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A NEW BROADBAND MICROSTRIP QUADRATURE HYBRID WITH VERY FLAT PHASE RESPONSE

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Abstract—A new broadband microstrip branch-line quadrature hybrid with very flat phase response is presented. The device is made by cascading four branch-line couplers with arbitrary power division. The novel design is based on the microstrip transposition of a broadband waveguide polariser [4]. Across a 32% bandwidth centred at 9.3 GHz, the RL and the IL are respectively −15 dB and −3 dB/−4 dB; the phase difference is very flat, i.e., 90° ± 1.5°.

1. INTRODUCTION

Quadrature branch-line hybrids are passive components widely used at microwave frequencies to design transmitting and receiving systems. These devices can be realized by using either waveguide or planar technology. In the latter case branch-line hybrids can be very small compared to their waveguide equivalents. In general, good performance and the required 90° phase response are normally achieved only within narrow bandwidths, around the central operational frequency. When the previous components are used in applications such as: radio astronomy correlation and pseudo-correlation receivers, broadband performances and very flat phase-response are required [5].

The new quadrature hybrid presented here, which works around 9.3 GHz, overcomes the above limits. The design is based on the transposition in microstrip technology of a combination of waveguide components used to build broadband polarizers [1–4].

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2. CONCEPT

Rectangular waveguide phase-shifters can be realized using dually polarized circular waveguides and a combination of 90° and 180° differential phase sections [1], i.e., using Quarter-Wave Sections (QWSs) and Half-Wave Sections (HWSs) respectively [2, 3]. In these designs, a rectangular-to-circular waveguide transition transforms a TE_{10} mode into a circular waveguide TE_{11} mode. A 45° oriented QWS converts it into an RHCP mode. An HWS, arbitrarily rotated by an angle ϑ, introduces a phase-shift equal to 2ϑ. A second QWS, parallel to the first one, brings back to the TE_{11} mode. A final circular-to-rectangular waveguide transition restores the TE_{10} mode. As proved in [3], this configuration provides a frequency-flat phase-shift response equal to twice the mechanical rotation angle, i.e., $\Delta\phi = 2\vartheta$.

It is possible to modify the above design removing the input and output transitions. This dually polarised device, $QWS_{45} \times HWS_{\vartheta} \times QWS_{45}$ introduces opposite phase-shifts on the two orthogonal x and y polarisations: $\Delta\phi = \pm 2\vartheta$. It is clear that such a device, with the internal HWS rotated by $\vartheta = 22.5°$, would introduce phase-shifts of $\Delta\phi = \pm 45°$ with an overall 90° phase-difference between the two orthogonal polarizations. The $QWS_{45} \times HWS_{22.5} \times QWS_{45}$ configuration behaves like a waveguide polarizer with a very flat phase response across the band. This polarizer can be rotated by 45° and can be efficiently used to convert x/y polarizations to RH/LH circular polarizations. A very broadband waveguide polarizer based on this idea has been recently developed [4].

The key idea of this paper is that the quadrature hybrid matrix is equivalent to the waveguide polarizer matrix rotated by 45°. This is true because there is a correspondence between the polarizer x and y directions and the Ports 1 and 4 of the hybrid (see Fig. 1). In particular we are going to design a new planar quadrature hybrid that is the microstrip transposition of the broadband waveguide polarizer discussed above.

3. DESIGN

In the optical system designs, lossless dually polarized devices can be modeled using the 2×2 Jones matrix formalism [6]. In a generic optical system, if the electric field vectors of the incoming and outgoing signals are defined respectively as $\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$ and $\vec{E}' = \begin{pmatrix} E'_x \\ E'_y \end{pmatrix}$, the Jones matrix $J = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ of the optical system relates them according to following equation:

$$\vec{E}' = J\vec{E} \rightarrow \begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}. \tag{1}$$

An ideal QWS with the fast axis parallel to the x axis has a matrix: $\mathrm{QWS}_{0^\circ} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$, whereas the matrix of a QWS rotated by $\vartheta = 45^\circ$ will be: $\mathrm{QWS}_{45^\circ} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$. This latter 4-port device splits both the x and y components of the incident field into two 3 dB output signals with $\pm 90^\circ$ phase-shifts. Similarly, a quadrature hybrid also is a four-port device that splits two input signals (on Port 1 and 4) into two 3 dB output signals (Port 2 and 3) with $\pm 90^\circ$ phase-shifts. In the first case the device has two dually polarized circular waveguides; in the second case there are four single mode waveguides or planar transmission lines (Fig. 1).

The broadband waveguide polarizer discussed in Section 2 has its fast axis aligned with the x axis, i.e., at 0°:

$$\mathrm{POL}_0 = \mathrm{QWS}_{45} \times \mathrm{HWS}_{22.5} \times \mathrm{QWS}_{45}. \tag{2}$$

In order to use the analogy with the quadrature hybrid, we need to rotate it by 45°:

$$\mathrm{POL}_{45} = \mathrm{QWS}_{90} \times \mathrm{HWS}_{67.5} \times \mathrm{QWS}_{90}. \tag{3}$$

In addition, the HWS can be made by cascading two QWSs:

$$\mathrm{HWS}_{67.5} = \mathrm{QWS}_{67.5} \times \mathrm{QWS}_{67.5} \tag{4}$$

and so the final Jones matrix of our device will be:

$$\mathrm{POL}_{45} = \mathrm{QWS}_{90} \times \mathrm{QWS}_{67.5} \times \mathrm{QWS}_{67.5} \times \mathrm{QWS}_{90}. \tag{5}$$

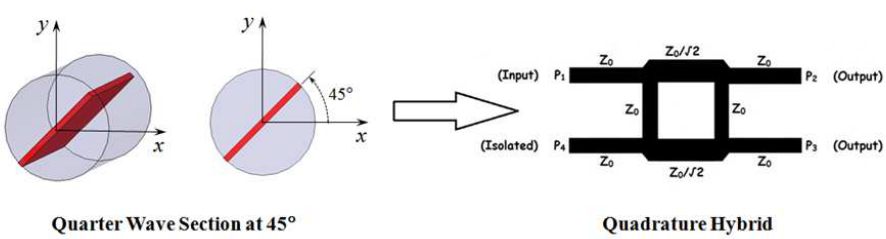

Figure 1. A planar quadrature hybrid is equivalent to a waveguide polarizer rotated by 45° setting a correspondence between the x and y polarized modes and port 1 and 4 of the hybrid.

In conclusion, in order to design a broadband microstrip hybrid we need to design two circuits with their scattering matrices equivalent to a QWS rotated by 90° and 67.5° and then cascade them following the configuration in equation [5].

In microwave engineering, N-port networks are described by using the scattering matrix S. This relates the voltage wave vector incident on port n, $[V_n^-]$, to the voltage wave vector reflected from port n, $[V_n^+]$, as shown in the following equation:

$$\left[V_n^-\right] = [S]\left[V_n^+\right] \rightarrow \begin{bmatrix} V_1^- \\ V_2^- \\ \dots \\ V_n^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1n} \\ S_{21} & S_{22} & \dots & S_{2n} \\ \dots & \dots & \dots & S_{3n} \\ S_{n1} & S_{n2} & \dots & S_{nn} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \dots \\ V_n^+ \end{bmatrix} \tag{6}$$

where the elements $S_{ij(i\neq j)}$ represent the relation between the voltage wave incoming on port j and the voltage wave outgoing on port i; the elements S_{ii} represent the relation between the voltage wave incoming and outgoing on the same port i. Considering the definition of S_{ij}, it is possible to find a correspondence between some of its elements (those related to the transmissions) and the Jones matrix elements. Referring to the Jones and scattering matrix four-port networks shown in Fig. 2, it is possible to write the following relationships:

$$a = \frac{E'_x}{E_x} \quad \leftrightarrow \quad \frac{V_2^-}{V_1^+} = S_{21} = S_{12} \tag{7a}$$

$$b = \frac{E'_x}{E_y} \quad \leftrightarrow \quad \frac{V_2^-}{V_4^+} = S_{24} = S_{42} \tag{7b}$$

$$c = \frac{E'_y}{E_x} \quad \leftrightarrow \quad \frac{V_3^-}{V_1^+} = S_{31} = S_{13} \tag{7c}$$

$$d = \frac{E'_y}{E_y} \quad \leftrightarrow \quad \frac{V_3^-}{V_4^+} = S_{34} = S_{43}. \tag{7d}$$

The previous relations (7) allow finding the scattering matrices which represent the two rotated QWSs required to design the quadrature hybrid. Knowing that the Jones matrix of a QWS rotated by an arbitrary angle ϑ has the following elements:

$$a = \cos^2\vartheta + j\sin^2\vartheta \tag{8a}$$

$$b = (1-j)\cos\vartheta\sin\vartheta \tag{8b}$$

$$c = (1-j)\cos\vartheta\sin\vartheta \tag{8c}$$

$$d = \sin^2\vartheta + j\cos^2\vartheta, \tag{8d}$$

it is possible to find the scattering matrix of the equivalent four-port

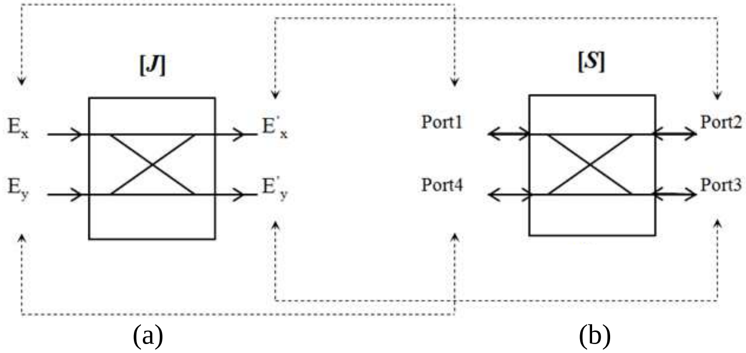

Figure 2. Four-port network associated to an optical device using the Jones formalism (a) and that associated to a microwave device using the scattering matrix formalism (b).

device:

$$[S] = \begin{bmatrix} 0 & a & c & 0 \\ a & 0 & 0 & c \\ b & 0 & 0 & b \\ 0 & d & d & 0 \end{bmatrix}. \tag{9}$$

The form of this matrix is the same as that of a branch-line coupler with arbitrary power division. There are many different designs available in literature for these kinds of devices [7–11], we have accurately chosen two. Furthermore, the design of the quadrature hybrid, described in this paper, is realized by cascading two pairs of the branch-line couplers chosen from literature.

4. MODELLING

The circuit schematics of the two branch-line couplers which have the same scattering matrices of the QWSs of interest are shown in Fig. 3. These devices, described in [7] and [8], identify respectively the $\mathrm{QWS}_{67.5°}$ and the $\mathrm{QWS}_{90°}$.

In [7], the arbitrary power division is obtained by controlling the characteristic impedances of the lines of device through the following equations:

$$Z_1 = Z_3 = \sqrt{\frac{d_1^2}{d_1^2 + d_2^2}} \tag{10a}$$

$$Z_2 = Z_4 = \frac{d_1}{d_2}, \tag{10b}$$

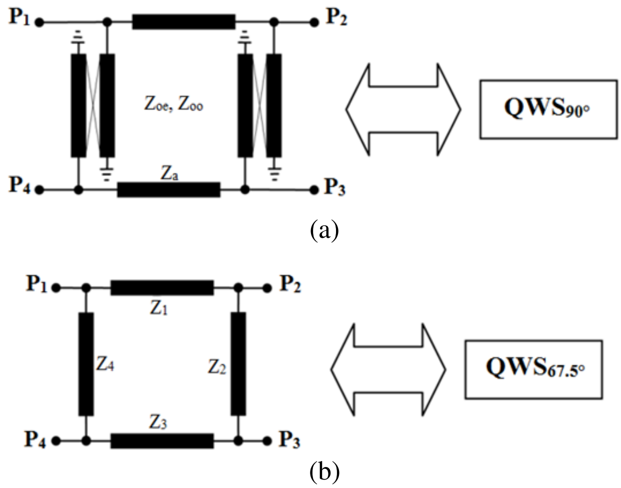

Figure 3. Correspondence between the branch-line couplers with arbitrary power division and the two QWSs of interest: $\mathrm{QWS}_{67.5^\circ}$ and QWS_{90°.

where d_1 and d_2 are respectively $|S_{21}|$ and $|S_{31}|$.

In [8], the large power division is obtained by replacing the vertical lines of the branch-line with the shorted parallel coupled-line sections through the following equations:

$$\frac{Z_a}{Z_0} = \frac{d}{\sqrt{1+d^2}} \tag{11a}$$

$$\frac{2}{Z_0(Y_{oo} - Y_{oe})} = d, \tag{11b}$$

where $d = 10^{\Delta/20}$, $\Delta(dB) = |S_{21}| - |S_{31}|$ and Z_0 is the port impedance.

All the transmission lines are a quarter of wavelength long, whereas their impedances are different as reported in Table 1.

A Rogers-RT Duroid 5880 substrate with a dielectric constant of 2.22 and a thickness of 0.25 mm has been used for the final microstrip device.

Figures 4, 5 and 6 show the performances of the $\mathrm{QWS}_{67.5^\circ}$ and QWS_{90° models in terms of insertion loss and phase difference. The insertion losses reveal the two different power divisions used to obtain the QWSs, while the phase differences are those typical of a branch-line coupler.

Figure 7 shows the final layout of the proposed branch-line coupler where the four devices have been connected together by using six lines

Table 1. Branch-line couplers line impedances.

$\mathbf{QWS}_{67.5°}$	$\mathbf{Z}_1 = 42.9\,\Omega$	$\mathbf{Z}_2 = 83.5\,\Omega$	$\mathbf{Z}_3 = 42.9\,\Omega$	$\mathbf{Z}_4 = 83.5\,\Omega$
$\mathbf{QWS}_{90°}$	$\mathbf{Z}_a = 49.75\,\Omega$	$\mathbf{Z}_{oe} = 55.24\,\Omega$	$\mathbf{Z}_{oo} = 45.45\,\Omega$	

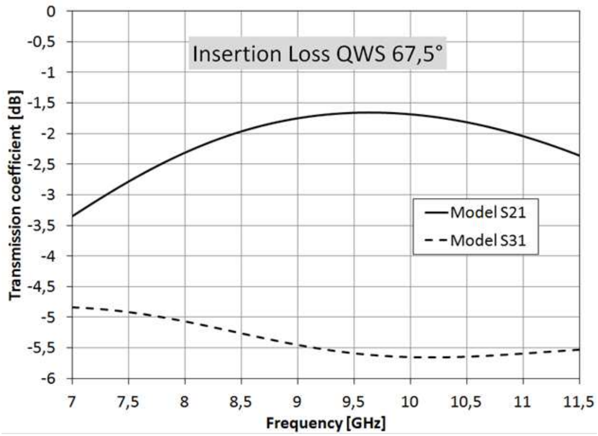

Figure 4. Insertion loss of $QWS_{67.5°}$.

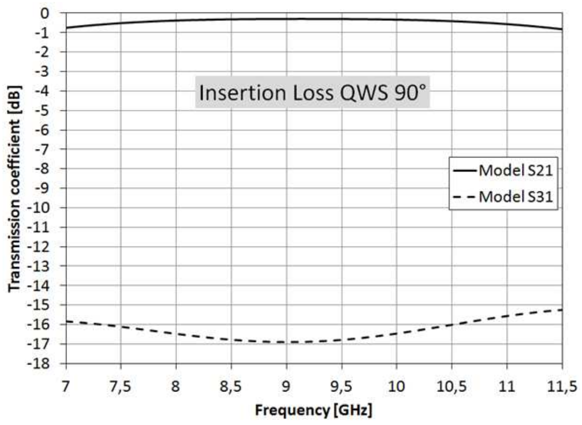

Figure 5. Insertion loss of $QWS_{90°}$.

of $50\,\Omega$ impedance and a quarter wavelength in length. The device is designed to have its central operational frequency at 9.3 GHz.

The modelling was carried out by using the finite-element analysis commercial software Ansoft HFSS [12]. The expected performances of the device within a 33% band are: $\mathrm{RL} = -20\,\mathrm{dB}$, $\mathrm{IL} = -3\,\mathrm{dB} \div -4\,\mathrm{dB}$ and phase difference $= 90° \pm 2°$.

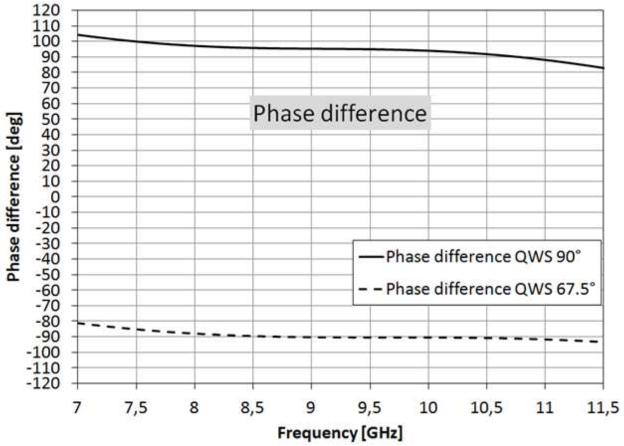

Figure 6. Phase difference of $QWS_{67.5^\circ}$ and QWS_{90°.

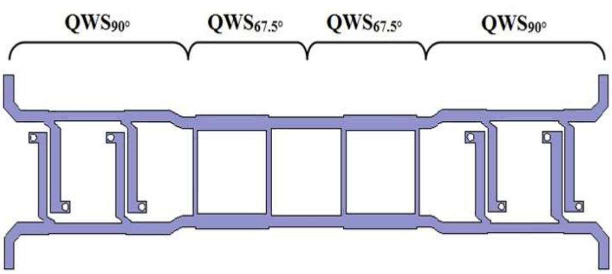

Figure 7. Branch-line coupler proposed.

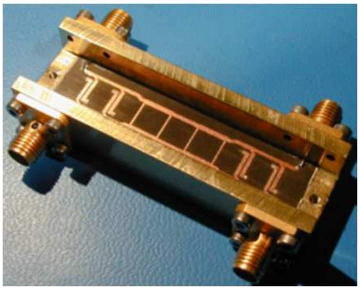

Figure 8. Branch-line coupler prototype.

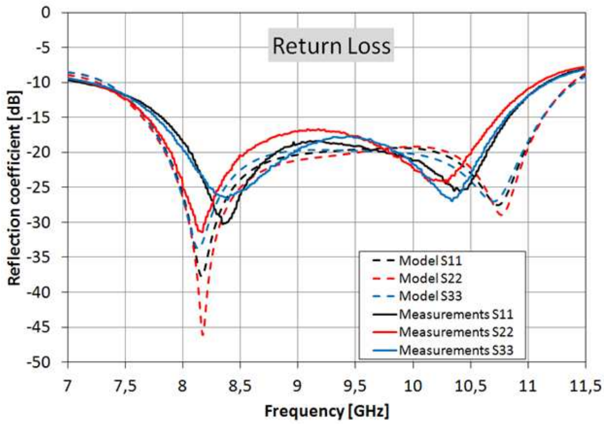

Figure 9. Measured and simulated return loss.

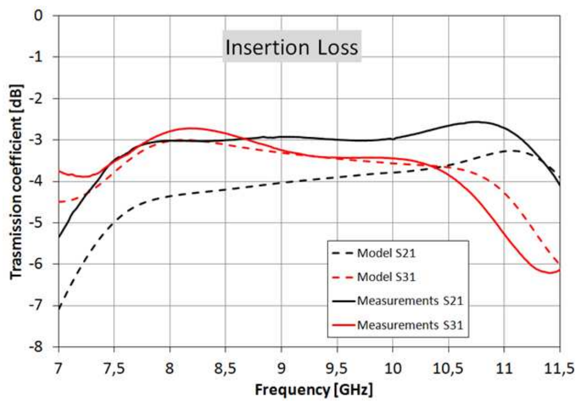

Figure 10. Measured and simulated insertion loss.

5. EXPERIMENTAL RESULTS

The branch-line coupler was manufactured by the Trackwise Company [13]. A prototype, with dimensions $53.3 \times 13\,\text{mm}^2$, is showed in Fig. 8. The measurements were carried out using a Rohde & Schwarz ZVA40 Vector Network Analyzer (VNA). The results of the measurements and their comparison with the models are reported in Figs. 9–11. Across a 32% bandwidth the Return Loss is lower than $-15\,\text{dB}$, whereas the Insertion Loss is between $-3\,\text{dB}$ and $-4\,\text{dB}$. Part of these losses is due to the quarter wavelength microstrip lines used to connect the four branch-line couplers and part of them is due to the QWS_{90° and $\text{QWS}_{67.5^\circ}$ Insertion Losses. Across the same bandwidth,

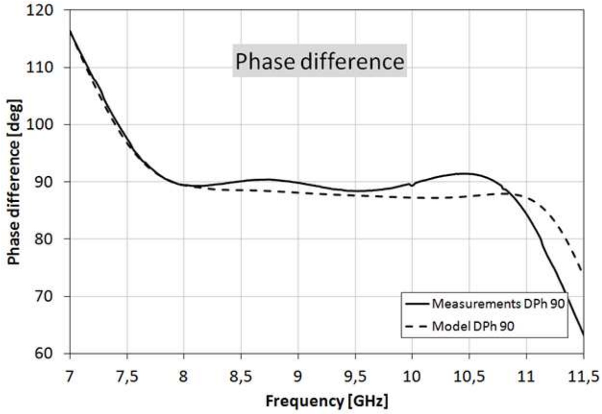

Figure 11. Measured and simulated differential phase-shift.

the phase-difference is very flat: $90° \pm 1.5°$. These results show good performances in terms of phase difference and transmission.

6. CONCLUSION

A new broadband branch-line quadrature hybrid has been designed, manufactured and tested. The new hybrid is made with a combination of four branch-line couplers with arbitrary power divisions. The design is the microstrip transposition of the design of a broadband waveguide polariser. The device performances are very good, especially in terms of phase flatness. Across a 32% bandwidth centred at 9.3 GHz, the measured RL, IL and phase-difference are respectively: -15 dB, -3 dB/-4 dB and $90° \pm 1.5°$. The very good agreement between measured results and simulations confirm the validity of the theoretical approach.

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