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ANALYTICAL MODEL FOR ELECTROMAGNETIC RADIATION BY BARE-WIRE STRUCTURES

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Abstract—This paper presents a simple analytical model to estimate the radiated field for broadband Power line communication (PLC) or metallic wire structures. In our approach, we avoid to discretize the line and compute the current for each segment (dipole). We consider only near and far end currents and their derivatives (voltages) to express analytically the radiated electromagnetic field. The case of multiple conductor power line is considered with simplified hypothesis: cables are not insulated and the surrounding media is homogenous. The basic electromagnetic equations are formulated and applied to the line to provide analytical expressions able to compute fields in near and far zones which is not usually treated. The main purpose of this paper is to provide an analytical model applied to bare wires corresponding to classical outdoor transmission lines. The advantage of this method is that we do not need to know the current along the line to calculate the radiated fields; therefore, in our study we use only the currents and voltages at the terminations. The calculation time is strongly reduced compared to dipoles conventional method. Results obtained from the proposed closed-form formulation agree with Feko simulation. For indoor configurations, cables are usually insulated and the surrounding media is no more homogeneous; this case is treated with a generalized approach and will be proposed in future paper.

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1. INTRODUCTION

Recently, power lines were used to transfer data in wide frequency range. Thus, power Line Communications (PLC) system aims to provide users with communications means using the already existed and widely distributed power line network [1, 2]. However, one of the main disadvantages of PLC technology is related to electromagnetic interference (EMI) problems, as overhead power distribution lines at the frequency range from 1 to 30 MHz behave as transmitting antennas [3, 4], thus representing an EMI source. One limit of this technology is that conductors are not shielded as they are designed and optimized for the transmission of electric power frequency 50/60 Hz, i.e., they are inadequate for the transmission of information at higher frequencies in the 1–30 MHz band. Transmission in this frequency generates the radiation leading to the disruption of sensitive equipment to these radio waves [4]. Therefore an accurate assessment of radiation from wire structures is of interest.

The radiation assessment is one of the most important issues which strongly require an accurate modeling. The objective is to quantify the radiation generated by this transmission, using an advanced mathematical method to compute the electromagnetic fields. Conventional numerical methods such Finite Difference Method (FDM) or the Method of Moments (MOM) require a high mesh to determine the current and the radiated wire structure field [5–7]. In the case of differential methods, in addition to the necessity to mesh the wire structure, it is also necessary to mesh the surrounding space in order to calculate the electromagnetic field at the observation point. In the case of integral methods, only the source region has to be discretized. These methods are implemented in commercial codes such as Feko, Nec, or CST [5, 6]. These specific codes are not suitable for the treatment of geometric configurations involving wires of arbitrary shape. An extensive network as PLC (Power line communication) whose dimensions exceed several tens of wavelengths will be tedious to deal with existing codes due to the large volume of data and excessive computing time. Moreover, it is hard to install sensors to measure the radiated field and to predict the variation of the field in time and space. In this work, an analytical approach to evaluate electromagnetic field radiated from thin-wire structures is proposed. Simulation results based on antenna theory [9] are compared to our close-form approximation and shows quite good agreement.

2. PROPOSED APPROACH

The interest of our approach is to have analytical expression to easily interpret the results physically and aims to provide the analytical solution to obtain a rapid estimation of the phenomena. An important practical benefit of such an approach is that the field calculation is provided by simply knowing the values of the currents and voltages (derivative of the current) at the line ends.

The mathematical model for the assessment of the electromagnetic field radiation is based on the corresponding integral equation formulation in the frequency domain (FD). We know that the most fundamental quantity, and the one most difficult to determine theoretically, is the current distributions along the wire. Firstly, the spatial distribution of the current in the line is assumed to be known. We have adopted this hypothesis to develop an efficient mathematical method for determining the radiated field. This step is an essential building block that will allow us to generalize to the case of radiation from power cable in their realistic environment, taking into account the real distribution of electric current and the surrounding environments. We propose then an analytical formulation for the electric and magnetic fields in both spherical and cylindrical coordinates.

The synoptic schema shown in Figure 1 presented below, provides the procedure for the evaluation of electromagnetic radiation.

A software code was developed to allow the representation of EM field radiation at any observation point in free space in the near and far zone. The length of the antenna (wire), the current frequency and the change in index between the antenna and free space are parameters that affect the radiation diagram of the antenna and the radiated power.

The second part of this paper deals with the current distribution estimation, in order to determine the electromagnetic fields (EMF). Using the expressions derived in the first section, the transmission Line approximation can be used under certain circumstances.

Simulation results based on antenna theory are compared to our close-form EM field and shows good agreement. The result can be generalized to a network of PLC channels, corresponding to real and

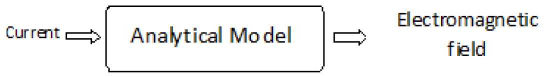

Figure 1. A simplified schema of the evaluation procedure of the field.

practical applications. The model is developed using the Frequency Domain (FD) which is useful to compute, with less consuming time, realistic scenarios of radiated field in far and near zones.

2.1. Modeling of Power Line Transmission Channel

Consider a line of length L placed along the z-axis. This line consists of a cylindrical copper conductor with its insulating sheath. The geometry of the line, characterized by the radius a, and length L is shown in Figure 2. The observation point is located at a certain distance R from the centre of the cable. We assume that $a \ll L$ and $\gamma a \ll 1$, where γ is the propagation constant along the wire and γ_0 is the propagation constant in the free space. The conductor is placed in the YZ plane and oriented along the z-axis. The line is subject to segmentation into an infinite number of elementary dipoles positioned in $M_0(0, 0, Z_0)$ and traversed by the current $I_{M0}(s, Z_0)$.

The electromagnetic field radiated consists of radial and tangential electric field component accompanied by an azimuthally component of the magnetic field. In the cylindrical system the corresponding electric field components can be expressed in terms of axially dependent magnetic vector potential [7]:

$$E_Z = -\frac{j\omega}{\beta_0^2}\left(\frac{\partial^2 A_z}{\partial z^2} + \beta_0^2 A_z\right) \quad (1)$$

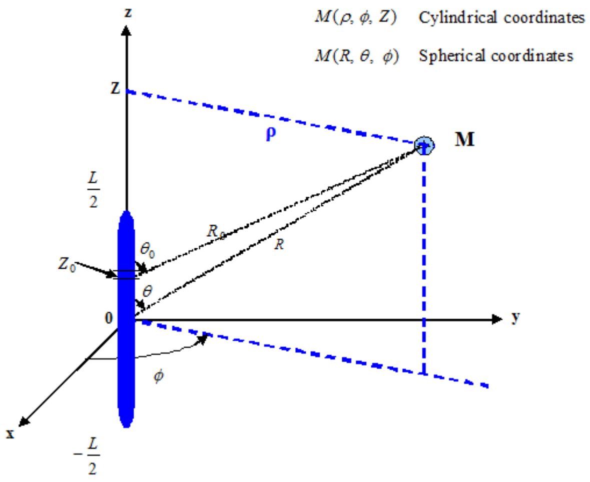

Figure 2. A view of a radiating antenna in free space.

$$E_\rho = -\frac{j\omega}{\beta_0^2}\frac{\partial^2 A_z}{\partial\rho\partial z} \tag{2}$$

while the magnetic field is given by:

$$H_\varphi = -\frac{1}{\mu_0}\frac{\partial A_z}{\partial\rho} \tag{3}$$

where the vector potential can be written by the following particular integral:

$$\overrightarrow{A_z(p)} = \frac{\mu_0}{4\pi}\int_0^L I(p,z)\frac{e^{-\gamma_0 R(z)}}{R(z)}dz\vec{z}. \tag{4}$$

Here, $I(s, z_0)$ is the current distribution along the conductor, $s = j\omega$ the Laplace variable, $R(z) = \sqrt{\rho^2 + (z - z_0)}$ the distance between the elementary dipole and the observation point M, $\gamma_0 = j\beta_0 = \frac{s}{c}$ the propagation constant in free space, and $\gamma = j\beta_1 = \frac{s}{v_1}$ the propagation constant of the PLC. c is the speed of light, v_1 the speed in line, and L the length of the line.

In this section, we have briefly sketched the equations of the EMF proposed by King [9], whatever the spatial current distribution. King presents a practical theoretical method for determining current distribution and integral equations to approximate the EMF. Rather than discretizing the integral equations into "two-term" and "three-term" theories developed in his book, he treats them by analytical means. These theories are presented as powerful alternatives to applying general-purpose numerical methods. Because the final two- and three-term formulas are quite simple in form, they require less running time when programmed in a computer. In addition, the analytical methods present a physical basis for understanding changes in the characteristics of the antenna as the parameters are changed.

2.2. Analytical Calculation of Electromagnetic Fields

Maxwell's equations relate the electromagnetic fields to the potential vector. The choice of cylindrical coordinates allows a simplified formulation in the case of our wire antenna. We develop the partial derivatives and we project the result on the spherical coordinates. Then we get the expression Fields H, E and E under an integral form.

The formulas for the electromagnetic field, expressed in spherical coordinates are:

$$\overrightarrow{H_\phi} = -\frac{1}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sin\theta_0 \frac{\partial G_0}{\partial R_0} I(Z_0) dZ_0 \overrightarrow{e_\phi} \tag{5}$$

$$\overrightarrow{E_R} = \frac{\eta}{4\pi\gamma}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\begin{array}{l}\left(\frac{\partial^2 G_0}{\partial R_0^2}-\gamma^2 G_0\right)\cos(\theta-\theta_0)\cos\theta_0\\ -\left(\frac{1}{R_0}\frac{\partial G_0}{\partial R_0}-\gamma^2 G_0\right)\sin(\theta-\theta_0)\sin\theta_0\end{array}\right]I(Z_0)dZ_0\overrightarrow{e_R} \quad (6)$$

$$\overrightarrow{E_\theta} = \frac{-\eta}{4\pi\gamma}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\begin{array}{l}\left(\frac{1}{R_0}\frac{\partial G_0}{\partial R_0}-\gamma^2 G_0\right)\cos(\theta-\theta_0)\sin\theta_0\\ +\left(\frac{\partial^2 G_0}{\partial R_0^2}-\gamma^2 G_0\right)\sin(\theta-\theta_0)\cos\theta_0\end{array}\right]I(Z_0)dZ_0\overrightarrow{e_\theta} \quad (7)$$

The components of the electromagnetic field expressed in the cylindrical coordinates, derived from the vector potential, are:

$$\vec{H}_\phi = -\frac{1}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\sin\theta_0\frac{\partial G_0}{\partial R_0}I(z_0)dz_0\vec{e_\phi} \quad (8)$$

$$\vec{E}_\rho = \frac{\eta}{8\pi\gamma}\int_{-\frac{L}{2}}^{\frac{L}{2}}\sin 2\theta_0\left(\frac{\partial^2 G_0}{\partial R_0^2}-\frac{1}{R_0}\frac{\partial G_0}{\partial R_0}\right)I(z_0)dz_0\vec{e_\rho} \quad (9)$$

$$\vec{E}_z = \frac{\eta}{4\pi\gamma}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(\cos^2\theta_0\frac{\partial^2 G_0}{\partial R_0^2}+\frac{\sin^2\theta_0}{R_0}\frac{\partial G_0}{\partial R_0}-\gamma^2 G_0\right)I(z_0)dz_0\vec{e_z} \quad (10)$$

where $I(z_0, s)$ is the current distribution along the conductor, $s = j\omega$ the Laplace variable, and $G(\gamma_0, R)$ the Green function.

The analytical solution provides the expressions for the electromagnetic field in cylindrical and spherical coordinates, whatever the Green function is. It is clear that these expressions depend on the antenna geometry, the power distribution and the actual Green function.

To further simplify Equations (8)–(10), we perform additional analytical effort [11]. We obtain

$$\vec{H}_\phi = \frac{-1}{4\pi\rho}\left\{\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(IG''_{-1}-I''G_{-1}\right)dz_0+\left[I\cos\theta_0 G'_{-1}+I'G_{-1}\right]_{-\frac{L}{2}}^{\frac{L}{2}}\right\}\vec{e}_\phi \quad (11)$$

$$\vec{E}_\rho = \frac{\eta}{4\pi\rho\gamma}\left\{\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(I'G''_{-1}-I'''G_{-1}\right)dz_0\right.$$
$$\left.+\left[I'\cos\theta_0 G'_{-1}+I''G_{-1}-I\sin^2\theta_0 G''_{-1}+I\sin^2\theta_0 G_0\right]_{-\frac{L}{2}}^{\frac{L}{2}}\right\}\vec{e}_\rho \quad (12)$$

$$\vec{E}_z = \frac{\eta}{4\pi\gamma}\left(\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(I''-\gamma^2 I\right)G_0 dz_0-\left[I\cos\theta_0 G'_0+I'G_0\right]_{-\frac{L}{2}}^{\frac{L}{2}}\right)\vec{e_z} \quad (13)$$

$G_{-1} = \int R_0 G_0 dR_0$, $G'_{-1} = \frac{\partial G_{-1}}{\partial R_0}$ et $G''_{-1} = \frac{\partial^2 G_{-1}}{\partial R_0^2}$, $G'_0 = \frac{\partial G_0}{\partial R_0}$, $I' = \frac{\partial I}{\partial z_0}$ et $I'' = \frac{\partial^2 I}{\partial z_0^2}$. The first term is represented by an integral whose kernel depends on the current and its derivatives. The proposed formulation is valid regardless of the Green function. The second term in (11)–(13) depends on the conditions at the antenna ends.

The representation, given by Equations (11)–(13), shows that a part of the radiation depends on the conditions at the ends, regardless the current distribution along the line.

These expressions (11)–(13) are valid regardless of the medium surrounding the radiating antenna.

It has been already emphasized that the estimation of the radiation from a wire requires appropriate modeling to determine the current distribution. Later on in this study, we will compare the analytical equation with the formulation of King in far areas to validate our model, also comparison will be made with the formulation of the radiating dipole.

2.3. Comparison with an Elementary Dipole

In the first step, and to validate our code, we compare our results with those of an elementary dipole in cylindrical coordinates. Here, for the fields of an elementary dipole we use the expressions proposed by R. W. King [9].

$$\overrightarrow{H_\phi} = \frac{IdZ_0}{4\pi}\left(\gamma + \frac{1}{R}\right)\frac{e^{-\gamma R}}{R}\sin\theta\overrightarrow{e_\phi} \tag{14}$$

$$\overrightarrow{E_R} = \frac{\eta IdZ_0}{4\pi}\left(\frac{2}{R} + \frac{2}{\gamma R^2}\right)\frac{e^{-\gamma R}}{R}\cos\theta\overrightarrow{e_R} \tag{15}$$

$$\overrightarrow{E_\theta} = \frac{\eta IdZ_0}{4\pi}\left(\gamma + \frac{1}{R} + \frac{1}{\gamma R^2}\right)\frac{e^{-\gamma R}}{R}\sin\theta\overrightarrow{e_\theta} \tag{16}$$

The electric field expressed in cylindrical coordinates is derived by projecting the spherical coordinates, we have:

$$E_\rho = E_R\sin(\theta) + E_\theta\cos(\theta) \tag{17}$$

$$E_z = E_R\cos(\theta) - E_\theta\sin(\theta) \tag{18}$$

The dipole is assumed as a conductor of negligible size where the current can be assumed constant over the length, as given in Figure 3.

The electric field of the dipole is shown in Figure 4.

In the case of free space, where the Green function is known, the kernel of the integral in Equations (11), (12), (13) developed before is easier to calculate and the integral term becomes evanescent. The Green function is well known and given by:

$$G(\gamma, R_0) = \frac{e^{-\gamma R_0}}{R_0} = \frac{e^{-j\beta_0 R_0}}{R_0} \tag{19}$$

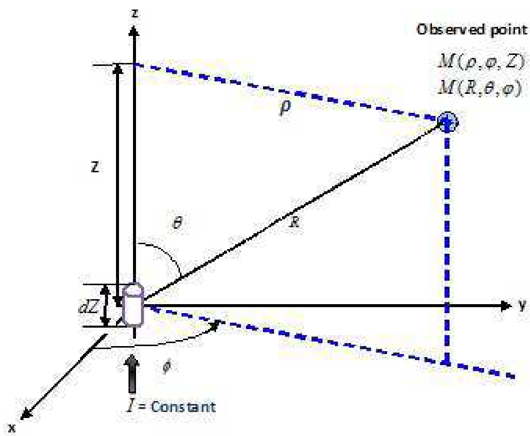

Figure 3. Schema of the radiating dipole.

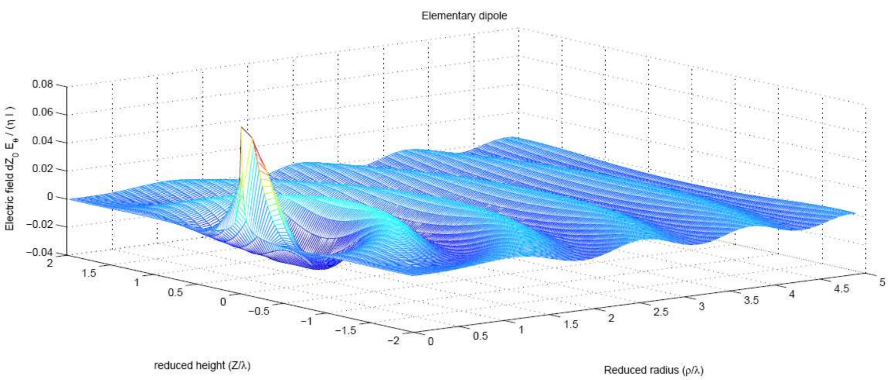

Figure 4. Total electric field for $\frac{L}{\lambda} = 0.01$ of an elementary dipole.

The expressions of the electric and magnetic field become:

$$\vec{H}_\phi = \frac{-1}{4\pi\rho\gamma}\left\{\int_{-\frac{L}{2}}^{\frac{L}{2}} \left(I''-\gamma^2 I\right)e^{-\gamma R_0}dz_0+\left[\left(I\gamma\cos\theta_0-I'\right)e^{-\gamma R_0}\right]_{-\frac{L}{2}}^{\frac{L}{2}}\right\}\vec{e}_\phi \quad (20)$$

$$\vec{E}_\rho = \frac{\eta}{4\pi\rho\gamma^2}\left\{\int_{-\frac{L}{2}}^{\frac{L}{2}} \left(I''-\gamma^2 I\right)'e^{-\gamma R_0}dz_0 + \left[\left(\gamma I'\cos\theta_0-I''+I\gamma^2\sin^2\theta_0+I\frac{\gamma}{R_0}\sin^2\theta_0\right)e^{-\gamma R_0}\right]_{-\frac{L}{2}}^{\frac{L}{2}}\right\}\vec{e}_\rho \quad (21)$$

$$\vec{E}_z = \frac{\eta}{4\pi\gamma}\left(\int_{-\frac{L}{2}}^{\frac{L}{2}} \left(I'' - \gamma^2 I\right) G_0 dz_0 - \left[I \cos\theta_0 G_0' + I' G_0\right]_{-\frac{L}{2}}^{\frac{L}{2}}\right)\vec{e_z} \quad (22)$$

The advantage of the representation given by (20)–(22) will be stressed out later on.

These formulations are a step in our research; they are compared with those of the radiating dipole model Equations (14)–(16), still for the case of free space.

Considering the assumptions of the model radiating dipole (constant current and small L), the electric field of our model is shown in the following Figure 5.

A good agreement can be easily noted between The electric field obtained by using our approach and the field obtained from the formulation of the dipole. they are shown respectively in the Figures 4 and 5. Only the real part has been represented, there is a good correlation between the two graphs in the same work zone. (With the aim of simplifying the integral equations and treat entities without unity, we have used reduced parameters $\frac{L}{\lambda}$).

2.4. Comparison with the Version of the King Model

We propose to treat the case of a wire with dielectric sheath in free space (known as antennas). The most advanced developments have been proposed by R. W. P. King [9] in the case of far field radiation ($\beta_0 R \gg 1$) from a nude antenna. We have improved this formulation in the case of a wire with its sheath dielectric [8]. To validate our model, the simulation results obtained by using our approach are compared with those given by King for the far zone.

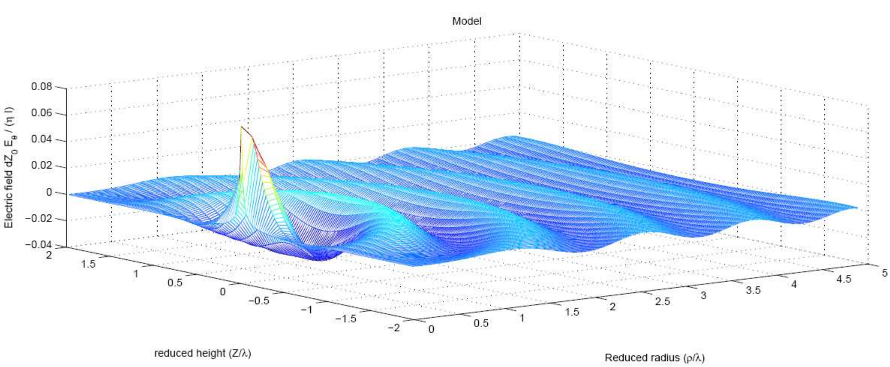

Figure 5. Total electric field for $\frac{L}{\lambda} = 0.01$ obtained by the proposed approach.

At sufficiently great distances from the antenna ($R > L$ and $\beta_0 R \gg 1$), the expression for the field is reduced to a simple form known as the radiation or far field. It is given by Equation (23):

$$\overrightarrow{E_\theta} = -\frac{\eta \sin\theta}{4\pi\gamma} \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(\frac{1}{R_0} \frac{\partial G_0}{\partial R_0} - \gamma^2 G_0 \right) I\left(Z_0\right) dZ_0 \overrightarrow{e_\theta} \tag{23}$$

In free space, the Green function is $G_0 = G\left(\gamma, R_0\right) = \frac{e^{-\gamma R_0}}{R_0}$, with $\gamma = j\beta_0$.

The expression becomes

$$\overrightarrow{E_\theta} = \frac{\eta\gamma \sin\theta}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(1 + \frac{1}{\gamma R_0} + \frac{1}{\left(\gamma R_0\right)^2} \right) \frac{e^{-\gamma R_0}}{R_0} I\left(Z_0\right) dZ_0 \overrightarrow{e_\theta} \tag{24}$$

The distance R_0 from an arbitrary point on the antenna to the field point is given in terms of R and Z_0 by the cosine law, namely Figure 6, $R_0 = R - Z_0 \cos\theta$.

In the radiation zone, $R > Z_0$, the phase variation of $\frac{e^{-\gamma R_0}}{R_0}$ is replaced by a linear phase variation given by $\frac{e^{-j\beta_0 R + j\beta_0 Z_0 \cos\theta}}{R}$. The amplitude $\frac{1}{R_0}$ of $\frac{e^{-j\beta_0 R_0}}{R_0}$ is a slowly varying function of Z_0 and is replaced by $\frac{1}{R}$, where R is the distance between the center of the antenna and the observation point. By using the above

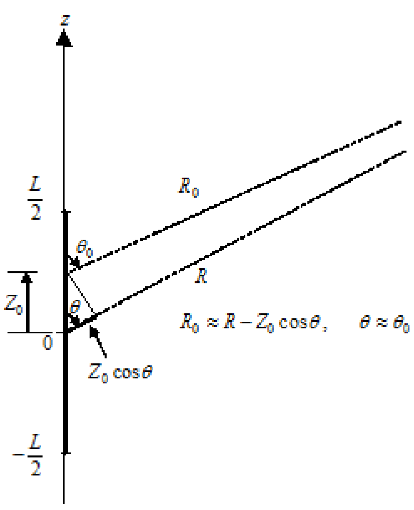

Figure 6. Coordinate system for calculations in the far zone.

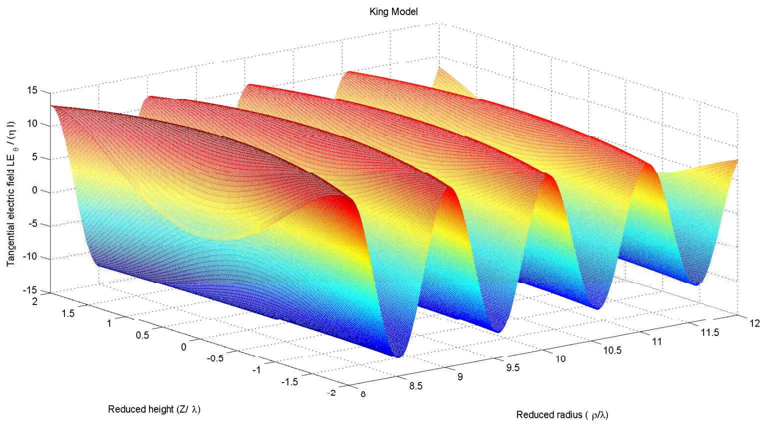

Figure 7. Tangential field in far zone obtained by King's formulation ($\frac{L}{\lambda} = 1$).

approximations, Equation (24) can be written as

$$\overrightarrow{E_\theta} = \frac{\eta(j\beta_0)\sin\theta}{4\pi}\frac{e^{-j\beta_0 R}}{R}\int_{-\frac{L}{2}}^{\frac{L}{2}} I\left(Z_0\right) e^{j\beta_0 Z_0\cos\theta} dZ_0 \overrightarrow{e_\theta} \tag{25}$$

where $\eta = 120\pi\,\Omega$.

If the current distribution is sinusoidal and given by the following expression (26)

$$I(Z_0) = I_{\max}\sin\beta_0\left(\frac{L}{2} - |Z_0|\right) \tag{26}$$

the tangential electric field in remote area may be expressed by:

$$E_\theta = \eta\frac{jI_{\max}}{2\pi}\left\{\frac{\cos\left(\beta_0\frac{L}{2}\cos\theta\right) - \cos\left(\beta_0\frac{L}{2}\right)}{\sin\theta}\right\}\frac{e^{-j\beta_0 R}}{R} \tag{27}$$

Considering the same current distribution as used by R. W. King, the far electric field obtained by our approach (Figure 8), is consistent with field obtained by King formulation (Figure 7).

Again, only the real part has been represented, which shows good spatial correlation of the simulated results.

2.5. Note

The classic expression of the electromagnetic field proposed by R. W. King is often limited to the term $\frac{1}{R}$, i.e., that the terms in $(\frac{1}{R})^2$ and $(\frac{1}{R})^3$ are neglected. This approximation is valid in far zone.

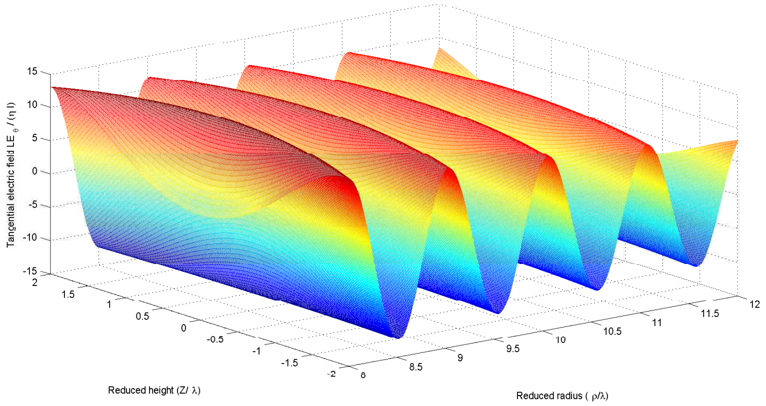

Figure 8. Tangential field in far zone obtained by the proposed approach ($\frac{L}{\lambda} = 1$).

In the near zone, the formulations for the electromagnetic field are more complicated. They include the terms in $(\frac{1}{R})^2$ and $(\frac{1}{R})^3$.

The need to determine the electromagnetic field at near zone requires other formulation. The integral formulations in Equations (20)–(22), will be used to express the electromagnetic field radiated by a wire antenna.

2.6. First Conclusion

Our study focuses on the radiation phenomenon which is described by the electromagnetic field. The radiation can be easily determined if the current distribution along the conductors are known. An analytical model based on antenna theory is presented to study the mechanisms of radiation for a wire system. Knowing the distribution of the current, the fields are obtained from the proposed analytical equations derived from Maxwell's equations.

The innovation here is a new formulation that will enable computation of the field in the near zone. A survey of the literature shows that it is almost impossible to find analytical formula for this kind of near field calculation. By comparing the simulation results obtained by our approach, with those obtained by King's expressions, and the formulation of the dipole radiation, we find good agreement between them. These comparison show the reliability of the code developed. This step is an essential building block to generalize to the case of radiation from wire structure.

In the next section it is given the calculation of the EMF based on a sinusoidal current derived from antenna theory (AT). In an attempt to simplify the analytical expression obtained in the first step, we use an equivalent current distribution derived by using transmission Line (TL) model. The basic electromagnetic equations are reformulated to provide analytical expressions able to compute fields in near and far zone which is not usually treated. The results of the analytical expressions are compared with those provided by Feko software.

In the study to follow, we will use TL theory to determine the current distribution and we will use the formulation given by (20), (21), (22) for the calculation of the radiated field.

3. APPLICATION IN THE CASE OF TRANSMISSION LINE

This second part of the paper deals with the calculation of the current distribution. It is well known that we need to know the current distribution to compute the EMF. In the formulation developed in the first part, an important practical benefit of such an approach is that a part of the field calculation is provided by simply knowing the values of the current and its derivatives at the line ends; without knowing the distribution of the current along the line.

The transmission line approximation, although sufficiently accurate for differential mode calculations, is not directly applicable to simulate the EMC behavior since it neglects the antenna-mode currents that are significant contributors to the radiated emissions.

Antenna theory should be used to get the electromagnetic fields, therefore in some special cases (in some frequency range), TL theory shows good results, sufficient to estimate the EM fields. Vukicevic [13] gives a new novel approach to evaluate the antenna-mode currents using a modified transmission line theory. Here, an integral equation of the antenna-mode currents along a two-wire transmission line is derived. It is shown that, when the line cross-sectional dimensions are electrically small, the integral equation reduces to a pair of transmission line-like equations with equivalent line parameters (per-unit-length inductance and capacitance). The derived equations make possible the computation of the antenna mode currents using a traditional transmission line code with appropriate parameters.

In order to improve the model developed in the first part, we subsequently analyzed cases where the current distribution along the conductors is known to be obtained by using TL theory. On the basis of the known current and its derivative at the ends of a multiple wire configuration, obtained using the TL theory, the proposed approach yields an analytical formalism for the determination of the radiated electromagnetic field. This analytical formalism is readily extended to multiple wires structures, as well. This may be very useful for the calculation of the field in the far and near zones, as it is less computationally demanding.

3.1. Theoretical Current Distribution Formalism

When we observe the formulations (20)–(22), we realize that, if TL theory is used the first term which is represented by an integral whose kernel depends on the current and its derivatives vanish and disappear ($I'' - \gamma^2 I = 0$). Finally, the second term, which contains the current and its derivatives at the line ends, remains. Since this situation occurred, thus we can apply this formulation in the case of bare-cable. After analysis, we found that TL theory gives approximately the same result as antenna theory. In this case TL could replace the AT theory to estimate the electromagnetic field.

If the surrounding medium is considered to be homogeneous, i.e., $\gamma = \gamma_0$. The eigenvalues of $\hat{Y}\hat{Z}$ are $\gamma_j = \gamma_0$ and in this case the term $I'' - \gamma^2 I = 0$ ($\hat{Z}$ and $\hat{Y}$ are the per-unit-length impedance matrix and admittance matrix) [10]. Next, we present a mathematical models used to express the electromagnetic radiation if the wires are bare. This configuration corresponds to the so-called outdoor overhead wires and the expressions (20)–(22) of the electromagnetic fields become:

$$\vec{H}_\phi = \frac{1}{4\pi\rho}\left[e^{-\gamma R}\left(-\frac{I'}{\gamma} + I\cos\theta\right)\right]_0^L \vec{e}_\phi \tag{28}$$

$$\vec{E}_\rho = \frac{\eta_0}{4\pi\rho}\left[\frac{e^{-\gamma R}}{\gamma R}\left(I\sin^2\theta - \gamma R\cos\theta\left(-\frac{I'}{\gamma} + I\cos\theta\right)\right)\right]_0^L \vec{e}_\rho \tag{29}$$

$$\vec{E}_z = \frac{\eta_0\gamma}{4\pi}\left[\frac{e^{-\gamma R}}{\gamma R}\left(\left(1 + \frac{1}{\gamma R}\right) I\cos\theta - \frac{I'}{\gamma}\right)\right]_0^L \vec{e}_z \tag{30}$$

The field calculation is provided by simply knowing the values of the current and its derivatives at the line ends; without knowing the distribution of the current along the line. These expressions are obtained in the case of TL theory current distribution.

3.2. Validation of the Proposed Theory

To verify the validity of the proposed model we compare the radiated field obtained from equation presented in part (3.1) with the radiated field provided by Feko software.

To illustrate our approach, an example of comparison is shown in Figure 9 corresponding to geometry of multiple conductor transmission line (MTL) structure. The case of uninsulated lines is considered, while the conductors are considered to be perfectly conducting and placed in homogeneous media. The wires have uniform cross section and they are parallel to each other, as well as to the ground plane. The length of the wires structure is 400 m; it is fed by a unit voltage source of 10 MHz frequency. This source is placed between the conductor number 1 and the neutral. The resistance load is equal to 20 Ω by phase.

Even if the current distribution is not needed to estimate the field by using the expression provided above, they are obtained by considering TL theory hypothesis. In this example, AT and TL theory give almost the same current distribution. Figure 10 shows that the current distribution along the line number 1 obtained by using both theories is almost the same.

The currents obtained by TL theory are compared to the current derived from antenna theory, obtained by simulation with Feko. The same result is obtained in the other lines.

Our study is not limited to prove coincidence of the currents, but also extended to know if the EMF generated by our model is the same as the one generated by Feko software [12].

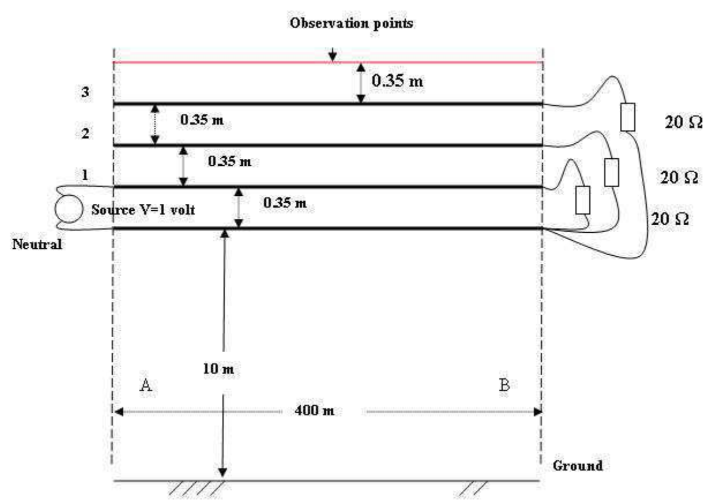

Figure 9. A cross section MTL configuration.

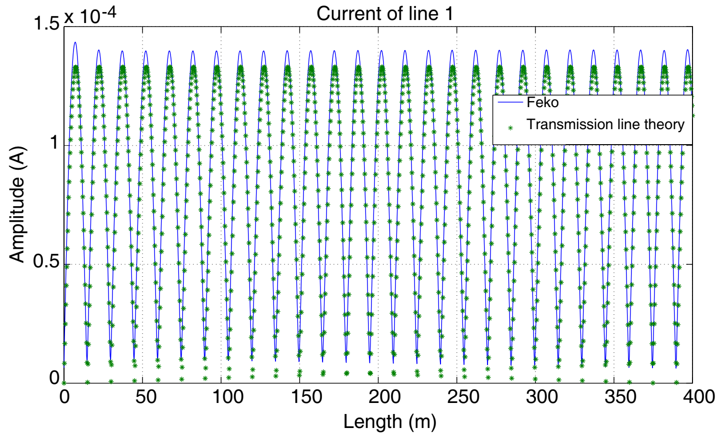

Figure 10. Current distribution along the first wire (Transmission line theory compared to antenna theory).

3.3. Performance and Results

For the the four-conductor line presented in Figure 9, the total magnetic and electric field generated can be readily evaluated by the sum of magnetic and electric fields radiated by each conductor. The expressions for fields become:

$$\vec{H}_\phi = \sum_{1\leq i\leq n} \frac{1}{4\pi\rho_i}\left[e^{-\gamma R_i}\left(-\frac{I_i{}'}{\gamma}+I_i\cos\theta_i\right)\right]_0^L \vec{e}_\phi \tag{31}$$

$$\vec{E}_\rho = \sum_{1\leq i\leq n} \frac{\eta_0}{4\pi\rho_i}\left[\frac{e^{-\gamma R_i}}{\gamma R_i}\left(I_i\sin^2\theta_i - \gamma R_i\cos\theta_i\left(-\frac{I_i{}'}{\gamma}+I_i\cos\theta_i\right)\right)\right]_0^L \vec{e}_\rho \tag{32}$$

Similarly, the total tangential electric field is given by the following sum:

$$\vec{E}_z = \sum_{1\leq i\leq n} \frac{\eta_0\gamma}{4\pi}\left[\frac{e^{-\gamma R_i}}{\gamma R_i}\left(\left(1+\frac{1}{\gamma R_i}\right)I_i\cos\theta_i - \frac{I_i{}'}{\gamma}\right)\right]_0^L \vec{e}_z \tag{33}$$

where I_i and θ_i are respectively the current and the angle at both ends of the line i.

By comparing the results obtained by using the expressions with the results obtained by FEKO software, we get almost the same results. Figure 11 shows the radiated electric and magnetic field respectively computed via the numerical approach and our closed form. The results correspond to the height of 1.4 m above the neutral.

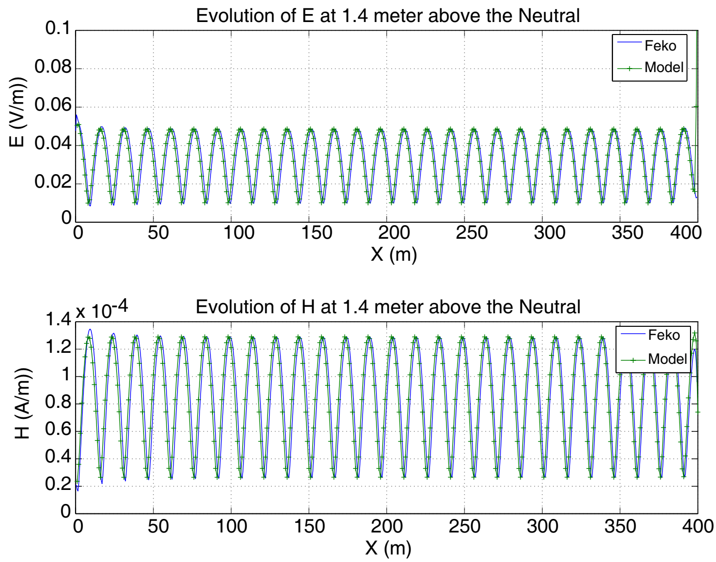

Figure 11. A Cross section of the electric and magnetic field respectively.

It can be stated that the results obtained by the proposed analytical model agree satisfactorily with those obtained from AT theory [12].

4. CONCLUSION

In this paper, we present an analytical model to estimate the radiated field by an outdoor PLC. The proposed novel closed-form formula is valid in both near and far field zones.

An important feature of the model is that the radiated fields are expressed only in terms of current and its derivatives (voltages) at the line ends. When compared to conventional methods, mostly arising from the antenna theory, the computational cost is significantly reduced.

This model is very useful to derive the electromagnetic field with less computation. It is valid in the case of aerial cables where the propagation constants are equal to the speed of the light. The current distribution, which is no more needed, is derived from TL theory.

Simulation results obtained from the proposed closed-form are compared to the results obtained from AT theory. The results computed via both approaches seem to be in a satisfactory agreement.

In the next studies, our objective is to extend our model to more realistic current distribution. By using antenna theory, the derived current can be expressed as a sum of complex sine waves and will be the base of new general formulas. The analytical model enables one to readily calculate the fields without requirement of wire structure discretization or involving the propagation effect over the domain of interest.

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