

RCS CHARACTERIZATION OF STEALTH TARGET USING χ^2 DISTRIBUTION AND LOGNORMAL DISTRIBUTION

Weiqliang Shi*, Xiao-Wei Shi, and Le Xu

National Laboratory of Science and Technology on Antennas and Microwaves, Xidian University, Xi'an 710071, China

Abstract—The radar backscatter from complex targets, such as aircrafts and ships, tends to vary rapidly with aspect or time. To describe the radar cross section distribution characteristics of such targets, statistic terms are often used. In this paper, we first give a brief introduction of χ^2 distribution and lognormal distribution model. And complete form of lognormal distribution is proposed which can be used when ratio of mean to median is less than 1 as stealth targets. The significance of the parameters is discussed in detail aiming to find a characterization standard. As an example, the statistic characteristics of the radar cross section data of a stealth aircraft are analyzed with Swerling 1, 3 distribution, χ^2 distribution and lognormal distribution. The applicability of the distributions is shown with error-of-fit and test of goodness of fit.

1. INTRODUCTION

RCS (Radar Cross Section) is an important evaluation criterion of aircraft's stealth performance and radar detection technique. Radar target usually consists of multiple scatterers. The relative positions of the scatterers change rapidly. And the target body vibrates randomly in its motion. The envelope of the backscatter from complex target varies rapidly with aspect angle [1]. Fluctuation of RCS is random without regular pattern. Because it is difficult to predict the exact nature of RCS, statistical models [2] are used for RCS characterization.

There are several RCS distribution models existing, including χ^2 distribution, Rice distribution and lognormal distribution model. The conventional lognormal distribution can be applied only when mean to

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* Corresponding author: Wei-Qiang Shi (weiqiangshi_xd@sina.com).

median ratio of RCS data is greater than 1. We propose its complete form which can be used when ratio of mean to median is less than 1. Then the significance of distribution parameters is studied to obtain a standard of target characterization. After that an example of a stealth aircraft is given using Swerling 1, 3 distribution, χ^2 distribution and lognormal distribution. And with the error-of-fit and fitting goodness test, the applicability of the distributions is discussed.

2. χ^2 AND LOGNORMAL DISTRIBUTION MODEL

Among the existing RCS distribution models, χ^2 distribution and lognormal distribution are worthwhile to promote. The complete form of lognormal distribution is proposed after a brief introduction of χ^2 distribution and lognormal distribution.

2.1. χ^2 Distribution Model

The PDF (Probability Distribution Function) of χ^2 distribution [3, 4] is as Eq. (1).

$$p(\sigma) = \frac{k}{(k-1)!\bar{\sigma}} \left(\frac{k\sigma}{\bar{\sigma}}\right)^{k-1} e^{-\frac{k\sigma}{\bar{\sigma}}}, \quad \sigma > 0 \quad (1)$$

In Eq. (1), σ is RCS variable and $\bar{\sigma}$ is mean value of RCS data. The advantage of χ^2 distribution is that its variable parameter includes only k named double number of degrees of freedom. Parameter k can be positive integer or non-integer. So its fitting to different distributions is very good. When k equals 1, 2, N , $2N$ and ∞ , χ^2 distribution is simplified as the original five RCS distributions (i.e., Swerling 1–4 and Marcum distributions [5]).

2.2. Lognormal Distribution Model

The PDF of conventional lognormal distribution [6] is as Eq. (2).

$$p(\sigma) = \frac{1}{\sigma\sqrt{4\pi\ln\rho}} e^{-\frac{(\ln\frac{\sigma}{\sigma_0})^2}{4\ln\rho}}, \quad \sigma > 0 \quad (2)$$

$$\rho = \bar{\sigma}/\sigma_0 \quad (3)$$

In Eq. (2), σ_0 is the median value of RCS data, and ρ , the ratio of mean to median shown in Eq. (3), is the ratio of the mean value of the RCS data to the median value. The advantage of lognormal distribution is that its distribution curve has a good form which can be applied to different complex electrically large targets.

2.3. Complete Form of Lognormal Distribution

PDF of lognormal distribution shown in Eq. (2) is limited that ratio of mean to median must be greater than 1. For stealth targets, mean value of its RCS data decreases with median almost unchanged. So the ratio of mean to median tends to be smaller than conventional targets and could be less than 1. In this case lognormal distribution is no longer applicable, such as the RCS data in window 3 shown in Section 4.3 of which ratio of mean to median is 0.8055. The complete form of lognormal distribution is provided in Eq. (4).

$$p(\sigma) = \begin{cases} \frac{1}{\sigma\sqrt{4\pi \ln \rho}} e^{-\frac{(\ln \frac{\sigma}{\sigma_0})^2}{4 \ln \rho}}, & \sigma > 0, \quad \rho > 1 \\ \frac{1}{\sigma\sqrt{-4\pi \ln \rho}} e^{\frac{(\ln \frac{\sigma}{\sigma_0})^2}{4 \ln \rho}}, & \sigma > 0, \quad \rho < 1 \end{cases} \quad (4)$$

3. PARAMETER ANALYSIS

In this section, the significance of the distribution parameters is studied. How the distribution changes with its parameters is shown with its probability curve plotted. Based on the analysis, we try to find a characterization standard for different targets.

3.1. Double Number of Degrees of Freedom k

Figure 1 shows χ^2 distribution curves with $\bar{\sigma} = 0.1 \text{ m}^2$ and k ranges from 0.8 to 20.0. As k increases, peaks of the PDF curves appear

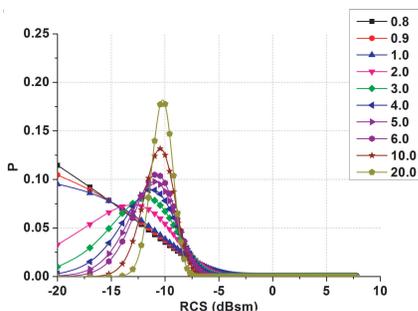


Figure 1. χ^2 distribution curves when $\bar{\sigma} = 0.1 \text{ m}^2$ (-10 dBsm) and $k = 0.8, 0.9, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 10.0$ and 20.0 .

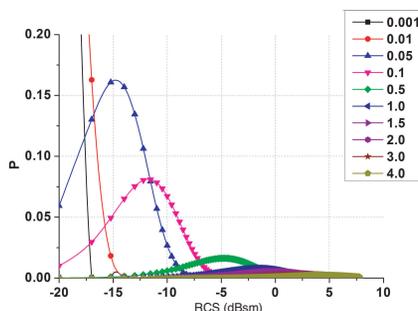


Figure 2. χ^2 distribution curves when $k = 3$ and $\bar{\sigma} = 0.001, 0.01, 0.05, 0.1, 0.5, 1.0, 1.5, 2.0, 3.0$ and 4.0 m^2 .

around the mean value and become higher. Parameter k describes the contribution of the main scatterer.

3.2. Mean $\bar{\sigma}$

Figure 2 shows χ^2 distribution curves when $k = 3$ and $\bar{\sigma} = 0.001, 0.01, 0.05, 0.1, 0.5, 1.0, 1.5, 2.0, 3.0$ and 4.0 . The peaks of the PDF curves just appear on the mean value. Therefore, mean value of RCS data determines the position of the distribution peak.

3.3. Ratio of Mean to Median ρ

Figure 3 shows lognormal distribution curves when $\sigma_0 = 0.1 \text{ m}^2$ (-10 dBsm) and ρ ranges from 0.90 to 1.35 . RCS value tends to be in the region around the median and the peak value would become larger if mean to median ratio shown in Eq. (3) approached 1. We find that the ratio of mean to median of RCS data of stealth targets is smaller than conventional targets. The typical value of parameter ρ ranges from 1.4 to 4.0 for conventional targets. And for stealth targets the value of ρ is always smaller than 1.1 . Examples will be shown later. Therefore, this can be a characterization standard for different targets.

3.4. Median σ_0

Figure 4 shows lognormal distribution curves when $\rho = 1.1$ and σ_0 ranges from 0.001 to 4.0 m^2 . Median mainly determines the position of distribution curve peak, which is like the mean value in χ^2 distribution.

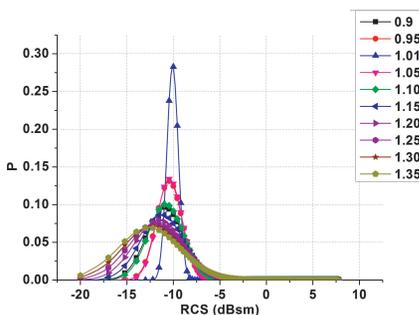


Figure 3. Lognormal distribution curves when $\sigma_0 = 0.1 \text{ m}^2$ (-10 dBsm) and $\rho = 0.90, 0.95, 1.01, 1.05, 1.10, 1.15, 1.20, 1.25, 1.30$ and 1.35 .

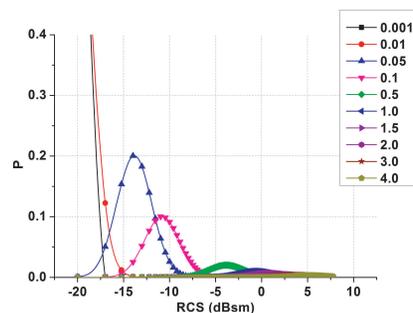


Figure 4. Lognormal distribution curves when $\rho = 1.1$ and $\sigma_0 = 0.001, 0.01, 0.05, 0.1, 0.5, 1.0, 1.5, 2.0, 3.0$ and 4.0 m^2 .

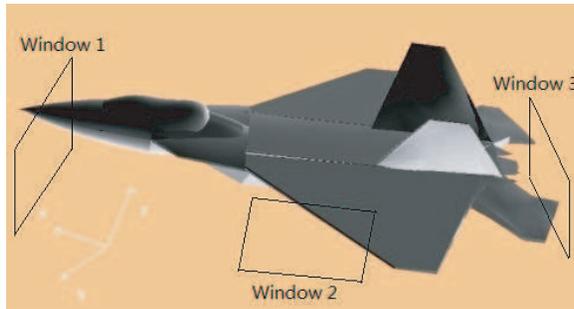


Figure 5. Target model.

4. EXAMPLE

4.1. Study Object

As an example, a 1:10 scaled NURBS model [7, 8] of a stealth aircraft is shown in Fig. 5. Length of the model in XYZ axis is $1.6656 \times 1.1300 \times 0.4033$ m. The wing area is 0.78 m^2 , and height of the empennage is 0.27 m. According to the electromagnetic scale relation of perfectly conducting full-size target and its model [6], RCS of real target can be obtained from its model's RCS using Eq. (5).

$$\sigma = \sigma' + 20 \lg s \quad (\text{dBsm}) \quad (5)$$

In Eq. (5), s is the scale of real target to its model. As $s = 10$, the RCS data of real target is 20 dBsm greater than its model in the same aspect angle. Their statistic characteristics are identical. The configuration of the target is shown in Fig. 5.

4.2. Study Method

PO (Physical Optics) method [9–11] with PTD (Physical Theory of Diffraction) revision to edge diffraction is used to evaluate the monostatic RCS of the target for HH polarization. PO method is an efficient high frequency method to calculate the scattering of the electrically large target surfaces. And PTD is adopted to calculate the diffraction of the boundaries [12, 13]. Backscattering RCS values are calculated over three windows shown in Fig. 5 with the coordinate original at the geometrical center of the target, (1). $75^\circ < \theta < 105^\circ$, $-15^\circ < \phi < 15^\circ$; (2). $75^\circ < \theta < 105^\circ$, $75^\circ < \phi < 105^\circ$; (3). $75^\circ < \theta < 105^\circ$, $165^\circ < \phi < 195^\circ$ at steps of 0.375° under frequency 4.5 GHz. Plots of backscattering RCS for the target in three windows at HH polarization are provided in Figs. 6, 7 and 8.

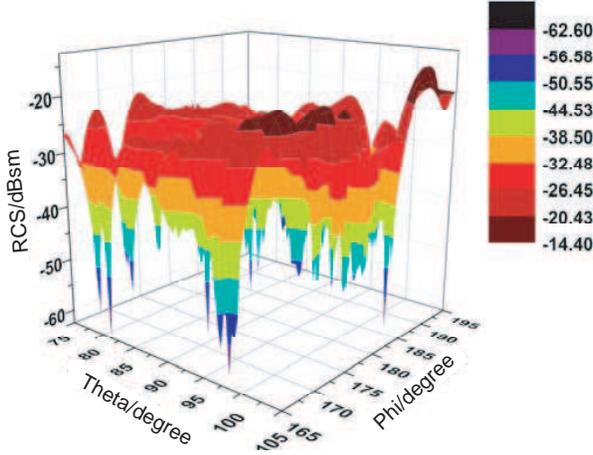


Figure 6. RCS data in window $75^\circ < \theta < 105^\circ$, $-15^\circ < \phi < 15^\circ$.

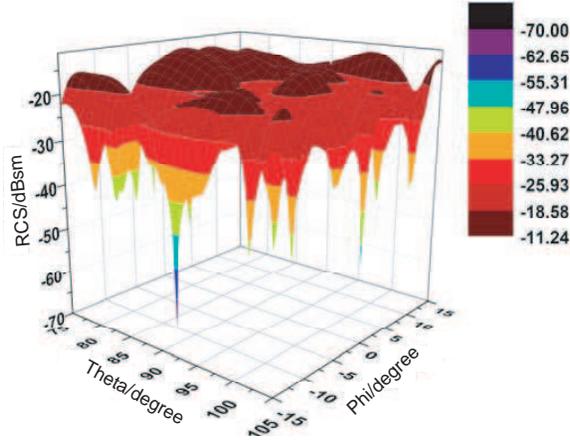


Figure 7. RCS data in window $75^\circ < \theta < 105^\circ$, $75^\circ < \phi < 105^\circ$.

After statistical analysis is conducted, the probability distribution of the RCS data is fitted using χ^2 distribution and lognormal distribution. Error-of-fit is defined as Eq. (6).

$$E = \sum_i [(P_e(i) - P_m(i))]^2 \quad (6)$$

P_e is the RCS statistical distribution and P_m is the fitting result of these distributions.

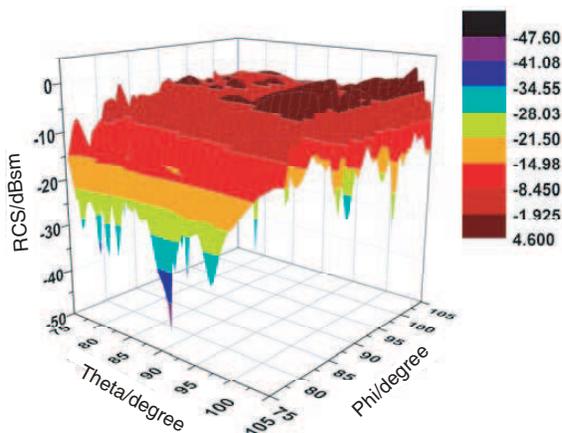


Figure 8. RCS data in window $75^\circ < \theta < 105^\circ$, $165^\circ < \phi < 195^\circ$.

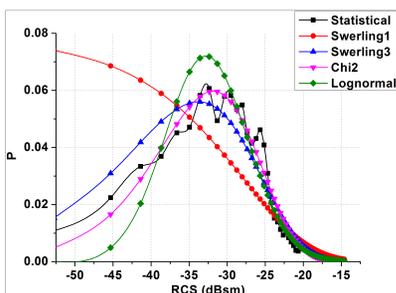


Figure 9. Distribution of window $75^\circ < \theta < 105^\circ$, $-15 < \phi < 15^\circ$.

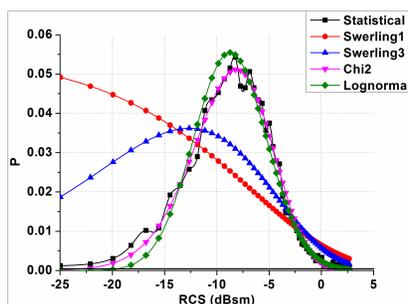


Figure 10. Distribution of window $75^\circ < \theta < 105^\circ$, $75^\circ < \phi < 105^\circ$.

4.3. Results

The fitting results are shown in Figs. 9, 10 and 11. And the parameters are provided in Table 1. The errors-of-fit are in Table 2.

4.4. Result Analysis

Kolmogorov-Smirnov test [14,15] is adopted as a method of fitting goodness test to determine which distribution is better to describe this aircraft's RCS distribution. The test equation is shown in Eq. (7). The test measures the absolute error between the CDF (Cumulative Distribution Function) $F(x)$ of sample data and the CDF $F'(x)$ of a

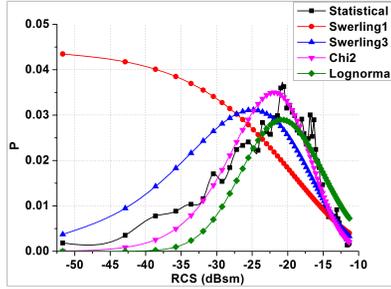


Figure 11. Distribution of window $75^\circ < \theta < 105^\circ$, $165^\circ < \phi < 195^\circ$.

Table 1. Distribution parameters in window 1, 2 and 3.

	Window 1	Window 2	Window 3
Swerling 1	$\bar{\sigma} = 0.0409$	$\bar{\sigma} = 0.4656$	$\bar{\sigma} = 0.1129$
Swerling 3	$\bar{\sigma} = 0.0409$	$\bar{\sigma} = 0.4656$	$\bar{\sigma} = 0.1129$
Chi-2	$\bar{\sigma} = 0.0409$ $k = 2.6765$	$\bar{\sigma} = 0.4656$ $k = 6.3515$	$\bar{\sigma} = 0.1129$ $k = 3.4535$
Lognormal	$\sigma_0 = 0.0367$ $\rho = 1.1132$	$\sigma_0 = 0.3325$ $\rho = 1.0839$	$\sigma_0 = 0.0806$ $\rho = 0.8055$

Table 2. Errors-of-fit in window 1, 2 and 3.

	Window 1	Window 2	Window 3
Swerling 1	0.0125	0.0180	0.0107
Swerling 3	0.0017	0.0083	0.0024
Lognormal	0.0026	0.0006	0.0015
Chi-2	0.0008	0.0002	0.0009

specified distribution to be tested. If the test result is too large, it can be determined that the distribution used is not applicable for the data. The results of Kolmogorov-Smirnov test are shown in Table 3.

$$D = \max |F(x) - F'(x)| \quad (7)$$

From the fitting curves, errors-of-fit and Kolmogorov-Smirnov test results, we can see that χ^2 distribution and lognormal distribution are better than Swerling 1 and Swerling 3 distribution for the RCS data of this target. And the fitting result of χ^2 distribution is a little better than lognormal distribution from the errors-of-fit and the Kolmogorov-Smirnov test results. Ratio of mean to median of the RCS

Table 3. Kolmogorov-Smirnov test result.

	Window 1	Window 2	Window 3
Swerling 1	0.1746	0.2721	0.2650
Swerling 3	0.0505	0.1826	0.1307
Lognormal	0.0461	0.0252	0.1232
Chi-2	0.0295	0.0123	0.0498

data in window 3 is 0.8055, which is less than 1. This case verifies the complete form of lognormal distribution model proposed in Section 2.3. Swerling 1 and 3 distribution models are no longer applicable due to their large errors and bad fitting results.

From the parameters provided in Table 1, the ratio of mean to median of three windows is 1.1132, 1.0839 and 0.8055, which are quite smaller than the typical value of conventional targets, 1.4–4.0. This indicates that the stealth performance of this aircraft is good.

5. CONCLUSION

The fluctuating characteristic of RCS data is complex. The complete form of lognormal distribution is proposed as the ratio of mean to median of RCS data in window 3 is less than 1 following a brief introduction of χ^2 distribution and lognormal distribution. After that, significance of the distribution parameters is analyzed. The fitting results of the RCS data in three aspect windows of a stealth aircraft are provided as examples. χ^2 distribution and lognormal distribution are more applicable to describe the RCS distribution of this target. And χ^2 distribution is a little better than lognormal distribution for the example RCS data. Analysis of parameters shows that the stealth performance of this aircraft is good.

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