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DERIVATION OF AN 'INVERSE LIÉNARD-WIECHERT EFFECT' FROM THE LORENTZ FORCE AND ITS APPLICATION TO THE WIRELESS POWER TRANSFER

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Abstract—Our work investigates the well-known Lorentz formula (in its original form) for the EM force. It allows the prediction of one effect of the action of the EM force on a moving charge. We use this effect to explain Tesla's mechanism of wireless power transfer between resonant coils.

1. DERIVATION OF THE FORMULA FOR THE EM FORCE ACTING ON THE MOVING CHARGE

The Lorentz formula for EM force is accepted as being independent of the Maxwell equations. While the Maxwell equations appear in textbooks in the form originally introduced by their author, the Lorentz formula appears in modern textbooks in a somewhat simplified form. But originally, Lorentz introduced his famous formula for the EM force for an *element* of the charge [1]

$$\bar{f} = \bar{d} + \left[\bar{v} \times \bar{h}\right], \tag{1}$$

but not for the *entire charge*

$$\bar{F} = q\left(\bar{E} + \left[\bar{v} \times \bar{H}\right]\right). \tag{2}$$

So, following Lorentz, we will express the EM force acting on the single charge in the form

$$\bar{F} = \int \rho\left(\bar{E} + \left[\bar{v} \times \bar{B}\right]\right) d^3r, \tag{3}$$

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where integration is over the entire space, and the boundaries of the charge are determined by function ρ describing the charge density. (As this was the original definition of the formula for the EM force†).

Lorentz's approach is logical, and it establishes a duality between the Maxwell equations containing the current and charge densities in the *rhs* of equations and the EM force acting from the fields upon the charge.

Our aim is to consider a possible effect connected with the representation of the elementary charge as a particle having finite (nonzero) size. The finite extent of the charge radiating EM waves leads to well-known Liénard-Wiechert (L-W) potentials‡. Since there is an amplification of the source charge potential due to the factor $1/[1 - \bar{v} \cdot \bar{r}/cr]$ and this effect is caused by the finite size of the source charge, it can be expected from the duality of the Maxwell equations and the EM force that a similar effect can also be found for the EM fields which influence the test charge, i.e., the EM force.

It should be noted that the L-W potentials are derived for EM fields propagating at the speed of light. It is assumed that the charge continuously emits EM waves which further propagate in space as concentric diverging waves. If our intent is to consider the inverse effect, we should analyze how these waves act on the test charge (the detector). Thus, such an analysis requires consideration of an EM wave passing through the charge. To extract the effect in the clearest form, one should consider the incident EM wave to be of a very small duration (we note it as the elementary wave).

We consider an 'elementary' EM wave to be described by

$$\bar{E} = \bar{E}_0 \cdot [\theta(x - ct - \Delta) - \theta(x - ct)]\,, \tag{4}$$

where $\theta(x)$ is the Heaviside step function. We integrate over the region containing the test charge to obtain

$$\begin{aligned} \bar{F} &= \int \rho(\bar{r} - \bar{v}t)\bar{E} d^3 r \\ &= \bar{E}_0 \int \rho(x - vt; y; z) \cdot [\theta\,(x - ct - \Delta) - \theta(x - ct)]\, dxdydz. \end{aligned} \tag{5}$$

If we integrate with respect to y and z and define

$$\rho'(x) = \int \rho(x, y, z) dydz, \tag{6}$$

† Original paper of Lorentz [2] and Eq. (XIV) of Schott's textbook [3].

‡ The generally accepted explanation why the Coulomb potential of the charge at rest transforms to the Liénard-Wiechert potentials of the moving charge is given in [4, 429–433].

we obtain

$$\bar{F} = \bar{E}_0 \int \rho'(x - vt) \cdot [\theta(x - ct - \Delta) - \theta(x - ct)]\, dx. \tag{7}$$

If we consider the wave to be of infinitesimal thickness $\Delta \to 0$, we can use the approximation

$$[\theta(x - ct - \Delta) - \theta(x - ct)] \cong \Delta \frac{d\theta(x - ct)}{dx} = \Delta \cdot \delta(x - ct)\,, \tag{8}$$

to the first order in Δ and express Eq. (12) in the form

$$\begin{aligned} \bar{F} &= \bar{E}_0 \int \rho'(x - vt) \cdot [\theta(x - ct - \Delta) - \theta(x - ct)]\, dx \\ &= \bar{E}_0 \cdot \Delta \cdot \rho' \left(ct - \frac{(\bar{v} \cdot \bar{r})t}{r} \right). \end{aligned} \tag{9}$$

Here, we use the requirement that the thickness of the elementary wave is substantially smaller than the size of the elementary charge.

This analysis does not, however, offer a result appropriate for experimental verification. Here we should note that during experimentation it is not possible using conventional methods to detect, and therefore it is impossible to measure, this force directly. Therefore, the force change produced by the motion of the particle must be measured by an indirect method. So when we compare the results of action of the EM wave on charges: one of them moving and the other being at rest, we should evaluate what velocity each charge acquires interacting with the EM wave. Therefore we consider the impulse, assuming that action of the traveling EM wave on the elementary charge cannot essentially change the velocity of the latter.

$$\delta \bar{u} = \frac{1}{m} \int_{t_0}^{t_1} \bar{F}(t) dt, \quad \text{where } t_1 = t_0 + \Delta, \tag{10}$$

where m is the relativistic mass of the charge.

Because the area occupied by the charge is very small we can assume, without loss of generality, that $t_0 = -\infty$, $t_1 = +\infty$. We thus obtain, for the detector charge:

$$\delta \bar{u}_{v=0} = \frac{\bar{E}_0 \Delta}{cm} \int_{-\infty}^{+\infty} \rho(ct) d(ct) = \frac{q \bar{E}_0}{mc}. \tag{11}$$

For the moving charge,

$$\delta \bar{u}_v = \frac{\bar{E}_0 \Delta}{cm} \int_{-\infty}^{+\infty} \rho \left(ct - \frac{(\bar{v} \cdot \bar{r})t}{r} \right) d(ct)$$

$$= \frac{E_0 \Delta}{\left(c - \frac{(\bar{v} \cdot \bar{r})}{r}\right) m} \int_{-\infty}^{+\infty} \rho \left(ct - \frac{(\bar{v} \cdot \bar{r}) t}{r} \right) d \left(c - \frac{(\bar{v} \cdot \bar{r})}{r} \right) t = \frac{\left(q \bar{E}_0 / mc\right)}{1 - \frac{(\bar{v} \cdot \bar{r})}{cr}}. \quad (12)$$

The above result gives validity to the supposition that the EM force of the incident wave is 'effectively increased' when it acts on the charge moving in the direction of the wave vector.

Obviously, the effect of amplification of the action of the incident EM wave on moving charges is caused by 'effectively increasing' the size of these charges in the direction of their motion. It allows one to describe under what conditions the predicted effect can be verified.

A dominant requirement is that the velocity of the charge should be comparable to the speed of light. Despite the fact that for classical electrons such a requirement is generally realized only in accelerators, this requirement may be satisfied in specific electrical circuits. In UWB technology short current pulses carry some amount of the charge and such pulses propagate in the wire with the speed $0,5c$ to $0,9c$, depending on neighboring elements of the wire [5]. In order to have maximum effect the pulse should move in the direction of propagation of the EM wave, so this effect can be essential in the near zone where the longitudinal component of the EM field is still comparable to the transversal components.

2. THE EXPERIMENT

It should be noted that the drift velocity of the electrons of conductivity is very small, so the effect of amplification of the current pulse propagating in the wire is expected to be very small. But the mechanism of amplification does not depend on the velocity of each separate electron. Both the L-W effect and the inverse L-W effect are caused by an effective extension of the region where the charge interacts with the EM field. Because the E field accelerates the extra charge carried by the current pulse, an essential parameter for the process of amplification is the length of time the pulse is within a region of traveling EM waves.

When the current pulse moves in a monopole of the antenna, it is in the field of the EM wave's minimal expansion during time T_1 (the directions of the traveling pulse and the EM wave coinciding), this pulse acquires more energy than another pulse that moves transversally to the direction of propagation of the EM wave because the time of interaction T_2 of the latter is shorter.

To test the effect experimentally, the following method was chosen: In order to achieve maximum accuracy of measurements, a coil was used rather than a monopole antenna, in which the current pulse is stopped and then reflected from the end. The radiator of the EM field in the near zone is another (primary) coil. The level of the power transferred from the primary to the secondary coil was measured. Because such an experimental setup is similar to the one used by Tesla during his famous Colorado Springs experiments [6], in the first series of our experiment we used Tesla's parameters for the coils: Coils with dimensions of 7 inches in diameter and 4.5 feet in length were tuned to a resonant frequency of 1.7 MHz. The distance between the coils was changed from 1 meter to 15 meters. The axes of symmetry of both coils were parallel. The power in the secondary coil was measured by the bolometric method having an accuracy of 1% or greater. The power detected in the secondary coil dropped from 84% of the power level in the primary coil at a distance between the coils of 1 m to 25% at a distance of 15 m.

3. FOUR FACTORS THAT CAN ACCOUNT FOR THE EXPERIMENTAL EFFECT

We consider factors that can account for the observed effect. We take into account four such factors:

- Radiation by the primary coil as a magnetic antenna [7],
- The inductance connection between the currents in the primary and secondary coils [8, 9],
- Action of the longitudinal component of the EM field in the near field zone of the antenna [10],
- Amplification of the current in the secondary coil due to the inverse L-W effect.

First, we estimate a factor of the radiation of the primary coil as a magnetic antenna. To eliminate this factor experimentally, a second set of measurements with the coils perpendicular to one another was made. The results are given in the second row of Table 1. Even with

Table 1. The measurement data of the power in the secondary coil. 1st series: the axes of the coils are parallel. 2nd series: the axes of the coils are turned by 90 degrees in angle.

Distance, m	1	3	5	7	10	15
1st series	84%	81%	75%	70%	50%	25%
2nd series	3%	1%	no	no	no	no

a distance of 3 m between the coils, the effective area of the receiving coil acting as an antenna should be of the same order as in the earlier experiment with the parallel axes; with the perpendicular orientation the power levels drop substantially. This allows us to conclude that this effect is not due to the radiation of the transversal waves. The second reason why this factor is negligibly small is because the receiving coil is located within the near field zone of the radiating coil where normally non-radiating components of the EM fields dominate the radiating components. The parameters of the experiment: the radius of the coils $r = 0.18$ m, the distance between the coils $y = 1$ to 15 m and the wavelength $\lambda \approx 300$ m, correspond to the near field zone.

Next, we estimate the factor of the resonant connection. For the coils having parallel axes the mutual inductance is (in SI units) [11, 6]

$$M \approx \frac{\pi}{16}\mu_0 \frac{d^2}{a^2} w^2 \frac{d}{2\sqrt{a^2+y^2}} \left[\frac{\sqrt{a^2+y^2}}{y} - 1 + \frac{d^2}{8\sqrt{a^2+y^2}} - \ldots \right], \quad (13)$$

where w is the number of turns, d the diameter of the coils, a their length, y the distance between the axes of the coils, and $\mu_0 = 4\pi \cdot 10^{-7}$ H/m the permeability of the vacuum. So for the dimensions of the coils used in the experiment, even for $w = 10^5$ the mutual inductance at a distance $y \approx 10$ m would less than 1%.

Although inductive near-field coupling between small coils of radius r separated by a distance y such that $r \ll y \ll \lambda$ is generally small (proportional to $(r/\lambda) \cdot (r/y)^3$) it can be resonantly enhanced if the coils are either self-resonant or connected to external resonating circuits [9]. We note that the effect of the external resonating circuits is an essential component of advanced systems for the wireless transfer of power [8, 12]. In the experiments performed, the coils are tuned to resonance so we should estimate the influence of resonance on efficiency. In our experiments, different configurations of the coils are used to demonstrate the results of the resonant amplification of the transmitted power. A common feature of each of our models is that the power in the secondary coil P_s is described by the expression

$$P_s = \frac{M}{\sqrt{L_p \cdot L_s}} \cdot F(R_p, R_s; L_p, L_s, M), \quad (14)$$

where $F\left(R_p,\ R_s;\ L_p,\ L_s, M\right)$ is some function of the resistance and inductance of the primary $(R_p,\ L_p)$ and secondary $(R_s,\ L_s)$ coils correspondingly; and the parameter M enters into the function F additively with other inductances. It allows us to demonstrate why the power transferred by means of the magnetic resonance drops with the distance y.

Table 2. Comparison of the experimental and calculated power in the receiving antenna.

y	1	3	5	7	10	15
$P_{experiment}$	0.84	0.81	0.75	0.70	0.50	0.25
$P \sim P_0/y$	0.84	0.28	0.168	0.120	0.084	0.056
$P \sim P_0/y^2$	0.84	0.093	0.034	0.017	0.008	0.004

One can find from (13) that the mutual inductance decreases with the distance y as $1/y^2$. So P_s displays the same distance interdependence, which is confirmed in the experiments (see, for example, [12]). However, if we compare the experimental data (the first row of Table 1) with calculated dependences $P_s \sim P_0/y$ and $P_s \sim P_0/y^2$, where P_0 is the power level detected in the secondary coil at $y = 1$ m (Table 2) we see that power in the secondary coil decreases with the distance slower even than $1/y$. Such a gradual reduction can be explained by the fact that for the distances to 15 m the primary coil cannot be treated as a point source of the EM field. However, the data of Table 2 allows us to conclude unambiguously that the experimental results cannot be explained by the magnetic resonance between the coils alone.

Since the experimental findings are greater than would be expected from transverse EM fields, we should consider the possible involvement of longitudinal EM fields. An experiment on propagation of the longitudinal component of the E field was made by Tzonchev where the signal was detected by the receiving antenna at the axis of symmetry of the radiating antenna. According to [10] at a distance greater than 3 m between the ends of the radiating dipoles, a signal was too weak to be detected. So the third factor cannot serve to explain the experimental results in the first row of Table 1. However, we need to explain why the fourth factor can serve as evidence for a longitudinal E field.

The key difference in the experiments is that in [10] an incident EM wave acts on the electrons in the receiving antenna when they are initially at rest. So the EM wave is not able to generate a current pulse moving with a velocity v close the speed of light. Therefore Tzonchev did not create the inverse L-W effect. In our experiment, the incident EM wave acted on the current pulses which initially have a velocity $v \approx c$.

Here, we give arguments to support our opinion that the inverse L-W effect is responsible for the experimental results of the 1st series. To demonstrate this, we compare our experimental results with corresponding results of the wireless power transfer experiments based on magnetic coupling resonance. A level of the transferred power can be described in general form as

$$\frac{P_{sec}}{P_{pri}} = \frac{k}{(R/L)^4}, \tag{15}$$

where P_{sec}, P_{pri} the power in the secondary and primary coils, correspondingly. k is the coefficient of transfer, R the distance between the coils and L the size of the radiating coil. So R/L is the dimensionless distance. Because the experiments are made in the near zone, $E \sim 1/R^2$, $P_{sec} \sim E^2$ so $P_{sec} \sim 1/R^4$. In [13], $\Delta P_{magn} = P_{sec}/P_{pri} = 40\%$ of the power was transferred to a distance of $R_{magn}/L_{magn} = 8$. In our experiment, $\Delta P_{long} = 25\%$ of the power was transferred to the distance of 15 m which corresponds to $15/\sqrt{1.3 \cdot 0.18} = 31$ dimensionless distances of the mean size $(\sqrt{L \cdot d})$ of our antenna. So the ratio of the coefficients of transfer is

$$\frac{k_{long}}{k_{mang}} = \frac{\Delta P_{long}}{\Delta P_{magn}} \cdot \left(\frac{(R/L)_{long}}{(R/L)_{magn}}\right)^4 = \left[\frac{0.25}{0.4} \cdot \left(\frac{31}{8}\right)^4\right] > 100. \tag{16}$$

The high efficiency produced in our experiments cannot be explained within the existing framework of wireless power transfer. But the denominator $1 - (\bar{v} \cdot \bar{r}/(cr)$ at $v \approx c$ in Eq. (12) can provide sufficient amplification of the transferred power.

In order to confirm that the inverse L-W effect is responsible for the larger-than-expected wireless power transfer, we conducted the same experiments as described above with the coils positioned in a different orientation. Specifically, the secondary coil was rotated physically by 90 degrees in the longitudinal plane so the axes of the coils were perpendicular (Fig. 1). Because the radiated E field from the primary coil propagates transversally to the axis of the coil, in the

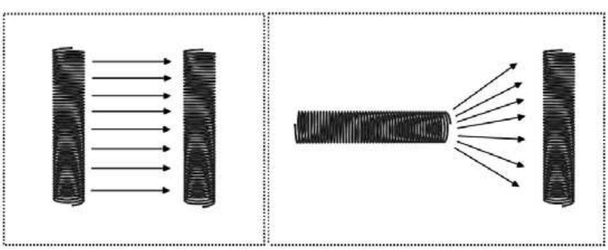

Figure 1. Schematic drawing of the coil alignment in 1st and 2nd series of the experiments. The arrows indicate the direction of radiated power between the transmitting and receiving coils.

1st alignment the value of the denominator is great because of $\bar{v}||\bar{r}$. In the 2nd alignment, the vector $\bar{v}$ is parallel to the vector $\bar{r}$ only in a very narrow area of the secondary coil. In such a setup, the longitudinal component of the E field cannot be longitudinally oriented with respect to the current pulses generated in the secondary coil, so the incident EM wave cannot act on the current pulse as Eq. (12) predicts. No measurable power was detected at the distance of 5 m between the coils for this coil alignment. Because the only difference in the two experiments is the orientation of the coils, we believe that the inverse L-W effect is the primary contributing factor for the increased wireless power transfer.

In the authors' point of view this effect is a plausible explanation for Teslas experimental finds in which there was an unusually high transfer of EM energy between two modes [6]. We note that in this work our analysis of the inverse L-W effect is entirely qualitative. The quantitative estimates of the effect should be explored in further work.

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