoulli map are susceptible to LQWL, while the CCM is robust to LQWL. The reason is because both Logistic map and Bernoulli map is one-dimensional chaotic map with finite dimension, while the CCM has approximate infinite dimension. As pseudo-random sequence, the period of CCM is much longer than that of Logistic and Bernoulli maps.

4.2. Performance of FM Radar Signal Based on CCM

The auto-correlation of radar signal reflects the range resolution [26, 27], which can be expressed as:

$$R(\tau) = E\left\{s(t)s^*(t+\tau)\right\} \tag{10}$$

where τ is the time delay. The discrete form of (10) is:

$$R(m) = E\left\{s(n)s^*(n+m)\right\} \tag{11}$$

In order to evaluate the performance of FM radar signal based on CCM, we conduct simulation for two cases as before: (a) without considering LQWL, (b) set the LQWL to be 14-bit. In the simulation, the signal's time duration is 10 μs, number of samples is 1000, and the corresponding bandwidth is 100 MHz. Owing to chaotic sequence's pseudo-random property, we realize FM radar signal based on chaotic map with random initial condition 50 times and then average these auto-correlations to guarantee simulation's reliability.

The simulation results are shown in Figure 6, which clearly shows that when we neglect the effect of LQWL, the auto-correlation's PSLRs for Logistic, Bernoulli, and CCM are −11.2 dB, −27.1 dB and −28.1 dB, respectively. However, when the LQWL is set to be 14-bit, the PSLRs of the three FM radar signals become to −12 dB, −14.7 dB and −27.8 dB, respectively, and evenly, there appears grating lobes in Logistic map case. Obviously, the performances of both Logistic map and Bernoulli map based FM signals degraded remarkably, but the performance of CCM based FM signal keeps unchanged. It is to say the Logistic map and Bernoulli map based FM signals are greatly affected by the LQWL effect, but the CCM based FM signal is almost unaffected.

In the following, we will further use power spectral density to evaluate the performances of FM signals. According to Weiner-Khintchine Theorem, the power spectral density of a signal is the

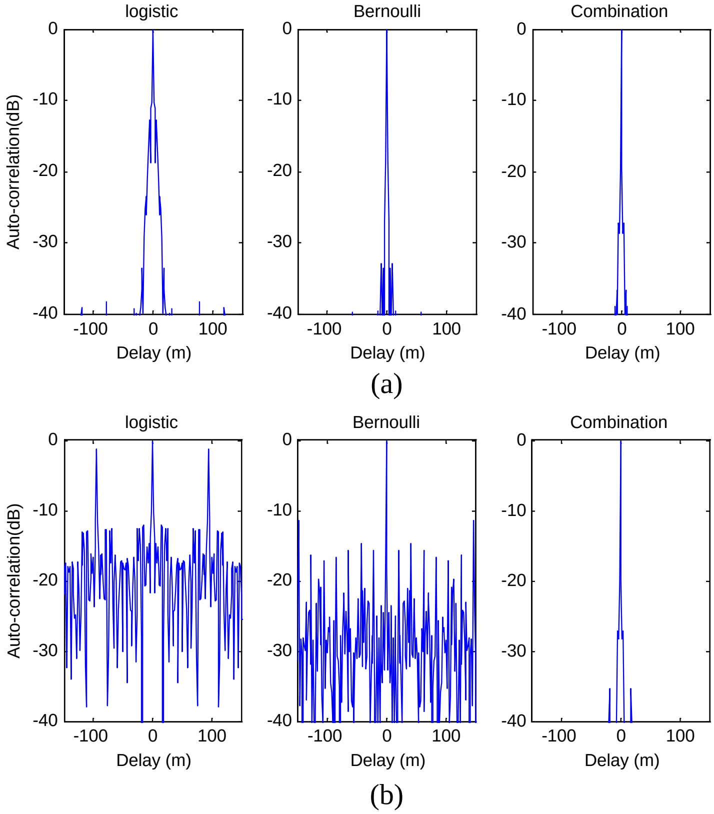

Figure 6. Auto-correlations of the FM signals based on the Logistic, Bernoulli and Combination chaotic maps. (a) Without considering LWL. (b) LWL is set to be 14-bit.

Fourier transform of its auto-correlation. For getting reliable results, the periodogram is used to estimate the power spectrum. We first generate FM radar signals with random initial value 50 times, and then average the auto-correlations. After performing Fourier transform to the averaged auto-correlation, we get the following periodogram:

$$P(f) = DFT\left\{\frac{1}{M}\cdot\sum_{k=1}^{M} R_k\right\} \tag{12}$$

where R_k is the kth auto-correlation of FM radar signal. The simulation results are presented in Figure 7. Figure 7(a) is the power spectra without considering the LQWL, and Figure 7(b) is the power spectra when the LQWL is set to be 14-bit.

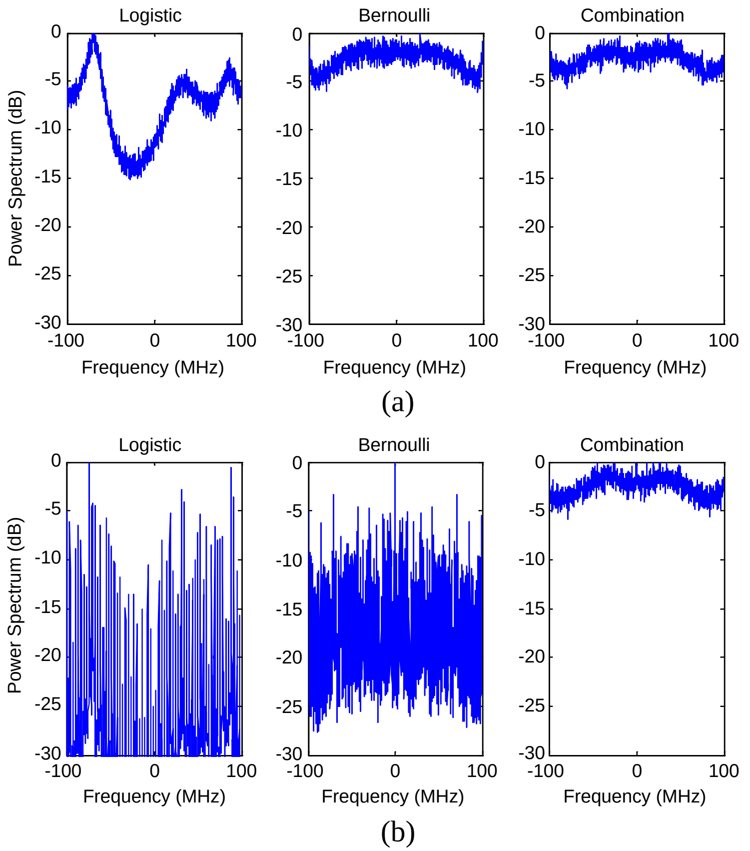

Figure 7. Power spectra of the FM signals based on the Logistic, Bernoulli and Combination chaotic maps. (a) Without considering LWL. (b) LWL is set to be 14-bit.

From Figure 7 one can see that when neglecting LQWL effect, the power spectrum of Bernoulli map based FM signal is as good as that of CCM, and the power spectrum fluctuate within 5 dB, while the power spectrum of Logistic map based FM radar signal fluctuates over 15 dB. When the LQWL is set to be 14-bit, the power spectra of both Logistic and Bernoulli maps based on FM signals fluctuate much more dramatically. In fact, the power spectrum of Logistic map case fluctuates over 30 dB, and the power spectrum of Bernoulli map case fluctuates about 25 dB. However, the power spectrum of CCM case is almost unchanged with fluctuation within 5 dB as before.

To further show how the signal bandwidth affects the power

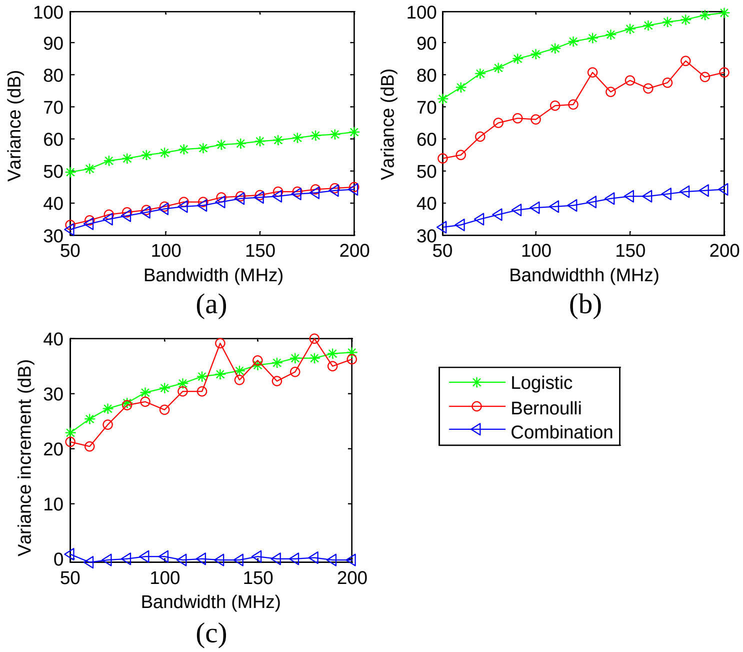

Figure 8. The power spectra of radar signals based on Logistic, Bernoulli and Combination chaotic maps. (a) Without considering LWL. (b) The LWL is set to be 14-bit. (c) Variance increment of power spectra between (a) and (b).

spectra quantitatively, we calculate the variance of power spectra. As we know, the smaller the variance is, the more flat the power spectrum should be; and on the contrary, the bigger the variance is, the less flat the power spectrum should be. In the simulation, the signal duration is 10 µs with bandwidth varies from 50 MHz to 200 MHz at step of 10 MHz. Same as above, we calculate them for two cases, i.e., (a) without considering LQWL, (b) the LQWL is set be 14-bit. The simulation results are presented in Figure 8.

Figure 8 shows that, when neglecting LQWL effect, the variance of power spectrum of Bernoulli map based FM signal is close to that of CCM, while the power spectrum variance of the Logistic map based FM signal is the biggest. When the LQWL is set to be 14-bit, the variance variations of both the Logistic and Bernoulli maps based signal power spectra increase as the signal bandwidth increases, but it is almost kept unchanged for CCM based signal. This means that the CCM based FM signal is very appropriate for wide-band radar.

5. CONCLUSION

A CCM is proposed in order to overcome the drawback of bad resistant capability of LQWL to one-dimensional chaotic sequence. The CCM is constructed by using the Logistic map and the Bernoulli map. In fact, it can also be constructed by using other chaotic maps. We use the Logistic map to produce the parameters for Bernoulli map, and then combine the Bernoulli map generated sequences with different parameters to get the final CCM sequence.

The good performance of the proposed CCM is analytically shown and tested through simulation. The results indicate that the CCM has approximate infinite dimension because it has more complicated phase space structure, and its chaotic property is much better. As a pseudo-random sequence, the CCM generated sequence has much longer period than that generated by Logistic and Bernoulli maps when LQWL is considered.

We further show through simulation that the CCM based FM radar signal has superior performances. The PSLR of its auto-correlation is much lower and stable even when LQWL of 14-bit is used. The power spectrum is very flat, and at the same time the power spectrum variance is very stable as the signal bandwidth increases, so it is very suitable for realizing wide-band signal and achieving high resolution radar imaging. In the future, the proposed signal model will be implemented into an existing experimental radar system and tested by real experiment.

REFERENCES

1. Lorenz, E. N., "Deterministic nonperiodic flow," *Journal of the Atmospheric Sciences*, Vol. 20, 130–141, 1963.
2. Jiang, T., S. Qiao, Z. Shi, L. Peng, J. Huangfu, W. Z. Cui, W. Ma, and L. Ran, "Simulation and experimental evaluation of the radar signal performance of chaotic signals generated from a microwave colpitts oscillator," *Progress In Electromagnetics Research,* Vol. 90, 15–30, 2009.
3. Ding, K. and R. Yang, "Point target imaging simulation using chaotic signals," *IEEE International Radar Conference*, 847–850, 2005.
4. Yang, Y., J. Zhang, and C. Liu, "Chaotic FM signals for SAR jamming imaging," *1st Asian and Pacific Conference on Synthetic Aperture Radar*, 87–89, 2007.
5. Harman, S. A., A. J. Fenwick, and C. Williams, "Chaotic signals in radar?," *3rd European Radar Conference*, 49–52, 2006.

6. Xiang, L. and J. Zeng, "The chaotic signal design for MIMO radar," *International Conference on Environmental Science and Information Application Technology (ESIAT),* 611–614, 2010.
7. Callegari, S., R. Rovatti, and G. Setti, "Chaos-based FM signals: Application and implementation issues," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Vol. 50, 1141–1147, 2003.
8. Qiao, S., Z. G. Shi, T. Jiang, and L. X. Ran, "A new architecture of UWB radar utilizing microwave chaotic signals and chaos synchronization," *Progress In Electromagnetics Research*, Vol. 75, 225–237, 2007.
9. Yang, J., Z. K. Qiu, X. Li, and Z. W. Zhuang, "Uncertain chaotic behaviours of chaotic-based frequency- and phase-modulated signals," *IET Signal Processing*, Vol. 5, 748–756, 2011.
10. Shi, Z. G., S. Qiao, K. S. Chen, W. Z. Cui, W. Ma, T. Jiang, and L. X. Ran, "Ambiguity functions of direct chaotic radar employing microwave chaotic Colpitts oscillator," *Progress In Electromagnetics Research*, Vol. 77, 1–14, 2007.
11. Chen, B., J. Tang, Y. Zhang, P. Cai, J. Huang, and G. Q. Huang, "Chaotic signals with weak-structure used for high resolution radar imaging," *International Conference on Communications and Mobile Computing*, 325–330, 2009.
12. Yang, J., Z.-K. Qiu, L. Nie, and Z.-W. Zhuang, "Frequency modulated radar signals based on high dimensional chaotic maps," *10th IEEE International Conference on Signal Processing (ICSP)*, 1923–1926, 2010.
13. Deng, Y., Y. Hu, and X. Geng, "Hyper chaotic logistic phase coded signal and its sidelobe suppression," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 46, 672–686, 2010.
14. Chua, M. Y. and V. C. Koo, "FPGA-based chirp generator for high resolution UAV SAR," *Progress In Electromagnetics Research*, Vol. 99, 71–88, 2009.
15. Wang, M., F. Guo, H. S. Qu, and S. Li, "Combined random number generators: A review," *IEEE 3rd International Conference on Communication Software and Networks (ICCSN),* 443–447, 2011.
16. Feng, T., C. Chen, and Y Hu, "A novel method of designing chaotic spread-spectrum sequence based on combined Tent-map," *China-Japan Joint Microwave Conference Proceedings (CJMW)*, 1–4, 2011.
17. Huang, J.-H. and L. Yang, "A block encryption algorithm

combined with the Logistic mapping and SPN structure," *2nd International Conference on Industrial and Information Systems (IIS),* 156–159, 2010.

18. Flores, B. C., E. A. Solis, and G. Thomas, "Assessment of chaos-based FM signals for range-Doppler imaging," *IEE Proceedings — Radar, Sonar and Navigation*, Vol. 150, 313–322, 2003.
19. Schuster, H. G., *Deterministic Chaos: An Introduction,* 4th, Revised and Enlarged Edition, Wiley-VCH Verlag, Weinheim, 2005.
20. Ashtari, A., G. Thomas, H. Garces, and B. C. Flores, "Radar signal design using chaotic signals," *International Waveform Diversity and Design Conference,* 353–357, 2007.
21. Ashtari, A., G. Thomas, W. Kinsner, and B. C. Flores, "Sufficient condition for chaotic maps to yield chaotic behavior after FM," *IEEE Transactions on Aerospace and Electronic Systems,* Vol. 4, 1240–1248, 2008.
22. Yanmin, G. L. H., "Method of generating infinite dimensional pseudo-random sequence based on combination chaotic map," *Statistics and Decision*, Vol. 10, 16–19, 2010 (in Chinese).
23. Liu, L., J. Hu, Z. He, C. Han, and C. Lu, "Chaotic signal reconstruction with application to noise radar system," *International Symposium on Intelligent Signal Processing and Communication Systems (ISPACS)*, 1–4, 2010.
24. Sobhy, M. I. and A. E. R. Shehata, "Methods of attacking chaotic encryption and countermeasures," *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP),* Vol. 2, 1001–1004, 2001.
25. Son, J. S., G. Thomas, and B. C. Flores, *Range-Doppler Radar Imaging and Motion Compensation*, Artech House Publishers, 2001.
26. Park, J. I. and K. T. Kim, "A comparative study on ISAR imaging algorithms for radar target identification," *Progress In Electromagnetics Research*, Vol. 108, 155–175, 2010.
27. Axelsson, S. R. J., "Noise radar using random phase and frequency modulation," *IEEE International Proceedings Geoscience and Remote Sensing Symposium (IGARSS)*, Vol. 7, 4226–4231, 2003.