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WAVE PROPAGATION AND FRESNEL COEFFICIENTS FOR THREE LAYERED UNIAXIALLY ANISOTROPIC MEDIA WITH ARBITRARILY ORIENTED OPTIC AXES

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Abstract—Fresnel coefficients for three-layered uniaxially anisotropic media with arbitrarily oriented optic axes have been obtained using half-space reflection and transmission coefficients. The optic axes of the anisotropic media are assumed to be tilted at different angles $((\psi_i, \chi_i), i = 1, 2)$. This gives arbitrary orientation for anisotropic medium in the stratified configuration. The half space coefficients are derived at the boundary surfaces of two different media including isotropic-anisotropic, anisotropic-anisotropic, anisotropic-isotropic interfaces with the application of boundary conditions. The interface between two media for the three-layered media leads to four different cases of wave propagation, which are analyzed in detail. The results are compared with the existing results in the limiting cases analytically. The results presented in this paper can be used to establish dyadic Green's functions, which can be used to calculate radiation and scattering from the stratified structure.

1. INTRODUCTION

The material characteristics are very important in the development of high performance devices at the microwave ranges. Recent advances in technology dictate use of materials with superior electrical and magnetic properties. Anisotropic materials are among the materials

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that have such properties [1–3] and provide design flexibility to have better performing microwave devices.

Composite and layered structures involving anisotropic materials have been commonly employed in electromagnetic problems including scattering and radiation [4–13]. The better device performance is obtained if the electromagnetic properties of the medium are known. Electromagnetic properties of a medium in layered or single layer configuration is best-understood with the calculation of Fresnel reflection and transmission coefficients for the structure under consideration. In addition, wave amplitudes for the existing polarization in each layer can also be found using Fresnel coefficients. However, it is mathematically tedious to obtain closed form of Fresnel coefficients for multi-layered media. In the multilayer configuration, n layer has $(n + 1)$ boundaries, which gives $(2n + 2)$ linear equations. If some of the layers are anisotropic, this increases the complexity of the problem. There has been some study on the derivation of Fresnel coefficients for multilayer structures in the literature for isotropic and anisotropic media. The closed form expressions for two-layered uniaxially anisotropic media with an arbitrarily oriented optic axis when the optic axis is tilted only in one direction is given in [5]. In [5], two-layer reflection and transmission coefficients are obtained in terms of half space coefficients using matrix method. The two-layer configuration in [5] has isotropic-anisotropic and anisotropic-isotropic interfaces. A generalized form of Snell's law for anisotropic media is given in [14] where refraction angles are obtained numerically when optic axis is normal to the boundary. Concept of refraction in a more general anisotropic media which can have negative or positive permeability and permittivity tensors have been investigated in [15]. There has been some study on the derivation of Fresnel coefficients for also layered and single layer biaxially anisotropic media, metamaterials, bianisotropic and gyrotropic materials [16–22].

In this paper, Fresnel coefficients for three-layered uniaxially anisotropic media with arbitrarily oriented optic axes are obtained using half-space reflection and transmission coefficients. The optic axis of the anisotropic medium in each layer is assumed to be tilted in preferred horizontal and azimuthal directions $((\psi_i, \chi_i)$, $i = 1, 2)$. This approximates more closely to the crystallographic optic axes of sea ice [23]. The method is applied between isotropic-anisotropic, anisotropic-anisotropic, and anisotropic-isotropic interfaces using continuity of tangential components of electric and magnetic field vectors. Wave amplitudes in each layer, reflection and transmission coefficients for an incident wave with an arbitrary polarization have been derived. The results are compared in the limiting cases with the existing results obtained in [5] and agreement has been seen on all of them. The study presented in this paper can be used in radiation, scattering and device design problem to analyze electromagnetic properties of the medium.

2. FORMULATION OF THE PROBLEM

The geometry of the three-layered anisotropic media with arbitrarily oriented optic axes is illustrated in Fig. 1. The optic axes for the uniaxially anisotropic media in Regions 1 and 2 are rotated by χ with

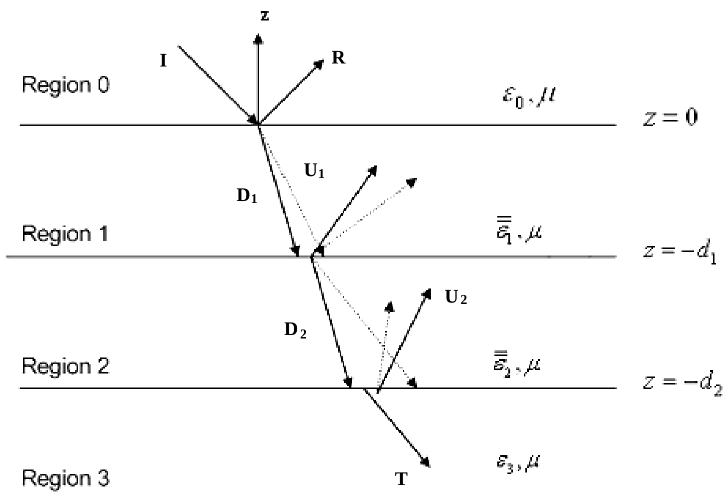

Figure 1. Geometry of three-layered anisotropic media.

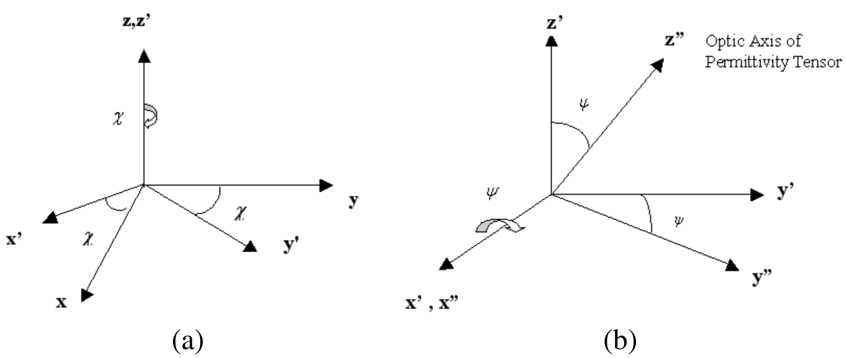

Figure 2. Geometry of uniaxially anisotropic medium.

respect to the z-axis and tilted by ψ with respect to x-axis as shown in Figs. 2(a) and 2(b), respectively. The permittivity tensor $\bar{\bar{\varepsilon}}$ of the anisotropic medium in each layer after its optic axis is tilted with respect z-axis and x-axis can be represented as

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \tag{1}$$

where

$$\varepsilon_{11} = \varepsilon + (\varepsilon_z - \varepsilon) \sin^2 \psi \sin^2 \chi \tag{2}$$
$$\varepsilon_{12} = \varepsilon_{21} = (\varepsilon_z - \varepsilon) \sin^2 \psi \sin \chi \cos \chi \tag{3}$$
$$\varepsilon_{13} = \varepsilon_{31} = (\varepsilon_z - \varepsilon) \sin \psi \cos \psi \sin \chi \tag{4}$$
$$\varepsilon_{22} = \varepsilon + (\varepsilon_z - \varepsilon) \sin^2 \psi \cos^2 \chi \tag{5}$$
$$\varepsilon_{23} = \varepsilon_{32} = (\varepsilon_z - \varepsilon) \sin \psi \cos \psi \cos \chi \tag{6}$$
$$\varepsilon_{33} = \varepsilon \sin^2 \psi + \varepsilon_z \cos^2 \psi \tag{7}$$

where ε and ε_z are the permittivity of constants of uniaxially anisotropic medium in x, y and z directions without rotation and defined in the following permittivity tensor

$$\bar{\bar{\varepsilon}}^{(o)} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

We assume two sets of medium parameters, (ε_1, ε_{1z}, χ_1, ψ_1), and (ε_2, ε_{2z}, χ_2, ψ_2), for regions 1 and 2, respectively.

The boundary conditions at the interfaces ($z = 0$, $z = -d_1$ and $z = -d_2$) of the three layered structure are :

$$\begin{aligned} \widehat{z} \times \bar{E}_0 &= \widehat{z} \times \bar{E}_1 && \text{at} \quad z = 0 \\ \widehat{z} \times \nabla \times \bar{E}_0 &= \widehat{z} \times \nabla \times \bar{E}_1 && \text{at} \quad z = 0 \\ \widehat{z} \times \bar{E}_1 &= \widehat{z} \times \bar{E}_2 && \text{at} \quad z = -d_1 \\ \widehat{z} \times \nabla \times \bar{E}_1 &= \widehat{z} \times \nabla \times \bar{E}_2 && \text{at} \quad z = -d_1 \\ \widehat{z} \times \bar{E}_2 &= \widehat{z} \times \bar{E}_3 && \text{at} \quad z = -d_2 \\ \widehat{z} \times \nabla \times \bar{E}_2 &= \widehat{z} \times \nabla \times \bar{E}_3 && \text{at} \quad z = -d_2 \end{aligned} \tag{8}$$

Field vectors in each region have certain polarization and accompanying transmission and reflection coefficients. For instance, the incident wave coming from isotropic region, Region 0, upon uniaxially anisotropic region, Region 1, can be horizontally or vertically polarized wave and can be expressed as

$$\bar{E}_0 = \hat{h}_0(-k_{0z}) e^{i(\bar{\kappa}_0 \cdot \bar{r})} + \hat{h}_0(k_{0z}) R_{01HH} e^{i(\bar{k}_0 \cdot \bar{r})} + \hat{v}_0(k_{0z}) R_{01HV} e^{i(\bar{k}_0 \cdot \bar{r})} \tag{9}$$

for horizontally polarized incident wave or

$$\bar{E}_0 = \hat{v}_0(-k_{0z})e^{i(\bar{\kappa}_0 \cdot \bar{r})} + \hat{v}_0(k_{0z})R_{01VV}e^{i(\bar{k}_0 \cdot \bar{r})} + \hat{h}_0(k_{0z})R_{01VH}e^{i(\bar{k}_0 \cdot \bar{r})} \quad (10)$$

for vertically polarized incident wave. $\bar{k}_0$ and $\bar{\kappa}_0$ are wave vectors for upward and downward propagating waves and $\hat{h}_0(\pm k_{0z})$ and $\hat{v}_0(\pm k_{0z})$ are the unit vectors for horizontally and vertically polarized waves and given by

$$\bar{k}_0 = k_x\hat{x} + k_y\hat{y} + k_{oz}\hat{z} \quad (11)$$

$$\bar{\kappa}_0 = k_x\hat{x} + k_y\hat{y} - k_{oz}\hat{z} \quad (12)$$

$$\hat{h}_0(\pm k_{0z}) = \frac{\hat{z} \times \bar{k}_0}{k_\rho} = \frac{(-\hat{x}k_y + \hat{y}k_x)}{k_\rho} \quad (13)$$

$$\hat{v}_0(k_{0z}) = \frac{\hat{h}_0(k_{0z}) \times \bar{k}_0}{k_0} = \frac{1}{k_0}\left[\frac{k_{oz}(\hat{x}k_x + \hat{y}k_y)}{k_\rho} - \hat{z}k_\rho\right] \quad (14a)$$

$$\hat{v}_0(-k_{0z}) = \frac{\hat{h}_0(-k_{0z}) \times \bar{\kappa}_0}{k_0} = \frac{1}{k_0}\left[-\frac{k_{oz}(\hat{x}k_x + \hat{y}k_y)}{k_\rho} - \hat{z}k_\rho\right] \quad (14b)$$

where $k_\rho = \sqrt{k_x^2 + k_y^2}$, $k_0 = |\bar{k}_0| = \omega\sqrt{\mu_0\varepsilon_0}$. Field vectors in other regions are analyzed in detail in Section 3 based on the wave polarization. It is clear at this point that field vectors have unknown reflection and transmission coefficients for the corresponding polarization as shown in Equations (9) and (10) and these coefficients can be calculated by application of boundary conditions such as the one given in Equation (8). The solution of six linear equations given in (8) leads to a total of twenty four reflection and transmission coefficients as given by Equations (15) and (16). Hence, Equations (15) and (16) represent reflection and transmission coefficients for three-layered uniaxially anisotropic medium with arbitrarily oriented optic axes.

$$R = \begin{bmatrix} R_{HH} & R_{VH} \\ R_{HV} & R_{VV} \end{bmatrix}, \quad T = \begin{bmatrix} X_{HH} & X_{VH} \\ X_{HV} & X_{VV} \end{bmatrix}, \quad D_1 = \begin{bmatrix} A_{Ho} & A_{Vo} \\ A_{He} & A_{Ve} \end{bmatrix} \quad (15)$$

$$U_1 = \begin{bmatrix} B_{Ho} & B_{Vo} \\ B_{He} & B_{Ve} \end{bmatrix}, \quad D_2 = \begin{bmatrix} C_{Ho} & C_{Vo} \\ C_{He} & C_{Ve} \end{bmatrix}, \quad U_2 = \begin{bmatrix} D_{Ho} & D_{Vo} \\ D_{He} & D_{Ve} \end{bmatrix} \quad (16)$$

Here, the subscripts H and V refer to horizontal and vertical polarizations in isotropic medium, and the subscripts o and e refer to ordinary and extraordinary waves in uniaxial medium. It is analytically quite time consuming to obtain the closed form of the coefficients in (15) and (16). Therefore, we adopt the solution method proposed in [4] which is based on representation of Fresnel reflection and transmission coefficients in terms of half-space coefficients. In the following section, the derivation of half-space coefficients is presented.

3. HALF-SPACE REFLECTION AND TRANSMISSION COEFFICIENTS

Let the amplitude vectors of the incident and reflected waves in Region 0 be A and B, respectively as shown in Fig. 3. $C(E)$ and $D(F)$ are amplitude vectors of the downward and upward propagating waves in Region 1 (2), respectively. G is amplitude vector of the transmitted wave in Region 3. Then, these vectors satisfy the following matrix equations.

$$\begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} R_{01} & T_{10} \\ T_{01} & R_{10} \end{bmatrix} \begin{bmatrix} A \\ D \end{bmatrix} \tag{17}$$

$$\begin{bmatrix} D \\ E \end{bmatrix} = \begin{bmatrix} R_{12} & T_{21} \\ T_{12} & R_{21} \end{bmatrix} \begin{bmatrix} C \\ F \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} F \\ G \end{bmatrix} = \begin{bmatrix} R_{23} \\ T_{23} \end{bmatrix} E \tag{19}$$

where R_{01}, T_{01}, R_{10}, T_{10}, R_{12}, T_{12}, R_{21}, T_{21}, R_{23} and T_{23} are the half-space Fresnel reflection and transmission coefficient matrices. It should be noted that R_{12}, T_{12}, R_{21}, T_{21}, R_{23} and T_{23} also account for the phase difference because the boundary is shifted. From the above

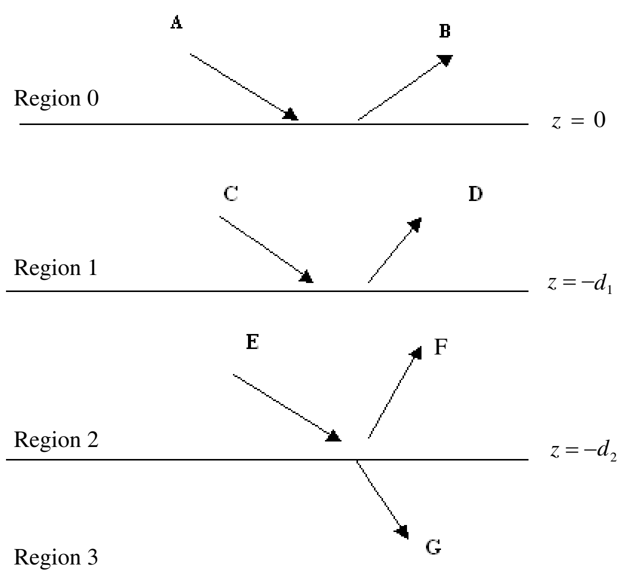

Figure 3. Amplitude vectors in each region of the three-layer problem.

matrix relationship, we obtain the following equations.

$$\begin{array}{lll} B = RA & C = D_1 A & D = U_1 A \\ E = D_2 A & F = U_2 A & G = TA \end{array} \tag{20}$$

where R, T, D_1, U_1, D_2, U_2 are given by Equations (15) and (16). Using Equations (17)–(20), we find

$$R = (R_{01} + T_{10} U_1) A \tag{21}$$

$$U_1 = \left\{ R_{12} + T_{21} R_{23} (\mathrm{I} - R_{21} R_{23})^{-1} T_{12} \right\} D_1 \tag{22}$$

$$D_1 = (\mathrm{I} - R_{10}\ [\cdot])^{-1} T_{01} \tag{23}$$

$$U_2 = R_{23} (\mathrm{I} - R_{21} R_{23})^{-1} T_{12} D_1 \tag{24}$$

$$D_2 = (\mathrm{I} - R_{21} R_{23})^{-1} T_{12} D_1 \tag{25}$$

$$T = T_{23} (\mathrm{I} - R_{21} R_{23})^{-1} T_{12} D_1 \tag{26}$$

where

$$[\cdot] = \left[R_{12} + T_{21} R_{23} (\mathrm{I} - R_{21} R_{23})^{-1} T_{12} \right] \tag{27}$$

R^{-1} indicates the inverse of matrix R and I is *a unit matrix*. The half-space reflection and transmission coefficients can now be expressed as follows.

$$R_{01} = \begin{bmatrix} R_{01HH} & R_{01VH} \\ R_{01HV} & R_{01VV} \end{bmatrix} \tag{28}$$

$$T_{01} = \begin{bmatrix} X_{Ho} & X_{Vo} \\ X_{He} & X_{Ve} \end{bmatrix} \tag{29}$$

$$R_{10} = \begin{bmatrix} R_{oo} & R_{eo} \\ R_{oe} & R_{ee} \end{bmatrix} \tag{30}$$

$$T_{10} = \begin{bmatrix} X_{oH} & X_{eH} \\ X_{oV} & X_{eV} \end{bmatrix} \tag{31}$$

$$R_{12} = \begin{bmatrix} e^{ik_{1z}^o d_1} R_{12oo} e^{ik_{1z}^o d_1} & e^{-ik_{1z}^{ed} d_1} R_{12eo} e^{ik_{1z}^o d_1} \\ e^{ik_{1z}^o d_1} R_{12oe} e^{ik_{1z}^{eu} d_1} & e^{-ik_{1z}^{ed} d_1} R_{12ee} e^{ik_{1z}^{eu} d_1} \end{bmatrix} = \begin{bmatrix} R_{oo1} & R_{eo1} \\ R_{oe1} & R_{ee1} \end{bmatrix} \tag{32}$$

$$T_{12} = \begin{bmatrix} e^{ik_{1z}^o d_1} X_{12oo} e^{-ik_{2z}^o d_1} & e^{-ik_{1z}^{ed} d_1} X_{12eo} e^{-ik_{2z}^o d_1} \\ e^{ik_{1z}^o d_1} X_{12oe} e^{ik_{2z}^{ed} d_1} & e^{-ik_{1z}^{ed} d_1} X_{12ee} e^{ik_{2z}^{ed} d_1} \end{bmatrix} = \begin{bmatrix} t_{oo1} & t_{eo1} \\ t_{oe1} & t_{ee1} \end{bmatrix} \tag{33}$$

$$R_{21} = \begin{bmatrix} e^{-ik_{2z}^o d_1} R_{21oo} e^{-ik_{2z}^o d_1} & e^{-ik_{2z}^{eu} d_1} R_{21eo} e^{-ik_{2z}^o d_1} \\ e^{-ik_{2z}^o d_1} R_{21oe} e^{ik_{2z}^{ed} d_1} & e^{-ik_{2z}^{eu} d_1} R_{21ee} e^{ik_{2z}^{ed} d_1} \end{bmatrix} = \begin{bmatrix} R_{oo2} & R_{eo2} \\ R_{oe2} & R_{ee2} \end{bmatrix} \tag{34}$$

$$T_{21} = \begin{bmatrix} e^{-ik_{2z}^o d_1} X_{21oo} e^{ik_{1z}^o d_1} & e^{-ik_{2z}^{eu} d_1} X_{21eo} e^{ik_{1z}^o d_1} \\ e^{-ik_{2z}^o d_1} X_{21oe} e^{ik_{1z}^{eu} d_1} & e^{-ik_{2z}^{eu} d_1} X_{21ee} e^{ik_{1z}^{eu} d_1} \end{bmatrix} = \begin{bmatrix} t_{oo2} & t_{eo2} \\ t_{oe2} & t_{ee2} \end{bmatrix} \tag{35}$$

$$R_{23}=\begin{bmatrix} e^{ik_{2z}^o d_2}R_{23oo}e^{ik_{2z}^o d_2} & e^{-ik_{2z}^{ed} d_2}R_{23eo}e^{ik_{2z}^o d_2} \\ e^{ik_{2z}^o d_2}R_{23oe}e^{ik_{2z}^{eu} d_2} & e^{-ik_{2z}^{ed} d_2}R_{23ee}e^{ik_{2z}^{eu} d_2} \end{bmatrix}=\begin{bmatrix} R_{oo3} & R_{eo3} \\ R_{oe3} & R_{ee3} \end{bmatrix} \tag{36}$$

$$T_{23}=\begin{bmatrix} e^{ik_{2z}^o d_2}X_{23oH}e^{-ik_{3z}^{d_2}} & e^{-ik_{2z}^{ed} d_2}X_{23eH}e^{-ik_{3z}^{d_2}} \\ e^{ik_{2z}^o d_2}X_{23oV}e^{-ik_{3z}^{d_2}} & e^{-ik_{2z}^{ed} d_2}X_{23eV}e^{-ik_{3z}^{d_2}} \end{bmatrix}=\begin{bmatrix} t_{oH} & t_{eH} \\ t_{oV} & t_{eV} \end{bmatrix} \tag{37}$$

The exponents in (32)–(37) carry phase factors due to the shift at the interfaces. Half-space coefficients in Equations (28)–(37) agree with those of three-layer isotropic-anisotropic case of Nghiem et al. [24]. Substituting Equations (28)–(37) into Equations (21)–(26), we finally obtain the following:

$$\begin{aligned} R_{\beta\alpha} = {} & R_{01\beta\alpha}+X_{\beta o}(M_{oo}P_{oo}+M_{oe}P_{eo})X_{o\alpha}+X_{\beta o}(M_{oo}P_{oe}+M_{oe}P_{ee})X_{e\alpha} \\ & +X_{\beta e}(M_{eo}P_{oo}+M_{ee}P_{eo})X_{o\alpha}+X_{\beta e}(M_{eo}P_{oe}+M_{ee}P_{ee})X_{e\alpha} \end{aligned} \tag{38}$$

$$\begin{aligned} X_{\beta\alpha} = {} & X_{\beta o}\left\{(M_{oo}t_{oo1}+M_{oe}t_{oe1})L_{oo}+(M_{oo}t_{oe1}+M_{oe}t_{ee1})L_{eo}\right\}t_{o\alpha} \\ & +X_{\beta o}\left\{(M_{oo}t_{oe1}+M_{oe}t_{ee1})L_{ee}+(M_{oo}t_{oo1}+M_{oe}t_{eo1})L_{oe}\right\}t_{e\alpha} \\ & +X_{\beta e}\left\{(M_{eo}t_{oo1}+M_{ee}t_{eo1})L_{oo}+(M_{eo}t_{oe1}+M_{ee}t_{ee1})L_{eo}\right\}t_{o\alpha} \\ & +X_{\beta e}\left\{(M_{eo}t_{oe1}+M_{ee}t_{ee1})L_{ee}+(M_{eo}t_{oo1}+M_{ee}t_{eo1})L_{oe}\right\}t_{e\alpha} \end{aligned} \tag{39}$$

$$A_{\beta\gamma} = X_{\beta o}M_{o\gamma} + X_{\beta e}M_{e\gamma} \tag{40}$$

$$B_{\beta\gamma} = X_{\beta o}(M_{oo}P_{o\gamma} + M_{oe}P_{e\gamma}) + X_{\beta e}(M_{eo}P_{o\gamma} + M_{ee}P_{e\gamma}) \tag{41}$$

$$C_{\beta\gamma} = X_{\beta o}N_{o\gamma} + X_{\beta e}N_{e\gamma} \tag{42}$$

$$D_{\beta\gamma} = X_{\beta o}(N_{oo}R_{o\gamma3} + N_{oe}R_{e\gamma3}) + X_{\beta e}(N_{eo}R_{o\gamma3} + N_{ee}R_{e\gamma3}) \tag{43}$$

where α, $\beta = H$ (horizontal) or V (vertical), $\gamma = o$ (ordinary) or e (extraordinary) and others are defined as follows.

$$L_{oo} = \frac{\mathrm{I} - S_{ee1}}{N_1} \tag{44}$$

$$L_{eo} = \frac{S_{eo1}}{N_1} \tag{45}$$

$$L_{oe} = \frac{S_{oe1}}{N_1} \tag{46}$$

$$L_{ee} = \frac{S_{oo1}}{N_1} \tag{47}$$

$$S_{oo1} = R_{oo3}R_{oo2} + R_{oe3}R_{eo2} \tag{48}$$

$$S_{eo1} = R_{eo3}R_{oo2} + R_{ee3}R_{eo2} \tag{49}$$

$$S_{oe1} = R_{oo3}R_{oe2} + R_{oe3}R_{ee2} \tag{50}$$

$$S_{ee1} = R_{eo3}R_{oe2} + R_{ee3}R_{ee2} \tag{51}$$

$$N_1 = (\mathrm{I} - S_{oo1})(\mathrm{I} - S_{ee1}) - S_{oe1}S_{eo1} \tag{52}$$

$$S_{oo2} = R_{oo3}t_{oo2} + R_{oe3}t_{eo2} \tag{53}$$

$$S_{eo2} = R_{eo3}t_{oo2} + R_{ee3}t_{eo2} \tag{54}$$
$$S_{oe2} = R_{oo3}t_{oe2} + R_{oe3}t_{ee2} \tag{55}$$
$$S_{ee2} = R_{eo3}t_{oe2} + R_{ee3}t_{ee2} \tag{56}$$
$$P_{oo} = R_{oo1}+(t_{oo1}L_{oo}+t_{oe1}L_{eo})S_{oo2}+(t_{oo1}L_{oe} + t_{oe1}L_{ee})S_{eo2} \tag{57}$$
$$P_{eo} = R_{eo1}+(t_{eo1}L_{oo}+t_{ee1}L_{eo})S_{oo2}+(t_{eo1}L_{oe}+t_{ee1}L_{ee})S_{eo2} \tag{58}$$
$$P_{oe} = R_{oe1}+(t_{oo1}L_{oo}+t_{oe1}L_{eo})S_{oe2}+(t_{oo1}L_{oe}+t_{oe1}L_{ee})S_{ee2} \tag{59}$$
$$P_{ee} = R_{ee1}+(t_{oe1}L_{oo}+t_{ee1}L_{eo})S_{oe2}+(t_{eo1}L_{oe}+t_{ee1}L_{ee})S_{ee2} \tag{60}$$
$$M_{oo} = \frac{\mathrm{I} - Q_{ee}}{N_2} \tag{61}$$
$$M_{eo} = \frac{Q_{eo}}{N_2} \tag{62}$$
$$M_{oe} = \frac{Q_{oe}}{N_2} \tag{63}$$
$$M_{ee} = \frac{\mathrm{I} - Q_{oo}}{N_2} \tag{64}$$
$$Q_{oo} = P_{oo}R_{oo} + P_{oe}R_{eo} \tag{65}$$
$$Q_{eo} = P_{eo}R_{oo} + P_{ee}R_{eo} \tag{66}$$
$$Q_{oe} = P_{oo}R_{oe} + P_{oe}R_{ee} \tag{67}$$
$$Q_{ee} = P_{eo}R_{oe} + P_{ee}R_{ee} \tag{68}$$
$$N_2 = (\mathrm{I} - Q_{oo})(\mathrm{I} - Q_{ee}) - Q_{oe}Q_{eo} \tag{69}$$
$$N_{oo} = (M_{oo}t_{oo1} + M_{oe}t_{eo1})L_{oo} + (M_{oo}t_{oe1} + M_{oe}t_{ee1})L_{eo} \tag{70}$$
$$N_{eo} = (M_{eo}t_{oo1} + M_{ee}t_{eo1})L_{oo} + (M_{eo}t_{oe1} + M_{ee}t_{ee1})L_{eo} \tag{71}$$
$$N_{oe} = (M_{oo}t_{oe1} + M_{oe}t_{ee1})L_{ee} + (M_{oo}t_{oo1} + M_{oe}t_{eo1})L_{oe} \tag{72}$$
$$N_{ee} = (M_{eo}t_{oe1} + M_{oe}t_{ee1})L_{ee} + (M_{eo}t_{oo1} + M_{ee}t_{eo1})L_{oe} \tag{73}$$

The above coefficients are checked for the limiting cases like those of [5]. In the following sub-sections, we consider the four cases of anisotropic-anisotropic interfaces according to the medium location (upper, lower) and the incident wave polarization (o- or e-wave).

3.1. Ordinary Wave Incidence from Upper Anisotropic Medium (Region 1) upon Lower Anisotropic Medium (Region 2)

When an ordinary wave $\widehat{o}(-k_{1z}^{o})$ is incident from upper anisotropic medium (Region 1) upon lower anisotropic medium (Region 2), the reflected wave will consist of upward ordinary wave $\widehat{o}(k_{1z}^{o})$ and extraordinary wave $\widehat{e}(k_{1z}^{eu})$ in Region 1 and the transmitted wave will consist of downward ordinary wave $\widehat{o}(-k_{2z}^{o})$ and extraordinary wave

(k_{2z}^{ed}) in Region 2. We express $\bar{E}_i$, $i = 1$, 2, in each region as follows:

$$\bar{E}_1 = \widehat{o}\left(-k_{1z}^{o}\right) + R_{12oo}\widehat{o}\left(k_{1z}^{o}\right) + R_{12oe}\widehat{e}\left(k_{1z}^{eu}\right) \tag{74}$$

$$\bar{E}_2 = X_{12oo}\widehat{o}\left(-k_{2z}^{o}\right) + R_{12oe}\widehat{e}\left(k_{2z}^{ed}\right) \tag{75}$$

where R_{12oe} is the reflection coefficient of reflected extraordinary wave for the ordinary wave incidence from Region 1 to Region 2 and X_{12oe} is the transmission coefficient of the transmitted extraordinary wave for ordinary wave incidence. R_{12oo} and X_{12oo} are ordinary wave reflection and transmission coefficients, respectively. Using

$$\widehat{z} \times \bar{E}_1 = \widehat{z} \times \bar{E}_2 \quad \text{and} \quad \widehat{z} \times \nabla \times \bar{E}_1 = \widehat{z} \times \nabla \times \bar{E}_2 \quad \text{at} \quad z = 0,$$

we obtain the following equations:

$$R_{12oo}\cdot\alpha_{i1}+R_{12oe}\cdot\alpha_{i2}+X_{12oo}\cdot\alpha_{i3}+X_{12oe}\cdot\alpha_{i4}=\alpha_{i5}, \quad i = 1, 2, 3, 4 \tag{76}$$

The equations above can be expressed as $A \cdot B = C$ for simplicity, where

$$A=\begin{bmatrix}\alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14}\\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24}\\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34}\\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44}\end{bmatrix} \tag{77}$$

$$\alpha_{11}=\frac{k_{1z}^{o}\sin\psi_1\cos\chi_1 - k_y\cos\psi_1}{g'_{u1}} \tag{78}$$

$$\alpha_{12}=\frac{k_x E\left(k_{1z}^{eu}\right) - k_1^2\sin\psi_1\sin\chi_1}{D'_U\left(k_{1z}^{eu}\right)} \tag{79}$$

$$\alpha_{13}=\frac{k_{2z}^{o}\sin\psi_2\cos\chi_2 + k_y\cos\psi_2}{g'_{d2}} \tag{80}$$

$$\alpha_{14}=-\frac{k_x E\left(k_{2z}^{ed}\right) - k_2^2\sin\psi_2\sin\chi_2}{D'_U\left(k_{2z}^{ed}\right)} \tag{81}$$

$$\alpha_{21}=\frac{\left(k_y^2 + k_{1z}^{o}\right)\sin\psi_1\sin\chi_1 - k_x\left(k_y\sin\psi_1\cos\chi_1 + k_{1z}^{o}\cos\psi_1\right)}{g'_{u1}} \tag{82}$$

$$\alpha_{22}=\frac{k_1^2\left(k_{1z}^{eu}\sin\psi_1\cos\chi_1 - k_y\cos\psi_1\right)}{D'_U\left(k_{1z}^{eu}\right)} \tag{83}$$

$$\alpha_{23}=\frac{-\left(k_y^2+k_{2z}^{o2}\right)\sin\psi_2\sin\chi_2 + k_x\left(k_y\sin\psi_2\cos\chi_2 - k_{2z}^{o}\cos\psi_2\right)}{g'_{d2}} \tag{84}$$

$$\alpha_{24}=\frac{k_2^2\left(k_y\cos\psi_2 - k_{2z}^{ed}\sin\psi_2\cos\chi_2\right)}{D'_U\left(k_{2z}^{ed}\right)} \tag{85}$$

$$\alpha_{31}=\frac{k_x\cos\psi_1 - k_{1z}^{o}\sin\psi_1\cos\chi_1}{g'_{u1}} \tag{86}$$

$$\alpha_{32} = \frac{k_y E\left(k_{1z}^{eu}\right) - k_1^2 \sin\psi_1 \cos\chi_1}{D'_U\left(k_{1z}^{eu}\right)} \tag{87}$$

$$\alpha_{33} = -\frac{k_{2z}^o \sin\psi_2 \sin\chi_2 + k_x \cos\psi_2}{g'_{d2}} \tag{88}$$

$$\alpha_{34} = -\frac{k_y E(k_{2z}^{ed}) - k_2^2 \sin\psi_2 \cos\chi_2}{D'_U(k_{2z}^{ed})} \tag{89}$$

$$\alpha_{41} = \frac{\left(k_x^2 + k_{1z}^{o2}\right) \sin\psi_1 \cos\chi_1 - k_y\left(k_x \sin\psi_1 \sin\chi_1 + k_{1z}^o \cos\psi_1\right)}{g'_{u1}} \tag{90}$$

$$\alpha_{42} = -\frac{k_1^2\left(k_{1z}^{eu} \sin\psi_1 \sin\chi_1 - k_x \cos\psi_1\right)}{D'_U\left(k_{1z}^{eu}\right)} \tag{91}$$

$$\alpha_{43} = -\frac{\left(k_x^2 + k_{2z}^{o2}\right) \sin\psi_2 \cos\chi_2 + k_y\left(k_x \sin\psi_2 \sin\chi_2 - k_{2z}^o \cos\psi_2\right)}{g'_{d2}} \tag{92}$$

$$\alpha_{44} = -\frac{k_2^2\left(k_x \cos\psi_2 - k_{2z}^{ed} \sin\psi_2 \sin\chi_2\right)}{D'_U\left(k_{2z}^{ed}\right)} \tag{93}$$

$$E(Z) = k_x \sin\psi_j \sin\chi_j + k_y \sin\psi_j \cos\chi_j + Z\cos\psi_j, \;\; j = 1 \text{ or } 2 \tag{94}$$

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} R_{12oo} \\ R_{12oe} \\ X_{12oo} \\ X_{12oe} \end{bmatrix} \tag{95}$$

$$C = \begin{bmatrix} \alpha_{15} \\ \alpha_{25} \\ \alpha_{35} \\ \alpha_{45} \end{bmatrix} = \frac{1}{g'_{d1}} \begin{bmatrix} k_{1z}^o \sin\psi_1 \cos\chi_1 + k_y \cos\psi_1 \\ -\left(k_y^2 + k_{1z}^{o2}\right)\sin\psi_1 \sin\chi_1 + k_x\left(k_y \sin\psi_1 \cos\chi_1 - k_{1z}^o \cos\psi_1\right) \\ -k_{1z}^o \sin\psi_1 \sin\chi_1 - k_x \cos\chi_1 \\ -\left(k_x^2 + k_{1z}^{o2}\right)\sin\psi_1 \cos\chi_1 - k_y\left(k_x \sin\psi_1 \sin\chi_1 - k_{1z}^o \cos\psi_1\right) \end{bmatrix} \tag{96}$$

Note that elements of the matrix C are from $\widehat{o}(-k_{1z}^o)$ of $\bar{E}_1$. Using the Gauss elimination method [25], we can express the reflection and transmission coefficients as follows:

$$X_{12oe} = \frac{J_E N_E - L_E M_E}{J_E P_E - K_E M_E} \tag{97}$$

$$X_{12oo} = \frac{L_E}{J_E} - c_{14} X_{12oe} \tag{98}$$

$$R_{12oe} = \frac{1}{b_{12}} \left\{ b_{15} - b_{14} X_{12oe} - b_{13} X_{12oo} \right\} \tag{99}$$

$$R_{12oo} = \frac{1}{\alpha_{11}}\left\{\alpha_{15} - \alpha_{14}X_{12oe} - \alpha_{13}X_{12oo} - \alpha_{12}R_{12oe}\right\} \quad (100)$$

where

$$K_E = \left(\alpha_{34} - \alpha_{31}\frac{\alpha_{14}}{\alpha_{11}}\right)b_{12} - \left(\alpha_{32} - \alpha_{31}\frac{\alpha_{12}}{\alpha_{11}}\right)b_{14} \quad (101)$$

$$J_E = \left(\alpha_{33} - \alpha_{31}\frac{\alpha_{13}}{\alpha_{11}}\right)b_{12} - \left(\alpha_{32} - \alpha_{31}\frac{\alpha_{12}}{\alpha_{11}}\right)b_{13} \quad (102)$$

$$L_E = \left(\alpha_{35} - \alpha_{31}\frac{\alpha_{15}}{\alpha_{11}}\right)b_{12} - \left(\alpha_{32} - \alpha_{31}\frac{\alpha_{12}}{\alpha_{11}}\right)b_{15} \quad (103)$$

$$N_E = \left(\alpha_{45} - \alpha_{41}\frac{\alpha_{15}}{\alpha_{11}}\right)b_{12} - \left(\alpha_{42} - \alpha_{31}\frac{\alpha_{12}}{\alpha_{11}}\right)b_{15} \quad (104)$$

$$M_E = \left(\alpha_{43} - \alpha_{41}\frac{\alpha_{13}}{\alpha_{11}}\right)b_{12} - \left(\alpha_{42} - \alpha_{41}\frac{\alpha_{12}}{\alpha_{11}}\right)b_{13} \quad (105)$$

$$P_E = \left(\alpha_{434} - \alpha_{41}\frac{\alpha_{14}}{\alpha_{11}}\right)b_{12} - \left(\alpha_{42} - \alpha_{41}\frac{\alpha_{12}}{\alpha_{11}}\right)b_{14} \quad (106)$$

$$b_{12} = \alpha_{22} - \alpha_{21}\frac{\alpha_{12}}{\alpha_{11}} \quad (107)$$

$$b_{13} = \alpha_{23} - \alpha_{21}\frac{\alpha_{13}}{\alpha_{11}} \quad (108)$$

$$b_{14} = \alpha_{24} - \alpha_{21}\frac{\alpha_{14}}{\alpha_{11}} \quad (109)$$

$$b_{15} = \alpha_{25} - \alpha_{21}\frac{\alpha_{15}}{\alpha_{11}} \quad (110)$$

$$c_{14} = \frac{K_E}{J_E} \quad (111)$$

3.2. Extraordinary Wave Incidence from Upper Medium (Region 1) upon Lower Medium (Region 2)

Similar to the previous case, for extraordinary wave incident from upper anisotropic medium to lower anisotropic medium, the reflected wave will consist of upward ordinary and extraordinary waves in Region 1, and the transmitted wave has downward ordinary and extraordinary waves in Region 2. We have electric fields in each region as follows:

$$\bar{E}_1 = \widehat{e}\left(k_{1z}^{ed}\right) + R_{12eo}\widehat{o}\left(k_{1z}^{o}\right) + R_{12ee}\widehat{e}\left(k_{1z}^{eu}\right) \quad (112)$$

$$\bar{E}_2 = X_{12eo}\widehat{o}\left(-k_{2z}^{o}\right) + X_{12ee}\widehat{e}\left(k_{2z}^{ed}\right) \quad (113)$$

For extraordinary wave incident from Region 1 to Region 2, $\widehat{e}(k_{1z}^{ed})$ replaces $\widehat{o}(-k_{1z}^{o})$ in Region 1 in the previous case. By using the same boundary conditions at $z=0$, we can express the equations in a matrix form as before, i.e., $A \cdot B = C \cdot A$ is the same as before but B and C are changed as follows:

$$B = \begin{bmatrix} R_{12eo} \\ R_{12ee} \\ X_{12ee} \\ X_{12eo} \end{bmatrix} \tag{114}$$

$$C = \begin{bmatrix} \alpha_{15} \\ \alpha_{25} \\ \alpha_{35} \\ \alpha_{45} \end{bmatrix} = \frac{1}{D_U'\left(k_{1z}^{eu}\right)} \begin{bmatrix} -k_x E\left(k_{1z}^{ed}\right) + k_1^2 \sin\psi_1 \sin\chi_1 \\ k_1^2\left(k_y \cos\psi_1 - k_{1z}^{ed} \sin\psi_1 \cos\chi_1\right) \\ -k_x E\left(k_{1z}^{ed}\right) + k_1^2 \sin\psi_1 \cos\chi_1 \\ -k_1^2\left(k_x \cos\psi_1 - k_{1z}^{ed} \sin\psi_1 \sin\chi_1\right) \end{bmatrix} \tag{115}$$

We notice that since α_{i5}, $i = 1, 2, 3$, and 4, have been changed into new values, b_{15}, L_E and N_E which are functions of α_{i5} should have different values though they have the same form as before. The Fresnel coefficients are given by:

$$X_{12ee} = \frac{J_E P_E' - M_E K_E'}{J_E P_E - M_E K_E} \tag{116}$$

$$X_{12eo} = \frac{K_E'}{J_E} - c_{14} X_{12ee} \tag{117}$$

$$R_{12ee} = \frac{1}{b_{12}}\left\{b_{15} - b_{14} X_{12ee} - b_{13} X_{12eo}\right\} \tag{118}$$

$$R_{12eo} = \frac{1}{\alpha_{11}}\left\{\alpha_{15} - \alpha_{14} X_{12ee} - \alpha_{13} X_{12eo} - \alpha_{12} R_{12ee}\right\} \tag{119}$$

where

$$P_E' = \left(\alpha_{45} - \alpha_{41}\frac{\alpha_{15}}{\alpha_{11}}\right) b_{12} - \left(\alpha_{42} - \alpha_{41}\frac{\alpha_{12}}{\alpha_{11}}\right) b_{15} \tag{120}$$

$$K_E' = \left(\alpha_{35} - \alpha_{31}\frac{\alpha_{15}}{\alpha_{11}}\right) b_{12} - \left(\alpha_{32} - \alpha_{31}\frac{\alpha_{12}}{\alpha_{11}}\right) b_{15} \tag{121}$$

3.3. Ordinary Wave Incidence from Lower Medium (Region 2) to Upper Medium (Region 1)

Now we consider the wave is incident from the lower anisotropic medium (Region 2) upon upper anisotropic medium (Region 1). When the incident wave is an ordinary wave $\widehat{o}(k_{2z}^{o})$, we can formulate the electric fields in each region as follows:

$$\bar{E}_1 = X_{21oo}\widehat{o}\left(k_{1z}^{o}\right) + X_{21oe}\widehat{e}\left(k_{1z}^{eu}\right) \tag{122}$$

$$\bar{E}_2 = \widehat{o}\left(k_{2z}^{o}\right) + R_{21oo}\widehat{o}\left(-k_{2z}^{o}\right) + R_{21oe}\widehat{e}\left(k_{2z}^{ed}\right) \tag{123}$$

We can express the boundary conditions as $A \cdot B = C$ with the same A but different B and C, which are given below.

$$B = \begin{bmatrix} X_{21oo} \\ X_{21oe} \\ R_{21oo} \\ R_{21oe} \end{bmatrix} \tag{124}$$

$$C = \begin{bmatrix} \alpha_{15} \\ \alpha_{25} \\ \alpha_{35} \\ \alpha_{45} \end{bmatrix} = \frac{1}{g'_{u2}} \begin{bmatrix} k_{2z}^{o}\sin\psi_2\cos\chi_2 - k_y\cos\psi_2 \\ \left(k_y^2 + k_{2z}^{o2}\right)\sin\psi_2\sin\chi_2 - k_x\left(k_y\sin\psi_2\cos\chi_2 + k_{2z}^{o}\cos\psi_2\right) \\ -k_{2z}^{o}\sin\psi_2\sin\chi_2 + k_x\cos\psi_1 \\ \left(k_x^2 + k_{2z}^{o2}\right)\sin\psi_2\cos\chi_2 - k_y\left(k_x\sin\psi_2\sin\chi_2 + k_{2z}^{o}\cos\psi_2\right) \end{bmatrix} \tag{125}$$

The Fresnel coefficients are obtained as follows:

$$R_{21oe} = \frac{J_E M'_E - J'_E M_E}{J_E P_E - K_E M_E} \tag{126}$$

$$R_{21oo} = \frac{J'_E}{J_E} - c_{14} R_{21oe} \tag{127}$$

$$X_{21oe} = \frac{1}{b_{12}}\left\{b_{15} - b_{14}R_{21oe} - b_{13}R_{21oo}\right\} \tag{128}$$

$$X_{21oo} = \frac{1}{\alpha_{11}}\left\{\alpha_{15} - \alpha_{14}R_{21oe} - \alpha_{13}R_{21oo} - \alpha_{12}X_{21oe}\right\} \tag{129}$$

where

$$J'_E = \left(\alpha_{35} - \alpha_{31}\frac{\alpha_{15}}{\alpha_{11}}\right)b_{12} - \left(\alpha_{32} - \alpha_{31}\frac{\alpha_{12}}{\alpha_{11}}\right)b_{15} \tag{130}$$

$$M'_E = \left(\alpha_{45} - \alpha_{41}\frac{\alpha_{15}}{\alpha_{11}}\right)b_{12} - \left(\alpha_{42} - \alpha_{41}\frac{\alpha_{12}}{\alpha_{11}}\right)b_{15} \tag{131}$$

3.4. Extraordinary Wave Incidence from Lower Medium (Region 2) to Upper Medium (Region 1)

Finally, for the extraordinary wave incidence from the lower layer (Region 2) to the upper layer (Region 1), we can express the electric fields in each region as follows:

$$\bar{E}_1 = X_{21eo}\widehat{o}\left(k_{1z}^{o}\right) + X_{21ee}\widehat{e}\left(k_{1z}^{eu}\right) \tag{132}$$

$$\bar{E}_2 = \widehat{e}\left(k_{2z}^{eu}\right) + R_{21eo}\widehat{o}\left(-k_{2z}^{o}\right) + R_{21ee}\widehat{e}\left(k_{2z}^{ed}\right) \tag{133}$$

The equations of the boundary conditions can be expressed as $A \cdot B = C$, where A is the same as before, but B and C are changed as follows:

$$B = \begin{bmatrix} X_{21eo} \\ X_{21ee} \\ R_{21eo} \\ R_{21ee} \end{bmatrix} \tag{134}$$

$$C = \begin{bmatrix} \alpha_{15} \\ \alpha_{25} \\ \alpha_{35} \\ \alpha_{45} \end{bmatrix} = \frac{1}{D'_U(k_{2z}^{eu})} \begin{bmatrix} k_x E\left(k_{2z}^{eu}\right) - k_2^2 \sin\psi_2 \sin\chi_2 \\ -k_2^2\left(k_y \cos\psi_2 - k_{2z}^{eu} \sin\psi_2 \cos\chi_2\right) \\ k_y E\left(k_{2z}^{eu}\right) - k_2^2 \sin\psi_2 \cos\chi_2 \\ k_2^2\left(k_x \cos\psi_2 - k_{2z}^{eu} \sin\psi_2 \sin\chi_2\right) \end{bmatrix} \tag{135}$$

The reflection and transmission coefficients are given by

$$R_{21ee} = \frac{J_E P''_E - M_E K''_E}{J_E P_E - M_E K_E} \tag{136}$$

$$R_{21eo} = \frac{K''_E}{J_E} - c_{14} R_{21ee} \tag{137}$$

$$X_{21ee} = \frac{1}{b_{12}}\left\{b_{15} - b_{14}R_{21ee} - b_{13}R_{121eo}\right\} \tag{138}$$

$$X_{21eo} = \frac{1}{\alpha_{11}}\left\{\alpha_{15} - \alpha_{14}R_{21ee} - \alpha_{13}R_{21eo} - \alpha_{12}X_{21ee}\right\} \tag{139}$$

where

$$K''_E = \left(\alpha_{35} - \alpha_{31}\frac{\alpha_{15}}{\alpha_{11}}\right) b_{12} - \left(\alpha_{32} - \alpha_{31}\frac{\alpha_{12}}{\alpha_{11}}\right) b_{15} \tag{140}$$

$$P''_E = \left(\alpha_{45} - \alpha_{41}\frac{\alpha_{15}}{\alpha_{11}}\right) b_{12} - \left(\alpha_{42} - \alpha_{41}\frac{\alpha_{12}}{\alpha_{11}}\right) b_{15} \tag{141}$$

4. LIMITING CASES

Now we check our results of the reflection and transmission coefficients in several limiting cases. In some cases, they reduce to a simpler form. We also want to check our results of arbitrarily oriented uniaxial medium with those of the tilted uniaxial medium [5]. There are four steps to obtain them because the results at each step are another Fresnel coefficients for different cases. First we set $\chi_i = 0$, $i = 1$ or 2. Secondly, we will let $\chi_1 = \chi_2 = 0$ and thirdly we will make

$\chi_1 = \chi_2 = 0$ and $\psi_i = 0$ and $\varepsilon_i = \varepsilon_{iz}$, $i = 1$ or 2. This limiting case reduces to the isotropic-anisotropic problem. Finally, $\chi_i = \psi_i = 0$ and $\varepsilon_i = \varepsilon_{iz}$, $i = 1$, 2 for the isotropic case.

4.1. Limiting Case 1: $\chi_i = 0$, $i = 1$, or 2

We set $\chi_1 = 0$ and find that

$$g'_{u1} \rightarrow g_{u1} \tag{142}$$

$$g'_{d1} \rightarrow g_{d1} \tag{143}$$

$$D'_U\left(k_{1z}^{ed}\right) \rightarrow D_U^{\left(k_{1z}^{ed}\right)} \tag{144}$$

$$D'_U\left(k_{1z}^{eu}\right) \rightarrow D_U^{\left(k_{1z}^{eu}\right)} \tag{145}$$

4.1.1. Ordinary Wave Incidence from Upper Medium (Region 1) upon Lower Medium (Region 2)

In this case, we find that

$$b_{12} \rightarrow K_2\left(k_1, k_{1z}^{eu}, \Psi_1, \Psi_1\right) \tag{146}$$

$$b_{15} \rightarrow k_x k_y \tag{147}$$

$$\alpha_{32} - \alpha_{31}\frac{\alpha_{13}}{\alpha_{11}} \rightarrow -K_1\left(k_{1z}^{eu}, k_1, \Psi_1, \Psi_1\right) \tag{148}$$

$$\alpha_{35} - \alpha_{31}\frac{\alpha_{15}}{\alpha_{11}} \rightarrow k_x \sin\Psi_1 \cos\Psi_1 \tag{149}$$

$$\alpha_{42} - \alpha_{41}\frac{\alpha_{14}}{\alpha_{11}} \rightarrow k_x N_1\left(k_1, k_{1z}^{eu}, \Psi_1, \Psi_1\right) \tag{150}$$

$$\alpha_{45} - \alpha_{41}\frac{\alpha_{15}}{\alpha_{11}} \rightarrow -2k_{1z}^{o}\left(k_y^2\cos^2\Psi_1 - \left(k_x^2 + k_{1z}^{o2}\right)\sin^2\Psi_1\right) \tag{151}$$

Then, coefficients reduce to

$$X_{12oe} = \frac{J_E N_E - k_x L_e M_E}{J_E P_E - K_E M_E} \tag{152}$$

$$X_{12oo} = -k_x\frac{L_e}{J_E} - \frac{K_E}{J_E}X_{12oe} \tag{153}$$

$$R_{12oe} = \frac{D_U\left(k_{1z}^{eu}\right)}{K_2\left(k_1, k_{1z}^{eu}, \Psi_1, \Psi_1\right)}\left\{2\frac{k_x k_y k_{1z}^{o}}{g_{d1}} - \frac{b'_{14}}{D'_U\left(k_{2z}^{ed}\right)}X_{12oe} + \frac{b'_{13}}{g'_{d2}}X_{12oo}\right\} \tag{154}$$

$$R_{12oo} = \frac{g_{u1}}{\left(k_{1z}^{o}\sin\psi_1 - k_y\cos\psi_1\right)} \left\{\begin{array}{c} \frac{\left(k_{1z}^{o}\sin\psi_1 + k_y\cos\psi_1\right)}{g'_{d1}} + \frac{k_x E\left(k_{2z}^{ed}\right) - k_2^2\sin\psi_2\sin\chi_2}{D'_U\left(k_{2z}^{ed}\right)}X_{12oe} \\ -\frac{\left(k_{2z}^{o}\sin\psi_2\cos\chi_2 + k_y\cos\psi_2\right)}{g'_{d2}}X_{12oo} - \frac{k_x\left(k_y\sin\psi_1 + k_{1z}^{eu}\cos\psi_1\right)}{D_U\left(k_{1z}^{eu}\right)}R_{12oe} \end{array}\right\} \tag{155}$$

4.1.2. Extraordinary Wave Incidence from Upper Medium (Region 1) upon Lower Medium (Region 2)

In this case, we obtain

$$b_{15} \rightarrow -K_2\left(k_1, k_{1z}^{ed}, \Psi_1, \Psi_1\right) \tag{156}$$

$$\alpha_{45} - \alpha_{41}\frac{\alpha_{15}}{\alpha_{11}} \rightarrow k_x N_1\left(k_1, k_{1z}^{ed}, \Psi_1, \Psi_1\right) \tag{157}$$

$$\alpha_{35} - \alpha_{31}\frac{\alpha_{15}}{\alpha_{11}} \rightarrow -K_1\left(k_{1z}^{ed}, k_1, \Psi_1, \Psi_1\right) \tag{158}$$

$$K_E' \rightarrow K_e' \tag{159}$$

$$P_E' \rightarrow k_x P_e' \tag{160}$$

$$X_{12ee}=\frac{k_x J_E P_e' - K_e' M_E}{J_E P_E - K_E M_E} \tag{161}$$

$$X_{12eo}=\frac{K_e'}{J_E}-\frac{K_E}{J_E}X_{12ee} \tag{162}$$

$$R_{12ee}=\frac{D_U\left(k_{1z}^{eu}\right)}{K_2\left(k_1, k_{1z}^{eu}, \Psi_1, \Psi_1\right)}\left\{-\frac{K_2\left(k_1, k_{1z}^{ed}, \Psi_1, \Psi_1\right)}{D_U'\left(k_{2z}^{ed}\right)}-\frac{b_{14}'}{D_U'\left(k_{2z}^{ed}\right)}X_{12ee}+\frac{b_{13}'}{g_{d2}'}X_{12eo}\right\} \tag{163}$$

$$R_{12eo}=\frac{g_{u1}}{\left(k_{1z}^o \sin\psi_1 - k_y\cos\psi_1\right)}\left\{\begin{array}{l} -\frac{k_x\left(k_y\sin\psi_1+k_{1z}^{ed}\cos\psi_1\right)}{D_U\left(k_{1z}^{ed}\right)}+\frac{k_x E\left(k_{2z}^{ed}\right)-k_2^2\sin\psi_2\sin\chi_2}{D_U'\left(k_{2z}^{ed}\right)}X_{12ee} \\ -\frac{\left(k_{2z}^o\sin\psi_2\cos\chi_2+k_y\cos\psi_2\right)}{g_{d2}'}X_{12eo}-\frac{k_x\left(k_y\sin\psi_1+k_{1z}^{eu}\cos\psi_1\right)}{D_U\left(k_{1z}^{eu}\right)}R_{12ee}\end{array}\right\} \tag{164}$$

4.1.3. Ordinary Wave Incidence from Lower Medium (Region 2) to upper Medium (Region 1)

The following coefficients are obtained as

$$R_{21oe}=\frac{J_E M_E' - J_E' M_E}{J_E P_E - K_E M_E} \tag{165}$$

$$R_{21oo}=\frac{J_E'}{J_E}-\frac{K_E}{J_E}R_{21oe} \tag{166}$$

$$X_{21oe}=\frac{D_U\left(k_{1z}^{eu}\right)}{K_2\left(k_1, k_{1z}^{eu}, \Psi_1, \Psi_1\right)}\left\{\frac{b_{15}'}{g_{u2}'}-\frac{b_{14}'}{D_U'\left(k_{2z}^{ed}\right)}R_{21oe}+\frac{b_{13}'}{g_{d2}'}R_{21oo}\right\} \tag{167}$$

$$X_{21oo} = \frac{g_{u1}}{(k_{1z}^o \sin\psi_1 - k_y \cos\psi_1)} \left\{ \begin{array}{c} \frac{\left(k_{2z}^o \sin\psi_2 \cos\chi_2 - k_y \cos\psi_2\right)}{g'_{u2}} + \frac{k_x E\left(k_{2z}^{ed}\right) - k_2^2 \sin\psi_2 \sin\chi_2}{D'_U\left(k_{2z}^{ed}\right)} R_{21oe} \\ -\frac{\left(k_{2z}^o \sin\psi_2 \cos\chi_2 + k_y \cos\psi_2\right)}{g'_{d2}} R_{21oo} - \frac{k_x\left(k_y \sin\psi_1 + k_{1z}^{eu}\cos\psi_1\right)}{D_U\left(k_{1z}^{eu}\right)} X_{21oe} \end{array} \right\} \quad (168)$$

4.1.4. Extraordinary Wave Incidence from Lower Medium (Region 2) to upper Medium (Region 1)

We obtain,

$$R_{21ee} = \frac{(J_E P''_E - K''_E M_E)}{(J_E P_E - K_E M_E)} \quad (169)$$

$$R_{21eo} = \frac{K''_E}{J_E} - \frac{K_E}{J_E} R_{21ee} \quad (170)$$

$$X_{21ee} = D'_U\left(k_{1z}^{eu}\right) \left\{ \frac{b''_{15}}{D'_U\left(k_{2z}^{eu}\right)} - \frac{b'_{14}}{D'_U\left(k_{2z}^{ed}\right)} R_{21ee} + \frac{b'_{13}}{g'_{d2}} R_{21ee} \right\} \quad (171)$$

$$X_{21eo} = \frac{g_{u1}}{(k_{1z}^o \sin\psi_1 - k_y \cos\psi_1)} \left\{ \begin{array}{c} \frac{k_x E\left(k_{2z}^{eu}\right) - k_2^2 \sin\psi_2 \sin\chi_2}{D'_U\left(k_{2z}^{eu}\right)} + \frac{k_x E\left(k_{2z}^{ed}\right) - k_2^2 \sin\psi_2 \sin\chi_2}{D'_U\left(k_{2z}^{ed}\right)} R_{21ee} \\ -\frac{k_{2z}^o \sin\psi_2 \cos\chi_2 + k_y \cos\psi_2}{g'_{d2}} R_{21eo} - \frac{k_x\left(k_y \sin\psi_1 + k_{1z}^{eu}\cos\psi_1\right)}{D'_U\left(k_{1z}^{eu}\right)} X_{21ee} \end{array} \right\} \quad (172)$$

where b'_{13}, b'_{14} and b'_{15} (and b''_{15}) are the values of b_{13}, b_{14} and b_{15} with $\chi_1 = 0$, respectively.

4.2. Limiting Case 2: $\chi_1 = \chi_2 = 0$

Two optic axes in the two anisotropic media are in the same plane when both $\boldsymbol{\chi_1}$ and $\boldsymbol{\chi_2}$ are zero. We then have the following coefficients.

$$X_{12oe} = -2\frac{D_U\left(k_{2z}^{ed}\right)}{g_{d1}} k_{1z}^o k_x \frac{J_e N_e - L_e M_e}{k_x^2 J_e P_e - K_e M_e} \quad (173)$$

$$X_{12oe} = 2\frac{g_{d2}}{g_{d1}} k_{1z}^o \frac{L_e}{J_e} - \frac{g_{d2}}{D_U\left(k_{2z}^{ed}\right)} \frac{K_e}{J_e} X_{12oe} \quad (174)$$

$$R_{12oe} = \frac{D_U\left(k_{1z}^{eu}\right)}{K_2\left(k_1, k_{1z}^{eu}, \Psi_1, \Psi_1\right)} \left\{ \begin{array}{c} 2\frac{k_x k_y k_{1z}^o}{g_{d1}} + \frac{K_2\left(k_2, k_{2z}^{ed}, \Psi_1, \Psi_2\right)}{D_U\left(k_{2z}^{ed}\right)} X_{12oe} \\ -k_x \frac{k_y\left(k_{1z}^o + k_{2z}\right)\cos(\Delta\psi) + \left(k_y^2 - k_{1z}^o k_{2z}^o\right)\sin(\Delta\psi)}{g_{d2}} X_{12oo} \end{array} \right\} \quad (175)$$

$$R_{12oo}=\frac{g_{u1}}{(k_{1z}^{o}\sin\psi_1-k_y\cos\psi_1)}\left\{\begin{array}{c}\frac{k_{1z}^{o}\sin\psi_1+k_y\cos\psi_1}{g_{d1}}+k_x\frac{k_y\sin\psi_2+k_{2z}^{ed}\cos\psi_2}{D_U\left(k_{2z}^{ed}\right)}X_{12oe}\\-\frac{k_{2z}^{o}\sin\psi_2+k_y\cos\psi_2}{g_{d2}}X_{12oo}-\frac{k_x\left(k_y\sin\psi_1+k_{1z}^{eu}\cos\psi_1\right)}{D_U\left(k_{1z}^{eu}\right)}R_{12oe}\end{array}\right\} \tag{176}$$

$$X_{12ee}=\frac{D_U\left(k_{2z}^{ed}\right)}{D_U\left(k_{1z}^{ed}\right)}\frac{k_x^2J_eP_e'-M_eK_e'}{k_x^2J_eP_e-K_eM_e} \tag{177}$$

$$X_{12oe}=\frac{g_{d2}}{D_U\left(k_{1z}^{ed}\right)}\frac{K_e'}{k_xJ_e}-\frac{g_{d2}}{D_U\left(k_{2z}^{ed}\right)}\frac{K_e}{k_xJ_e}X_{12ee} \tag{178}$$

$$R_{12ee}=\frac{D_U\left(k_{1z}^{eu}\right)}{K_2\left(k_1,k_{1z}^{eu},\Psi_1,\Psi_1\right)}\left\{\begin{array}{c}-\frac{K_2\left(k_1,k_{1z}^{ed},\Psi_1,\Psi_1\right)}{D_U\left(k_{1z}^{ed}\right)}+\frac{K_2\left(k_2,k_{2z}^{ed},\Psi_1,\Psi_2\right)}{D_U\left(k_{2z}^{ed}\right)}X_{12ee}\\-k_x\frac{k_y\left(k_{1z}^{o}+k_{2z}^{o}\right)\cos(\Delta\psi)+\left(k_y^2-k_{1z}^{o}k_{2z}^{o}\right)\sin(\Delta\psi_2)}{g_{d2}}X_{12eo}\end{array}\right\} \tag{179}$$

$$R_{12eo}=\frac{g_{u1}}{\left(k_{1z}^{o}\sin\psi_1-k_y\cos\psi_1\right)}\left\{\begin{array}{c}-\frac{k_x(k_y\sin_{psi_1}+k_{1z}^{ed}\cos\psi_1)}{D_U\left(k_{1z}^{ed}\right)}+\frac{k_x\left(k_y\sin\psi_2+k_{2z}^{ed}\cos\psi_2\right)}{D_U\left(k_{2z}^{ed}\right)}X_{12ee}\\-\frac{k_{2z}^{o}\sin\psi_2+k_y\cos\psi_2}{g_{d2}}X_{12eo}-\frac{k_x\left(k_y\sin\psi_1+k_{1z}^{eu}\cos\psi_1\right)}{D_U\left(k_{1z}^{eu}\right)}R_{12ee}\end{array}\right\} \tag{180}$$

$$R_{21oe}=-\frac{D_U\left(k_{2z}^{ed}\right)}{g_{u2}}k_x\frac{J_eM_e'-J_e'M_e}{k_x^2J_eP_e-K_eM_e} \tag{181}$$

$$R_{21oo}=\frac{g_{d2}}{g_{u2}}\frac{J_e'}{J_e}-\frac{g_{d2}}{D_U(k_{2z}^{ed})}\frac{K_e}{k_xJ_e}R_{21oe} \tag{182}$$

$$X_{21oe}=\frac{D_U\left(k_{1z}^{eu}\right)}{K_2\left(k_1,k_{1z}^{eu},\Psi_1,\Psi_1\right)}\left\{\begin{array}{c}-k_x\frac{k_y\left(k_{1z}^{o}-k_{2z}^{o}\right)\cos(\Delta\psi)+\left(k_y^2+k_{1z}^{o}k_{2z}^{o}\right)\sin(\Delta\psi)}{g_{u2}}+\frac{K_2\left(k_2,k_{2z}^{ed},\Psi_1,\Psi_2\right)}{D_U'\left(k_{2z}^{ed}\right)}R_{21oe}\\-k_x\frac{k_y\left(k_{1z}^{o}+k_{2z}^{o}\right)\cos(\Delta\psi)+\left(k_y^2-k_{1z}^{o}k_{2z}^{o}\right)\sin(\Delta\psi)}{g_{d2}}R_{21oo}\end{array}\right\} \tag{183}$$

$$X_{21oo}=\frac{g_{u1}}{(k_{1z}^{o}\sin\psi_1-k_y\cos\psi_1)}\left\{\begin{array}{c}\frac{k_{2z}^{o}\sin\psi_2-k_y\cos\psi_2}{g_{u2}}+k_x\frac{k_y\sin\psi_2+k_{2z}^{ed}\cos\psi_2}{D_U\left(k_{2z}^{ed}\right)}R_{21oe}\\-\frac{k_{2z}^{o}\sin\psi_2+k_y\cos\psi_2}{g_{d2}}R_{21oo}-k_x\frac{k_y\sin\psi_1+k_{1z}^{eu}\cos\psi_1}{D_U\left(k_{1z}^{eu}\right)}X_{21oe}\end{array}\right\} \tag{184}$$

$$R_{21ee} = \frac{D_U\left(k_{2z}^{ed}\right)}{D_U\left(k_{2z}^{eu}\right)} \frac{k_x^2 J_e P_e'' - M_e K_e''}{k_x^2 J_e P_e - K_e M_e} \tag{185}$$

$$R_{21eo} = \frac{-g_{d2}}{D_U\left(k_{2z}^{eu}\right)} \frac{K_e''}{k_x J_e} - \frac{g_{d2}}{D_U\left(k_{2z}^{ed}\right)} \frac{K_e}{k_x J_e} R_{21ee} \tag{186}$$

$$X_{21ee} = \frac{D_U\left(k_{1z}^{eu}\right)}{K_2\left(k_1, k_{1z}^{eu}, \Psi_1, \Psi_1\right)} \left\{ \begin{array}{c} \frac{K_2\left(k_2, k_{2z}^{eu}, \Psi_1, \Psi_2\right)}{D_U\left(k_{2z}^{eu}\right)} + \frac{K_2\left(k_2, k_{2z}^{ed}, \Psi_1, \Psi_2\right)}{D_U\left(k_{2z}^{ed}\right)} R_{21ee} \\ -k_x \frac{k_y\left(k_{1z}^o + k_{2z}^o\right)\cos(\Delta\psi) + \left(k_y^2 - k_{1z}^o k_{2z}^o\right)\sin(\Delta\psi)}{g_{d2}} R_{21eo} \end{array} \right\} \tag{187}$$

$$X_{21eo} = \frac{g_{u1}}{\left(k_{1z}^o \sin\psi_1 - k_y \cos\psi_1\right)} \left\{ \begin{array}{c} \frac{k_x\left(k_y \sin_{psi_2} + k_{2z}^{eu}\cos\psi_2\right)}{D_U\left(k_{2z}^{eu}\right)} + k_x \frac{k_y \sin\psi_2 + k_{2z}^{ed}\cos\psi_2}{D_U\left(k_{2z}^{ed}\right)} R_{21ee} \\ -\frac{k_{2z}^o \sin\psi_2 + k_y\cos\psi_2}{g_{d2}} R_{21eo} - \frac{k_x\left(k_y \sin\psi_1 + k_{1z}^{eu}\cos\psi_1\right)}{D_U\left(k_{1z}^{eu}\right)} X_{21ee} \end{array} \right\} \tag{188}$$

where

$$g_{di} = \left[\left(k_{iz}^{o2}\sin\psi_i + k_y\cos\psi_i\right)^2 + k_x^2\right]^{1/2}, \quad i = 1, 2 \tag{189}$$

$$g_{ui} = \left[\left(k_{iz}^{o2}\sin\psi_i - k_y\cos\psi_i\right)^2 + k_x^2\right]^{1/2}, \quad i = 1, 2 \tag{190}$$

$$D_U(k_{iz}^e) = \left\{\left[(k_y\sin\psi_i + k_{iz}^e\cos\psi_i)^2 - \omega^2\mu\varepsilon_i\right] \left[k_x^2 + k_y^2 + k_{iz}^{e2} - \omega^2\mu(\varepsilon_i + \varepsilon_{iz})^2\right]\right\}^{1/2}, \quad i = 1, 2 \tag{191}$$

$$J_e = -\left(k_{1z}^o\sin\psi_1\cos\psi_2 + k_{2z}^o\sin\psi_2\cos\psi_1\right) K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) + K_1\left(k_{1z}^{eu}, k_1, \psi_1, \psi_1\right)\left[k_y(k_{1z}^o + k_{2z}^o)\cos(\Delta\psi) + \left(k_y^2 - k_{1z}^o k_{2z}^o\right)\sin(\Delta\psi)\right] \tag{192}$$

$$L_e = -\sin\psi_1\cos\psi_1 K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) + k_y K_1\left(k_{1z}^{eu}, k_1, \psi_1, \psi_1\right) \tag{193}$$

$$N_e = \left[k_y^2\cos^2\psi_1 - \left(k_x^2 + k_{1z}^{o2}\right)\sin^2\psi_1\right] K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) + k_x^2 k_y N_1\left(k_{1z}^{eu}, k_1, \psi_1, \psi_1\right) \tag{194}$$

$$K_e = K_1\left(k_{2z}^{ed}, k_2, \psi_1, \psi_2\right) \times K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) - K_1\left(k_{1z}^{eu}, k_1, \psi_1, \psi_1\right) \times K_2\left(k_2, k_{2z}^{ed}, \psi_1, \psi_2\right) \tag{195}$$

$$M_e = \left\{ \begin{array}{c} k_y^2 \left(k_{1z}^o + k_{2z}^o\right) \cos\psi_1 \cos\psi_2 \\ -k_y \left[\left(k_x^2 + k_{1z}^{o2} + k_{1z}^o k_{2z}^o\right) \sin\psi_1 \cos\psi_2 \right. \\ \left. -\left(k_x^2 + k_{2z}^{o2} + k_{1z}^o k_{2z}^o\right) \cos\psi_1 \sin\psi_2 \right] \\ -\left(k_{1z}^o + k_{2z}^o\right)\left(k_x^2 + k_{1z}^o k_{2z}^o\right) \sin\psi_1 \sin\psi_2 \end{array} \right\}$$

$$\times K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) + k_x^2 \left[k_y \left(k_{1z}^o + k_{2z}^o\right) \cos(\Delta\psi) + \left(k_y^2 - k_{1z}^o k_{2z}^o\right) \sin(\Delta\psi)\right] \times N_1\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) \quad (196)$$

$$P_e = N_1\left(k_2, k_{2z}^{ed}, \psi_1, \psi_2\right) \times K_2(k_1, k_{1z}^{eu}, \psi_1, \psi_1) - N_1\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) \times K_2\left(k_2, k_{2z}^{ed}, \psi_1, \psi_2\right) \quad (197)$$

$$P'_e = N_1\left(k_1, k_{1z}^{ed}, \psi_1, \psi_1\right) \times K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) - N_1\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) \times K_2\left(k_1, k_{1z}^{ed}, \psi_1, \psi_1\right) \quad (198)$$

$$K'_e = K_1\left(k_{1z}^{ed}, k_1, \psi_1, \psi_1\right) \times K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) - K_1\left(k_{1z}^{eu}, k_1, \psi_1, \psi_1\right) \times K_2\left(k_1, k_{1z}^{ed}, \psi_1, \psi_1\right) \quad (199)$$

$$P''_e = N_1\left(k_2, k_{2z}^{eu}, \psi_1, \psi_2\right) \times K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) - N_1\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) \times K_2\left(k_2, k_{2z}^{eu}, \psi_1, \psi_2\right) \quad (200)$$

$$K''_e = K_1\left(k_{2z}^{eu}, k_2, \psi_1, \psi_2\right) \times K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) - K_1\left(k_{1z}^{eu}, k_1, \psi_1, \psi_1\right) \times K_2\left(k_2, k_{2z}^{eu}, \psi_1, \psi_2\right) \quad (201)$$

$$M'_e = \left\{ \begin{array}{c} -k_y^2 \left(k_{1z}^o - k_{2z}^o\right) \cos\psi_1 \cos\psi_2 \\ +k_y \left[\left(k_x^2 + k_{1z}^{o2} - k_{1z}^o k_{2z}^o\right) \sin\psi_1 \cos\psi_2 \right. \\ \left. -\left(k_x^2 + k_{2z}^{o2} - k_{1z}^o k_{2z}^o\right) \cos\psi_1 \sin\psi_2 \right] \\ +\left(k_{1z}^o - k_{2z}^o\right)\left(k_x^2 - k_{1z}^o k_{2z}^o\right) \sin\psi_1 \sin\psi_2 \end{array} \right\}$$

$$\times K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) + k_x^2 \left[k_y \left(k_{1z}^o + k_{2z}^o\right) \cos(\Delta\psi) + \left(k_y^2 - k_{1z}^o k_{2z}^o\right) \sin(\Delta\psi)\right] \times N_1\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) \quad (202)$$

$$J'_e = \left(k_{1z}^o \sin\psi_1 \cos\psi_2 - k_{2z}^o \sin\psi_2 \cos\psi_1\right) K_2\left(k_1, k_{1z}^{eu}, \psi_1, \psi_1\right) - K_1\left(k_{1z}^{eu}, k_1, \psi_1, \psi_1\right)\left[k_y \left(k_{1z}^o - k_{2z}^o\right) \cos(\Delta\psi) + \left(k_y^2 + k_{1z}^o k_{2z}^o\right) \sin(\Delta\psi)\right] \quad (203)$$

$$\Delta\psi = \psi_1 - \psi_2 \quad (204)$$

where

$$K_1(X, Y, \psi_1, \psi_2) = k_\rho^o X \cos\psi_1 \cos\psi_2 + k_{1z}^o \left(Y^2 - k_y^2\right) \sin\psi_1 \sin\psi_2 - k_y\left[\left(Y^2 - k_\rho^2\right) \cos\psi_2 \sin\psi_1 + k_{1z}^o X \sin\psi_1 \cos\psi_2\right] \quad (205)$$

$$K_2(X, Y, \psi_1, \psi_2) = \left(X^2 k_y^2 + k_x^2 k_{1z}^o Y\right) \cos\psi_1 \cos\psi_2 - k_y \left[\left(X^2 k_{1z}^o - k_x^2 Y\right)\right.$$

$$+k^o_{1z}\left(Y^2-k_y^2\right)\sin\psi_1\cos\psi_2+\left(X^2Y-k_x^2k^o_{1z}\right)$$
$$\cos\psi_1\sin\psi_2\big]+\left(X^2k^o_{1z}Y+k_x^2k_y^2\right)\sin\psi_1\sin\psi_2 \quad (206)$$
$$N_1(X,Y,\psi_1,\psi_2)=k_y\left(X^2-k^o_{1z}Y\right)\cos\psi_1\cos\psi_2$$
$$+\left[Y\left(k_x^2+k^{o2}_{1z}\right)-X^2k^o_{1z}\right]\sin\psi_1\cos\psi_2$$
$$-k_y^2k^o_{1z}\cos\psi_1\sin\psi_2+k_y\left(k_x^2+k^{o2}_{1z}\sin\psi_1\sin\psi_2\right) \quad (207)$$

4.3. Limiting Case 3: $\chi_1 = \chi_2 = 0$, $\psi_i = 0$, $\varepsilon_i = \varepsilon_{iz}$, $i = 1$ or 2 (Isotropic — Anisotropic)

Results agree with the results reported in [5] for isotropic and anisotropic interface. Twelve coefficients out of sixteen are checked analytically and all sixteen coefficients are checked numerically.

4.4. Limiting Case 4: $\chi_i = \psi_i = 0$, $\varepsilon_i = \varepsilon_{iz}$, $i = 1$ and 2 (Isotropic — Isotropic)

The ordinary and extraordinary waves coincide with horizontally (TE) and vertically (TM) polarized waves. The coefficients for the cross polarized terms, R_{ijHV}, $R_{ij\,VH}$, X_{ijHV}, $X_{ij\,VH}$, are found to be equal to zero, $R_{ijHV} = R_{ij\,VH} = X_{ijHV} = X_{ij\,VH} = 0$.

5. CONCLUSION

Fresnel coefficients for three-layered anisotropic media have been obtained. The optic axes of two anisotropic media have been assumed to be in arbitrary direction. The fields are expressed in terms of two characteristic waves, i.e, the extraordinary wave and the ordinary wave. Using the matrix method, the coefficients of the three-layer problem are expressed in terms of half-space Fresnel reflection and transmission coefficients. The half-space coefficients are obtained analytically for the anisotropic-anisotropic interface, and are checked for four limiting cases. It is shown that the results in the two limiting cases agree with the existing results. Results presented here can be applied to dyadic Green's functions [26] and to radiation and scattering problems in the presence of multi-layer uniaxially anisotropic media.

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