AN EXACT FORMULATION FOR THE REFLECTION COEFFICIENT FROM ANISOTROPIC MULTILAYER STRUCTURES WITH ARBITRARY BACKING

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Abstract—This paper is concerned with the theory of wave propagation in biaxial anisotropic media. Consider a multilayered planar structure composed of media with electric and magnetic anisotropy, surrounded by two half spaces. Exat relations for reflection coefficient from this structure can be useful for arriving at the intended applications. In this paper, by matching of transverse field components at the bounderies, we will arrive at exact recursive relations for reflection coefficient of the structure. In the previous works, the magnetic and electric anisotropy were not taken into consideration at the same time, or complex relations were arrived. But using this novel method, those complexities will not appear and both electric and magnetic anisotropy are take into consideration. Moreover, we will not set any limits on the right half-space so the right most half-space may be a PEC, PMC, PEMC, surface impedance, dielectric or a metamaterial. Finally, the last section of the paper confirms the validity of the relations arrived at and as an interesting application; the zero reflection condition will be obtained.

1. INTRODUCTION

In recent years, more and more attention has been paid to the interaction of fields and waves with composite materials, especially anisotropic ones. This is because of the extensive use of these materials in designing and analyzing antennas, angle filters, polarizers and high efficiency microwave and millimeter wave instruments, and also their use in the propagation of radio frequency waves and in reduction of

the radar cross section. Moreover, developments in polymer synthesis techniques for manufacturing chiral materials, in addition to the artificial manufacturing of composite materials, have employed these usages in practice.

"Anisotropic materials" refers to a group of complex media in which constitutive relations are not as simple as those used in the conventional materials, and the electric displacement vector $(\bar{D})$ and the magnetic flux density vector $(\bar{B})$, in the most general case, are related to the electric and the magnetic fields by four tensors. In recent years, numerous researches have been done in this area [1–7].

Initial works on the subject were based on 4×4 characteristic matrix of a single anisotropic slab [8, 9]. Later efforts include generalization of the problem for stratified structures by different methods [10–12]. Morgan et al. paid attention to a numerical solution, and introduced an efficient and simple algorithm for this case [13]. Others proposed various techniques based on eigenvalue computation, Ricatti differential equation and transmission line method [14–18] which are more complex. The characteristic matrix algorithm [8, 9] had a serious drawback and showed instability for thick layers compared to wavelength. To avoid this instability which was due to the numerical finite difference algorithm, the use of hybrid matrix of the structure is suggested [19].

The above mentioned methods are complex or cannot be used for all of the problems. In this paper, we propose an exact analytical method which is simple and accurate.

Orfanidis in [20] has proposed a simple method for the case in which only one of the electric and the magnetic biaxial anisotropy exists, but we aim to study the propagation of waves in anisotropic media in which both the electric and the magnetic biaxial anisotropy exist at the same time.

The structure that is considered here is a multilayer planar structure composed of anisotropic materials and surrounded by two half spaces. We will write the field components in each layer, and, by matching the transverse field components with one another, we will arrive at recursive relations for reflection and transmission coefficients. We will also not set any limits on the right half-space throughout this procedure.

At last, validity of the relations has been verified through a numerical example and as an interesting application — the zero reflection condition — has been provided using these relation.

2. FORMULATING THE PROBLEM AND DERIVING THE RELATIONS

Consider a plane wave with the single frequency ω, composed of TE and TM polarizations, propagating in z direction. The wave is incident, from left half-space in Figure 1, on a multilayer structure composed of biaxial anisotropic materials.

The waves in this half-plane — region 0 — are sum of incident and reflected waves.

For TE mode:

$$\begin{cases} \mathrm{E}_{yl}^0 = \left(\mathrm{E}_0 e^{-jk_{z0}z} + C_0 e^{+jk_{z0}z} \right) e^{-jk_x x} \\[2mm] H_{xl}^0 = -\dfrac{k_{z0}}{\omega\mu_0} \left(\mathrm{E}_0 e^{-jk_{z0}z} - C_0 e^{+jk_{z0}z} \right) e^{-jk_x x} \\[2mm] H_{zl}^0 = \dfrac{k_x}{\omega\mu_0} \left(\mathrm{E}_0 e^{-jk_{z0}z} + C_0 e^{+jk_{z0}z} \right) e^{-jk_x x} \end{cases} \tag{1}$$

For TM mode:

$$\begin{cases} H_{yl}^0 = \left(H_0 e^{-jk_{z0}z} + D_0 e^{+jk_{z0}z} \right) e^{-jk_x x} \\[2mm] E_{xl}^0 = \dfrac{k_{z0}}{\omega\varepsilon_0} \left(H_0 e^{-jk_{z0}z} - D_0 e^{+jk_{z0}z} \right) e^{-jk_x x} \\[2mm] E_{zl}^0 = -\dfrac{k_x}{\omega\varepsilon_0} \left(H_0 e^{-jk_{z0}z} + D_0 e^{+jk_{z0}z} \right) e^{-jk_x x} \end{cases} \tag{2}$$

In the above relations $e^{+jk_{z0}z}$ is for incident wave and $e^{+jk_{z0}z}$ is for reflected wave.

The dispersion relation in this area is:

$$k_{z0}^2 + k_x^2 = k_0^2 \tag{3}$$

All layers are without sources and are composed of lossless linear anisotropic materials with constitutive parameters similar to the following:

$$\bar{\bar{\varepsilon}}_l = \varepsilon_0 \begin{bmatrix} \varepsilon_{lx} & 0 & 0 \\ 0 & \varepsilon_{ly} & 0 \\ 0 & 0 & \varepsilon_{lz} \end{bmatrix} \tag{4}$$

$$\bar{\bar{\mu}}_l = \mu_0 \begin{bmatrix} \mu_{lx} & 0 & 0 \\ 0 & \mu_{ly} & 0 \\ 0 & 0 & \mu_{lz} \end{bmatrix} \tag{5}$$

The medium $N + 1$ may be a PEC, PMC, PEMC, surface impedance, dielectric, or metamaterials. All of these possibilities are taken into consideration throughout this paper.
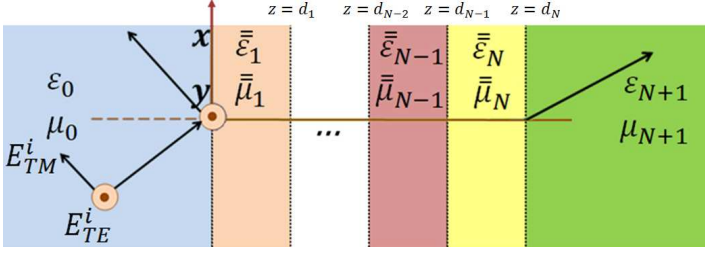
Figure 1. The structure of the multilayer anisotropic problem.

Supposing $e^{+j\omega t}$, the solution of the wave equation in layer l, which would be some of two waves with orthogonal TE and TM polarizations, is thus [5]:

TE mode:

$$\begin{cases} \mathrm{E}_{yl} = \left(A_l e^{-jk^I_{zl}z} + C_l e^{+jk^I_{zl}z} \right) e^{-jk_x x} \\[2mm] H_{xl} = -\dfrac{k^I_{zl}}{\omega\mu_0\mu_{lx}} \left(A_l e^{-jk^I_{zl}z} - C_l e^{+jk^I_{zl}z} \right) e^{-jk_x x} \\[2mm] H_{zl} = \dfrac{k_x}{\omega\mu_0\mu_{lz}} \left(A_l e^{-jk^I_{zl}z} + C_l e^{+jk_{zl}z} \right) e^{-jk_x x} \end{cases} \quad (6)$$

For TM mode:

$$\begin{cases} H_{yl} = \left(B_l e^{-jk^{II}_{zl}z} + D_l e^{+jk^{II}_{zl}z} \right) e^{-jk_x x} \\[2mm] E_{xl} = \dfrac{k^{II}_{zl}}{\omega\varepsilon_0\varepsilon_{lx}} \left(B_l e^{-jk^{II}_{zl}z} - D_l e^{+jk^{II}_{zl}z} \right) e^{-jk_x x} \\[2mm] E_{zl} = -\dfrac{k_x}{\omega\varepsilon_0\varepsilon_{lz}} \left(B_l e^{-jk^{II}_{zl}z} + D_l e^{+jk^{II}_{zl}z} \right) e^{-jk_x x} \end{cases} \quad (7)$$

In which the subscripts k^I_{zl} and k^{II}_{zl} refer to wave numbers for TE and TM mode, respectively. $e^{-jk_x x}$, in the above relations, is obtained from phases matching and is identical for all layers.

From the dispersion relation [5] for each layer, we have:

$$k^{I2}_{zl} + \frac{\mu_{lx}}{\mu_{lz}} k^2_x = k^2_0 \varepsilon_{ly}\mu_{lx} \quad (8)$$

$$k^{II2}_{zl} + \frac{\varepsilon_{lx}}{\varepsilon_{lz}} k^2_x = k^2_0 \varepsilon_{lx}\mu_{ly} \quad (9)$$

In the region $l = 0$ we have the following:

$$A_0 = E_0 \quad (10)$$

$$B_0 = H_0 \quad (11)$$

$$C_0 = R_{TE} \cdot E_0 \quad (12)$$

$$D_0 = R_{TM} \cdot H_0 \tag{13}$$

In which R_{TE} and R_{TM} are reflection coefficient for TE and TM polarization in the right most half-space, respectively.

And in the region $l = N + 1$ we have:

$$A_{N+1} = T_{TE} \cdot E_0 \tag{14}$$
$$B_{N+1} = T_{TM} \cdot H_0 \tag{15}$$
$$C_{N+1} = 0 \tag{16}$$
$$D_{N+1} = 0 \tag{17}$$

In which T_{TE} and T_{TM} are transmission coefficient for TE and TM polarization in the left most half-space, respectively.

The problem we are concerned with has $N + 1$ boundaries, and according to the relations above, we have $4N + 4$ unknown parameters. Therefore, we need $4N + 4$ equations for solving them, which are obtained from the boundary conditions- at each boundary we obtain 4 equations.

We write down the boundary condition at $z = d_l$ — the boundary between the regions l and $l + 1$:

$$A_l e^{-jk_{zl}^I d_l} + C_l e^{+jk_{zl}^I d_l} = A_{(l+1)} e^{-jk_{z(l+1)}^I d_l} + C_{(l+1)} e^{+jk_{z(l+1)}^I d_l} \tag{18}$$

$$\frac{k_{zl}^I}{\mu_{lx}} \left(A_l e^{-jk_{zl}^I d_l} - C_l e^{+jk_{zl}^I d_l} \right)$$
$$= \frac{k_{z(l+1)}^I}{\mu_{(l+1)x}} \left(A_{(l+1)} e^{-jk_{z(l+1)}^I d_l} - C_{(l+1)} e^{+jk_{z(l+1)}^I d_l} \right) \tag{19}$$

$$B_l e^{-jk_{zl}^{II} d_l} + D_l e^{+jk_{zl}^{II} d_l} = B_{(l+1)} e^{-jk_{z(l+1)}^{II} d_l} + D_{(l+1)} e^{+jk_{z(l+1)}^{II} d_l} \tag{20}$$

$$\frac{k_{zl}^{II}}{\varepsilon_{lx}} \left(B_l e^{-jk_{zl}^{II} d_l} - D_l e^{+jk_{zl}^{II} d_l} \right)$$
$$= \frac{k_{z(l+1)}^{II}}{\varepsilon_{(l+1)x}} \left(B_{(l+1)} e^{-jk_{z(l+1)}^{II} d_l} - D_{(l+1)} e^{+jk_{z(l+1)}^{II} d_l} \right) \tag{21}$$

The boundary condition at $z = 0$ — the boundary between the regions 0 and 1 — will be:

$$E_0 + R_{TE} E_0 = A_1 + C_1 \tag{22}$$

$$\frac{k_{z0}}{\mu_0}(E_0 - R_{TE} E_0) = \frac{k_{zl}^I}{\mu_0 \mu_{1x}}(A_1 - C_1) \tag{23}$$

$$H_0 + R_{TM} H_0 = B_1 + D_1 \tag{24}$$

$$\frac{k_{z0}}{\varepsilon_0}(H_0 - R_{TM} H_0) = \frac{k_{zl}^{II}}{\varepsilon_0 \varepsilon_{1x}}(B_1 - D_1) \tag{25}$$

We write down the boundary condition at $z = d_N$ — the boundary between the regions N and $N + 1$:

$$A_N e^{-jk_{zN}^I d_N} + C_N e^{+jk_{zN}^I d_N} = T_{TE} \cdot E_0 e^{-jk_{z(N+1)}^I d_N} \quad (26)$$

$$\frac{k_{zN}^I}{\mu_{Nx}} \left(A_N e^{-jk_{zN}^I d_N} - C_N e^{+jk_{zN}^I d_N} \right) = \frac{K_{z(N+1)}^I}{\mu_{(N+1)x}} \left(T_{TE} \cdot E_0 e^{-jk_{z(N+1)}^I d_N} \right) \quad (27)$$

$$B_N e^{-jk_{zN}^{II} d_N} + D_N e^{+jk_{zN}^{II} d_N} = T_{TM} \cdot H_0 e^{-jk_{z(N+1)}^{II} d_N} \quad (28)$$

$$\frac{k_{zN}^{II}}{\varepsilon_{Nx}} \left(B_N e^{-jk_{zN}^{II} d_N} - D_N e^{+jk_{zN}^{II} d_N} \right) = \frac{K_{z(N+1)}^{II}}{\varepsilon_{(N+1)x}} \left(T_{TM} \cdot H_0 e^{-jk_{z(N+1)}^{II} d_N} \right) \quad (29)$$

In the above equations k_z^I and k_z^{II} can be derived from Equations (8) and (9).

To find the unknown parameters, we solve the obtained equations using two methods.

The first method is to turn the equations into matrix form, which can be seen below.

$$\begin{bmatrix} R_{TE} \\ R_{TM} \\ A_1 \\ C_1 \\ B_1 \\ D_1 \\ \ldots \\ A_N \\ C_N \\ B_N \\ D_N \\ T_{TE} \\ T_{TM} \end{bmatrix} = \begin{bmatrix} E_0 & 0 & -1 & -1 & 0 & 0 & \ldots \\ -E_0 & 0 & -p_{01}^I & p_{01}^I & 0 & 0 \\ 0 & H_0 & 0 & 0 & -1 & -1 & \ldots \\ 0 & -H_0 & 0 & 0 & -p_{01}^{II} & p_{01}^{II} \\ & & \ldots & & & & \ldots & \ldots \end{bmatrix}^{-1} \cdot \begin{bmatrix} -E_0 \\ -E_0 \\ -H_0 \\ -H_0 \\ 0 \\ \ldots \\ 0 \end{bmatrix} \quad (30)$$

where:

$$p_{l(l+1)}^I = \frac{\mu_{lx}}{k_{zl}^I} \cdot \frac{k_{z(l+1)}^I}{\mu_{(l+1)x}} \quad (31)$$

$$p_{l(l+1)}^{ll} = \frac{\varepsilon_{lx}}{k_{zl}^{II}} \cdot \frac{k_{z(l+1)}^{II}}{\varepsilon_{(l+1)x}} \quad (32)$$

The second method for finding the values of the variables is to use recursive relations, which is more suitable for being used in

programming.

$$\frac{C_l}{A_l} = \frac{R^I_{l(l+1)} e^{-j2k^I_{z(l+1)}d_l} + \frac{C_{(l+1)}}{A_{(l+1)}}}{e^{-j2k^I_{z(l+1)}d_l} + R^I_{l(l+1)} \frac{C_{(l+1)}}{A_{(l+1)}}} \cdot e^{-j2k^I_{zl}d_l} \tag{33}$$

$$\frac{D_l}{B_l} = \frac{R^{II}_{l(l+1)} e^{-j2k^{II}_{z(l+1)}d_l} + \frac{D_{(l+1)}}{B_{(l+1)}}}{e^{-j2k^{II}_{z(l+1)}d_l} + R^{II}_{l(l+1)} \frac{D_{(l+1)}}{B_{(l+1)}}} \cdot e^{-j2k^{II}_{zl}d_l} \tag{34}$$

Therefore, we have:

$$R^I_{l(l+1)} = \frac{1 - p^I_{l(l+1)}}{1 + p^I_{l(l+1)}} \tag{35}$$

$$R^{II}_{l(l+1)} = \frac{1 - p^{II}_{l(l+1)}}{1 + p^{II}_{l(l+1)}} \tag{36}$$

In which the subscripts I and II are for TE and TM mode, respectively.

The initial value that is used in the above recursive relations is obtained from the boundary condition at the boundary between the regions N and $N + 1$, which we will examine in five different cases.

(I) If we have a dielectric or a metamaterial half-space in the last layer, the boundary condition at the last boundary will be applied to the above relations thus:

$$\frac{C_{(N+1)}}{A_{(N+1)}} = 0 \tag{37}$$

$$\frac{D_{(N+1)}}{B_{(N+1)}} = 0 \tag{38}$$

(II) If the last layer is a PEC, we should apply the condition in the following way:

$$\frac{C_N}{A_N} = -e^{-j2k^I_{zN}d_N} \tag{39}$$

$$\frac{D_N}{B_N} = e^{-j2k^{II}_{zN}d_N} \tag{40}$$

(III) And if it is a PMC, we apply the condition as below:

$$\frac{C_N}{A_N} = e^{-j2k^I_{zN}d_N} \tag{41}$$

$$\frac{D_N}{B_N} = -e^{-j2k^{II}_{zN}d_N} \tag{42}$$

(IV) In case the last layer is a PEMC, $\frac{C_N}{A_N}$ and $\frac{D_N}{B_N}$ will be related to each other through the relations below:

$$\frac{k_{zN}^I}{\mu_0\mu_{Nx}}\left(A_N e^{-jk_{zN}^I d_N} - C_N e^{+jk_{zN}^I d_N}\right)$$

$$+ M \cdot \left\{ \frac{k_{zN}^{II}}{\varepsilon_0\varepsilon_{Nx}}\left(B_N e^{-jk_{zN}^{II} d_N} - D_N e^{+jk_{zN}^{II} d_N}\right)\right\} = 0 \quad (43)$$

$$\left(B_N e^{-jk_{zN}^{II} d_N} + D_N e^{+jk_{zN}^{II} d_N}\right) + M \cdot \left(A_N e^{-jk_{zN}^I d_N} + C_N e^{+jk_{zN}^I d_N}\right) = 0 \quad (44)$$

$$\left(B_N e^{-jk_{zN}^{II} d_N} + D_N e^{+jk_{zN}^{II} d_N}\right) + M \left(A_N e^{-jk_{zN}^I d_N} + C_N e^{+jk_{zN}^I d_N}\right) = 0 \quad (45)$$

(V) And in case the last layer has the surface impedance η_s, the relations below hold true:

$$\frac{k_{zN}^{II}}{\varepsilon_0\varepsilon_{Nx}}\left(B_N e^{-jk_{zN}^{II} d_N} - D_N e^{+jk_{zN}^{II} d_N}\right)$$

$$= -\eta_s \cdot \left(B_N e^{-jk_{zN}^{II} d_N} + D_N e^{+jk_{zN}^{II} d_N}\right) \quad (46)$$

$$\left(A_N e^{-jk_{zN}^I d_N} + C_N e^{+jk_{zN}^I d_N}\right)$$

$$= \eta_s \cdot \frac{k_{zN}^I}{\mu_0\mu_{Nx}}\left(A_N e^{-jk_{zN}^I d_N} - C_N e^{+jk_{zN}^I d_N}\right) \quad (47)$$

Now, we have relations with which we can analyze multilayer structures which are composed of electric and magnetic anisotropic materials and the last layer of which is a PEC, PMC, PEMC, surface impedance, or a dielectric.

3. NUMERICAL EXAMPLES AND VALIDATION

To test the validity of the relations that were presented in the previous section, we solve a problem, as an example, by the program written on the basis of our recursive relations, and compare the results with those of the program using the state space method [18, 19]. These programs are written using Matlab software.

In this example, we present a multilayer structure with $N = 4$ layers and the left half-space is a free space.

The structure of this problem is in the form $n_0|n_H n_L n_H n_L|n_S$, in which $n_0 = 1$, $n_H = 2.32$, $n_L = 1.46$ and $n_S = 1.6$. The thicknesses of the layers are:

$$d_H = d_L = \lambda_0/2 \quad (48)$$

λ_0 indicates the wave length of the free space at the central frequency.

Respectively, the layers have the constitutive parameters below in a periodic form:

$$\overline{\overline{\varepsilon}}_H = \varepsilon_0 \begin{bmatrix} n_H^2 & 0 & 0 \\ 0 & n_H^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (49)$$

$$\overline{\overline{\mu}}_H = \mu_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (50)$$

$$\overline{\overline{\varepsilon}}_L = \varepsilon_0 \begin{bmatrix} n_L^2 & 0 & 0 \\ 0 & n_L^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (51)$$

$$\overline{\overline{\mu}}_L = \mu_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (52)$$

The central frequency is $f_0 = 12\,\text{GHz}$, and the angle of the incidence is $\theta = 45°$.

The results can be observed below. The answer arrived at through the state space method can be compared with the one arrived at through our recursive relations in Figure 2. Excellent agreement is
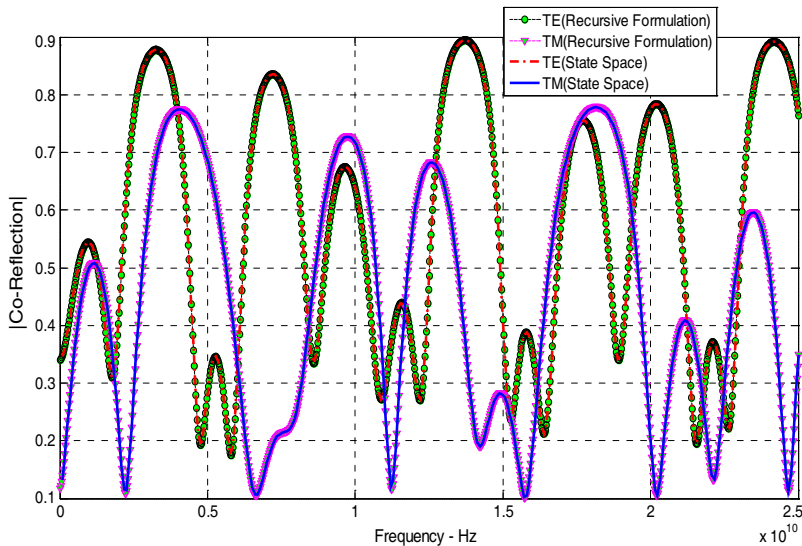
Figure 2. The results obtained from the state space relations and the derived relations.

observed from the curves in Figure 2 which confirms the validity of our relations.

4. ACHIEVING ZERO REFLECTION

In this section, as an application, we use the relations arrived at, for considering one of the important problems concerning radar cross section, i.e., zero reflection.

The above mentioned quality can be employed in different applications. As an example for the visible light spectrum, sturdy structures can be built that are transparent against visible light and can be used in buildings with transparent walls. And as an example for the electromagnetic spectrum, machines can be manufactured that are transparent against electromagnetic waves, such as military tools and vehicles made, instead of metal, of sturdy multilayer materials that are transparent against radar waves.

Now, using the relations arrived at, if we wish to design a single layer slab of thickness d which, by being placed in free space, will enable us to achieve zero reflection in both polarizations, we should assign the following to recursive relations (33) and (34):

$$l = 0, 1, \qquad \frac{C_2}{A_2} = \frac{D_2}{B_2} = 0, \qquad \frac{C_0}{A_0} = \frac{D_0}{B_0} = 0 \tag{53}$$

which will lead to relations below:

$$\frac{C_1}{A_1} = R_{12}^I e^{-j2k_{z1}^I d} \tag{54}$$

$$\frac{D_1}{B_1} = R_{12}^{II} e^{-j2k_{z1}^{II} d} \tag{55}$$

$$\frac{C_0}{A_0} = \frac{R_{01}^I + R_{12}^I e^{-j2k_{z1}^I d}}{1 + R_{01}^I R_{12}^I e^{-j2k_{z1}^I d}} = 0 \tag{56}$$

$$\frac{D_0}{B_0} = \frac{R_{01}^{II} + R_{12}^I e^{-j2k_{z1}^{II} d}}{1 + R_{01}^{II} R_{12}^I e^{-j2k_{z1}^{II} d}} = 0 \tag{57}$$

Therefore, we must have:

$$R_{01}^I + R_{12}^I e^{-j2k_{z1}^I d} = 0 \tag{58}$$

$$R_{01}^{II} + R_{12}^{II} e^{-j2k_{z1}^{II} d} = 0 \tag{59}$$

Or:

$$R_{01}^I = -R_{12}^I \cos\left(2k_{z1}^I d\right), \qquad 0 = -R_{12}^I \sin\left(2k_{z1}^I d\right) \tag{60}$$

$$R_{01}^{II} = -R_{12}^{II} \cos\left(2k_{z1}^{II} d\right), \qquad 0 = -R_{12}^{II} \sin\left(2k_{z1}^{II} d\right) \tag{61}$$

For the relations above to hold for all angles of incidence, we must have:

$$R_{01}^{I} = R_{12}^{I} = 0 \tag{62}$$

$$R_{01}^{II} = R_{12}^{II} = 0 \tag{63}$$

From relations (35), (36), (31), and (32), we have:

$$R_{12}^{I} = \frac{1 - p_{12}^{I}}{1 + p_{12}^{I}}, \qquad R_{12}^{II} = \frac{1 - p_{12}^{II}}{1 + p_{12}^{II}},$$

$$R_{01}^{I} = \frac{1 - p_{01}^{I}}{1 + p_{01}^{I}}, \qquad R_{01}^{II} = \frac{1 - p_{01}^{II}}{1 + p_{01}^{II}} \tag{64}$$

$$p_{12}^{I} = \frac{\mu_{1x}}{k_{z1}^{I}} \cdot \frac{k_{z2}}{\mu_2}, \qquad p_{12}^{II} = \frac{\varepsilon_{1x}}{k_{z1}^{II}} \cdot \frac{k_{z2}}{\varepsilon_2},$$

$$p_{01}^{I} = \frac{1}{k_{z0}} \cdot \frac{k_{z1}^{I}}{\mu_{1x}}, \qquad p_{01}^{II} = \frac{1}{k_{z0}} \cdot \frac{k_{z1}^{II}}{\varepsilon_{1x}} \tag{65}$$

And from relations (8) and (9), we know that:

$$k_{z2} = k_0 \left(\varepsilon_2 \mu_2 - \sin^2(\theta) \right)^{1/2} \tag{66}$$

$$k_{z0} = k_0 \cos(\theta) \tag{67}$$

$$k_{z1}^{I} = k_0 \left(\varepsilon_{1y}\mu_{1x} - \frac{\mu_{1x}}{\mu_{1y}} \sin^2(\theta) \right)^{1/2} \tag{68}$$

$$k_{z1}^{II} = k_0 \left(\varepsilon_{1x}\mu_{1y} - \frac{\varepsilon_{1x}}{\varepsilon_{1z}} \sin^2(\theta) \right)^{1/2} \tag{69}$$

After doing some calculations, we arrive at:

$$R_{12}^{I} = \frac{\mu_2 \left(\mu_{1z}\varepsilon_{1y} - \sin^2(\theta) \right)^{1/2} - (\mu_{1z}\mu_{1x})^{1/2} \left(\varepsilon_2\mu_2 - \sin^2(\theta) \right)^{1/2}}{\mu_2 \left(\mu_{1z}\varepsilon_{1y} - \sin^2(\theta) \right)^{1/2} + (\mu_{1z}\mu_{1x})^{1/2} \left(\varepsilon_2\mu_2 - \sin^2(\theta) \right)^{1/2}} \tag{70}$$

$$R_{12}^{II} = \frac{\varepsilon_2 \left(\varepsilon_{1z}\mu_{1y} - \sin^2(\theta) \right)^{1/2} - (\varepsilon_{1z}\varepsilon_{1x})^{1/2} \left(\mu_2\varepsilon_2 - \sin^2(\theta) \right)^{1/2}}{\varepsilon_2 \left(\varepsilon_{1z}\mu_{1y} - \sin^2(\theta) \right)^{1/2} + (\varepsilon_{1z}\varepsilon_{1x})^{1/2} \left(\mu_2\varepsilon_2 - \sin^2(\theta) \right)^{1/2}} \tag{71}$$

$$R_{01}^{I} = \frac{\cos(\theta)(\mu_{1z}\mu_{1x})^{1/2} - \left(\mu_{1z}\varepsilon_{1y} - \sin^2(\theta) \right)^{1/2}}{\cos(\theta)(\mu_{1z}\mu_{1x})^{1/2} + \left(\mu_{1z}\varepsilon_{1y} - \sin^2(\theta) \right)^{1/2}} \tag{72}$$

$$R_{01}^{II} = \frac{\cos(\theta)(\varepsilon_{1z}\varepsilon_{1x})^{1/2} - \left(\varepsilon_{1z}\mu_{1y} - \sin^2(\theta) \right)^{1/2}}{\cos(\theta)(\varepsilon_{1z}\varepsilon_{1x})^{1/2} + \left(\varepsilon_{1z}\mu_{1y} - \sin^2(\theta) \right)^{1/2}} \tag{73}$$

To meet the relations (62) and (63), we must have:

$$R_{12}^I = 0 \;\rightarrow\; (\mu_2)^2(\mu_{1z}\varepsilon_{1y}-\sin^2(\theta))=(\mu_{1z}\mu_{1x})(\varepsilon_2\mu_2-\sin^2(\theta)) \quad (74)$$

$$R_{12}^{II} = 0 \;\rightarrow\; (\varepsilon_2)^2\left(\varepsilon_{1z}\mu_{1y}-\sin^2(\theta)\right)=(\varepsilon_{1z}\varepsilon_{1x})\left(\mu_2\varepsilon_2-\sin^2(\theta)\right) \quad (75)$$

$$R_{01}^I = 0 \;\rightarrow\; \mu_{1x}\mu_{1z}\cos^2(\theta)=\mu_{1z}\varepsilon_{1y}-\sin^2(\theta) \quad\quad\quad\quad\quad (76)$$

$$R_{01}^{II} = 0 \;\rightarrow\; \varepsilon_{1x}\varepsilon_{1z}\cos^2(\theta)=\varepsilon_{1z}\mu_{1y}-\sin^2(\theta) \quad\quad\quad\quad\quad (77)$$

Or more concisely:

$$\left(\mu_2^2 - 1\right)\cos^2(\theta) = \varepsilon_2\mu_2 - 1 \quad\quad\quad (78)$$

$$\left(\varepsilon_2^2 - 1\right)\cos^2(\theta) = \mu_2\varepsilon_2 - 1 \quad\quad\quad (79)$$

$$\left(\mu_{1x}\mu_{1z} - 1\right)\cos^2(\theta) = \mu_{1z}\varepsilon_{1y} - 1 \quad\quad\quad (80)$$

$$\left(\varepsilon_{1x}\varepsilon_{1z} - 1\right)\cos^2(\theta) = \varepsilon_{1z}\mu_{1y} - 1 \quad\quad\quad (81)$$

For relations (78), (79), (80), and (81) to hold for all angles, we must have:

$$\mu_2^2 - 1 = \varepsilon_2^2 - 1 = \varepsilon_2\mu_2 - 1 = 0 \quad\quad\quad (82)$$

$$\mu_{1x}\mu_{1z} - 1 = \mu_{1z}\varepsilon_{1y} - 1 = \varepsilon_{1x}\varepsilon_{1z} - 1 = \varepsilon_{1z}\mu_{1y} - 1 = 0 \quad (83)$$

Or:

$$\mu_2 = \varepsilon_2 = \pm 1 \quad\quad\quad (84)$$

$$\varepsilon_{1x} = \mu_{1y} = \frac{1}{\varepsilon_{1z}} \quad\quad\quad (85)$$

$$\mu_{1x} = \varepsilon_{1y} = \frac{1}{\mu_{1z}} \quad\quad\quad (86)$$

This means that the matrix of the constitutive parameters of the slab must be thus:

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{bmatrix} \frac{1}{\varepsilon_{1z}} & 0 & 0 \\ 0 & \frac{1}{\mu_{1z}} & 0 \\ 0 & 0 & \varepsilon_{1z} \end{bmatrix} \quad\quad\quad (87)$$

$$\overline{\overline{\mu}} = \mu_0 \begin{bmatrix} \frac{1}{\mu_{1z}} & 0 & 0 \\ 0 & \frac{1}{\varepsilon_{1z}} & 0 \\ 0 & 0 & \mu_{1z} \end{bmatrix} \quad\quad\quad (88)$$

In order to have no limitations for the incident waves in the plane ZX, the tensors must stay the same after the rotation of the axes around the z axis; therefore, the values of x and y must be equal in

the tensors. So the single layer structure which has no reflection in any incidence has the parameters below:

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & 1/p \end{bmatrix} \tag{89}$$

$$\bar{\bar{\mu}} = \mu_0 \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & 1/p \end{bmatrix} \tag{90}$$

In which P can have any value and in principle there is nothing to limit it. It is, however, possible to find practical limitations for it, which should be considered in examining the realization of anisotropic media.

Assuming $p = 2$, the following demonstrates the result obtained from the recursive relations which show zero reflection.
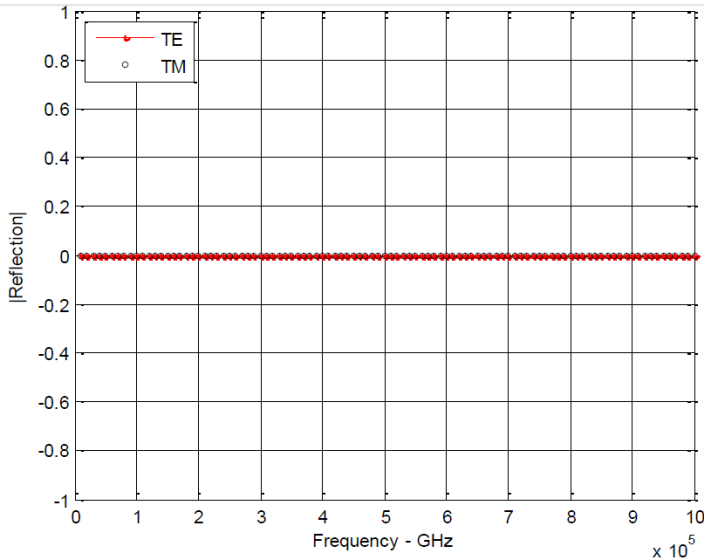
Figure 3. Reaching zero reflection in all frequencies.

As it can be observed in Figure 3 we have reached zero reflection in all frequencies for both polarizations.

5. CONCLUSIONS

One of the issues that arise in dealing with anisotropic media is the complex relations resulting from the tensor notation of parameters; in

this paper an exact recursive formulation has been represented for the computation of fields in media with electric and magnetic anisotropy backed by PEC, PMC, PEMC, surface impedance, dielectric, or metamaterial, which can make the computations related to these media much easier.

At last, validity of the relations has been verified and an interesting application — the zero reflection condition — has been provided.

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