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SCATTERING OF AN AXIAL GAUSSIAN BEAM BY A CONDUCTING SPHEROID WITH NON-CONFOCAL CHIRAL COATING

Huayong Zhang*, Yufa Sun, Tongqing Liao, and Rongqing Hui

Key Lab of Intelligent Computing and Signal Processing, Ministry of Education, Anhui University, Hefei 230039, China

Abstract—Within the generalized Lorenz–Mie theory framework, an analytic solution to the scattering by a conducting spheroid with non-confocal chiral coating, for incidence of an axial Gaussian beam, is presented. To overcome the difficulty of non-confocal boundary conditions, a theoretical procedure is developed by virtue of a transformation between the spherical and spheroidal vector wave functions. Numerical results of the normalized differential scattering cross section are shown for chiral-coated conducting spheroids.

1. INTRODUCTION

Some investigations have been carried out on the electromagnetic wave scattering by spheroids. Aside from providing exact analytic solutions for a large number of real objects which can be modeled by spheroids or layered spheroids with appropriate axial ratios, they are valuable in the evaluation of approximate and numerical solutions. By using the method of separation of variables for the vector wave functions in the spheroidal coordinate system, Asano and Yamamoto studied the scattering of a linearly polarized plane electromagnetic wave by a homogeneous isotropic spheroid with any size and refractive index [1, 2]. Sebak and Sinha examined the same case, but for a conducting spheroidal object with a confocal dielectric coating [3]. Layered spheroids have been treated by the extended boundary condition method (EBCM) [4]. Li et al. provided a solution of the electromagnetic radiation from a prolate spheroidal antenna enclosed

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* Corresponding author: Huayong Zhang (huayongzhang74@yahoo.com.cn).

in a confocal radome [5]. For an incident shaped beam, Barton has calculated the intensity distributions internal and external to a layered spheroid with arbitrary illumination, in which the method used is that of describing the incident fields by surface integrals [6]. Within the framework of the generalized Lorenz-Mie theory (GLMT), Han et al. investigated the electromagnetic scattering of a shaped beam by a spheroid at parallel incidence [7, 8], and the general case was analyzed by Xu et al. of an arbitrarily oriented, located, and shaped beam scattered by a homogeneous spheroid [9, 10].

A chiral or optically active medium is characterized by different phase velocities for the right and left circularly polarized waves, and a linearly polarized wave undergoes a rotation of its polarization as it propagates inside this medium [11, 12]. The interaction of electromagnetic waves with chiral media has been studied over the years for many applications involving antennas and arrays, antenna radomes, microstrip substrates, and waveguides [13–16]. For normal incidence of a TE or TM plane wave, analytic solutions have been presented by Kluskens and Newman to the scattering by a layered chiral circular cylinder [17], and by Khatir et al. to the scattering by a chiral elliptic cylinder [18]. Based on the EBCM, a theoretical procedure is devised by Lakhtakia et al. to examine the plane wave scattering and absorption characteristics of chiral objects [19]. A software package was developed by Demir et al. to calculate the plane-wave scattering by a chiral sphere [20]. The method of moments technique has been introduced by Worasawate et al., and the bi-isotropic finite difference time domain technique by Semichaevsky et al. for the numerical analysis of electromagnetic scattering from a homogeneous chiral body [21, 22]. Yokota et al. studied the scattering of a Hermite-Gaussian beam by a chiral sphere by using the relations between the multipole fields and the conventional Hermite-Gaussian beam [23]. In our recent papers [24, 25], a great effort has been made to examine Gaussian beam scattering by a coated spheroid. In order to analyze theoretically the interaction between the chiral medium and shaped beam in the spheroidal coordinates, this paper, based on our previous works, is devoted to the presentation of axial Gaussian beam (a focused TEM_{00} mode laser beam) scattering by a chiral-coated conducting spheroid.

The paper is organized as follows. Section 2 provides the theoretical procedure for the determination of the scattered fields of an axial Gaussian beam by a conducting spheroid with a non-confocal chiral coating. In Section 3, numerical results of axial Gaussian beam scattering properties are presented. Section 4 is a conclusion.

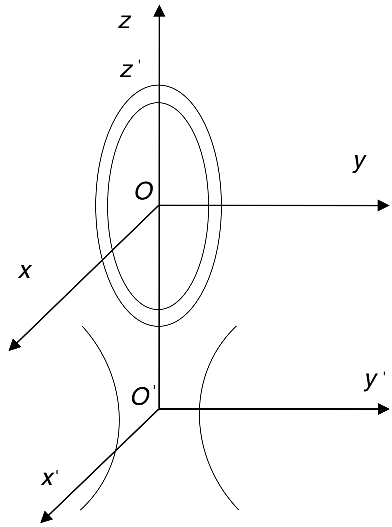

Figure 1. The Cartesian coordinate system $Oxyz$ is obtained by a translation of the Gaussian beam coordinate system $O'x'y'z'$ along the z' axis, and origin O is at $(0, 0, z_0)$ in $O'x'y'z'$. A chiral-coated conducting spheroid is natural to $Oxyz$.

2. FORMULATION

2.1. Expansion of Axial Gaussian Beam in Spheroidal Coordinates

As shown in Fig. 1, an incident Gaussian beam propagates in free space and from the negative z' to the positive z' axis of the Cartesian coordinate system $O'x'y'z'$, with the middle of its beam waist located at origin O'. The system $Oxyz$ is obtained by a translation of $O'x'y'z'$ along the z' axis, so that origin O is on the $O'z'$ axis, i.e., at $(0, 0, z_0)$ in $O'x'y'z'$. The center of a chiral-coated conducting spheroid is located at origin O, and the major axis is along the z axis (axial case). The conducting spheroid surface and the outer surface of the chiral coating are concentric, but not necessarily confocal. The semifocal distance, semimajor and semiminor axes are denoted by f_1, a_1 and b_1 for the conducting spheroid surface, and by f_2, a_2 and b_2 for the outer surface of the chiral coating. In this paper, we assume that the time-dependent part of the electromagnetic fields is $\exp(-i\omega t)$.

The electromagnetic fields of an axial Gaussian beam, for the TE mode, can be expanded in terms of the spheroidal vector wave functions attached to the chiral coating, as follows [1, 24, 25]:

$$\mathbf{E}^i = -E_0 \sum_{n=1}^{\infty} i^n G_n \left[\mathbf{M}_{e1n}^{r(1)}(k_0 f_2) + i\mathbf{N}_{o1n}^{r(1)}(k_0 f_2)\right] \tag{1}$$

$$\mathbf{H}^i = E_0 \frac{1}{\eta_0} \sum_{n=1}^{\infty} i^n G_n \left[i\mathbf{N}_{e1n}^{r(1)}(k_0 f_2) - \mathbf{M}_{o1n}^{r(1)}(k_0 f_2)\right] \tag{2}$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the free-space wave impedance, and the beam shape coefficients G_n can be expressed explicitly as

$$G_n = \frac{2}{N_{1n}(k_0 f_2)} \sum_{r=0,1}^{\infty}{}' d_r^{1n}(k_0 f_2) g_{r+1} \tag{3}$$

$$N_{1n}(k_0 f_2) = 2 \sum_{r=0,1}^{\infty}{}' \frac{(r+2)!}{(2r+3)r!} \left[d_r^{1n}(k_0 f_2)\right]^2 \tag{4}$$

where the prime over the summation sign indicates that the summation is over even values of r when $n-1$ is even and over odd values of r when $n-1$ is odd.

When the symmetrized Davis-Barton model of the Gaussian beam is used [26], the g_{r+1} coefficients (axial Gaussian beam shape coefficients in the spherical coordinates attached to $Oxyz$) in Eq. (3) can be computed by using the localized approximation as [27]

$$g_{r+1} = \frac{1}{1 + 2isz_0/w_0} \exp(ik_0 z_0) \exp\left[\frac{-s^2(r+1+1/2)^2}{1 + 2isz_0/w_0}\right] \tag{5}$$

where $s = \frac{1}{k_0 w_0}$, and w_0 is the beam waist radius.

In Eq. (4) $d_r^{1n}(k_0 f_2)$ are the expansion coefficients of the spheroidal angle functions [28].

For the TM mode, the corresponding expansions can be written as

$$\mathbf{E}^i = E_0 \sum_{n=1}^{\infty} i^n G_n \left[\mathbf{M}_{o1n}^{r(1)}(k_0 f_2) - i\mathbf{N}_{e1n}^{r(1)}(k_0 f_2)\right] \tag{6}$$

$$\mathbf{H}^i = -E_0 \frac{1}{\eta_0} \sum_{n=1}^{\infty} i^n G_n \left[i\mathbf{M}_{o1n}^{r(1)}(k_0 f_2) + \mathbf{N}_{e1n}^{r(1)}(k_0 f_2)\right] \tag{7}$$

2.2. Expansions of Electromagnetic Fields within the Chiral Coating

The constitutive relations for a chiral medium can be described by [11–14]

$$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} + i\kappa\sqrt{\mu_0 \varepsilon_0}\mathbf{H} \tag{8}$$

$$\mathbf{B} = \mu_0\mu_r\mathbf{H} - i\kappa\sqrt{\mu_0\varepsilon_0}\mathbf{E} \tag{9}$$

where κ is the chirality parameter.

The electromagnetic fields in a chiral medium ($\mathbf{E}$, $\mathbf{H}$) are the sum of the right-handed waves ($\mathbf{E}_+$, $\mathbf{H}_+$) and left-handed waves ($\mathbf{E}_-$, $\mathbf{H}_-$)

$$\begin{bmatrix}\mathbf{E}\\ \mathbf{H}\end{bmatrix} = \begin{bmatrix}\mathbf{E}_+\\ \mathbf{H}_+\end{bmatrix} + \begin{bmatrix}\mathbf{E}_-\\ \mathbf{H}_-\end{bmatrix} \tag{10}$$

where

$$\mathbf{E}_\pm = \pm i\eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}}\mathbf{H}_\pm = \pm i\eta\mathbf{H}_\pm \tag{11}$$

The uncoupled source-free wave equation in chiral media is

$$\nabla^2\begin{bmatrix}\mathbf{E}_+\\ \mathbf{E}_-\end{bmatrix} + \begin{bmatrix}k_+^2\mathbf{E}_+\\ k_-^2\mathbf{E}_-\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \tag{12}$$

where

$$k_\pm = k_0(\sqrt{\mu_r\varepsilon_r} \pm \kappa) \tag{13}$$

The curl and divergence of $(\mathbf{E}_+, \mathbf{E}_-)$ are given by

$$\nabla\times\begin{bmatrix}\mathbf{E}_+\\ \mathbf{E}_-\end{bmatrix} = \begin{bmatrix}k_+\mathbf{E}_+\\ -k_-\mathbf{E}_-\end{bmatrix} \tag{14}$$

$$\nabla\cdot\begin{bmatrix}\mathbf{E}_+\\ \mathbf{E}_-\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \tag{15}$$

To represent the fields within the chiral coating, it is necessary that the spherical vector wave functions be combined to form the right and left circularly polarized vector wave functions $\mathbf{m}^{r(j)}_{\substack{e\\o}mn}(k_+r)+\mathbf{n}^{r(j)}_{\substack{e\\o}mn}(k_+r)$ and $\mathbf{m}^{r(j)}_{\substack{e\\o}mn}(k_-r) - \mathbf{n}^{r(j)}_{\substack{e\\o}mn}(k_-r)$ ($j = 1$ or 3 according to the spherical Bessel functions of the first or third kind used in them) [29], which satisfy Eqs. (12), (14) and (15) in a source-free chiral region. Then, by following the expansions expressions of an axial Gaussian beam as in Eqs. (1), (2) or in Eqs. (6), (7), the electromagnetic fields within the chiral coating can be expanded in the form

$$\begin{aligned}
&\mathbf{E}^w\\
&= E_0\sum_{n=1}^{\infty} i^n\Big\{\delta_n\left[\mathbf{m}^{r(1)}_{e1n}(k_+r)+\mathbf{n}^{r(1)}_{e1n}(k_+r)\right] + \delta'_n\left[\mathbf{m}^{r(3)}_{e1n}(k_+r)+\mathbf{n}^{r(3)}_{e1n}(k_+r)\right]\\
&\quad +\chi_n\left[\mathbf{m}^{r(1)}_{o1n}(k_+r) + \mathbf{n}^{r(1)}_{o1n}(k_+r)\right] + \chi'_n\left[\mathbf{m}^{r(3)}_{o1n}(k_+r) + \mathbf{n}^{r(3)}_{o1n}(k_+r)\right]\\
&\quad +\tau_n\left[\mathbf{m}^{r(1)}_{e1n}(k_-r) - \mathbf{n}^{r(1)}_{e1n}(k_-r)\right] + \tau'_n\left[\mathbf{m}^{r(3)}_{e1n}(k_-r) - \mathbf{n}^{r(3)}_{e1n}(k_-r)\right]
\end{aligned}$$

$$+\gamma_n\Big[\mathbf{m}_{o1n}^{r(1)}(k_-r)-\mathbf{n}_{o1n}^{r(1)}(k_-r)\Big]+\gamma_n'\Big[\mathbf{m}_{o1n}^{r(3)}(k_-r)-\mathbf{n}_{o1n}^{r(3)}(k_-r)\Big]\Big\} \tag{16}$$

$$\mathbf{H}^w$$
$$=E_0\frac{i}{\eta}\sum_{n=1}^{\infty}i^n\Big\{-\delta_n\Big[\mathbf{m}_{e1n}^{r(1)}(k_+r)+\mathbf{n}_{e1n}^{r(1)}(k_+r)\Big]-\delta_n'\Big[\mathbf{m}_{e1n}^{r(3)}(k_+r)+\mathbf{n}_{e1n}^{r(3)}(k_+r)\Big]$$
$$-\chi_n\Big[\mathbf{m}_{o1n}^{r(1)}(k_+r)+\mathbf{n}_{o1n}^{r(1)}(k_+r)\Big]-\chi_n'\Big[\mathbf{m}_{o1n}^{r(3)}(k_+r)+\mathbf{n}_{o1n}^{r(3)}(k_+r)\Big]$$
$$+\tau_n\Big[\mathbf{m}_{e1n}^{r(1)}(k_-r)-\mathbf{n}_{e1n}^{r(1)}(k_-r)\Big]+\tau_n'\Big[\mathbf{m}_{e1n}^{r(3)}(k_-r)-\mathbf{n}_{e1n}^{r(3)}(k_-r)\Big]$$
$$+\gamma_n\Big[\mathbf{m}_{o1n}^{r(1)}(k_-r)-\mathbf{n}_{o1n}^{r(1)}(k_-r)\Big]+\gamma_n'\Big[\mathbf{m}_{o1n}^{r(3)}(k_-r)-\mathbf{n}_{o1n}^{r(3)}(k_-r)\Big]\Big\} \tag{17}$$

In order to apply the boundary conditions over the conducting spheroid surface and over the outer surface of the chiral coating, which are generally non-confocal, i.e., in different spheroidal coordinate systems, it is important that the fields expansions in Eqs. (16), (17) be transformed into their counterparts in terms of the spheroidal vector wave functions. We have found that a transformation between the spherical and spheroidal vector wave functions can be used to achieve this, which is of the form [28]

$$[\mathbf{m}(kr)\quad \mathbf{n}(kr)]^{r(j)}_{\substack{e\\o}1n}$$
$$=\frac{2n(n+1)}{2n+1}\sum_{l=1,2}^{\infty}{}'\frac{i^{l-n}}{N_{1l}(fk)}d_{n-1}^{1l}(kf)\,[\mathbf{M}(kf)\quad \mathbf{N}(kf)]^{r(j)}_{\substack{e\\o}1l} \tag{18}$$

where the superscript j takes, as already mentioned, the value 1 or 3 according to the radial functions $R_{mn}^{(j)}(kf)$ and $R_{mn}^{(j)}(kr)$ of the first or third kind.

Substituting Eq. (18) into Eq. (16), interchanging the summation order $\sum\limits_{n=1}^{\infty}\sum\limits_{l=1,2}^{\infty}{}'$ with $\sum\limits_{l=1}^{\infty}\sum\limits_{n=1,2}^{\infty}{}'$ and then replacing l by n and n by l, Eq. (16) or the expansion of the electric field within the chiral coating in terms of the spheroidal vector wave functions can be written as follows:

$$\mathbf{E}^w=E_0\sum_{n=1}^{\infty}i^n\Big\{A_n(k_+f)\Big[\mathbf{M}_{e1n}^{r(1)}(k_+f)+\mathbf{N}_{e1n}^{r(1)}(k_+f)\Big]+A_n'(k_+f)$$
$$\Big[\mathbf{M}_{e1n}^{r(3)}(k_+f)+\mathbf{N}_{e1n}^{r(3)}(k_+f)\Big]+B_n(k_+f)\Big[\mathbf{M}_{o1n}^{r(1)}(k_+f)+\mathbf{N}_{o1n}^{r(1)}(k_+f)\Big]$$
$$+B_n'(k_+f)_l\Big[\mathbf{M}_{o1n}^{r(3)}(k_+f)+\mathbf{N}_{o1n}^{r(3)}(k_+f)\Big]+E_n(k_-f)\Big[\mathbf{M}_{e1n}^{r(1)}(k_-f)$$

$$-\mathbf{N}_{e1n}^{r(1)}(k_-f)\Big] + E_n'(k_-f)\left[\mathbf{M}_{e1n}^{r(3)}(k_-f) - \mathbf{N}_{e1n}^{r(3)}(k_-f)\right] + F_n(k_-f)$$
$$\left[\mathbf{M}_{o1n}^{r(1)}(k_-f) - \mathbf{N}_{o1n}^{r(1)}(k_-f)\right] + F_n'(k_-f)_l\left[\mathbf{M}_{o1n}^{r(3)}(k_-f) - \mathbf{N}_{o1n}^{r(3)}(k_-f)\right]\Big\} \quad (19)$$

where

$$A_n(k_+f) = \frac{1}{N_{1n}(k_+f)} \sum_{l=1,2}^{\infty}{}' \frac{2l(l+1)}{2l+1} d_{l-1}^{1n}(k_+f)\delta_l \quad (20)$$

$$E_n(k_-f) = \frac{1}{N_{1n}(k_-f)} \sum_{l=1,2}^{\infty}{}' \frac{2l(l+1)}{2l+1} d_{l-1}^{1n}(k_-f)\tau_l \quad (21)$$

and $A_n'(k_+f)$ is obtained by replacing δ_l in Eq. (20) with δ_l', $B_n(k_+f)$ with χ_l, $B_n'(k_+f)$ with χ_l', and $E_n'(k_-f)$ in Eq. (21) with τ_l', $F_n(k_+f)$ with γ_l, $F_n'(k_+f)$ with γ_l'.

The corresponding expansion of the magnetic field within the chiral coating can also be obtained by following the same procedure.

2.3. Axial Gaussian Beam Scattering by a Chiral-coated Conducting Spheroid

By considering the expansions expressions of an axial Gaussian beam and of the fields within the chiral coating, the scattered fields can be expanded as follows:

$$\begin{aligned} \mathbf{E}^s &= E_0 \sum_{n=1}^{\infty} i^n \Big[\beta_n \mathbf{M}_{e1n}^{r(3)}(k_0f_2) + \beta_n' \mathbf{M}_{o1n}^{r(3)}(k_0f_2) \\ &\quad - i\alpha_n' \mathbf{N}_{e1n}^{r(3)}(k_0f_2) + i\alpha_n \mathbf{N}_{o1n}^{r(3)}(k_0f_2)\Big] \end{aligned} \quad (22)$$

$$\begin{aligned} \mathbf{H}^s &= \frac{E_0}{\eta_0} \sum_{n=1}^{\infty} i^n \Big[-i\beta_n \mathbf{N}_{e1n}^{r(3)}(k_0f_2) - i\beta_n' \mathbf{N}_{o1n}^{r(3)}(k_0f_2) \\ &\quad - \alpha_n' \mathbf{M}_{e1n}^{r(3)}(k_0f_2) + \alpha_n \mathbf{M}_{o1n}^{r(3)}(k_0f_2)\Big] \end{aligned} \quad (23)$$

The unknown expansion coefficients of the scattered fields α_n, α_n', β_n and β_n' in Eqs. (22), (23), and of the internal fields δ_n, δ_n', χ_n, χ_n', τ_n, τ_n', γ_n and γ_n' in Eqs. (16), (17) can be determined by using the boundary conditions, i.e., continuity of the tangential components of the electromagnetic fields over the surface $\zeta = \zeta_2$ and vanishing of the tangential components of the electric field over the surface $\zeta = \zeta_1$, with ζ_2 and ζ_1 as the radial coordinates of the outer surface of the chiral coating and of the conducting spheroid surface, respectively.

The boundary conditions at $\zeta = \zeta_2$ and $\zeta = \zeta_1$ are described by

$$\left.\begin{aligned} E_\eta^i + E_\eta^s = E_\eta^w, \quad E_\phi^i + E_\phi^s = E_\phi^w \\ H_\eta^i + H_\eta^s = H_\eta^w, \quad H_\phi^i + H_\phi^s = H_\phi^w \end{aligned}\right\} \quad \text{at} \quad \zeta = \zeta_2 \tag{24}$$

and

$$E_\eta^w = 0, \quad E_\phi^w = 0 \quad \text{at} \quad \zeta = \zeta_1 \tag{25}$$

By virtue of the fields expansions, Eq. (24) can be expressed as

$$\begin{aligned} &\sum_{n=1}^{\infty} i^n A_n(k_+ f_2) U_{1n}^{(1),t}(k_+ f_2) + \sum_{n=1}^{\infty} i^n A'_n(k_+ f_2) U_{1n}^{(3),t}(k_+ f_2) \\ &-\sum_{n=1}^{\infty} i^n i B_n(k_+ f_2) V_{1n}^{(1),t}(k_+ f_2) - \sum_{n=1}^{\infty} i^n i B'_n(k_+ f_2) V_{1n}^{(3),t}(k_+ f_2) \\ &+\sum_{n=1}^{\infty} i^n E_n(k_- f_2) U_{1n}^{(1),t}(k_- f_2) + \sum_{n=1}^{\infty} i^n E'_n(k_- f_2) U_{1n}^{(3),t}(k_- f_2) \\ &+\sum_{n=1}^{\infty} i^n i F_n(k_- f_2) V_{1n}^{(1),t}(k_- f_2) + \sum_{n=1}^{\infty} i^n i F'_n(k_- f_2) V_{1n}^{(3),t}(k_- f_2) \\ = &\sum_{n=1}^{\infty} i^n G_n \left[U_{1n}^{(1),t}(k_0 f_2) + V_{1n}^{(1),t}(k_0 f_2)\right] \\ &+\sum_{n=1}^{\infty} i^n \left[\beta_n U_{1n}^{(3),t}(k_0 f_2) + \alpha_n V_{1n}^{(3),t}(k_0 f_2)\right] \end{aligned} \tag{26}$$

$$\begin{aligned} &\sum_{n=1}^{\infty} i^n i A_n(k_+ f_2) V_{1n}^{(1),t}(k_+ f_2) + \sum_{n=1}^{\infty} i^n i A'_n(k_+ f_2) V_{1n}^{(3),t}(k_+ f_2) \\ &+\sum_{n=1}^{\infty} i^n B_n(k_+ f_2) U_{1n}^{(1),t}(k_+ f_2) + \sum_{n=1}^{\infty} i^n B'_n(k_+ f_2) U_{1n}^{(3),t}(k_+ f_2) \\ &-\sum_{n=1}^{\infty} i^n i E_n(k_- f_2) V_{1n}^{(1),t}(k_- f_2) - \sum_{n=1}^{\infty} i^n i E'_n(k_- f_2) V_{1n}^{(3),t}(k_- f_2) \\ &+\sum_{n=1}^{\infty} i^n F_n(k_- f_2) U_{1n}^{(1),t}(k_- f_2) + \sum_{n=1}^{\infty} i^n F'_n(k_- f_2) U_{1n}^{(3),t}(k_- f_2) \\ = &\sum_{n=1}^{\infty} i^n \left[\beta'_n U_{1n}^{(3),t}(k_0 f_2) + \alpha'_n V_{1n}^{(3),t}(k_0 f_2)\right] \end{aligned} \tag{27}$$

$$\begin{aligned}
&\sum_{n=1}^{\infty} i^n A_n(k_+f_2) X_{1n}^{(1),t}(k_+f_2) + \sum_{n=1}^{\infty} i^n A'_n(k_+f_2) X_{1n}^{(3),t}(k_+f_2) \\
&-\sum_{n=1}^{\infty} i^n iB_n(k_+f_2) Y_{1n}^{(1),t}(k_+f_2) - \sum_{n=1}^{\infty} i^n iB'_n(k_+f_2) Y_{1n}^{(3),t}(k_+f_2) \\
&+\sum_{n=1}^{\infty} i^n E_n(k_-f_2) X_{1n}^{(1),t}(k_-f_2) + \sum_{n=1}^{\infty} i^n E'_n(k_-f_2) X_{1n}^{(3),t}(k_-f_2) \\
&+\sum_{n=1}^{\infty} i^n iF_n(k_-f_2) Y_{1n}^{(1),t}(k_-f_2) + \sum_{n=1}^{\infty} i^n iF'_n(k_-f_2) Y_{1n}^{(3),t}(k_-f_2) \\
= &\sum_{n=1}^{\infty} i^n G_n[X_{1n}^{(1),t}(k_0f_2) + Y_{1n}^{(1),t}(k_0f_2)] \\
&+\sum_{n=1}^{\infty} i^n \left[\beta_n X_{1n}^{(3),t}(k_0f_2) + \alpha_n Y_{1n}^{(3),t}(k_0f_2)\right] \qquad (28)
\end{aligned}$$

$$\begin{aligned}
&\sum_{n=1}^{\infty} i^n iA_n(k_+f_2) Y_{1n}^{(1),t}(k_+f_2) + \sum_{n=1}^{\infty} i^n iA'_n(k_+f_2) Y_{1n}^{(3),t}(k_+f_2) \\
&+\sum_{n=1}^{\infty} i^n B_n(k_+f_2) X_{1n}^{(1),t}(k_+f_2) + \sum_{n=1}^{\infty} i^n B'_n(k_+f_2) X_{1n}^{(3),t}(k_+f_2) \\
&-\sum_{n=1}^{\infty} i^n iE_n(k_-f_2) Y_{1n}^{(1),t}(k_-f_2) - \sum_{n=1}^{\infty} i^n iE'_n(k_-f_2) Y_{1n}^{(3),t}(k_-f_2) \\
&+\sum_{n=1}^{\infty} i^n F_n(k_-f_2) X_{1n}^{(1),t}(k_-f_2) + \sum_{n=1}^{\infty} i^n F'_n(k_-f_2) X_{1n}^{(3),t}(k_-f_2) \\
= &\sum_{n=1}^{\infty} i^n \left[\beta'_n X_{1n}^{(3),t}(k_0f_2) + \alpha'_n Y_{1n}^{(3),t}(k_0f_2)\right] \qquad (29)
\end{aligned}$$

$$\begin{aligned}
&\sum_{n=1}^{\infty} i^n A_n(k_+f_2) V_{1n}^{(1),t}(k_+f_2) + \sum_{n=1}^{\infty} i^n A'_n(k_+f_2) V_{1n}^{(3),t}(k_+f_2) \\
&-\sum_{n=1}^{\infty} i^n iB_n(k_+f_2) U_{1n}^{(1),t}(k_+f_2) - \sum_{n=1}^{\infty} i^n iB'_n(k_+f_2) U_{1n}^{(3),t}(k_+f_2) \\
&+\sum_{n=1}^{\infty} i^n E_n(k_-f_2) V_{1n}^{(1),t}(k_-f_2) + \sum_{n=1}^{\infty} i^n E'_n(k_-f_2) V_{1n}^{(3),t}(k_-f_2)
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{n=1}^{\infty} i^n i F_n(k_- f_2) U_{1n}^{(1),t}(k_- f_2)+\sum_{n=1}^{\infty} i^n i F'_n(k_- f_2) U_{1n}^{(3),t}(k_- f_2)\\
=\ &\frac{\eta}{\eta_0}\sum_{n=1}^{\infty} i^n G_n\left[V_{1n}^{(1),t}(k_0 f_2)+U_{1n}^{(1),t}(k_0 f_2)\right]\\
&+\frac{\eta}{\eta_0}\sum_{n=1}^{\infty} i^n\left[\beta_n V_{1n}^{(3),t}(k_0 f_2)+\alpha_n U_{1n}^{(3),t}(k_0 f_2)\right] \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{n=1}^{\infty} i^n i A_n(k_+ f_2) U_{1n}^{(1),t}(k_+ f_2)+\sum_{n=1}^{\infty} i^n i A'_n(k_+ f_2) U_{1n}^{(3),t}(k_+ f_2)\\
&+\sum_{n=1}^{\infty} i^n B_n(k_+ f_2) V_{1n}^{(1),t}(k_+ f_2)+\sum_{n=1}^{\infty} i^n B'_n(k_+ f_2) V_{1n}^{(3),t}(k_+ f_2)\\
&-\sum_{n=1}^{\infty} i^n i E_n(k_- f_2) U_{1n}^{(1),t}(k_- f_2)-\sum_{n=1}^{\infty} i^n i E'_n(k_- f_2) U_{1n}^{(3),t}(k_- f_2)\\
&+\sum_{n=1}^{\infty} i^n F_n(k_- f_2) V_{1n}^{(1),t}(k_- f_2)+\sum_{n=1}^{\infty} i^n F'_n(k_- f_2) V_{1n}^{(3),t}(k_- f_2)\\
=\ &\frac{\eta}{\eta_0}\sum_{n=1}^{\infty} i^n\left[\beta'_n V_{1n}^{(3),t}(k_0 f_2)+\alpha'_n U_{1n}^{(3),t}(k_0 f_2)\right] \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{n=1}^{\infty} i^n A_n(k_+ f_2) Y_{1n}^{(1),t}(k_+ f_2)+\sum_{n=1}^{\infty} i^n A'_n(k_+ f_2) Y_{1n}^{(3),t}(k_+ f_2)\\
&-\sum_{n=1}^{\infty} i^n i B_n(k_+ f_2) X_{1n}^{(1),t}(k_+ f_2)-\sum_{n=1}^{\infty} i^n i B'_n(k_+ f_2) X_{1n}^{(3),t}(k_+ f_2)\\
&+\sum_{n=1}^{\infty} i^n E_n(k_- f_2) Y_{1n}^{(1),t}(k_- f_2)+\sum_{n=1}^{\infty} i^n E'_n(k_- f_2) Y_{1n}^{(3),t}(k_- f_2)\\
&+\sum_{n=1}^{\infty} i^n i F_n(k_- f_2) X_{1n}^{(1),t}(k_- f_2)+\sum_{n=1}^{\infty} i^n i F'_n(k_- f_2) X_{1n}^{(3),t}(k_- f_2)\\
=\ &\frac{\eta}{\eta_0}\sum_{n=1}^{\infty} i^n G_n\left[Y_{1n}^{(1),t}(k_0 f_2)+X_{1n}^{(1),t}(k_0 f_2)\right]\\
&+\frac{\eta}{\eta_0}\sum_{n=1}^{\infty} i^n\left[\beta_n Y_{1n}^{(3),t}(k_0 f_2)+\alpha_n X_{1n}^{(3),t}(k_0 f_2)\right] \qquad (32)
\end{aligned}
$$

$$\begin{aligned}
&\sum_{n=1}^{\infty} i^n i A_n(k_+f_2) X_{1n}^{(1),t}(k_+f_2) + \sum_{n=1}^{\infty} i^n i A'_n(k_+f_2) X_{1n}^{(3),t}(k_+f_2) \\
&+ \sum_{n=1}^{\infty} i^n B_n(k_+f_2) Y_{1n}^{(1),t}(k_+f_2) + \sum_{n=1}^{\infty} i^n B'_n(k_+f_2) Y_{1n}^{(3),t}(k_+f_2) \\
&- \sum_{n=1}^{\infty} i^n i E_n(k_-f_2) X_{1n}^{(1),t}(k_-f_2) - \sum_{n=1}^{\infty} i^n i E'_n(k_-f_2) X_{1n}^{(3),t}(k_-f_2) \\
&+ \sum_{n=1}^{\infty} i^n F_n(k_-f_2) Y_{1n}^{(1),t}(k_-f_2) + \sum_{n=1}^{\infty} i^n F'_n(k_-f_2) Y_{1n}^{(3),t}(k_-f_2) \\
=\ & \frac{\eta}{\eta_0} \sum_{n=1}^{\infty} i^n \left[\beta'_n Y_{1n}^{(3),t}(k_0f_2) + \alpha'_n X_{1n}^{(3),t}(k_0f_2)\right]
\end{aligned} \tag{33}$$

and Eq. (25) as

$$\begin{aligned}
&\sum_{n=1}^{\infty} i^n A_n(k_+f_1) U_{1n}^{(1),t}(k_+f_1) + \sum_{n=1}^{\infty} i^n A'_n(k_+f_1) U_{1n}^{(3),t}(k_+f_1) \\
&- \sum_{n=1}^{\infty} i^n i B_n(k_+f_1) V_{1n}^{(1),t}(k_+f_1) - \sum_{n=1}^{\infty} i^n i B'_n(k_+f_1) V_{1n}^{(3),t}(k_+f_1) \\
&+ \sum_{n=1}^{\infty} i^n E_n(k_-f_1) U_{1n}^{(1),t}(k_-f_1) + \sum_{n=1}^{\infty} i^n E'_n(k_-f_1) U_{1n}^{(3),t}(k_-f_1) \\
&+ \sum_{n=1}^{\infty} i^n i F_n(k_-f_1) V_{1n}^{(1),t}(k_-f_1) + \sum_{n=1}^{\infty} i^n i F'_n(k_-f_1) V_{1n}^{(3),t}(k_-f_1) = 0
\end{aligned} \tag{34}$$

$$\begin{aligned}
&\sum_{n=1}^{\infty} i^n i A_n(k_+f_1) V_{1n}^{(1),t}(k_+f_1) + \sum_{n=1}^{\infty} i^n i A'_n(k_+f_1) V_{1n}^{(3),t}(k_+f_1) \\
&+ \sum_{n=1}^{\infty} i^n B_n(k_+f_1) U_{1n}^{(1),t}(k_+f_1) + \sum_{n=1}^{\infty} i^n B'_n(k_+f_1) U_{1n}^{(3),t}(k_+f_1) \\
&- \sum_{n=1}^{\infty} i^n i E_n(k_-f_1) V_{1n}^{(1),t}(k_-f_1) - \sum_{n=1}^{\infty} i^n i E'_n(k_-f_1) V_{1n}^{(3),t}(k_-f_1) \\
&+ \sum_{n=1}^{\infty} i^n F_n(k_-f_1) U_{1n}^{(1),t}(k_-f_1) + \sum_{n=1}^{\infty} i^n F'_n(k_-f_1) U_{1n}^{(3),t}(k_-f_1) = 0
\end{aligned} \tag{35}$$

$$\sum_{n=1}^{\infty} i^n A_n(k_+f_1) X_{1n}^{(1),t}(k_+f_1) + \sum_{n=1}^{\infty} i^n A'_n(k_+f_1) X_{1n}^{(3),t}(k_+f_1)$$

$$-\sum_{n=1}^{\infty} i^n i B_n(k_+f_1)Y_{1n}^{(1),t}(k_+f_1) - \sum_{n=1}^{\infty} i^n i B'_n(k_+f_1)Y_{1n}^{(3),t}(k_+f_1)$$
$$+\sum_{n=1}^{\infty} i^n E_n(k_-f_1)X_{1n}^{(1),t}(k_-f_1) + \sum_{n=1}^{\infty} i^n E'_n(k_-f_1)X_{1n}^{(3),t}(k_-f_1)$$
$$+\sum_{n=1}^{\infty} i^n i F_n(k_-f_1)Y_{1n}^{(1),t}(k_-f_1) + \sum_{n=1}^{\infty} i^n i F'_n(k_-f_1)Y_{1n}^{(3),t}(k_-f_1) = 0 \quad (36)$$

$$\sum_{n=1}^{\infty} i^n i A_n(k_+f_1)Y_{1n}^{(1),t}(k_+f_1) + \sum_{n=1}^{\infty} i^n i A'_n(k_+f_1)Y_{1n}^{(3),t}(k_+f_1)$$
$$+\sum_{n=1}^{\infty} i^n B_n(k_+f_1)X_{1n}^{(1),t}(k_+f_1) + \sum_{n=1}^{\infty} i^n B'_n(k_+f_1)X_{1n}^{(3),t}(k_+f_1)$$
$$-\sum_{n=1}^{\infty} i^n i E_n(k_-f_1)Y_{1n}^{(1),t}(k_-f_1) - \sum_{n=1}^{\infty} i^n i E'_n(k_-f_1)Y_{1n}^{(3),t}(k_-f_1)$$
$$+\sum_{n=1}^{\infty} i^n F_n(k_-f_1)X_{1n}^{(1),t}(k_-f_1) + \sum_{n=1}^{\infty} i^n F'_n(k_-f_1)X_{1n}^{(3),t}(k_-f_1) = 0 \quad (37)$$

The parameters $U_{1n}^{(j),t}$, $V_{1n}^{(j),t}$, $X_{1n}^{(j),t}$, $Y_{1n}^{(j),t}$ ($j = 1$ or 3 depending on the usage of the radial functions $R_{mn}^{(j)}(kf)$ in them of the first or third kind) are provided by Asano and Yamamoto in [1], and Eqs. (26)–(37) are valid for each of $t \geq 0$. We can truncate the infinite system consisting of Eqs. (26)–(37) by setting $n = 1, 2, \ldots, N$, $t = 0, 1, \ldots, N-1$, N being a suitable large number for a convergent solution, and then the total number of unknown expansion coefficients of the scattered and internal fields is $12N$. To determine the unknown coefficients, each summation in Eqs. (26)–(37) is written in a matrix form. The summation $\sum\limits_{n=1}^{N} i^n A_n(k_+f_2)U_{1n}^{(1),t}(k_+f_2)$ in Eq. (26), for example, is written as $[i^n U_{1n}^{(1),t}(k_+f_2)]_{N\times N}[A_n(k_+f_2)]_{N\times 1}$. By doing so, Eqs. (26)–(37) give a $12N \times 12N$ system of equations. Then, the standard numerical techniques, such as an iterative procedure or inverse matrix method, may be employed to solve for the unknown coefficients [1, 24, 25].

3. NUMERICAL RESULTS

Of practical interest is the behavior of the scattered wave at relatively large distances from the scatterer (far field), which can be deduced by taking the asymptotic form of $\mathbf{E}^s$, as $k_0 f_2 \zeta \to \infty$. When neglected terms of order higher than $1/r$ in $\mathbf{M}^{r(3)}_{\substack{e\\o}1n}(k_0 f_2)$ and $\mathbf{N}^{r(3)}_{\substack{e\\o}1n}(k_0 f_2)$, as $k_0 f_2 \zeta \to \infty$, the asymptotic form of the scattered electric field $\mathbf{E}^s$ can be obtained from Eq. (22), which is of the form [1, 24, 25]

$$E^s_\eta = -E_0 \frac{i\lambda}{2\pi r} \exp\left(i\frac{2\pi}{\lambda} r\right) T_1(\theta, \phi) \tag{38}$$

$$E^s_\phi = E_0 \frac{i\lambda}{2\pi r} \exp\left(i\frac{2\pi}{\lambda} r\right) T_2(\theta, \phi) \tag{39}$$

where

$$\begin{aligned} T_1(\theta, \phi) = \sum_{n=1}^{\infty} \Bigg\{ & \left[\alpha_n \frac{dS_{1n}(k_0 f_2, \cos\theta)}{d\theta} + \beta_n \frac{S_{1n}(k_0 f_2, \cos\theta)}{\sin\theta}\right] \sin\phi \\ & - \left[\alpha'_n \frac{dS_{1n}(k_0 f_2, \cos\theta)}{d\theta} + \beta'_n \frac{S_{1n}(k_0 f_2, \cos\theta)}{\sin\theta}\right] \cos\phi \Bigg\} \end{aligned} \tag{40}$$

$$\begin{aligned} T_2(\theta, \phi) = \sum_{n=1}^{\infty} \Bigg\{ & \left[\alpha_n \frac{S_{1n}(k_0 f_2, \cos\theta)}{\sin\theta} + \beta_n \frac{dS_{1n}(k_0 f_2, \cos\theta)}{d\theta}\right] \cos\phi \\ & + \left[\alpha'_n \frac{S_{1n}(k_0 f_2, \cos\theta)}{\sin\theta} + \beta'_n \frac{dS_{1n}(k_0 f_2, \cos\theta)}{d\theta}\right] \sin\phi \Bigg\} \end{aligned} \tag{41}$$

By virtue of the asymptotic form of $\mathbf{E}^s$, we can have the differential scattering cross section which is defined by [1, 24, 25]

$$\sigma(\theta, \phi) = 4\pi r^2 \left|\frac{\mathrm{E}^s}{E_0}\right|^2 = \frac{\lambda^2}{\pi} \left(|T_1(\theta, \phi)|^2 + |T_2(\theta, \phi)|^2\right) \tag{42}$$

In this paper, the normalized differential scattering cross section $\pi\sigma(\theta, \phi)/\lambda^2$ is thereafter evaluated in the spherical coordinate system attached to the coating. In the following calculations, the incident Gaussian beam is assumed to be TE polarized, and the middle of its beam waist coincides with the center (origin O) of the chiral-coated spheroid ($z_0 = 0$).

Figure 2 shows the normalized differential scattering cross section $\pi\sigma(\theta, \phi)/\lambda^2$ for incidence of an axial Gaussian beam with $w_0 = 2\lambda$ on a chiral-coated conducting spheroid.

Figure 3 shows the ratio $|T_1(\theta, 0)|/|T_2(\theta, 0)|$ as a function of θ for the same model as in Fig. 2. From Fig. 3 we can see that $|T_1(\theta, 0)|$ and $|T_2(\theta, 0)|$ are of the same order in magnitude in the angle regions between 90° and 130°, i.e., the amplitude of the η component of the scattered electric field is comparable with that of the ϕ component. But for the same model with a dielectric coating ($\kappa = 0$), the former, to our computations, is too small to be neglected compared with the latter.

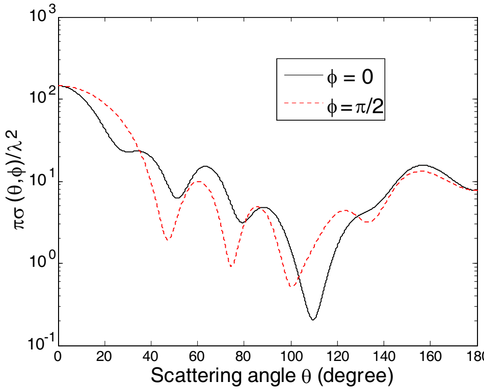

Figure 2. Normalized differential scattering cross sections $\pi\sigma(\theta, 0)/\lambda^2$ and $\pi\sigma(\theta, \frac{\pi}{2})/\lambda^2$ for a chiral-coated conducting spheroid ($k_0a_1 = 6$, a_1/b_1 =2, $k_0a_2 = 9.14$, a_2/b_2 =2, $\varepsilon_r = 4$, $\mu_r = 1$, $\kappa = 0.5$) illuminated by an axial Gaussian beam with $w_0 = 2\lambda$.

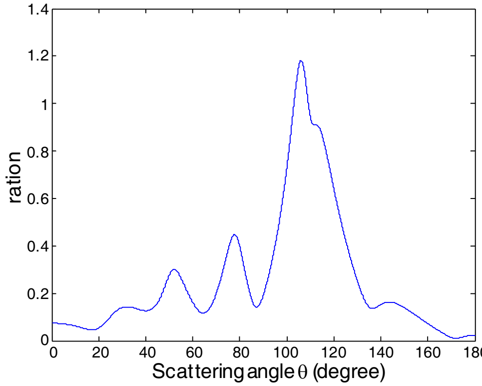

Figure 3. Ratio $|T_1(\theta, 0)|/|T_2(\theta, 0)|$ for the same model as in Fig. 2.

This is due to the chirality of the medium of the coating, which will rotate the plane of incident polarized electromagnetic waves passing through the chiral coating.

A comparison is shown in Fig. 4 between the normalized differential scattering cross section $\pi\sigma(\theta, 0)/\lambda^2$ for a conducting spheroid with a non-confocal chiral coating and that for the same model but with a dielectric coating, both illuminated by an axial Gaussian beam with $w_0 = 2\lambda$.

Figure 4 indicates that there is no obvious difference between the

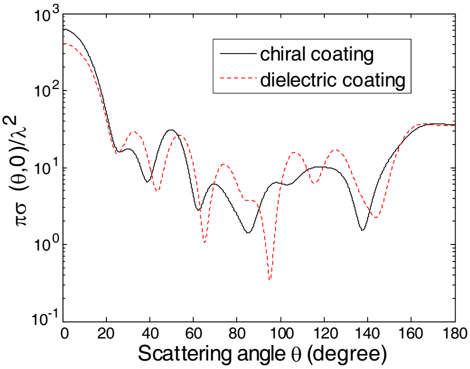

Figure 4. Comparison between the normalized differential scattering cross section $\pi\sigma(\theta, 0)/\lambda^2$ for a chiral-coated conducting spheroid (solid line) ($k_0a_1 = 6.28$, a_1/b_1 =2, $k_0a_2 = 9.42$, a_2/b_2 =2, $\varepsilon_r = 4$, $\mu_r = 1$, $\kappa = 0.5$) and that for the same model but with a dielectric coating (dotted line) ($\kappa = 0$), both illuminated by an axial Gaussian beam of $w_0 = 2\lambda$.

two curves, both in the forwardscattering and in the backscattering results. But up to now, numerical results are only obtained for a limited number of theoretical models. From those computational results, it is probably reasonable to conclude that, for incidence of an axial Gaussian beam, the chirality of the medium of the coating has a more significant influence on the polarization properties than on the intensities of the scattered fields.

4. CONCLUSION

An approach to compute the scattering of an axial Gaussian beam by a conducting spheroid with a non-confocal chiral coating is provided within the GLMT framework. In fact, this scattering problem becomes complicated due to a coupling within the chiral coating between the right and left circularly polarized waves characterized by different phase velocities. Numerical results show that the chirality of the medium of the coating has a greater impact on the polarization properties of the scattered fields in a certain angle regions, owing to the optical activity in the chiral medium. As a result, this study extends the Gaussian beam scattering by a conducting spheroid with a dielectric coating to the case of a chiral-coated one, and is suggestive and useful for interpretation of shaped beam scattering phenomena for chiral-coated conducting objects.

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