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A NOVEL INTERACTING MULTIPLE MODEL PARTICLE FILTER FOR MANEUVERING TARGET TRACKING IN CLUTTER

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Abstract—In this paper, a novel interactive multiple model particle filter (IMMPF) is developed after a Bayesian estimator for maneuvering target tracking in clutter is derived theoretically. In this new algorithm, base state estimation and modal state estimation are completely separated to control the number of particles in each maneuvering mode. Only continuous-valued particles are used to numerically implement the procedure of Bayesian base state estimation, whereas modal state is estimated analytically without dependence on the number of particles. Density mixing is performed by aggregation of the total particles and mixing associated weights. To prevent the exponentially growing number of particles with the time, a resampling step is included following the interaction step. Through MC simulations, the new IMMPF has been tested and shown to provide reliable performance improvements with different sample sizes and under various clutter conditions.

1. INTRODUCTION

Tracking maneuvering target, which might switch among multiple operating regimes, is usually a difficult Bayesian inference problem and has attracted significant attention in the signal processing community for many years [1–5]. The so-called multiple model (MM) approaches have been shown to be highly effective in many situations, among which especially the interacting multiple model (IMM) algorithm [6] has become almost the standard approach to maneuvering target tracking. IMM algorithm assumes a stochastic hybrid Markov system that behaves according to a finite number of switching models to

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characterize the targets motion modes. At every sampling time, several parallel corresponding elemental filters run and interact with each other. The output of IMM contains a continuous-valued "base state" and a discrete "modal state" that indicates in which mode the target is. When the nonlinearities are involved in the assumed motion models and/or measurement equation, extended Kalman Filter (EKF) and its variants, which rely on linearization of the nonlinear equations, can be used as the elemental filters in the IMM algorithm. However, if the system nonlinearities are severe, EKF and accordingly EKF-based IMM algorithms often give unreliable estimates.

Only since 1990s has the sampling importance resampling (SIR) based particle filter (PF) won proper recognition with the computational power being adequate for its implementation. Nowadays various PFs are widely applied to the problems involving highly nonlinearities and/or non-Gaussianity that are difficult for the conventional Kalman filter. In the Bayesian framework, the posterior density of the target state provides a complete statistical description of the state. However, an analytical expression for the posterior density is available only for some restricted situations. PF is a recursive numerical implementation of the exact Bayesian filtering scheme where the posterior distribution of the state is represented by a set of random samples with associated weights. It has been demonstrated that the standard PF has the ability to directly estimate the state of the stochastic hybrid Markov system through the utilization of the random sampled augmented hybrid particles that consist of both the base state and modal state in [7–9], where it was termed as multiple model PF (MMPF). The major drawback of straightforwardly applying the standard PF to maneuvering target tracking lies in that there is no control over the number of particles in a certain motion mode. More precisely, in such MMPF with hybrid particles, the number of particles in a specific mode is directly proportional to the model probability, and therefore low model probability might result in too few corresponding particles to accurately capture the true model-conditioned density of the target state. A brute-force approach to alleviate this sample degeneracy is to simply increase the total number of particles, which will lead to unnecessary large number of particles in the modes with high model probabilities and unreasonable increase in computational load. Other intelligent ways of dealing with this problem are to control or fix the number of particles in each mode just as the IMMPF do [10]. It must be mentioned that IMMPF is not the abbreviation for the simple combination of IMM and PF, i.e., IMM filter with PF as elemental filters. Essentially it is a PF for stochastic hybrid MM system with an mixing/interaction step at the beginning of each estimation cycle. It was argued in [11] that the interaction step is exactly the same special feature of IMM estimator and the optimal estimator and can be seen as the main reason for the success of IMM estimator. The IMMPF algorithm was first proposed in [10] where instead of a resampling step, a Gaussian sum density was used to fit the particle cloud and approximate the true model-conditioned posterior density of the state in order to implement the density mixing. This approach introduces additional approximations and needs a mixture reduction algorithm, which leads to increased computational complexity. To obtain a computationally cheaper PF for stochastic hybrid system, the mixing step was replaced by direct sampling from a weighted sum of distributions and posterior mode probabilities were calculated in an approximate form in [12]. In [13], an interaction resampling step was used to mix the model-conditioned densities. In that work, though mode switching is performed analytically, posterior model probabilities are estimated using the updated weights for base state. Therefore the model probabilities are not memorized in each estimation cycle. Note that the clutter issue was not addressed in the previous works [10, 12, 13].

For practical tracking problems, data association technique must be considered to select measurements for use from all validated measurements that may contain spurious measurements. In this paper, a novel IMMPF combined with probabilistic data association (PDA) [14] is developed for maneuvering target tracking in clutter. To fix the number of particles in each mode, we completely separate the base state estimation and modal state estimation. Base state estimation is performed numerically using the continuous-valued base state particles, whereas modal state is estimated analytically using the exact Bayesian approach. In our algorithm, mode switching is independent of the number of particles and associated weights in each model. However, the model probabilities need be memorized for Bayesian filtering recursion in the estimation cycle. The rest of the paper is organized as follows. In Section 2, the recursive Bayesian estimator is derived for maneuvering target state estimation in cluttered scenario. In Section 3, we develop a novel PF to implement the Bayesian estimator in Section 2. Monte Carlo simulation results are given in Section 4 to validate the proposed algorithm. Section 5 presents some concluding remarks.

2. BAYESIAN ESTIMATOR FOR MANEUVERING TARGET TRACKING IN CLUTTER

2.1. Tracking Model

Consider a nonlinear stochastic hybrid MM system described by

$$\mathbf{x}_k = f_k(\mathbf{x}_{k-1}, \mathbf{v}_{k-1}, M_k) \tag{1}$$

$$\mathbf{z}_k = h_k(\mathbf{x}_k) + \mathbf{w}_k \tag{2}$$

where k is the time index, $\mathbf{x}_k$ is the n_x dimensional base state, M_k, the modal state, denotes the model in effect during the time interval $(t_{k-1}, t_k]$, $\mathbf{z}_k$ is the n_z dimensional target-originated measurement, the process noise sequence $\mathbf{v}_k$ and measurement noise sequence $\mathbf{w}_k$ are assumed to be white with known probability density functions (pdf's). Without loss of generality, the measurement noise in target tracking model is usually assumed to be additive Gaussian noise with zero-mean and covariance matrix $\mathbf{R}_k$, i.e., $\mathbf{w}_k \backsim N(\mathbf{0}, \mathbf{R}_k)$, where $N(\mu, \Sigma)$ denotes the Gaussian distribution with mean μ and covariance Σ. The two noise sequence, the initial base state $\mathbf{x}_0$ and the mode sequence are independent of each other. The modal state M_k is a Markov chain with known initial model probabilities and time-homogeneous Markovian transition probability

$$Pr\left\{M_k = l\,|M_{k-1} = r\right\} = p_{rl}, \quad r, l = 1, \ldots, L \tag{3}$$

where $Pr\{\cdot|\cdot\}$ denotes a conditional probability, L is the number of possible models.

In cluttered scenario, instead of a single $\mathbf{z}_k$, a set of validated measurements denoted as $\mathbf{Z}_k = \{\mathbf{z}_{k,1}, \mathbf{z}_{k,2}, \ldots, \mathbf{z}_{k,m_k}\}$ are usually obtained from the sensor at the time k, where m_k is the number of validated measurements. The measurement set contains clutter measurements and, if detected, a target measurement with a probability of P_g. The probability that the target is detected at each scan is denoted as P_d. A typical model for clutter measurements is that they are uniformly spatially distributed within the validation region and independent across time assuming that the number of them accords with a Poisson distribution with mean λV_k, where λ is the spatial clutter density and V_k is the volume of validation region.

2.2. Bayesian State Estimation

In the Bayesian approach to above target tracking problem, the goal is to find a recursive way to compute the conditional pdf $p\left(\mathbf{x}_k, M_k = l|\mathbf{Z}^k\right)$, i.e., the posterior distribution of the augmented target state based on all available measurement history $\mathbf{Z}^k = \{\mathbf{Z}_1, \mathbf{Z}_2, \ldots, \mathbf{Z}_k\}$.

Before discussing the new algorithm, we would like to review briefly on the standard approach to apply the PF to our problem. Given the posterior distribution of the augmented target state $\mathbf{y}_{k-1} = [\mathbf{x}_{k-1}; M_{k-1}]$ at the previous time $k-1$, the posterior distribution at time k can be predicted via the Chapman-Kolmogorov equation and then corrected with the current measurements using Bayes' formula as follows [9] :

$$p(\mathbf{y}_k|\mathbf{Z}^{k-1}) = \int p(\mathbf{y}_k|\mathbf{y}_{k-1})p\left(\mathbf{y}_{k-1}|\mathbf{Z}^{k-1}\right)d\mathbf{y}_{k-1} \tag{4}$$

$$p(\mathbf{y}_k|\mathbf{Z}^k) = \frac{p\left(\mathbf{Z}_k|\mathbf{y}_k\right)p\left(\mathbf{y}_k|\mathbf{Z}^{k-1}\right)}{\int p\left(\mathbf{Z}_k|\mathbf{y}_k\right)p\left(\mathbf{y}_k|\mathbf{Z}^{k-1}\right)d\mathbf{y}_k} \tag{5}$$

where $p(\cdot|\cdot)$ represents a conditional pdf. Note that the augmented target state has two components, the continuous valued base state and the discrete valued modal state. For the problem under consideration, the transition density of the augmented target state is given by

$$p(\mathbf{y}_k|\mathbf{y}_{k-1}) = p(\mathbf{x}_k|\mathbf{x}_{k-1}, M_k)Pr\left\{M_k|M_{k-1}\right\} \tag{6}$$

where the state transition density $p(\mathbf{x}_k|\mathbf{x}_{k-1}, M_k = l)$ can be obtained from the dynamic Equation (1). The measurement likelihood $p(\mathbf{Z}_k|\mathbf{y}_k) = p(\mathbf{Z}_k|\mathbf{x}_k)$ in (5) will be given in (17). To numerically evaluate the Chapman-Kolmogorov-Bayes (CKB) filter recursion shown in (4) and (5), a SIR procedure with hybrid state particles drawn from the conditional distribution of the augmented state is usually used, which results in the dependence of mode probability on the relative number of particles in that mode.

To avoid using the augmented hybrid particles, the joint posterior density of base state and modal state $p(\mathbf{x}_k, M_k = l|\mathbf{Z}^k)$ can be rewritten as

$$p\left(\mathbf{x}_k, M_k = l|\mathbf{Z}^k\right) = p\left(\mathbf{x}_k|M_k = l, \mathbf{Z}^k\right)Pr\left(M_k = l|\mathbf{Z}^k\right). \tag{7}$$

The model-conditioned posterior pdf $p\left(\mathbf{x}_k|M_k = l, \mathbf{Z}^k\right)$ of the base state can be obtained as follows

$$\begin{aligned} p(\mathbf{x}_k|M_k = l, \mathbf{Z}^k) &= \frac{p\left(\mathbf{Z}_k|M_k = l, \mathbf{x}_k\right)p\left(\mathbf{x}_k|M_k = l, \mathbf{Z}^{k-1}\right)}{\int p(\mathbf{Z}_k|M_k = l, \mathbf{x}_k)p\left(\mathbf{x}_k|M_k = l, \mathbf{Z}^{k-1}\right)d\mathbf{x}_k} \\ &= \frac{p(\mathbf{Z}_k|\mathbf{x}_k)p\left(\mathbf{x}_k|M_k = l, \mathbf{Z}^{k-1}\right)}{c^1_{k,l}} \end{aligned} \tag{8}$$

where $c^1_{k,l} = \int p(\mathbf{Z}_k|M_k = l, \mathbf{x}_k)p(\mathbf{x}_k|M_k = l, \mathbf{Z}^{k-1})d\mathbf{x}_k$ is the normalization factor and the assumption that sensor measurement model is independent of the target maneuvering mode is used in the derivation of the second equation. The second term in the numerator of the right-hand side (RHS) of (8) is the prediction of the base state conditioned on model M_k and is given by

$$\begin{aligned} & p\left(\mathbf{x}_k|M_k=l,\mathbf{Z}^{k-1}\right) \\ &= \int p\left(\mathbf{x}_k|\mathbf{x}_{k-1},M_k=l\right)p\left(\mathbf{x}_{k-1}|M_k=l,\mathbf{Z}^{k-1}\right)d\mathbf{x}_{k-1} \end{aligned} \tag{9}$$

The second term under the above integral can be rewritten, using the total probability theorem with respect to all the possible models at time $k-1$, as

$$p\left(\mathbf{x}_{k-1}|M_k=l,\mathbf{Z}^{k-1}\right) = \sum_{r=1}^{L} p(\mathbf{x}_{k-1}|M_{k-1}=r,\mathbf{Z}^{k-1})\mu_{k-1}^{r|l} \tag{10}$$

where $p\left(\mathbf{x}_{k-1}|M_{k-1} = r,\mathbf{Z}^{k-1}\right)$ is the given model-conditioned posterior pdf of the base state at the previous time $k-1$, $\mu_{k-1}^{r|l}$ is the mixing probability and is given by

$$\begin{aligned} \mu_{k-1}^{r|l} &= Pr\left(M_{k-1}=r|M_k=l,\mathbf{Z}^{k-1}\right) \\ &= \frac{Pr\left(M_k=l|M_{k-1}=r,\mathbf{Z}^{k-1}\right)Pr\left(M_{k-1}=r|\mathbf{Z}^{k-1}\right)}{Pr\left(M_k=l|\mathbf{Z}^{k-1}\right)} \\ &= \frac{1}{\mu_{k|k-1}^{l}}p_{rl}\mu_{k-1|k-1}^{r} \end{aligned} \tag{11}$$

where $\mu_{k-1|k-1}^{r} = Pr(M_{k-1}=r|\mathbf{Z}^{k-1})$ is the given model probability at the previous time $k-1$, $\mu_{k|k-1}^{l} = Pr(M_k=l|\mathbf{Z}^{k-1})$, given by

$$\mu_{k|k-1}^{l} = \sum_{r=1}^{L} p_{rl}\mu_{k-1|k-1}^{r}, \tag{12}$$

is the predicted model probability. The first term in the numerator of the RHS of (8) is the measurement likelihood conditioned on the base state $\mathbf{x}_k$. If the current measurement $\mathbf{Z}_k$ is available, it can be rewritten, by expanding it over the association hypotheses, as

$$\begin{aligned} p(\mathbf{Z}_k|\mathbf{x}_k) &= \sum_{j=0}^{m_k} p(\mathbf{Z}_k|\theta_{k,j},m_k,\mathbf{x}_k)p(\theta_{k,j}|m_k,\mathbf{x}_k) \\ &= \sum_{j=0}^{m_k} p(\mathbf{Z}_k|\theta_{k,j},m_k,\mathbf{x}_k)p(\theta_{k,j}|m_k) \end{aligned} \tag{13}$$

where $\theta_{k,0}$ denotes the hypothesis that all validated measurements are due to clutter, $\theta_{k,j}, j = 1, 2, \ldots, m_k$ denotes the hypothesis that measurement $\mathbf{z}_{k,j}$ is due to the target with the remaining measurements due to clutter, $p(\theta_{k,j}|m_k)$ is the prior probability of $\theta_{k,j}$ conditioned on the number of measurements. In PDA approach, it was shown in [15] that

$$p(\theta_{k,j}|m_k) = \begin{cases} (1-P_dP_g)\gamma(m_k)[P_dP_g + (1-P_dP_g)\gamma(m_k)]^{-1}, & j=0 \\ \frac{1}{m_k}P_dP_g[P_dP_g + (1-P_dP_g)\gamma(m_k)]^{-1}, & j=1,\ldots,m_k \end{cases} \tag{14}$$

where $\gamma(m_k) = \frac{\mu_F(m_k)}{\mu_F(m_k-1)}$ and $\mu_F(m_k)$ is probability mass function (pmf) for the number of clutter measurements. Substituting the assumed Poisson pmf with mean λV_k into (14) yields

$$p(\theta_{k,j}|m_k) = \begin{cases} \frac{(1-P_dP_g)\lambda V_k}{(1-P_dP_g)\lambda V_k + m_k P_d P_g}, & j=0 \\ \frac{P_dP_g}{(1-P_dP_g)\lambda V_k + m_k P_d P_g}, & j=1,\ldots,m_k \end{cases}. \tag{15}$$

Since the clutter measurements are assumed to be uniformly spatially distributed within the validation region, the first component of the summation in (13) is given by

$$p(\mathbf{Z}_k|\theta_{k,j}, m_k, \mathbf{x}_k) = \begin{cases} (1/V_k)^{m_k}, & j=0 \\ P_g^{-1}(1/V_k)^{m_k-1}N(e_{k,j}; \mathbf{0}, \mathbf{R}_k), & j=1,\ldots,m_k \end{cases} \tag{16}$$

where $e_{k,j} = z_{k,j} - h_k(\mathbf{x}_k)$. Substituting (15) and (16) into (13) gives

$$p(\mathbf{Z}_k|\mathbf{x}_k) = \frac{1 - P_dP_g + P_d/\lambda \cdot \sum_{j=1}^{m_k} N(e_{k,j}; \mathbf{0}, \mathbf{R}_k)}{[1 - P_dP_g + m_kP_dP_g/(\lambda V_k)]V_k^{m_k}}. \tag{17}$$

The posterior model probability $Pr(M_k = l|\mathbf{Z}^k)$ in (7) can be calculated by

$$\begin{aligned} &Pr\left(M_k = l|\mathbf{Z}^k\right) \\ &= \frac{p\left(\mathbf{Z}_k|M_k = l, \mathbf{Z}^{k-1}\right) Pr\left(M_k = l|\mathbf{Z}^{k-1}\right)}{\sum_{l=1}^{L} p\left(\mathbf{Z}_k|M_k = l, \mathbf{Z}^{k-1}\right) Pr\left(M_k = l|\mathbf{Z}^{k-1}\right)} \\ &= \frac{p\left(\mathbf{Z}_k|M_k = l, \mathbf{Z}^{k-1}\right) \mu_{k|k-1}^{l}}{c_k^2} \end{aligned} \tag{18}$$

where $c_k^2 = \sum_{l=1}^{L} p(\mathbf{Z}_k|M_k = l, \mathbf{Z}^{k-1})Pr(M_k = l|\mathbf{Z}^{k-1})$ is the normalization factor. The model likelihood $p(\mathbf{Z}_k|M_k = l, \mathbf{Z}^{k-1})$ in

(18) can be formulated, also by expanding it over the association hypotheses, as

$$\begin{aligned} & p\left(\mathbf{Z}_k | M_k = l, \mathbf{Z}^{k-1}\right) \\ = & \sum_{j=0}^{m_k} p\left(\mathbf{Z}_k | \theta_{k,j}, M_k = l, m_k, \mathbf{Z}^{k-1}\right) p\left(\theta_{k,j} | M_k = l, m_k, \mathbf{Z}^{k-1}\right) \\ = & \sum_{j=0}^{m_k} p\left(\mathbf{Z}_k | \theta_{k,j}, M_k = l, m_k, \mathbf{Z}^{k-1}\right) p\left(\theta_{k,j} | m_k\right). \end{aligned} \tag{19}$$

In view of the assumed uniform distribution for the clutter measurements, the first term in above summation can be given by

$$\begin{aligned} & p\left(\mathbf{Z}_k | \theta_{k,j}, M_k = l, m_k, \mathbf{Z}^{k-1}\right) \\ = & \begin{cases} (1/V_k)^{m_k}, & j = 0 \\ P_g^{-1}(1/V_k)^{m_k-1} p\left(\mathbf{z}_{k,j} | \theta_{k,j}, M_k = l, m_k, \mathbf{Z}^{k-1}\right), & j = 1, \ldots, m_k \end{cases} \end{aligned} \tag{20}$$

where

$$\begin{aligned} & p\left(\mathbf{z}_{k,j} | \theta_{k,j}, M_k = l, m_k, \mathbf{Z}^{k-1}\right) \\ = & \int p(\mathbf{z}_{k,j} | \theta_{k,j}, m_k, \mathbf{x}_k) p\left(\mathbf{x}_k | M_k = l, \mathbf{Z}^{k-1}\right) d\mathbf{x}_k \end{aligned} \tag{21}$$

Substituting (15), (20) and (21) into (19) yields

$$\begin{aligned} & p\left(\mathbf{Z}_k | M_k = l, \mathbf{Z}^{k-1}\right) \\ = & \frac{P_d}{m_k}[(1 - P_d P_g)\lambda V_k / m_k + P_d P_g]^{-1} V_k^{1-m_k} \left[\frac{\lambda(1 - P_d P_g)}{P_d} \right. \\ & \left. + \sum_{j=1}^{m_k} \int p(\mathbf{z}_{k,j} | \theta_{k,j}, m_k, \mathbf{x}_k) p(\mathbf{x}_k | M_k = l, \mathbf{Z}^{k-1}) d\mathbf{x}_k \right] \end{aligned} \tag{22}$$

Hereto, both model-conditioned posterior pdf $p(\mathbf{x}_k | M_k = l, \mathbf{Z}^k)$ of the base state and posterior model probability $Pr(M_k = l | \mathbf{Z}^k)$ can be computed based on the available information. For output purpose, the posterior pdf of the base state is given by

$$\begin{aligned} p\left(\mathbf{x}_k | \mathbf{Z}^k\right) &= \sum_{l=1}^{L} p\left(\mathbf{x}_k, M_k = l | \mathbf{Z}^k\right) \\ &= \sum_{l=1}^{L} p\left(\mathbf{x}_k | M_k = l, \mathbf{Z}^k\right) Pr\left(M_k = l | \mathbf{Z}^k\right) \end{aligned} \tag{23}$$

3. PARTICLE FILTERING IMPLEMENTATION

Summarizing the development of previous section, the recursive equations of Bayesian state estimation can be numerically implemented as follows.

(i) At the beginning, randomly generate n initial (base state) particles $\mathbf{x}_0^{i,l}$ with equal weight $w_0^{i,l} = 1/n$ from each initial model-conditioned posterior pdf $p(\mathbf{x}_0|M_0 = l)$. Note that the total number of proposed particles is nL and n should be chosen as a trade-off between computational load and estimation accuracy.

(ii) For $k = 1, 2, \ldots$, do the following.

(a) Density mixing (Interaction). Compute the mixing probabilities and predicted model probabilities according to Equations (11) and (12). Then following Equation (10), the mixed density matched to mode $M_k = l$ can be approximated by

$$\hat{p}(\mathbf{x}_{k-1}|M_k = l, \mathbf{Z}^{k-1}) = \sum_{r=1}^{L}\left[\sum_{i=1}^{n} w_{k-1}^{i,r}\delta(\mathbf{x}_{k-1} - \mathbf{x}_{k-1}^{i,r})\right]\mu_{k-1}^{r|l}$$

$$= \sum_{i=1}^{n}\sum_{r=1}^{L}\mu_{k-1}^{r|l} w_{k-1}^{i,r}\delta\left(\mathbf{x}_{k-1} - \mathbf{x}_{k-1}^{i,r}\right). \quad (24)$$

where the notation $\hat{p}$ denotes the numerical approximation to the true density p by the corresponding particles, $\delta(\cdot)$ is the Dirac delta distribution, a special case of Gaussian distribution. Equation (24) indicates clearly that $\hat{p}(\mathbf{x}_{k-1}|M_k = l, \mathbf{Z}^{k-1})$ is an empirical density spanned by nL particles $\overline{\mathbf{x}}_{k-1}^{m,l} = \mathbf{x}_{k-1}^{i,r}$ with associated weights $\overline{w}_{k-1}^{m,l} = \mu_{k-1}^{r|l} w_{k-1}^{i,r}$, where $m = 1, 2, \ldots, nL$.

(b) Resampling. It is observed from (24) that the number of particles for the model-conditioned density has increased from n to nL at the end of interaction step. The growth in the number of particles at each estimation cycle will prevent the practicability of this filter. A reasonable approach to solve this issue is to perform resampling of $\hat{p}(\mathbf{x}_{k-1}|M_k = l, \mathbf{Z}^{k-1})$. Consequently, resample n particles $\widetilde{\mathbf{x}}_{k-1}^{i,l}$ with associated weights $\widetilde{w}_{k-1}^{i,l}$ from the empirical density $\hat{p}(\mathbf{x}_{k-1}|M_k = l, \mathbf{Z}^{k-1})$ on the basis of $\overline{w}_{k-1}^{m,l}$. Note that the notation $\widetilde{\mathbf{x}}$ and $\widetilde{w}$ denote the resampled particles and their corresponding weights.

(c) Model-conditioned updates. For each maneuvering mode $M_k = l$, do the following.

1. Perform the time propagation using the process Equation (1) and the measurement Equation (2):
$$\widetilde{\mathbf{x}}_k^{i,l} = f_k(\widetilde{\mathbf{x}}_{k-1}^{i,l}, \mathbf{v}_{k-1}^{i,l}, M_k = l) \tag{25}$$
$$\widetilde{\mathbf{z}}_k^{i,l} = h_k(\widetilde{\mathbf{x}}_k^{i,l}) \tag{26}$$
$$\widehat{\mathbf{z}}_k^{l} = \sum_{i=1}^{n} \widetilde{w}_{k-1}^{i,l} \widetilde{\mathbf{z}}_k^{i,l} \tag{27}$$
where $\mathbf{v}_{k-1}^{i,l}$ is randomly generated according to the known pdf of process noise, the notation $\widehat{\mathbf{z}}$ represents the weighted average of $\widetilde{\mathbf{z}}$, i.e., the priori estimate of the target-originated measurement.
2. Compute the relative likelihood $\psi_k^{i,l}$ of each particle $\widetilde{\mathbf{x}}_k^{i,l}$ based on the validated measurements $\mathbf{Z}_k$ following Equation (17):
$$\psi_k^{i,l} = 1 - P_d P_g + P_d/\lambda \cdot \sum_{j=1}^{m_k} N(z_{k,j} - \widetilde{\mathbf{z}}_k^{i,l}; \mathbf{0}, \mathbf{R}_k) \tag{28}$$
3. Generate the model-conditioned posterior particles and update the weights using sequential importance sampling (SIS) procedure:
$$\mathbf{x}_k^{i,l} = \widetilde{\mathbf{x}}_k^{i,l} \tag{29}$$
$$w_k^{i,l} = \psi_k^{i,l} \widetilde{w}_{k-1}^{i,l} \Bigg/ \sum_{t=1}^{n} \psi_k^{t,l} \widetilde{w}_{k-1}^{t,l} \tag{30}$$
where the importance density is assumed to be the prior density $p(\mathbf{x}_k|\mathbf{x}_{k-1}, M_k = l)$ for convenience.
4. Compute the relative model likelihood Λ_k^l according to Equation (22) where the density $p(\mathbf{z}_{k,j}|\theta_{k,j}, M_k = l, m_k, \mathbf{Z}^{k-1})$ is approximated by a continuous Gaussian mixture distribution instead of a discrete Dirac mixture distribution (a special case of Gaussian mixture) in order to obtain its accurate value for a specific $\mathbf{z}_{k,j}$:
$$\mathbf{S}_k^l = \sum_{i=1}^{n} \widetilde{w}_{k-1}^{i,l} (\widetilde{\mathbf{z}}_k^{i,l} - \widehat{\mathbf{z}}_k^l)(\widetilde{\mathbf{z}}_k^{i,l} - \widehat{\mathbf{z}}_k^l)^T + \mathbf{R}_k \tag{31}$$
$$\Lambda_k^l = \frac{\lambda(1 - P_d P_g)}{P_d} + \sum_{j=1}^{m_k} \sum_{i=1}^{n} \widetilde{w}_{k-1}^{i,l} N(z_{k,j}; \widetilde{\mathbf{z}}_k^{i,l} + \mathbf{w}_k^{i,l}, \mathbf{S}_k^l) \tag{32}$$

where $\mathbf{w}_k^{i,l}$ is randomly generated according to the known pdf of measurement noise.

(d) Model probability updates. According to Equation (18), the posterior model probability $\mu_{k|k}^l$ for $M_k = l$ is given by

$$\mu_{k|k}^l = \frac{1}{c_k^2}\mu_{k|k-1}^l \Lambda_k^l \tag{33}$$

(e) Output of the posterior base state estimates. The posterior density of the base state can be approximated, following Equation (23), by

$$\begin{aligned}\hat{p}\left(\mathbf{x}_k|\mathbf{Z}^k\right) &= \sum_{l=1}^{L}\sum_{i=1}^{n} w_k^{i,l}\delta\left(\mathbf{x}_k - \mathbf{x}_k^{i,l}\right)\mu_{k|k}^l \\ &= \sum_{i=1}^{n}\sum_{l=1}^{L} \mu_{k|k}^l w_k^{i,l}\delta\left(\mathbf{x}_k - \mathbf{x}_k^{i,l}\right)\end{aligned} \tag{34}$$

Equation (34) makes clear that now we have nL particles $\mathbf{x}_k^m = \mathbf{x}_k^{i,l}$ and associated weights $w_k^m = \mu_{k|k}^l w_k^{i,l}$ that are distributed according to the density $p\left(\mathbf{x}_k|\mathbf{Z}^k\right)$. Then we can compute any desired statistical measure of the density.

4. SIMULATION RESULTS

In this section, we illustrate the performance of the proposed algorithm by 100 Monte Carlo simulations for a radar tracking scenario with the target trajectory shown in Fig. 1. Here, an aircraft, starting from the initial position (20000 m, 20000 m), flies towards the radar at a constant course with a velocity of (−150 m/s, −150 m/s) for the first 60 s. Then it performs a 180° left turn for course change with a turn rate of 3°/s. After the turn, the straight and level flight is continued for another 60 s. The radar, located at (0 m, 0 m), provides range, range rate and bearing measurements for every sampling interval $T = 3$ s with the respective standard deviations of the measurement errors $\sigma_r = 50$ m, $\sigma_{\dot{r}} = 5$ m/s, $\sigma_b = 2$ mrad. The measurement errors of range and range rate are correlated with correlation coefficient $\rho = -0.2$. The target detection probability P_d and gate probability P_g are assumed to be 0.9 and 0.9989, respectively.

For comparison purpose, both our proposed IMMPF algorithm and the MMPF algorithm described in [9] are used to estimate the target state in cluttered scenario. In each of the two algorithms, we employ the systematic resampling approach proposed in [16]

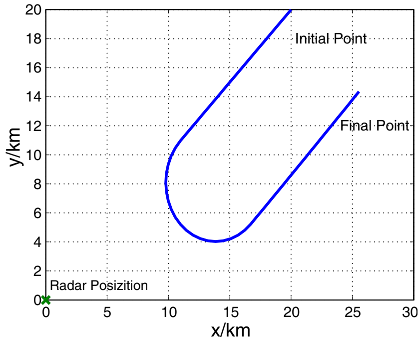

Figure 1. The target trajectory and radar position.

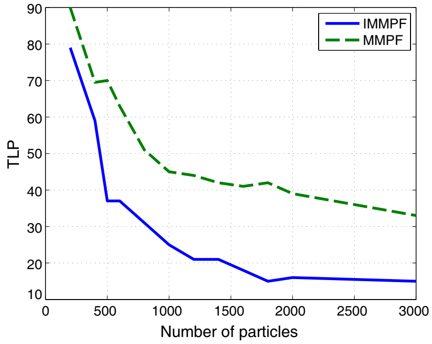

Figure 2. Comparisons of TLP for different sample sizes.

in the resampling step. The PDA technique is adopted to select measurements for use in the tracking filters in the experiment. Two second order WNA models [6] with different noise levels are used to model the target motion. The one with standard deviation $0.1\,\mathrm{m/s^2}$ is used to model the uniform motion and the other one with standard deviation $10\,\mathrm{m/s^2}$ for the maneuver. The initial mode probabilities are assumed to be equal and the Markovian mode transition probability matrix is set to be

$$\mathbf{\Pi} = \begin{bmatrix} 0.98 & 0.02 \\ 0.05 & 0.95 \end{bmatrix}. \tag{35}$$

Here track loss percentage (TLP) is used to assess the tracking quality since the TLP measure is more critical than the tracking error measure from a filter that is successfully following the target from the practical point of view. A track is considered to be lost if for 10 consecutive scans, the estimated target state falls out of the 5-sigma region centered around the true target position in the measurement space [17], that is,

$$[h_k(\mathbf{x}_{k|k}) - h_k(\mathbf{x}_k)]^T \mathbf{R}_k^{-1} [h_k(\mathbf{x}_{k|k}) - h_k(\mathbf{x}_k)] > 25. \tag{36}$$

The TLP is measured as the ratio of the number of MC runs for which the tracks are lost to the number of total MC runs performed. The performance comparisons of two competitive tracking algorithms are shown in Fig. 2, Fig. 3 and Fig. 4. In Fig. 2, we vary the sample size used by the PFs with the fixed spatial clutter density $\lambda = 2 \times 10^{-5}$ points/(scan·m· m/s·rad), whereas in Fig. 3, we vary the clutter density with the fixed 1000 particles in total. Obviously our proposed IMMPF algorithm always outperforms the MMPF algorithm, especially with a reasonable sample size (e.g., 500–2000 particles) and

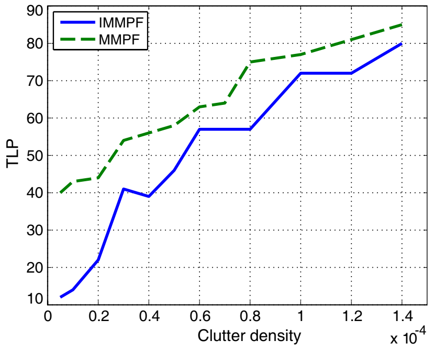

Figure 3. Comparisons of TLP for different clutter densities.

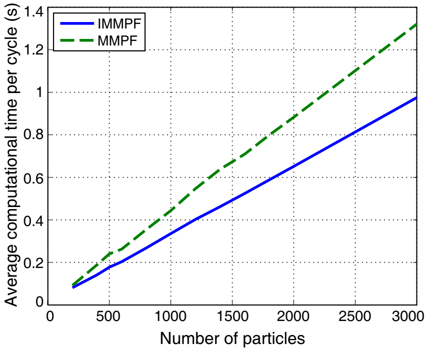

Figure 4. Comparisons of computational complexity per estimation cycle for different sample sizes.

in moderate cluttered scenarios. However, a brute-force increase in the total number of particles will contribute to narrow the performance gap between two PFs gradually.

The computational complexity of each PF, measured by means of average computational time on a dual-core 2.6 GHz Intel Pentium processor for per estimation cycle as function of the particle number, is illustrate in Fig. 4. It should be underlined that the computational complexity of a PF also depends on its practical implementations and the choices made in the importance sampling and resampling procedures. In a sense, Fig. 4 only offers the relative computational load comparison. Provided the same number of particles are used, the computational load of the proposed algorithm is reduced as expected because the modal state is estimated analytically and discrete valued particles are not needed in its filtering cycle. As the number of particles increases, the reduction in computational load will steadily grow larger.

5. CONCLUSIONS

We have addressed the problem of maneuvering target tracking with measurement origin uncertainty in this paper. First, a recursive Bayesian state estimation approach is described, and then a novel IMMPF has been proposed to implement it. The new algorithm overcomes the drawback with the approach of directly applying the standard PF to maneuvering target tracking by completely separating base state estimation and modal state estimation. The resampling step need be performed before the measurement update step to alleviate the growing number of the particles inherent to density mixing. Simulation results have exhibited a significant improvement of the new IMMPF over the prior algorithms for maneuvering target tracking in clutter.

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