Progress In Electromagnetics Research Letters, Vol. 36, 87–101, 2013

OPTIMIZATION TOWARDS BROADBAND CYLINDRICAL CLOAKS WITH LAYERED MAGNETIC MATERIALS

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Abstract—Inhomogeneous anisotropic cloaks can be approximated by more realizable homogeneous and isotropic material layers at the expense of their bandwidth and angular dependence. Aiming at applications to a monostatic Radar, we propose a scheme to design broadband cylindrical cloaks with minimized backscattering RCS. The cloak is composed of a few layers of concentric magnetic materials, with optimized parameters using a genetic algorithm (GA). We also examine extensively the parameters in the optimization, including the initial population and the relationship of required discretization with the operation frequency. It has been demonstrated that, through a proper designed optimization, the bandwidth can exceed 80% for non-dispersive cloaks and 4% for dispersive cloaks.

1. INTRODUCTION

Since it was proposed by Pendry et al. [1] and Leonhardt and Tyc [2], the transformation-based invisible cloak has been developed in various ways. The main practical difficulty in constructing a perfectly invisible cloak is the requirement of continuous inhomogeneous and highly anisotropic material with extreme permittivities and permeabilities, which are often absent in nature. In order to facilitate the physical realization, various approximation schemes were applied, including discretization [3, 4], simplification [4–6], and anisotropic-to-isotropic conversion [7]. Various coordinate transformation schemes [8–18] were applied, rendering a cloak to be conformal, free of singularity, with magnifying or minifying function, or totally electric/magnetic (for 2D model), with preliminary experiments or full wave simulations validation. However, the resulting cloak design is often narrowband

Received 4 September 2012, Accepted 15 November 2012, Scheduled 3 December 2012
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and suffers from reduced performance of invisibility. In order to increase the cloaking performance and further reduce the complexity in physical realization, novel cloaks were designed by utilizing optimizations [19–22] to the multilayered anisotropic cloaks. Based on the analytical solution of the multilayer cylinder, a Broyden-Fletcher-Goldfarb-Shanno (BFGS) optimization method was applied in the cloak design [19, 23]. Through minimizing the forward scattering, a 3-layer cylindrical cloak was devised with the overall scattering considerably lower than that of the original discretized cloak with 100 layers. Since BFGS is not a global optimization algorithm, the initial guess was found to play a vital role in the performance of the optimized cloak. Aiming at minimized total scattering, an optimized cloak comprised of a few anisotropic layers can be designed, by using a global optimization algorithm such as the genetic algorithm (GA) [20, 24], or the particle swarm optimization (PSO) algorithm [21, 22]. In such optimized cloaks, the extreme values in the original cloak can be avoided and the results from [21] indicated a trade-off between the operating frequency band and the RCS reduction. Other than optimized layered cloaks, transmission line cloak loaded with metallic wires also shows improved bandwidth [25].

An anisotropic material can be replaced by electrically thin and concentric bi-layered composite, as shown in [7]. Yao et al. applied this conversion to the optimized 3-layer cloak presented in [19] and studied the impact of the conversion parameters, such as the thickness and the number of repetition [23]. By applying Mie theory, physical insights of the scattering from the isotropic multilayered cloaks were obtained through eigenmode analysis [24]. The relationship of the invisibility with the number of layers of the isotropic cloaks was extensively investigated [24], and a minimum number of layers of spherical cloak was suggested by using optimization [26]. In [27, 28], the bi-layered non-magnetic isotropic cloak with minimum number of layers was directly optimized. However, most of these isotropic cloak optimizations are investigated under a single-frequency perspective.

Despite the previous extensive studies, design of cloaks with broadband feature remains a challenging problem. So, in this paper, an optimization scheme towards a broadband isotropic cloak is proposed. Numerical experiments show that in order to achieve broader bandwidth, more layers than previously studied become necessary, which in turn leads to new challenges in the optimization algorithm. As a consequence, the influence of corresponding optimization parameters is extensively studied in this paper. Moreover, in order to facilitate practical fabrication, optimized broadband dispersive cloaks with permeability described by Lorentz model were designed. With different type of loss considered, simulations show an approximate bandwidth of 4% can be achieved by these dispersive cloaks.

2. OPTIMIZATION OF IMPERFECT CLOAKS

A transformation-based cloak renders the using of inhomogeneous anisotropic media. By applying discretization and anisotropic-to-isotropic conversion [7], the cloak can be approximated by using bi-layered concentric homogeneous isotropic materials. For example, an anisotropic cylindrical material can be replaced by alternately arranging the two types of isotropic materials A and B with thickness d_A and d_B:

$$u_\theta = \frac{u_A + \eta u_B}{1+\eta}, \quad \frac{1}{u_r} = \frac{1}{1+\eta}\left(\frac{1}{u_A} + \frac{\eta}{u_B}\right) \tag{1}$$

where $\eta = \frac{d_B}{d_A}$ is the ratio of the thicknesses of the two materials.

In the following, a full wave scattering method, as described in [29], is applied to calculate the scattering from a multilayered cylindrical cloak. Without losing the generality, the analysis for TE polarization is presented as an example.

Consider an M-layered cylindrical structure with the cross section shown in Fig. 1. The electric field component satisfies

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{\rho}{\mu_{\phi m}}\frac{\partial E_{zm}}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial}{\partial\phi}\left(\frac{1}{\mu_{\rho m}}\frac{\partial E_{zm}}{\partial\phi}\right) + k_0^2\varepsilon_{zm}E_{zm} = 0 \tag{2}$$

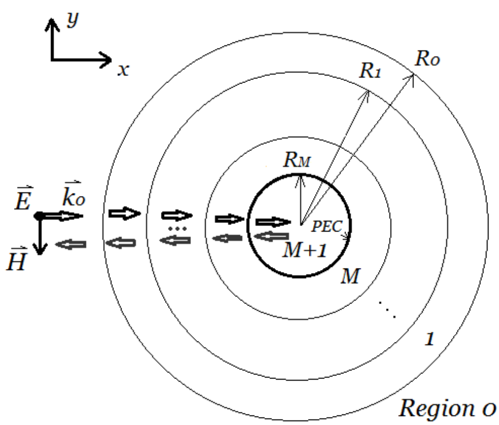


Figure 1. The cross section of an M-layered cylindrical structure.

with its solution being

$$E_{zm} = \sum_{n=-\infty}^{\infty} a_{mn} \left[J_{v_{mn}} \left(k_m \rho \right) + \tilde{r}_{m(m+1)n} H_{v_{mn}}^{(2)} \left(k_m \rho \right) \right] \exp(in\phi) \quad (3)$$

where $\nu_{mn} = n\sqrt{\mu_{\phi m}/\mu_{\rho m}}$, and $k_m = k_0\sqrt{\varepsilon_{zm}/\mu_{\phi m}}$. $J_{v_{mn}}$ and $H_{v_{mn}}^{(2)}$ denote the Bessel function of order v_{mn} and second kind Hankel function of order v_{mn} respectively. For an isotropic layer, v_{mn} equals to n; otherwise it is a fraction. a_{mn} is the coefficient to be determined. $\tilde{r}_{m(m+1)n}$ is the scattering coefficient on boundary R_m from region m to region $(m+1)$, which can be obtained through a recursive way:

$$\begin{aligned} \tilde{r}_{m(m+1)n} &= r_{m(m+1)n} + \tilde{t}_{(m+1)mn} \\ \tilde{t}_{(m+1)mn} &= t_{m(m+1)n} t_{(m+1)mn} \tilde{r}_{(m+1)(m+2)n} \\ &\quad / \left[1 - r_{(m+1)mn} \tilde{r}_{(m+1)(m+2)n} \right], \end{aligned} \quad (4)$$

and $\tilde{r}_{M(M+1)n} = -\frac{J_{\nu_{Mn}}(k_M R_M)}{H_{\nu_{Mn}}(k_M R_M)}$, since the inner-most boundary is PEC. In order to calculate the echo width of an object, which is defined by

$$\sigma(\varphi) = \lim_{\rho \to \infty} 2\pi\rho \frac{|E_z^s|^2}{|E_z^i|^2} \quad (5)$$

we can set $a_{0n} = 1$. Then the echo width is given by

$$\sigma(\varphi) = \frac{4}{k_0} \left| \sum_{n=-\infty}^{\infty} \tilde{r}_{01n} e^{jn\varphi} \right|^2 \quad (6)$$

3. NUMERICAL RESULTS

The inner and outer radii of the cloak are $R_a = 0.3\,\mathrm{m}$ and $R_b = 0.6\,\mathrm{m}$. The inner surface of the cloak is backed with a PEC, as illustrated in Fig. 1. The transformation-based cloak is first discretized into 10 homogeneous anisotropic layers. The thickness R_i for each layer can be evenly chosen, as in Section A and C, or can be taken as an optimization parameter, as in Section B below. After each layer being approximated by one pair of bi-layered concentric isotropic materials ($\eta = 1$), the cloak consisting of 20 layers is optimized using the genetic algorithm method. In order to facilitate the physical realization, the relative permittivity of the optimized cloak is fixed to be unity, and the search space for the relative permeability is [0.01, 1] for materials A and [1, P] for materials B, where P is empirically determined as 42 for the inner-most layer and 30 for the rest layers.

In many practical applications, the back scattering is of more interest than scatterings in other angles. Additionally, wider bandwidth is often preferable. Therefore, the sum of back-scattered echo widths of the cloak on discretized frequencies in a target band is chosen as the fitness function of GA. It is worth noting that in the fitness function if the backscattered echo width at a frequency point exceeds that of an ideal linear inhomogeneous, anisotropic cloak, it will be given excessive punishment. Also, if an echo width is lower than $-30\,\text{dB}$, it will be replaced by $-30\,\text{dB}$. Numerical experiments show that these treatments are effective in acquiring satisfactory optimization results. We used the default setting for GA in MATLAB tool box.

3.1. Initial Population by Various Cloak Schemes

Although GA is a global optimization algorithm, it is found through the numerical experiments that the initial guess plays an important

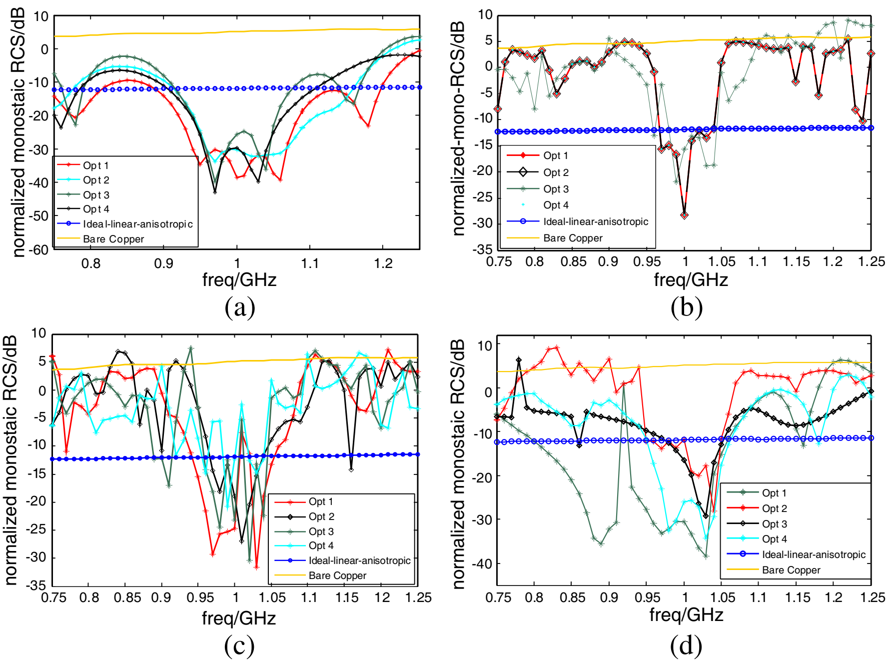


Figure 2. The monostatic echo widths of the optimized magnetic cloaks based on initial guesses of (a) simplified nonlinear cloak [18], (b) the ideal nonlinear cloak [1], (c) the nonlinear quasi-cloak [16], and (d) the default parameters provided by GA.

role in the optimized results. Even with the same initial parameters, GA provides various results in numerous numerical experiments due to its randomness nature. The failure in achieving optimal results is due to the incapability in GA, which arises from the large search space and its premature problem. In the following numerical experiments, GA optimizations are based on four different initial guesses: (a) simplified nonlinear cloak [18], (b) the ideal nonlinear cloak [1], (c) the nonlinear quasi-cloak [16], and (d) the default parameters provided by MATLAB GA toolbox. With each initial guess, optimization is performed for four times and the monostatic echo widths of the GA optimized cloaks are presented in Fig. 2. The central frequency is 1 GHz and the target bandwidth is 8% (80 MHz). The echo width of an inhomogeneous, anisotropic ideal cloak is used as reference in determining the operating frequency band of the optimized cloak. It is found that the GA optimized EM parameters are fairly close to the corresponding initial guess. The fact that $\varepsilon_z = 1$ in the optimized cloak implies that the optimized cloak resembles the most with the simplified cloak compared with the other cloaks. That explains why the optimized cloak with the initial guess of simplified cloak tends to give the lowest echo widths in the target frequency band. As mentioned earlier, the thickness R_i for each layer is evenly chosen. The optimization results (Opt 1) in Fig. 2(a) are provided in Table 1.

3.2. Bandwidth of the Optimized Cloak

According to the above results, in the following optimizations, the simplified cloak parameters are used as initial guess. Fig. 3 shows the echo widths of the GA optimized cloaks with central frequency of 1 GHz and different target bandwidths. It can be seen that 80% bandwidth can be achieved. The optimization results can be found in Table 2.

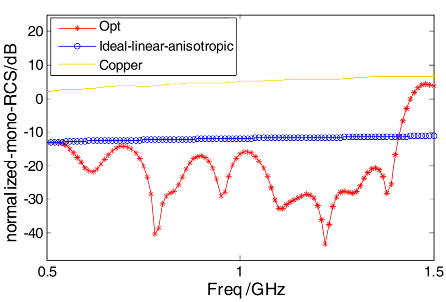


Figure 3. The monostatic echo widths from the optimized cloak targeting at 80% bandwidth, the ideal linear cloak, and the bare copper cylinder.

Table 1. Permeability of the optimized cloaks (Opt 1) based on simplified nonlinear one.

Layer	μ_a	μ_b
1	0.828	1.717
2	0.591	2.789
3	0.526	3.974
4	0.529	6.314
5	0.400	6.82
6	0.325	8.495
7	0.242	10.588
8	0.159	13.347
9	0.069	13.511
10	0.010	16.841

Table 2. Permeability of the optimized cloak targeting at 80% bandwidth.

Layer	μ_a	μ_b
1	0.890	1.968
2	0.584	2.300
3	0.637	3.974
4	0.518	5.367
5	0.596	7.312
6	0.362	8.587
7	0.242	10.502
8	0.216	12.429
9	0.075	14.651
10	0.014	17.839

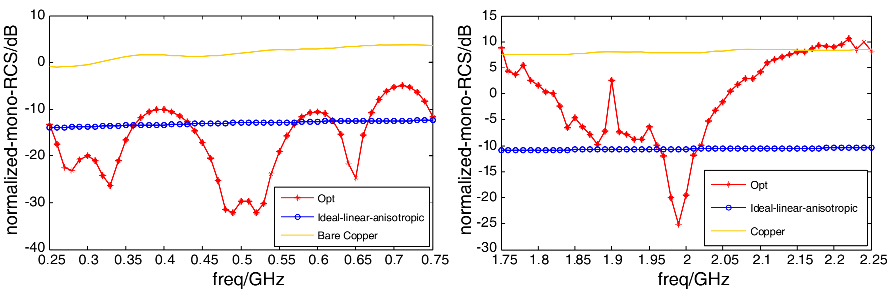
Table 3. Optimization parameters of the 10-layer cloak with central frequency of 2 GHz designed by optimizing (μ_i, ε_i).

Layer	μ_a	μ_b	ε_z
1	0.011	1.781	2.731
2	0.032	10.176	1.403
3	0.614	23.968	2.473
4	0.526	12.265	1.413
5	0.252	4.007	1.188
6	0.485	5.597	1.394
7	0.153	6.560	1.406
8	0.248	9.735	1.706
9	0.366	10.918	1.352
10	0.940	24.129	2.752

Next, we studied the parameters affecting the bandwidth in the optimized cloaks. By applying Bessel functions' properties, the infinite series in (6) can be truncated approximately, and $\sigma(\varphi)$ can be expressed explicitly as:

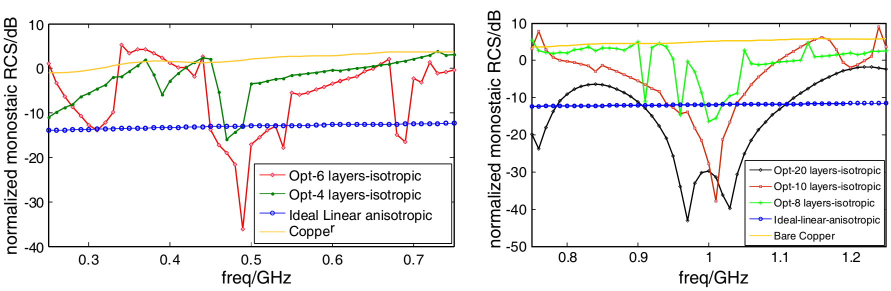
$$\sigma(\pi) \cong \frac{4}{k_0} \left| \sum_{n=-N}^{N} \tilde{r}_{01n} e^{jn\pi} \right|^2 = \left| \sum_{n=-N}^{N} f_n(\mu_i, \varepsilon_i, R_i) \right|^2 \quad (i = 1 \dots M) \tag{7}$$

where N increases as the frequency ω increasing. Therefore, in higher frequency band, where greater N is required, we need to use more optimization parameters to increase the dynamic range of f_n in order to find the optimal result. This can be done by increasing M [23, 24] or increase optimization parameters. Consider the following example: targeting at bandwidth of 80 MHz, and optimizing an equally discretized 20-layer cloak with center frequencies f_c (with wavelength λ_c) of 0.5 GHz and 2 GHz respectively. It can be seen that using the same number of layers, we can obtain narrower bandwidth in higher frequency (Ref. Fig. 4). On the other hand, a wider bandwidth requires finer discretization in the cloak. For example, if the target bandwidth



(a) (b)

Figure 4. The monostatic echo widths from the optimized cloak targeting at 80MHz bandwidth with the same number of layers ($M = 20$), the ideal linear cloak, and the bare copper cylinder. (a) Center frequency: 0.5 GHz. (b) Center frequency: 2 GHz.



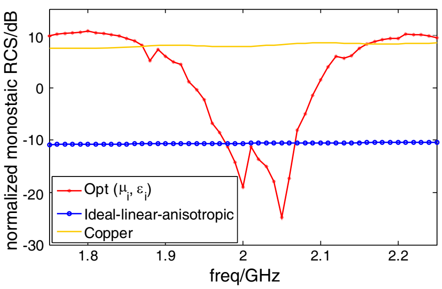
(a) (b)

Figure 5. The monostatic echo widths from the optimized cloak realized by different number of layers. Target bandwidth 80 MHz at (a) 0.5 GHz, (b) 1 GHz.The echo widths from the ideal linear cloak, and the bare copper cylinder are presented as references.

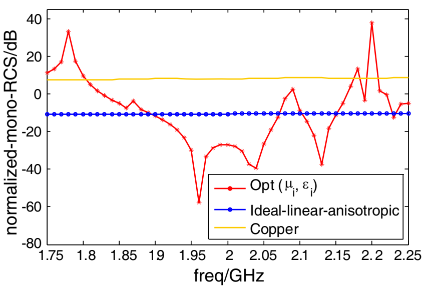
is 80 MHz, the thickness of each layer should not exceed $\lambda_c/10$, with the experiment results based on central frequencies of 0.5 and 1 GHz presented in Fig. 5. Alternatively, we can add optimization parameters ε_i or R_i for each layer. Figs. 6(a) and (b) show the results when using (μ_i, ε_i) and (μ_i, R_i) as optimization parameters respectively. Compared with Fig. 4(b), it can be seen that the bandwidth is obviously increased. The parameters of the optimized cloaks are given in Tables 3 and 4.

Table 4. Optimization results of the 10-layer cloak with central frequency of 2 GHz designed by optimizing (μ_i, R_i).

Layer	μ_a	μ_b	d_i
1	0.703	1.966	0.0302
2	0.591	2.789	0.0302
3	0.525	3.996	0.0278
4	0.47	5.316	0.0304
5	0.397	6.820	0.0274
6	0.333	8.544	0.0300
7	0.241	10.378	0.0302
8	0.153	12.347	0.0302
9	0.082	14.518	0.0333
10	0.017	16.81	0.0303



(a)



(b)

Figure 6. The monostatic echo widths from the optimized cloak designed using increased optimization parameters, with center frequency: 2 GHz, the ideal linear cloak, and the bare copper cylinder. (a) (μ_i, ε_i), (b) (μ_i, R_i) are used as the optimization parameters.

3.3. Bandwidth of Optimized Dispersive Cloak

In the analysis of aforementioned cloaks, we assumed that all required materials have broadband permeabilities. However, in reality, such permeabilities as prescribed in materials A are always dispersive with

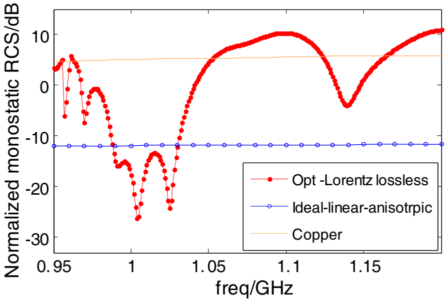


Figure 7. The monostatic echo widths from the optimized dispersive lossless cloaks, the ideal linear cloak, and the bare copper cylinder.

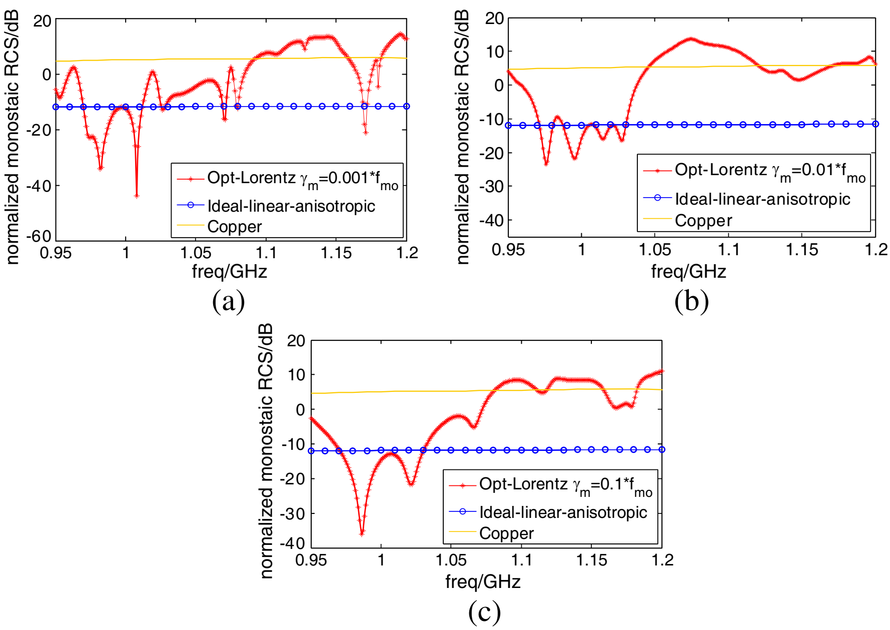


Figure 8. The monostatic echo widths from the optimized dispersive lossy cloaks, the ideal linear cloak, and the bare copper cylinder: (a) $\gamma_m = 0.001 * f_{mo}$; (b) $\gamma_m = 0.01 * f_{mo}$; (c) $\gamma_m = 0.1 * f_{mo}$.

μ_r. When we use SRR-type metamaterials [30], μ_r can be described by the Lorentz model [4]:

$$\mu_{eff\_m} = 1 - \frac{f_{mp}^2 - f_{mo}^2}{f^2 - f_{mo}^2 - i\gamma_m f} \tag{8}$$

where f_{mo} and f_{mp} refer to the magnetic resonant and plasma frequencies, and γ_m represents the losses of material in layer m. Here $f_{mp} = 1.02 \cdot f_{mo}$ is chosen as following the experimental results based on SRR-type metamaterials [21, 31, 32]. For materials B, whose permeability is greater than one, ferrite doped with various materials [33] could be a candidate bearing much weaker dispersion than materials A. Due to the strong dispersion in materials A, such a dispersive cloak usually present extremely narrow bandwidth [15]. Here, we adopted the same procedure and the same fitness functions with previous numerical experiments and use f_{mo} and μ_{b_m} as the optimization parameters with initial guess calculated using simplified nonlinear cloak model. In Fig. 7, the monostatic echo widths of the optimized dispersive lossless cloaks ($\gamma_m = 0$) are plotted against those of the ideal linear anisotropic cloak and bare copper cylinder. It can be clearly seen that roughly 4% bandwidth was achieved. The bandwidths of the optimized cloaks with various losses in the material are shown in Fig. 8. It turned out that 4% bandwidth can also be achieved by the optimized dispersive lossy cloaks. The parameters of the optimized dispersive cloaks are given in Tables 5–8.

Table 5. Optimization results of the dispersive cloak based on Lorentz model, $\gamma_m = 0$, designed by optimizing (f_{mo}, μ_b).

Layer	f_{mo}/GHz	μ_b
1	0.638	3.11
2	0.833	1.391
3	0.801	5.608
4	0.126	2.021
5	0.516	15.36
6	0.939	18.728
7	0.646	15.006
8	0.605	9.111
9	0.748	3.104
10	0.898	34.592

Table 6. Optimization results of the dispersive cloak based on Lorentz model, $\gamma_m = 0.001 * f_{mo}$ designed by optimizing (f_{mo}, μ_b).

Layer	f_{mo}	μ_b
1	0.5714	9.576
2	0.6404	28.997
3	0.1620	14.741
4	0.0849	8.849
5	0.0688	14.812
6	0.5078	23.018
7	0.8465	3.581
8	0.8682	15.380
9	0.3493	21.432
10	0.2958	32.885

Table 7. Optimization results of the dispersive cloak based on Lorentz model, $\gamma_m = 0.01 * f_{mo}$ designed by optimizing (f_{mo}, μ_b).

Layer	f_{mo}/GHz	μ_b
1	0.0771	4.395
2	0.2944	6.510
3	0.2021	9.185
4	0.8424	19.654
5	0.51881	17.470
6	0.1265	23.537
7	0.9500	15.006
8	0.3368	11.530
9	0.2098	10.559
10	0.8465	7.1934

Table 8. Optimization results of the dispersive cloak based on Lorentz model, $\gamma_m = 0.1 * f_{mo}$ designed by optimizing (f_{mo}, μ_b).

Layer	f_{mo}/GHz	μ_b
1	0.1993	3.948
2	0.9500	10.505
3	0.0872	17.434
4	0.3908	19.725
5	0.7181	5.726
6	0.9305	7.173
7	0.8777	3.803
8	0.8718	20.226
9	0.7372	21.742
10	0.6296	19.006

4. CONCLUSION

In this paper, a cylindrical electromagnetic cloak composed of layered magnetic materials is optimized using the genetic algorithm for broadband monostatic applications. Numerical experiments show that, due to limited global search capability of GA, the initial guess plays an important role in the optimization result. In this paper, different initial guesses are investigated by numerical experiments, and a best one in the proposed scheme is found to be a nonlinear simplified cloak, which presents the best similarity with the targeting optimized magnetic cloak. Under this initial guess, it is demonstrated that more than 80% bandwidth can be achieved at around 1 GHz by properly designed optimization. Further study on the frequency band property shows that, in order to optimize cloaks with higher working frequency or wider bandwidth, more optimization parameters are required to increase the dynamic range of the fitness functions. Considering the physical realization of materials A required by the optimization result, optimization on cloaks based on dispersive model (Lorentz model) was performed. Despite the strong sensitivity of the material parameters with respect to the frequency, broadband dispersive cloaks with 4% bandwidth were obtained. Numerical results show that for certain applications such as the backscattering reduction, a cloak can be optimized with significantly wider bandwidth through properly designed optimization scheme.

REFERENCES

1. Pendry, J. B., D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science*, Vol. 312, 1780–1782, 2006.
2. Leonhardt, U. and T. Tyc, "Broadband invisibility by non-Euclidean cloaking," *Science*, Vol. 323, 110–112, 2009.
3. Cummer, S. A., B.-I. Popa, D. Schurig, D. R. Smith, and J. B. Pendry, "Full-wave simulation of electromagnetic cloaking structures," *Phys. Rev. E*, Vol. 74, 036621, 2006.
4. Zhao, Y., C. Argyropoulos, and Y. Hao, "Full-wave finite-difference time-domain simulation of electromagnetic cloaking structures," *Opt. Express*, Vol. 16, 6717–6730, 2008.
5. Schurig, D., J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, Vol. 314, 977–980, 2006.
6. Cai, W. S., U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, "Optical cloaking with metamaterials," *Nat. Photonics*, Vol. 1, 224, 2007.
7. Huang, Y., Y. Feng, and T. Jiang, "Electromagnetic cloaking by layered structure of homogeneous isotropic materials," *Opt. Express*, Vol. 15, 11133–11141 2007.
8. Jiang, W. X., J. Y. Chin, Z. Li, Q. Cheng, R. Liu, and T. J. Cui, "Analytical design of conformally invisible cloaks for arbitrarily shaped objects," *Phys. Rev. E*, Vol. 77, 066607, 2008.
9. Hu, J., X. M. Zhou, and G. K. Hu, "Nonsingular two dimensional cloak of arbitrary shape," *Appl. Phys. Lett.*, Vol. 95, 011107, 2009.
10. Kohn, R. V., H. Shen, M. S. Vogelius, and M. I. Weinstein, "Cloaking via change of variables in electric impedance tomography," *Inverse Problems*, Vol. 24, 15016, 2008.
11. Isi'c, G., R. Gaji'c, B. Novakovi'c, Z. V. Popovi'c, and K. Hingerl, "Radiation and scattering from imperfect cylindrical electromagnetic cloaks," *Opt. Express*, Vol. 16, 1413–1422, 2008.
12. Liu, H. Y., "Virtual reshaping and invisibility in obstacle scattering," *Inverse Problems*, Vol. 25, 045006, 2009.
13. Li, J., H. Y. Liu, and H. Sun, "Enhanced approximate cloaking by SH and FSH lining," *Inverse Problems*, Vol. 28, 075011, 2012.
14. Liu, H. Y. and T. Zhou, "On approximate electromagnetic cloaking by transformation media," *SIAM J. Appl. Math.*, Vol. 71, 218–241, 2011.
15. Song, W. and X.-Q. Sheng, "A cloak scheme insusceptible to the change of material properties," *Journal of Electromagnetic Waves*

and Applications, Vol. 26, Nos. 2–3, 149–160, 2012.

16. Song, W., X.-H. Yang, and X.-Q. Sheng, "Scattering characteristics of 2-D imperfect cloaks with layered isotropic materials," *IEEE Antennas and Wireless Propagation Letters*, Vol. 11, 53–56, 2012.
17. Jiang, W. X., T. J. Cui, X. M. Yang, Q. Cheng, R. Liu, and D. R. Smith, "Invisibility cloak without singularity," *Appl. Phys. Lett.*, Vol. 93, 194102, 2008.
18. Yan, W., M. Yan, and M. Qiu, "Non-magnetic simplified cylindrical cloak with suppressed zeroth order scattering," *Appl. Phys. Lett.*, Vol. 93, 021909, 2008.
19. Popa, B.-I. and S. A. Cummer, "Cloaking with optimized homogeneous anisotropic layers," *Phys. Rev. A*, Vol. 79, 023806, 2009.
20. Xi, S., H. S. Chen, B. Zhang, B.-I. Wu, and J. A. Kong, "Route to low-scattering cylindrical cloaks with finite permittivity and permeability," *Phys. Rev. B*, Vol. 79, 155122, 2009
21. Ivsic, B., T. Komljenovic, and Z. Sipus, "Optimization of uniaxial multilayer cylinders used for invisible cloak realization," *IEEE Trans. on Antennas and Propagation*, Vol. 58, 3397–3401, 2010.
22. Ivsic, B., T. Komljenovic, and Z. Sipus, "Performance of uniaxial multilayer cylinders and spheres used for invisible cloak realization," *Proceedings of the 5th European Conference on Antennas and Propagation (EUCAP)*, 1092–1096, 2012.
23. Yao, H.-Y., C.-W. Qiu, and L.-W. Li, "Scattering characteristics from conducting cylinder with reconstructing electromagnetic cloaking layers," *Asia Pacific Microwave Conference, APMC*, 2009.
24. Qiu, C.-W., L. Hu, X. F. Xu, and Y. J. Feng, "Spherical cloaking with homogeneous isotropic multilayered structures," *Phys. Rev. E*, Vol. 79, 047602, 2009.
25. Danaeifar, M., M. Kamyab, A. Jafargholi, and M. Veysi, "Bandwidth enhancement of a class of cloaks incorporating metamaterials," *Progress In Electromagnetics Research Letters*, Vol. 28, 37–44, 2012.
26. Martins, T. C. and V. Dmitriev, "Spherical invisibility cloak with minimum number of layers of isotropic materials," *Microwave and Optical Technology Lett.*, Vol. 54, 2217–2220, 2012.
27. Yu, Z. Z., Y. J. Feng, X. F. Xu, J. M. Zhao, and T. Jiang, "Optimized cylindrical invisibility cloak with minimum layers of non-magnetic isotropic materials," *J. Phys. D: Appl. Phys.*,

Vol. 44, 185102, 2011.

28. Peng, L., L. Ran, and N. A. Mortensen, "The scattering of a cylindrical invisibility cloak: Reduced parameters and optimization," *J. Phys. D: Appl. Phys.*, Vol. 44, 135101, 2011.
29. Chew, W. C., *Waves and Fields in Inhomogeneous Media*, 2nd Edition, IEEE, New York, 1995.
30. Pendry, J. B., A. Holden, J. D. Robbins, and J. W. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Trans. on Microwave Theory and Tech.*, Vol. 47, No. 11, 2075–2084, 1999.
31. Hrabar, S., L. Benic, and J. Bartolic, "Simple experimental determinationof complex permittivity or complex permeability of SNG metamaterials," *Proc. 36th Eur. Microwave Conf.*, 1395–1398, Manchester, UK, 2006.
32. Hrabar, S., N. Engheta, and R. Ziolkowsky, "Waveguide experi-ments to characterize the properties of SNG and DNG metamaterials," *Metamaterials: Physics and Engineering Explorations*, Ch. 3, Wiley and IEEE, Hoboken/Piscataway, NJ, 2006.
33. Goldman, A., *Modern Ferrite Technology*, 2nd Edition, Ch. 15, Springer, Pittsburgh, PA, USA, 2006.