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ANALYZING TWO SLOTS TERMINATED WITH MICROWAVE NETWORK ON THE GROUND USING MULTI-MODE EXPANSION

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Abstract—In this paper, two slots which connect with waveguides and a microwave network are studied by using multi-modes technique. A TE incident plane wave is assumed. The moment method is employed to solve the problem. And the mode and triangular functions are used as basic and test functions. A different HM-pattern is obtained. It is found that the microwave network greatly affects the HM-pattern.

1. INTRODUCTION

In paper [1], two slots which are terminated by microwave network are studied by Lu and Harrington. The electromagnetic energy coupling between two regions is frequently encountered in real environment. A general case is the loaded apertures [1] which means the apertures are connected by waveguides and microwave networks. The single mode is considered by the authors in [1]. And the expansion functions of the mode function are plus functions. In fact, the single mode condition is much insufficient. There are lots of the configurations which can be viewed as terminated slots. And they are extensively studied [2–4]. Since it is very important in practice, a deeper study is needed. In this paper, we consider the multi-modes in the waveguide and use the triangle basis function as the expansion functions. The moment procedure is deduced carefully. And a computer program is developed to calculate the equivalence magnetic currents. The magnetic field pattern is used to compare with the solution which is obtained by using single mode technique. The results show that the characteristics are different in the multi-modes situation.

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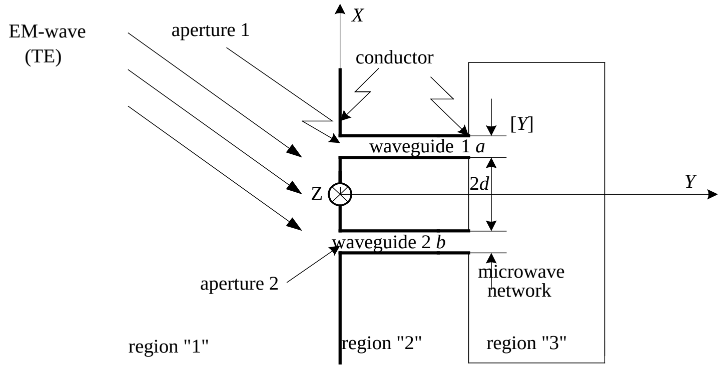

Figure 1. Two slots with microwave network.

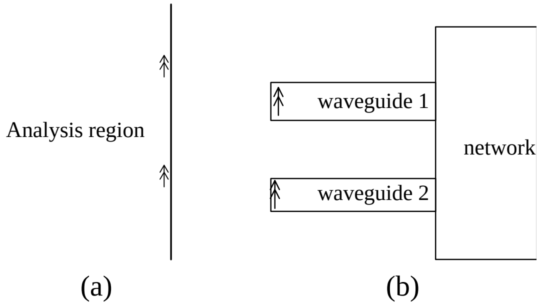

Figure 2. The equivalence problems. (a) Outer problem. (b) Inner problem.

2. THEORETICAL ANALYSIS

The configuration of the problem is shown in Fig. 1, the same in [1]. A TE plane wave illuminates from the negative y direction. Without loss of generality, the two slots are placed equally around the original point in the x direction. The analysis region is divided into three sub-regions. Region 1 is the opening area, by opening we mean the region is extended to infinity, and the region 2 is the waveguides, the region 3 the microwave network, as shown in the figure. The width of the apertures are a and b, and the distant between the inner edge of the apertures is $2d$. The microwave network is described by impedance matrix $[Y]$.

Equivalence principle [5] is used, and then an equivalence problem is obtained. As shown in the Fig. 2, an inner problem and an outer problem are constructed. The outer problem is half plane problem. It has two sources. One is the illuminating plane wave, and the other is the equivalence magnetic currents. Because the apertures are sealed with conductor, this problem can be further simplified to a free space problem using the image theory. So that the magnetic field in region 1 is

$$\mathbf{H}^1 = \mathbf{H}^{sc} + \sum_i^2 \mathbf{H}^1(\mathbf{M}_i) \tag{1}$$

where the superscript indicates the region index, the "*sc*" means that the illuminating plane wave is short-circuited by the conduct plane, and the subscript means the aperture index. The $\mathbf{H}(\mathbf{M})$ is the magnetic field radiated by $\mathbf{M}$ in the free space [6], here is:

$$\mathbf{H}^1(\mathbf{M}) = -\frac{k}{4\eta} \int_C 2\mathbf{M}(x') H_1^2(k|\boldsymbol{\rho} - \boldsymbol{\rho}'|) dx' \tag{2}$$

where the Green function is the second kind of first order Hankel function. The inner problem includes region 2 and region 3. In the region 2, the waveguides are shorted. And the equivalence magnetic currents are the same as that in region 1 except the minus sign. As discussed in [7], the equivalence currents are placed a distance away from the waveguide's beginning. The magnetic field in region 2 is:

$$\mathbf{H}^2 = \mathbf{H}^2(-\mathbf{M}_1) + \mathbf{H}^2(-\mathbf{M}_2) \tag{3}$$

In this situation, image theory is no more a simplified technique. Here, we use the mode expansion technique. The $\mathbf{H}(\mathbf{M})$ in (2) is no longer the simplified formula in (1). The field in the aperture 1 is affected by the source $\mathbf{M}_2$ through the waveguides and the microwave network, vice versa. And the minus sign in (3) indicate the continuous condition of the electric field. So, by using continuous condition of tangential component of the magnetic field, we have

$$\mathbf{H}_j^1 + \mathbf{H}^{sc} = \mathbf{H}_j^2 \tag{4}$$

$$\left\{\mathbf{H}_j^1(\mathbf{M}_1) + \mathbf{H}_j^1(\mathbf{M}_2)\right\} + \left\{\mathbf{H}_j^2(\mathbf{M}_1) + \mathbf{H}_j^2(\mathbf{M}_2)\right\} = -\mathbf{H}^{sc}, \quad j = 1, 2 \tag{5}$$

where j is the aperture index, in the above equations all vectors are tangential components to the aperture surfaces.

3. THE MOMENT METHOD SOLUTION

The moment procedure is described in detail in [1]. Here, we use the triangle basis function in stand of the plus basis.

3.1. Mode Function Expand by Triangle Basis Functions

The equivalence magnetic currents are expended as:

$$M_i = \sum_p V_{ip} M_{ip}, \quad i = 1, 2 \tag{6}$$

where the currents are in z direction, and M_{ip} is the pth mode of the ith waveguide, the same as the coefficients of the mode functions. While doing the calculations in the region 1, we further expand the mode functions into triangle basis functions:

$$M_{ip} = \sum_q A_{ipq} T_{iq} \tag{7}$$

where the T_{iq} is the triangle basis function,

$$T_{iq} = \begin{cases} \dfrac{x - x_q}{x_{q+1} - x_q}, & x_q < x < x_{q+1} \\ \dfrac{x - x_{q+2}}{x_{q+1} - x_{q+2}}, & x_{q+1} < x < x_{q+2} \end{cases} \tag{8}$$

From (7), the moment method [4] is applied,

$$\langle T_l, M_{ip} \rangle = \sum_q A_{ipq} \langle T_l, T_{iq} \rangle \tag{9}$$

The expansion coefficients are different for every mode in each aperture. And the simplified matrix equation for one mode at specific aperture is

$$[Z_{lq}][A_q] = [I_l] \tag{10}$$

where the element of the $[Z]$ is

$$Z_{lq} = \int_{C_i} T_l T_q dx = \begin{cases} \dfrac{x_{q+1} - x_q}{6}, & l = q - 1 \\ \dfrac{x_{q+2} - x_q}{3}, & l = q \\ \dfrac{x_{q+2} - x_{q+1}}{6}, & l = q + 1 \end{cases} \tag{11}$$

where C_i means the specify aperture. And the element of the excitation vector is

$$\begin{aligned} I_l = \langle T_l, \sin(ux) \rangle &= \int_{C_i} T_l \sin(ux) dx \\ &= \frac{1}{u^2} \left\{ \frac{\sin[u(x_{l+1} - x_0)] - \sin[u(x_l - x_0)]}{x_{l+1} - x_l} \right. \\ &\quad \left. - \frac{\sin[u(x_{l+2} - x_0)] - \sin[u(x_{l+1} - x_0)]}{x_{l+2} - x_{l+1}} \right\} \end{aligned} \tag{12}$$

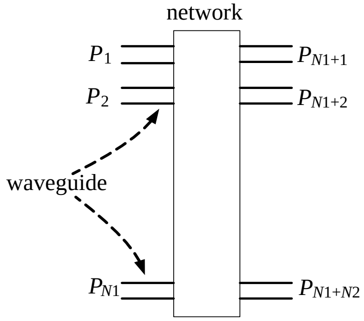

Figure 3. The multi-mode network.

where $u = \frac{n\pi}{w_i}$, n is the integer, and w_i is the width of the aperture, e.g., a or b, x_0 is the location of the aperture's edge. The Z_{lq} is the same for every mode, but the excitation vector $[I]$ is different. So it only needs to evaluate the I_l for each mode expansion.

3.2. Multi-mode Network

We consider the wave guide and microwave network bellow. The waveguides and the microwave network together can be viewed as a new network. The waveguide is viewed as a cascade network. Then, we have

$$[S] = [S]_{wg1} \oplus [S]_{network} \oplus [S]_{wg2} \tag{13}$$

where the scattering matrix of the multi-mode waveguide is

$$[S]_{wg} = \begin{bmatrix} e^{-\gamma_1 l} & & & \\ & e^{-\gamma_2 l} & & \\ & & . & \\ & & & e^{-\gamma_n l} \end{bmatrix} \tag{14}$$

And the $\oplus$ means cascade. The scattering matrix is a multi-port network as shown in Fig. 3. The modes in the same apertures do not affect each other directly. For every mode in aperture 1, when there is energy through the microwave network, it excites all modes in aperture 2, vice versa. There are also modes that cannot reach the microwave network, which do not affect the fields in the aperture. Then, by using the expression in [1] and the overall scattering matrix, the final matrix elements are evaluated.

4. EXAMPLES AND CONCLUSIONS

We consider a cavity as microwave network. The scattering matrix of the microwave network can be obtained by using mode matching method. The number of the waveguides and that of the cavity is n_1 and n_2. Then the connection between wave guide and the cavity is a discontinuous step with different mode's number. The connection formulas are

$$S_{11}^{all} = S_{11}^{1} + S_{12}^{1}(I - S_{11}^{2}S_{22}^{1})^{-1}S_{11}^{2}S_{21}^{1} \tag{15}$$

$$S_{12}^{all} = S_{12}^{1}(I - S_{11}^{2}S_{22}^{1})^{-1}S_{12}^{2} \tag{16}$$

$$S_{12}^{all} = S_{21}^{2}(I - S_{22}^{1}S_{11}^{2})^{-1}S_{21}^{1} \tag{17}$$

$$S_{22}^{all} = S_{22}^{2} + S_{21}^{2}(I - S_{22}^{1}S_{11}^{2})^{-1}S_{22}^{1}S_{12}^{2} \tag{18}$$

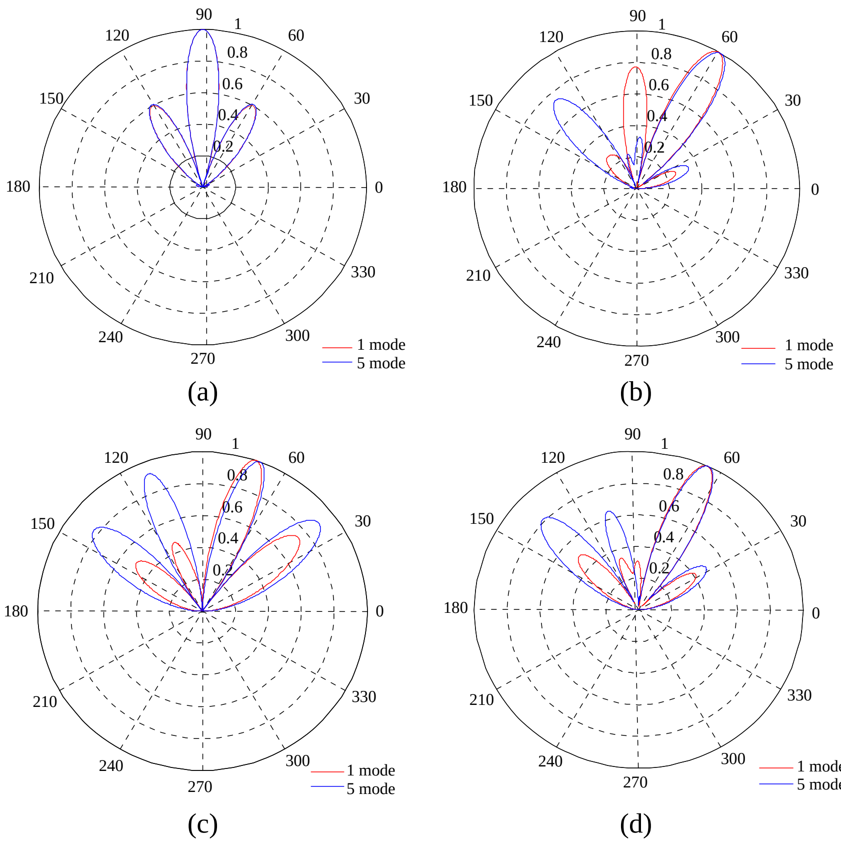

Figure 4. Different incident angle. (a) The incident angle is $\frac{\pi}{2}$. (b) The incident angle is $\frac{\pi}{3}$. (c) The incident angle is $\frac{\pi}{6}$. (d) The incident angle is $\frac{\pi}{12}$.

where S^1 is a $(2 * n_1) \times (2 * n_1)$ matrix, and the S^1_{ij} is $n_1 \times n_1$, S^2_{11} is $n_1 \times n_1$, S^2_{12} is $n_1 \times n_2$, S^2_{21} is $n_2 \times n_1$, and S^2_{22} is $n_2 \times n_2$. S^{all} is a $(n_1 + n_2) \times (n_1 + n_2)$ matrix. The scattering matrix of the microwave network contains three sub-networks, one is the waveguide cavity, and the other two are discontinuous steps. So, there are five sub-networks in the total network, which including the waveguides. The 'HM-pattern' is used to display the characteristics of the far field. And the comparisons are made between one and multi mode technique.

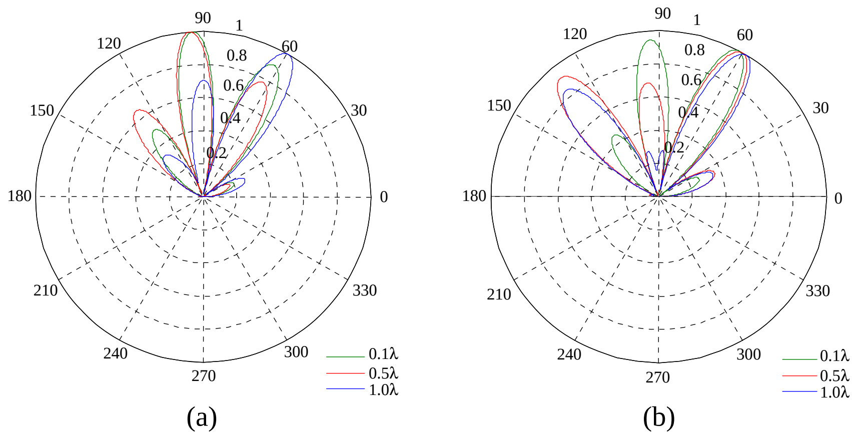

Figure 5. Deferent length of the waveguides. (a) One mode. (b) Five modes.

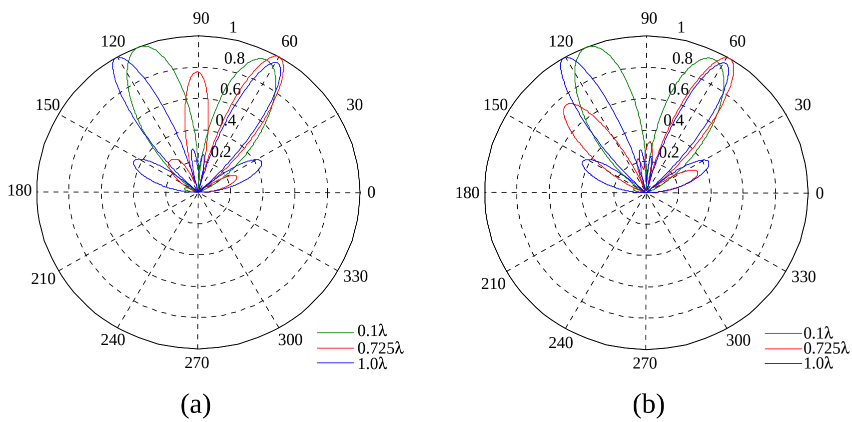

Figure 6. Deferent width of the slots. (a) One mode. (b) Five modes.

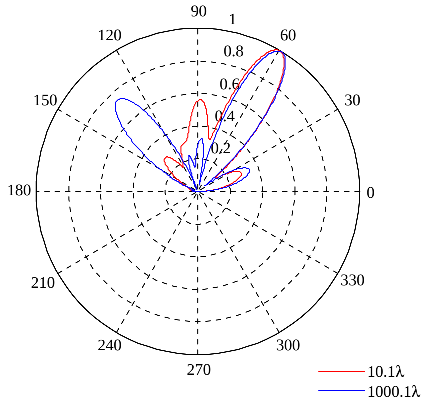

Figure 7. The cavity with different widths.

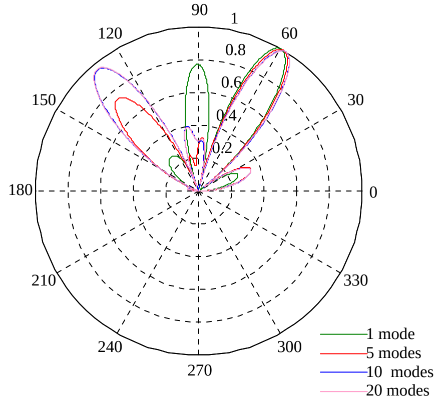

Figure 8. Using different number of modes.

4.1. Examples

Case 1: The incident wave is coming from different angle. The results are shown in Fig. 4. The widths of the two slots are 0.725λ. The distance between the slots is 1.0λ. The length of the waveguides is 0.3λ.

Case 2: The configuration is the same as case 1, except the length of the waveguides. And the incident angle of $\frac{\pi}{3}$ is selected. It is shown in Fig. 5.

Case 3: The widths of the slots are changed in this case. The length of the waveguides is 0.3λ, and the incident angle is the same as case 2. The results are shown in Fig. 6.

Case 4: In this case, we alter the microwave network by changing the width of the cavity, and let the other parameter unchanged. Fig. 7 shows the differences.

Case 5: Different number of the modes is used in the simulation, and the results are shown in Fig. 8.

4.2. Conclusions

In this paper, we consider multi-modes arriving at the microwave network rather than one-mode. It is found that big difference in the HM-pattern. When the incident wave comes from vertical angle, the patterns are all most the same. But with an oblique incident wave, the pattern changes a lot when using different number of modes, and it converges when the number is getting bigger. The effect of the microwave network can't be ignored again. It is not the same conclusion in [1].

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