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INFLUENCE OF THE PLASMA COLUMN CROSS-SECTION NON-CIRCULARITY ON THE EXCITATION OF THE AZIMUTHAL SURFACE WAVES IN ELECTRON CYCLOTRON FREQUENCY RANGE BY ANNULAR ELECTRON BEAM

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Abstract—The initial stage of interaction between an annular beam of electrons, which rotate along Larmor orbits in the gap between a localized plasma column and a metal waveguide with a circular cross-section of its walls, and the electromagnetic waves of the surface type, is studied theoretically. These waves are extraordinary polarized; they propagate along the azimuthal angle across an axial external steady magnetic field in the electron cyclotron frequency range. The numerical analysis shows that changing the shape of the plasma filling cross section leads to corrections to the eigen frequency of the surface waves but does not cause a disruption of the resonance beam-wave instability development. Moreover, the conditions are found when appropriate choice of the shape can lead to increasing the instability growth rate by dozens of percent.

1. INTRODUCTION

The interaction between the charged particle flows and the eigen waves of the plasma filled waveguides is used for a long time in plasma electronics to generate and enhance electromagnetic radiation [1–4]. The application of plasma in these devices allows us to reach the important goals: increasing the electric current limit, expanding the frequency range of the excited oscillations, better control of the excitation process, etc.. This requires studying the spectra of the eigen oscillations which could be excited in the devices and the transportation of the charged particle flows which interact with the

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oscillations [5–7]. Special attention is paid to the study of the processes of wave excitation in magnetized plasma waveguides. Using these systems allows: first, to protect the waveguide wall from the active interaction with plasma and charged particle beam; second, to get wider spectrum of the eigen frequencies in comparison with the devices without external magnetic field.

The electronic devices based on an annular beam are expected to have a higher efficiency than devices based on longitudinal ones. For example, the instability growth rates and the efficiency of the energy exchange in annular lasers based on free electrons are higher ($\gamma^{2/3}$ times higher) than the longitudinal ones [2]. Moreover, the efficiency of generators based on longitudinal beams is limited by the device length. In devices based on annular beams, the particles rotate along the Larmor orbits in the gap between the chamber wall and the plasma column. They transfer their energy to the electromagnetic waves until the particles reach the plasma surface as a result of their deceleration. In this case, the particles can pass a way that is much larger than the size of the devices based on the longitudinal beams. First, this allows reaching the higher efficiency of the devices based on annular beams in comparison with those based on longitudinal beams. Second, this allows developing the more compact electronic devices.

The charged particle beams can also be used to produce plasma in the discharge chambers which are filled initially by neutral gas. The plasma is produced due to the gas ionization and the electromagnetic waves can be generated during the process. The results of studying the microwave generation from the plasma structures produced by the charged particle beams were presented in [8, 9]. But the effect of the produced plasma shape on the generated radiation was not studied. Plasma was confined by the magnetic field but there is no guarantee that the plasma column cross section was circular.

Among other things, the eigen electromagnetic waves of surface type are used widely in the plasma electronic devices, particularly to design the plasma antenna [10–12]. As a positive result of surface waves (SW) application for developing the plasma-antenna systems, we should like to indicate paper [13] devoted to peculiarities of the HF SW radar operation as a highly effective device that can provide over-the-horizon surveillance of any vessels, which move above sea level. The excitation of surface type waves by charged particle beams in the devices is well known to have some interesting features.

That is why the dispersion properties of the surface electromagnetic waves with extraordinary polarization which propagate along the azimuthal angle near the boundary of the plasma column have been studied in [14]. These modes were called as Azimuthal Surface Waves

(ASW). A detailed analysis of how the parameters of the plasma-beam system (the plasma and beam densities, the azimuthal mode number, the external axial magnetic field value, the radius of the plasma cylinder, the width of the vacuum gap) influence the initial stage of beam instability of the ASW's propagation in the range of the electron cyclotron frequency (let's call them Low Frequency (LF) waves here) was carried out in [15].

The periodic spatial inhomogeneity of the medium along the direction of the electromagnetic wave propagation can enhance the efficiency of plasma electronic devices [16, 17]. Therefore LF ASW propagation in magnetized waveguides filled partially by plasmas with noncircular cross-section has been studied in [18]. It was shown that in this case the frequency spectrum and the spectrum composition of the wave packet are predetermined by the shape of the plasma column cross section. Additionally, the effect that the plasma cross section shape has on the LF ASW dispersion properties has been investigated in the case when the angular period of the wave perturbation is exactly twice the inhomogeneity period of the plasma-dielectric interface [19].

But the theory of the plasma-beam instability is not complete yet, since the interaction of the charged particle beam with the eigen waves of different waveguides depends, essentially, on large number of factors including the dispersion properties of the waves, the waves' polarization, the spatial distribution of their fields, the geometry and the design features of the waveguides. Therefore, the aim of this paper is to study the influence of the non-circularity of the plasma column cross-section on the excitation of the LF ASW and to examine the possibility of breaking down this beam-plasma instability in the case of LF ASW propagation in such corrugated waveguides. Here only the initial stage of resonance beam instability of the LF ASW is studied.

The paper is arranged as follow. The problem is formulated in Section 2. The results of the numerical analysis are presented in Section 3. The conclusions are drawn in Section 4.

2. FORMULATION OF THE PROBLEM

The studied waveguide consists of a cylindrical metal waveguide with a circular cross-section with a radius of $R_2 = b$, which is partially filled by a plasma column with a radius of $R_1(\varphi) < R_2$, whose surface has non-circular cross-section. The waveguide chamber is made of a perfect conductor (metal). The external homogeneous steady magnetic field is directed along the waveguide axis, $\mathbf{B}_0||\mathbf{Z}$. The waveguide is assumed to be uniform along the cylinder axis, $\partial/\partial z \equiv 0$. Electromagnetic waves with extraordinary polarization (the nonzero components are E_r, E_φ, B_z) propagate inside a cylindrical chamber of the waveguide (Figure 1). These extraordinary modes (X-modes) propagate strictly perpendicular to the magnetic field, for example, their magnetic component has the following dependence on the coordinates and time $H_z(r, \varphi, t) = H_z(r) \exp(im\varphi - i\omega t)$.

The plasma occupies the space $r < R_1(\varphi)$. Plasma density is assumed to be homogeneous to study just the effect of the cross section shape of the plasma-vacuum interface on the plasma-beam interaction. This assumption is particularly valid for the cases when the surface waves are used to produce and sustain gas discharges. In gas discharges, the plasma density is homogeneous in an edge layer whose width is of the order of the plasma's wave penetration depth [20]. The cases of meta-material plasmas and solid state plasmas correspond to the assumption as well [21, 22].

The radial coordinate of the plasma-vacuum interface $R_1(\varphi)$ has the following dependence on the azimuthal angle:

$$R_1(\varphi) = a \cdot \left[1 + \sum_{n=1}^{\infty} h_n \cos\left(n\varphi + \varphi_n\right) \right], \tag{1}$$

where a is the averaged radius of the plasma column. The parameters h_n describe the relative depth of the corrugation of the plasma-vacuum interface and are small parameters of the problem. It is clear that any shape of the plasma column cross section can be modeled by the expansion of its radius in the Fourier series. The individual terms $\propto \cos(n\varphi + \varphi_n)$ in (1) lead to independent effects on the ASW dispersion properties if the small parameters h_n are taken into consideration only up to quadratic terms (the higher order terms are neglected). That is why the following analysis will be done here for only one term

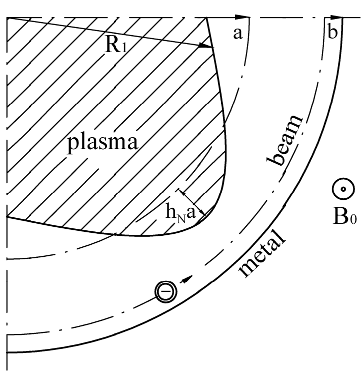

Figure 1. Schematic of the plasma-beam structure.

in (1): $R_1(\varphi) = a \cdot [1 + h_N \cos(N\varphi + \varphi_N)]$. This allows us to exclude the effects of other terms, but there are no fundamental problems associated with considering any finite number of terms from (1) in the studied dispersion equation. Moreover some values of the parameter N are of special physical interest: the term with $N = 1$ in (1) describes a violation of the coaxiality between the plasma column and the metal chamber, the term with $N = 2$ takes into account an ellipticity of the plasma column cross section, the cross section triangularity is taken into account by the term with $N = 3$, etc.. For instance, the effect of the plasma cross-section shape on sawtooth oscillations in the DIII-D tokamak plasmas is studied in [23]. The differences in nature between sawtooth perturbations, which propagate in plasma, that correspond to bean and oval cross section shapes, are found to be determined primarily by strong differences in electron heat transport for these corrugations.

The displacement vector and electric field are related by the permeability tensor of weakly collisional cold magnetized plasma. Let's write down two components of the permeability tensor which will be used below:

$$\varepsilon_{11} = \varepsilon_0 - \sum_\alpha \frac{\Omega_\alpha^2}{\omega^2 - \omega_\alpha^2} \equiv \varepsilon_1; \quad \varepsilon_{12} = i \sum_\alpha \frac{\omega_\alpha \Omega_\alpha^2}{\omega\left(\omega^2 - \omega_\alpha^2\right)} \equiv i\varepsilon_2. \tag{2}$$

Here Ω_α and ω_α are the plasma and cyclotron frequencies of the particle species α, respectively, and ε_0 is the dielectric constant of the metamaterial or of the crystal lattice of the semiconductor ($\varepsilon_0 > 1$), for gas plasmas $\varepsilon_0 = 1$.

The components of the ASW electric field can be expressed in terms of the magnetic field component H_z in the following way:

$$E_r = \frac{k}{k_\perp^2}\left(\mu\frac{\partial H_z}{\partial r} - \frac{i}{r}\frac{\partial H_z}{\partial \varphi}\right), \quad E_\varphi = \frac{k}{k_\perp^2}\left(i\frac{\partial H_z}{\partial r} + \frac{\mu}{r}\frac{\partial H_z}{\partial \varphi}\right). \tag{3}$$

Here $k = \omega/c$, the value $k_\perp^{-1}$ defines the penetration depth of the electromagnetic field into plasma, $k_\perp^2 = k^2\varepsilon_1(\mu^2 - 1)$, $\mu = \varepsilon_2/\varepsilon_1$. The considered surface waves can only propagate in frequency ranges where the value of $k_\perp^2$ is positive.

It is assumed that the plasma density is high enough to ensure that the inequality $\Omega_e^2 > \varepsilon_0\omega_e^2$ is valid. The inequality is realized surely in semiconductor plasmas but it can also be realized in laboratory gas plasmas when the external magnetic field is weak enough. In this case $k_\perp^2 > 0$ in the following frequency ranges: $\omega_{LH} < \omega < |\omega_e|$, $|\omega_e| < \omega < \omega_1 - |\omega_e|$, $\omega_{UH} < \omega < \omega_1$. Here ω_{LH} and ω_{UH} are the lower hybrid and upper hybrid frequencies respectively, $\omega_1 = 0.5\,|\omega_e| + \sqrt{\Omega_e^2 + \omega_e^2/4}$ is the cut-off frequency. In this paper, ASW propagating in the frequency ranges $\omega_{LH} < \omega < |\omega_e|$ and $|\omega_e| < \omega < \omega_1 - |\omega_e|$ are under consideration. Here, let' call them Low Frequency (LF) and High Frequency (HF) ranges, respectively.

It is assumed that an annular electron beam is injected into the gap $R_2 > r > R_1$ between the plasma column and the metal wall of the waveguide. The beam is modeled as a set of oscillators with the same transverse momentum $p_{\perp 0}$ and zero axial momentum, $p_z = 0$. The plasma-beam system is assumed to be compensated in respect to currents and charges. Such an electron beam is described by the equilibrium distribution function [7]:

$$f_0 = n_b \delta(p_\perp - p_{\perp 0}) \delta(p_z)/(2\pi\, p_{\perp 0}). \tag{4}$$

In (4), $p_{\perp 0} = m_e V_{\perp 0} \gamma$ is the transverse momentum of the electrons, $\gamma = \sqrt{1 + p_{\perp 0}^2 m_e^{-2} c^{-2}}$ the relativistic factor, and n_b the density of the beam electrons. The electrodynamical properties of the waveguide inner part occupied by the beam are described by the permeability tensor $\varepsilon_{ik}^{(b)}$. Three components of the tensor $\varepsilon_{ik}^{(b)}$, which will be used in the calculations below, are expressed as:

$$\varepsilon_{11}^{(b)} = 1 + \frac{\Omega_b^2}{\omega\gamma} \sum_{s=-\infty}^{+\infty} s^2 \left[\frac{\left(J_s^2(x)\right)'}{(s-y)\, k_\varphi V_{\perp 0}} + \frac{\omega J_s^2(x)}{(s-y)^2\, c^2 k_\varphi^2} \right]; \tag{5}$$

$$\begin{aligned} \varepsilon_{12}^{(b)} &= \frac{i\Omega_b^2}{\omega\, |\omega_e|} \sum_{s=-\infty}^{+\infty} s \left[\frac{\left(J_s(x)\, J_s'(x)\right)'}{s-y} + \frac{J_s(x)\, J_s'(x)}{(s-y)\, x} + \frac{J_s(x) J_s'(x) \omega V_{\perp 0}}{(s-y)^2 c^2 k_\varphi} \right] \\ &= -\varepsilon_{21}^{(b)}; \end{aligned} \tag{6}$$

$$\varepsilon_{22}^{(b)} = 1 + \frac{\Omega_b^2}{\omega\, |\omega_e|} \sum_{s=-\infty}^{+\infty} \left[\frac{2\left(J_s'(x)\right)^2}{s-y} + \frac{2x J_s'(x) J_s''(x)}{s-y} + \frac{\left(J_s'(x)\right)^2 V_{\perp 0}^2 y}{(s-y)^2\, c^2} \right]. \tag{7}$$

Here $\Omega_b^2 = 4\pi e^2 n_b m_e^{-1}$, $x = k_\varphi V_{\perp 0} \gamma / |\omega_e|$, $y = \omega\gamma / |\omega_e|$, $k_\varphi = |m|\, R_1^{-1}$, $J_s(x)$ is a Bessel function of the first kind, and a prime denotes the derivative of the function with respect to the argument.

Solving the set of Maxwell equations in the region occupied by the beam with the indicated components of the permeability tensor $\varepsilon_{ik}^{(b)}$ leads to the expressions for the ASW fields as a linear combination of Bessel functions of the first kind $J_m(\zeta)$, Bessel functions of the second kind $N_m(\zeta)$ and their derivatives with respect to their argument $\zeta = kr\sqrt{\psi_b}$. Here and below $\psi_b = \varepsilon_{22}^{(b)} + (\varepsilon_{12}^{(b)})^2 (\varepsilon_{11}^{(b)})^{-1}$.

The dispersion properties of the considered hybrid waveguide structure can be studied using the method of successive approximations. In the zero approximation, the plasma-vacuum interface is circular and coaxial with the metal chamber. Thus, ASW properties in this approximation can be described using the results presented in [14]. The ASW with different azimuthal numbers propagate independently when the curvature radius of the plasma-vacuum interface does not depend on the azimuthal angle. That is why it can be assumed that the electromagnetic perturbations propagate with a unique azimuthal mode number m in the zero approximation with respect to the small parameter h_N of the problem.

As a result of the periodic spatial inhomogeneity of the plasma-vacuum interface (see the expression (1)), the ASW propagate as wave packets in the present case. Each such packet consists of a fundamental harmonic, with a field which is proportional to $\exp(im\varphi - i\omega t)$ and an infinite number of satellite harmonics with fields which are proportional to $\exp[i(m \pm jN)\varphi - i\omega t]$ $(j = 1, 2, 3 \ldots)$. It is well known from the theory of wave propagation in media with periodical properties along some direction [24] that the amplitudes of the satellite harmonics are small values (the order of $(h_N)^j$) in comparison with the amplitude of the fundamental harmonic. A wave packet which consisted of the fundamental and the two nearest satellite $(j = 1)$ harmonics with fields which are proportional to $\exp[i(m \pm N)\varphi - i\omega\, t]$ has been considered in [18, 19] to study the dependence of the ASW dispersion properties on the shape of the noncircular cross section of the plasma-vacuum interface. Such an approach is based on the fact that taking into consideration the higher satellite harmonics gives a contribution to the eigen ASW frequency correction which is of the third and higher orders of the small parameter of the problem h_N.

Taking into consideration of the terms which are of the first order of h_N does not change the amplitude of the fundamental harmonic but leads to an appearance of nonzero small satellite harmonics of the wave field which change proportionally to $\exp[i(m \pm N)\varphi - i\omega t]$. In the framework of the second approximation in respect to the small parameter h_N, the amplitude of the fundamental harmonic obtains a correction of second order of h_N which together with the amplitude of the first satellite harmonic gives a correction of the second order of h_N to the dispersion relation, $D^{(2)}(\omega, m, N, \ldots) \propto h_N^2$. In such a way the dispersion relation obtained in the zero approximation, $D^{(0)}(\omega, m, \ldots) = 0$, transforms in the second approximation into the dispersion relation: $D^{(0)}(\omega, m, \ldots) + D^{(2)}(\omega, m, N, \ldots) = 0$. The expression for the term $D^{(2)}(\omega, m, N, \ldots)$ is too cumbersome to be presented here, but one can find it in detail in [18].

The following boundary conditions for the wave fields should be used to solve the problem of LF ASW propagation in the described waveguide:

- tangential component of the ASW electric field is equal to zero at the metal surface:

$$E_\varphi\ (r = R_2) = 0; \tag{8}$$

- tangential components of the electric and magnetic fields of the ASW are continuous at the noncircular plasma-vacuum interface:

$$\begin{aligned} E_\tau(r = R_1 + 0) &= E_\tau(r = R_1 - 0), \\ H_z(r = R_1 + 0) &= H_z(r = R_1 - 0); \end{aligned} \tag{9}$$

- the ASW fields are finite everywhere inside the waveguide, particularly at the waveguide axis:

$$H_z(r = 0) < \infty. \tag{10}$$

Applying these boundary conditions allows us to obtain the dispersion relation that describes the ASW excitation in the described waveguide:

$$\begin{aligned} &\frac{I'_m(k_\perp a)k}{k_\perp I_m(k_\perp a)} + \frac{\mu mka}{k_\perp^2 a^2} + D^{(2)}(\omega, m, N, \ldots) \\ &= \frac{im\varepsilon_{12}^{(b)}}{\varepsilon_{11}^{(b)}\zeta_1\sqrt{\psi_b}} - \frac{J'_m(\zeta_1) - \Phi N'_m(\zeta_1)}{\sqrt{\psi_b}\left[J_m(\zeta_1) - \Phi N_m(\zeta_1)\right]}, \end{aligned} \tag{11}$$

where $\Phi = [\frac{im\varepsilon_{12}^{(b)}}{\varepsilon_{11}^{(b)}\zeta_2} J_m(\zeta_2) - J'_m(\zeta_2)][\frac{im\varepsilon_{12}^{(b)}}{\varepsilon_{11}^{(b)}\zeta_2} N_m(\zeta_2) - N'_m(\zeta_2)]^{-1}$, $\zeta_1 = \zeta(a)$, $\zeta_2 = \zeta(b)$. The Equation (11) can be applied to the study the LF ASW excitation by the electron beam in the case of a noncircular cross section of the plasma column. It is analyzed here under the following resonance condition:

$$\omega = \omega_0 + \delta\omega = l\,|\omega_e|\,\gamma^{-1} + \delta\omega, \tag{12}$$

here l is an integer, ω_0 the eigen frequency of the LF ASW in the waveguide structure without the beam and the corrugation of the plasma-vacuum interface, and $\delta\omega$ the correction to the frequency caused by the interaction of the beam electrons with the plasma in the presence of the corrugation.

3. NUMERICAL ANALYSIS

The results of the numerical analysis of the Equation (11) are presented in Figures 2–5. The dashed line in all the figures shows the dependence of the ASW growth rate in the case of the circular cross section. The dependence of the resonance beam-plasma instability growth rate, normalized by the absolute value of the electron cyclotron frequency, on the effective wave number $k_{ef} = |m|c/(\Omega_e a)$ for the

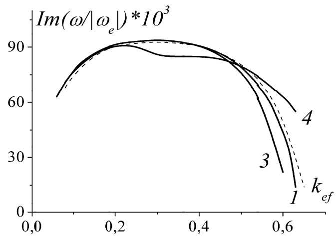

Figure 2. The LF ASW growth rate vs effective wave number; $m = -2$, $Z = 8$, $b = 1.1a$, $n_b = 10^{-3} n_{plasma}$, $h_N = 0.05$, $N = 1$, 3, 4.

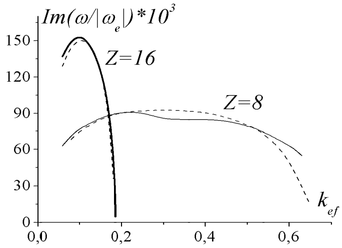

Figure 3. The same as in Figure 2, but for $N = 4$; $Z = 8$ and $Z = 16$.

ASW with the azimuthal number $m = -2$ is shown in Figure 2. The following parameters of the plasma-beam system have been used for the calculations: $Z \equiv \Omega_e/|\omega_e| = 8$, $b = 1.1a$, $h_N = 0.05$, $n_b = 0.001 n_{plasma}$. The numbers near the solid lines indicate the value of the corrugation parameter $N = 1$, 3, 4, which defines the number of the angular periods of the plasma-vacuum interface. The dependence of the ASW growth rate in the case $N = 2$ is not shown in the figure since the difference between the curves for the cases $N = 1$ and $N = 2$ is not visible. This difference exceeds 2.5% for large values of k_{ef} only when $k_{ef} > 0.6$. The effect of the noncircular shape of the plasma-vacuum interface in the case $N \neq 2|m|$ can be seen as a nonessential shift of the curve in the direction of the small values of k_{ef} (smaller values of k_{ef} correspond to larger radii of the plasma column and higher plasma densities). The shape of the curve becomes wider in the resonant case $N = 2|m|$. The calculations in the direction of the large values of k_{ef} are stopped when the values of the growth rate become small enough (more than ten times smaller) in comparison with its maximal value $\mathrm{Im}(\omega/|\omega_e|) = 0.094$. The left limit of the calculation range for k_{ef} was defined from the condition of when the method of successive approximations can be applied while taking into consideration the shape of the plasma column cross section: when $k_{ef} = 0.048$ the frequency correction caused by the noncircular shape of the plasma-vacuum interface reaches 19%.

It was shown in [19] that the largest effect of the plasma-vacuum interface curvature on the dispersion properties of the LF ASW is caused in the resonant case $N = 2|m|$, when the angular period of the wave perturbations is equal to just two periods of the inhomogeneity of the plasma-vacuum interface. This resonance is caused by the fact that

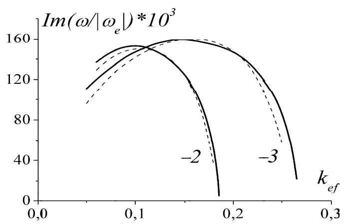

Figure 4. The same as in Figure 3, but for two azimuthal wave numbers $m = -2$ ($N = 4$) and $m = -3$ ($N = 6$); $Z = 16$.

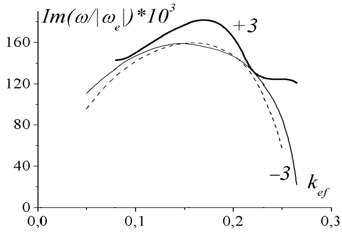

Figure 5. The same as in Figure 4, but for $m = +3$ and $m = -3$.

the LF ASW frequency does not depend on the sign of the azimuthal mode number m in plasmas without a magnetic field. An external magnetic field introduces differences in the dependences for the modes, which propagate with positive and negative azimuthal wave numbers. The difference in the frequencies of the LF ASW with opposite values of m is smaller for larger values of Z. The bold solid line in Figure 3 shows the dependence of the growth rate of the LF ASW with $m = -2$ for the case of a smaller external magnetic field in comparison with data for the Figure 2, $Z = 16$. The dependence of the LF ASW growth rate in the case $Z = 8$ is shown for comparison by the solid line. It can be seen that decreasing the magnetic field twice leads to the increase of the maximal value of the growth rate by 1.681 times, to the "bell" becoming narrower and its maximum position shifting to the smaller values of k_{ef}: from $k_{ef} = 0.225$ in the case $Z = 8$ to $k_{ef} = 0.1$ in the case $Z = 16$. The left limit of the calculation range, $k_{ef} = 0.06$, was defined once again from the condition that the method of successive approximations can be applied for taking into consideration the noncircular cross-section of the plasma-vacuum interface. The frequency correction caused by the noncircular shape of the plasma column reaches 23% at the left limit of the range. The effect of the noncircular cross-section of the plasma column on the LF ASW growth rates is seen as a nonessential (2%) increase of $\mathrm{Im}(\omega/|\omega_e|)$ in its maximum.

It was shown in [19] that the resonant effect of the noncircular cross section of the plasma column on the ASW dispersion properties is larger for larger values of the parameter N. That is why in this paper we studied the influence of the value N on the ASW growth rate while keeping the resonant condition $N = 2|m|$. The dependence of the LF ASW growth rates on the effective wave number is shown in Figure 4 for different values of N. The numbers correspond to the azimuthal wave numbers: $m = -3$ (with $N = 6$) and $m = -2$ (with $N = 4$). Other parameters of the beam-plasma system are the same as in Figure 3. Increasing $|m|$ leads to the "bell" becoming wider and the maximum shifting to the larger values of k_{ef}: from $k_{ef} = 0.1$ in the case $m = -2$ to $k_{ef} = 0.15$ in the case $m = -3$. The range of k_{ef} where the noncircular shape of the plasma-vacuum interface leads to a decrease of the growth rate can be seen also in Figure 4: this range is $0.12 < k_{ef} < 0.15$ in the case $m = -2$ and $k_{ef} \in [0.15; 0.205]$ in the case $m = -3$. It should be noted that one of the limits of these ranges corresponds to the k_{ef} for which the LF ASW frequency correction caused by the noncircular shape of the plasma-vacuum interface changes the sign. It is important to underline that the calculations in both cases $N = 4$ and $N = 6$ have been carried out for the same value of the corrugation depth $h_N = 0.05$. However, studying the dispersion properties of the waveguides with a noncircular cross section, one has to keep in mind the following important feature. Namely, the contribution of the terms with large values of the summation index n to the expression (1) can be neglected while modeling an arbitrary shape of the cross section due to a rapid decrease of the corrugation depth h_n that accompanies an increase of the value of n. The latter corresponds to the condition of applicability of the method of successive approximations. For example, when modeling the cross section of a square waveguide by the expression (1) the value of the corrugation depth h_n decreases proportionally to the inverse square of the summation index:

$$h_n = 4\sqrt{2}(-1)^{\frac{n}{4}} / \left[n^2 \ln\left(3 + 2\sqrt{2}\right) \right], \tag{13}$$

which allows one to apply the method of successive approximations correctly.

The effects of the noncircular cross section of the plasma column on the growth rates of the LF ASW with different signs of the azimuthal mode number $m = \pm 3$ are compared in Figure 5. The bold solid line shows the dependence of the growth rate of the LF ASW with the positive mode number $m = +3$ on the effective wave number. The thin solid line presents for comparison the dependence for the case of $m = -3$. At the limits of the k_{ef} calculation range the frequency correction for the LF ASW with $m = -3$ is small: it reaches 25% when $k_{ef} = 0.05$ and is less than 3% when $k_{ef} = 0.265$. For the LF ASW with the positive mode number $m = +3$, the correction is essential not only at the left limit, but also at right limit: it reaches 27% when $k_{ef} = 0.08$ and 17% when $k_{ef} = 0.265$.

4. CONCLUSIONS

The initial stage of the resonance beam-plasma instability of LF ASW propagating along a plasma column interface with a noncircular shape of its cross-section has been studied. The noncircular shape of the plasma column cross section essentially does not affect the LF ASW excitation by the annular electron beam. This is explained by the fact that the effective wave number range (the range of the beam- plasma system parameters) where the excitation of the LF ASW is effective [15] differs from the range within which the non-circularity of the plasma-vacuum interface has an essential effect on the LF ASW dispersion properties [18, 19]. This fact can be useful for technological purposes: small defects in gas discharge tubes do not decrease the efficiency of LF ASW excitation by the beam. It is correct even in the case when the angular period of the wave perturbations is just twice the period of inhomogeneity of the plasma-dielectric interface [19] and the non-circularity of the plasma column cross section has a most essential effect on the LF ASW dispersion properties.

In most cases, the non-circularity of the plasma column cross section even increases the growth rates of the resonance beam instability of the LF ASW. It is especially notable for waves with positive azimuthal mode numbers, $m > 0$.

The problem of wave interaction with the annular electron beam differs technologically from that with the linear electron beam. Designing electronic devices based on linear beams requires, usually, extensive calculations to provide an exact phase relation between the beam and the wave. That is why the corrugated line can have its corrugation period change along the direction of beam propagation. Such a method is not applicable to enhance wave- beam interaction for annular beams since a beam rotating along a Larmor orbit comes back again to the same part of the device.

The beam excitation of the LF ASW in the corrugated metal chamber is not studied here since it is well known that the effect of the corrugation of the plasma-vacuum interface on the wave dispersion properties is much larger than the effect of the metal chamber non-circularity. This feature of the LF ASW dispersion properties comes from the fact that the power of the LF ASW is concentrated just near the plasma-vacuum interface but not near the metal wall.

From a practical point of view, the LF ASW excited by annular electron beams could be applied to the of sustaining microwave gas discharges and as operating modes in plasma-antenna and plasma-radar systems. The efficiency of the gas discharges [20] and the plasma-antenna system (see, e.g., [25]) depends on the type of the operating mode and the stability of the process of these modes' excitation. Since accidental deviation of the plasma column cross-section from a circular one does not lead to an essential change of the ASW growth rate value, then these modes seem to be suitable for application in such devices as operating ones.

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