Progress In Electromagnetics Research B, Vol. 44, 367–382, 2012

PARAMETER IDENTIFIABILITY OF MONOSTATIC MIMO CHAOTIC RADAR USING COMPRESSED SENSING

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Abstract—Compressed sensing (CS) has attracted significant attention in the radar community. The better understanding of CS theory has led to substantial improvements over existing methods in CS radar. But there are also some challenges that should be resolved in order to benefit the most from CS radar, such as radar signal with low signal to noise ratio (Low-SNR). In this paper, we will focuses on monostatic chaotic multiple-inputmultiple-output (MIMO) radar systems and analyze theoretically and numerically the performance of sparsity-exploiting algorithms for the parameter estimation of targets at Low-SNR. The novelty of this paper is that it capitalizes on chaotic coded waveform to construct measurement operator incoherent with noise and singular value decomposition (SVD) to suppress noise. In order to improve the robustness of azimuth estimation interpolation method is applied to construction of sparse bases. The gradient pursuit (GP) algorithm for reconstruction is implemented at Low-SNR. Finally, the conclusions are all demonstrated by simulation experiments.

1. INTRODUCTION

Chaotic waveform radars transmit pseudorandom signals and apply coherent reception to achieve low probability of interception (LPI) and low probability of detection (LPD). Chaotic waveform radars have the unique property that allows them to achieve high resolution in both range and Doppler which can be independently controlled by varying the bandwidth and integration time respectively. They also have excellent resistance to jamming and interference. Another

Received 27 July 2012, Accepted 27 September 2012, Scheduled 1 October 2012
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advantage of chaotic waveform radars is their ability to efficiently share the frequency spectrum, thereby allowing a number of chaotic waveform radars to operate over the same frequency band with minimal cross-interference. This spectrally parsimonious feature can be used to integrate several chaotic waveform radars to a network centric platform. Therefore, this field has attracted more and more attention [1–4].

Unlike a conventional transmit beamforming radar system that uses highly correlated waveforms, a MIMO radar system transmits multiple independent waveforms via its antennas [5, 6]. A MIMO radar system is advantageous in both widely separated antennas scenario and collocated antennas scenario. In the first scenario, the transmit antennas are located far apart from each other relative to their distance to the target, which make the radar system offer considerable advantages for estimation of target parameters, such as location and velocity. In the second scenario, the transmit antennas and receive antennas are located close to each other relative to the target that all antennas view the same aspect of the target, which enables the MIMO radar to achieve superior resolution in terms of direction finding [7]. The latter scenario, which is adopted in this paper, performs direction finding (DF) for monostatic MIMO chaotic waveform radar using compressive sensing.

In this paper, the chaotic waveform radar concept is extended to an array of N_T transmit antenna and N_R receive antenna. When independent chaotic waveform sources are transmitted from each antenna the approach may be viewed as a special case of MIMO radar and direction finding may be derived. In this case, the monostatic MIMO chaotic radar is equipped with N_T transmit and N_R receive antennas that are close to each other relative to the target, so that the RCS does not vary between the different paths. In this case, the phase differences induced by transmit and receive antennas can be exploited to form a long virtual array with $N_T N_R$ elements. This enables the MIMO chaotic radar system to achieve superior spatial resolution as compared to a traditional chaotic waveform radar system.

CS is a new paradigm in signal processing that trades sampling frequency for computing power and allows accurate reconstruction of signals sampled at rates many times less than the conventional Nyquist frequency, received considerable attention recently and has been applied successfully in diverse fields. The theory of CS states that a K-sparse signal $\boldsymbol{\eta}$ of length N_η can be recovered exactly from few measurements with high probability via linear programming. Let $\boldsymbol{\Psi}$ denote the basis matrix that spans this sparse space, and $\boldsymbol{\Phi}$ a measurement matrix. The convex optimization problem arising from

CS is formulated as follows:

$$\min \|\boldsymbol{\eta}\|_1, \quad \text{subject to} \quad \mathbf{X} = \boldsymbol{\Phi\Psi\eta} + \mathbf{N} \tag{1}$$

where $\boldsymbol{\eta}$ is a sparse vector with K principal elements and the remaining elements can be ignored; $\boldsymbol{\Phi}$ is an matrix incoherent with $\boldsymbol{\Psi}$. $\mathbf{N}$ denotes the interference term which is a complex Gaussian noise vector.

CS techniques offer a framework for the detection and allocation of sparse signals for radar with a reduced number of samples [8, 9]. The application of compressive sensing to MIMO radar system was investigated in [10–12]. The problem discussed in [10] is of the targets angular separation and reduction of the physical array elements required for the system. [10] uses CS to reduce the number of real receiving elements so as to obtain a sparse MIMO array. The sensing matrix is obtained from the conventional digital beam forming matrix by selecting only a subset of rows corresponding the sparse MIMO receive channels. In [11], the DOA estimation for MIMO radar in a distributed scenario is proposed. The transmitted waveforms in MIMO radar are known at each receive antennas, so that each receive antenna can construct the basis matrix locally, without the knowledge of the received signal at other antennas. In [12], CS approach to accurately estimate properties (position, velocity) of multiple targets was exploited for MIMO radar. The sampled outputs of the matched filter at the receivers are used to estimate the positions and velocities of multiple targets using MIMO radar systems with widely separated antennas by employing sparse modeling and CS.

In this paper, we present one specific scenario in which the proposed system improves the performance of estimating target parameters and reducing sampling rate considerably at Low-SNR. Unlike the scenario considered in [10–12], this paper capitalizes on chaotic coded waveform to construct measurement operator at the transmitter and presents a new method of how the influence of deviation and noise in the MIMO radar can be reduced through the use of CS. Further, we provide SVD method and interpolation method for the noise suppression and robustness respectively for the proposed approach. We also provide simulation results to show that the proposed approach can accomplish superresolution in MIMO chaotic waveform radar systems at Low-SNR than existing methods, such as amplitude and phase estimation (APES), Capon method, generalized likelihood ratio (GLR) [6], and Bayes-ics in CS-based monostatic noise MIMO radar [13, 14].

2. SYSTEM ARCHITECTURE

In this section, we describe a signal model for the MIMO radar. The model focuses on the effect of the target spatial properties including azimuth, Doppler and range effects. For simplicity, we consider a monostatic MIMO radar system, shown in Figure 1, with an N_T-element transmit array and an N_R-element receive array, both of which are closely spaced uniform linear arrays (ULA). The targets and antennas all lie in the same plane. Assume that the inter-element spaces of the transmit and receive arrays are denoted by Δ_T and Δ_R, respectively. The targets appear in the far-field of transmit and receive arrays.

At the transmit site, N_T different waveforms transmitted are modeled as

$$s_j(t) = u_j(t) \exp(i2\pi f_c t) \tag{2}$$

where $1 \le j \le N_T$ and f_c is the carrier frequency of the waveform. It is assumed that the chaotic MIMO radars transmit N_T-array chaotic modulated signals, consisting of L pulses and with each pulse containing N sub-pulses. At the transmit site, N_T different bandlimited and chaotic logistic signal transmitted are modeled as

$$u_j(t) = \sum_{l=1}^{L} \sum_{n=1}^{N} A_j \mu_j(t) \exp\left(i2\pi\varphi_j^n t/T_s + \phi_j^{l,n}\right) \tag{3}$$

$$\mu_j(t) = \begin{cases} \mu_j^n, & (n-1)T_s < t \le nT_s \\ 0, & \text{else} \end{cases} \tag{4}$$

where $1 \le j \le N_T$. T_s is a subpulse width. NT_s is a single pulse width. $\{\mu_j^n\}_{n=1}^N$ are discrete chaotic pseudo-random sequences.

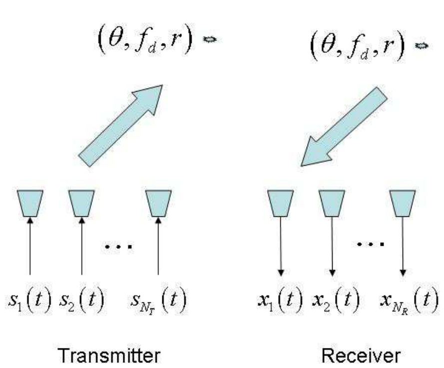

Figure 1. Monostatic MIMO chaotic radar scenario.

$\varphi_j^l \in \{1, 2, \ldots, L\}$ is the chaotic frequency-hopping code. $\phi_j^{l,n}$ is chaotic phase code, and LN is the length of the code.

Let $s_j(t)$ denote the waveform transmitted by the j-th transmit antenna.

$$\mathrm{a}_j(\theta) = \exp\left(i\left(2\pi/\lambda\right)(j-1)\Delta_T \sin\theta\right) \tag{5}$$

is the transmit array steering vector, where λ denotes the wavelength.

$$\mathbf{A}(\theta) = \left[a_1(\theta), a_2(\theta), \ldots, a_{N_T}(\theta)\right] \tag{6}$$

is the transmitted signal steering matrix.

$$b_j(\theta) = \exp\left(i\left(2\pi/\lambda\right)(j-1)\Delta_R \sin\theta\right) \tag{7}$$

is the steering vector of the receive array. We further define

$$\mathbf{B}(\theta) = \left[b_1(\theta), b_2(\theta), \ldots, b_{N_R}(\theta)\right] \tag{8}$$

as the received signal steering matrix.

The backscatter from a point target observed at the j-th receiver $(1 \leq j \leq N_R)$, is given by

$$x_j(t; \theta, r, f_d) = \mathbf{S}(t-\tau)\,\mathbf{e}^{i2\pi f_d t}\mathbf{A}^T(\theta) b_j(\theta)\boldsymbol{\eta} + \mathbf{N}_j(t) \tag{9}$$

where $\mathbf{S}(t) = [s_1(t), s_2(t), \ldots, s_{N_T}(t)]$, $(\cdot)^T$ denotes the transpose. $\boldsymbol{\eta}$ is the complex amplitude proportional to the radar-cross-section (RCS) of the point target, θ is the azimuth parameter, f_d is the Doppler parameter (corresponding to its radial velocity with respect to the radar), r is the range parameter, and $\mathbf{N}_j(t)$ denotes the complex Gaussian noise term. The unknown parameters, to be estimated from $x_j(t)$, are azimuth θ, Doppler f_d and range r.

3. COMPRESSED SENSING FOR MIMO CHAOTIC RADAR

3.1. Chaotic Waveforms

Chaotic sequences are usually generated using discrete chaotic maps, such as the logistic map, triangular map, and exponential map. The logistic map is one of the simplest and most widely studied

$$x^{k+1} = \beta x^k \left(1 - x^k\right) \tag{10}$$

where $x^k \in [0, 1]$, $\beta \in [3.57, 4]$, and α is called a bifurcation parameter. Depending on the value of β, the dynamics of this system can change dramatically, exhibiting periodicity or chaos. For $\beta \in [3.57, 4]$, the sequence is nonperiodic and nonconverging. The parameter $\beta = 4$ is adopted in this paper. The sequence $\{x^k\}$ is used as the chaotic sequences throughout this paper. To improve the random and the efficiency of chaotic Logistic coded algorithm, a new algorithm is proposed. This algorithm gives a new amplitude modulation method of chaotic sequences. Firstly, random numbers x_j^0 $(1 \leq j \leq N_T)$ from uniform distribution are used to generate N_T real chaos sequences by Logistic map. Then N_T chaotic sequences are sorted to generate an array of indices by ordinal numbers corresponding to the original sequences, and these index sequences are used for chaotic frequency-hopping modulation. Finally, the N_T chaotic sequences are used for chaotic amplitude modulation of waveform.

The main steps of the proposed algorithm are as follows:

1) Generate uniform distribution random numbers x_j^0 $(1 \leq j \leq N_T)$ in the interval $(0, 1)$ as the initial values.
2) Generate N_T chaotic sequences by formula (10), then take the last state values as the new initial value to generate N_T new chaotic sequence, and so on. After T_c $(T_c > LN)$ times' iteration, N_T different chaotic sequences are generated. Selecting the last LN elements of sequences, these new sequences are generated.
3) Sort N_T real chaotic sequences and generate N_T new integer sequences by ordinal numbers corresponding to the chaotic sequences, these new sequences are use to realize chaotic frequency-hopping modulation.
4) Use the N_T chaotic sequences to realize chaotic amplitude modulation by formula (4).

3.2. Compressed Sensing in Receiver

For simple but without loss of generality, the treatment of CS-MIMO chaotic radar focuses on the azimuth estimation at Low-SNR ignoring Doppler effects and range. The proposed approach for CS-MIMO chaotic waveform radar is based on two key observations. First, there exists a small number of targets, the unknown parameters θ are sparse in the angle space, i.e., $\boldsymbol{\eta} = [\eta_1, \eta_2, \ldots, \eta_{N_\eta}]^T$ is a sparse vector. A non-zero element with index j in $\boldsymbol{\eta}$ indicates that there is a target at the azimuth angles θ_j. Second, modulated version of the stationary chaotic waveform process transmitted as radar waveforms $s_j(t)$ $(1 \leq j \leq N_T)$ of the target form a measurement matrix

$$\boldsymbol{\Phi}_j = \left[s_1(t), s_2(t), \ldots, s_{N_T}(t)\right] \otimes \mathbf{I} \tag{11}$$

where $\boldsymbol{\Phi}_j$ is incoherent with the frequency base $\boldsymbol{\Psi}_j$ $(1 \leq j \leq N_R)$. $\otimes$ denotes kronecker product, and $\mathbf{I}$ is a $1 \times N_I$ matrix as shown in formula (16). By combining these observations we can both eliminate

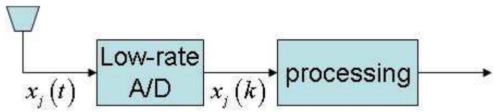

Figure 2. CS-based MIMO chaotic waveform radar receivers for the transmitters.

the matched filter in the radar receiver and lower the receiver A/D converter bandwidth using CS principles. Consider a new design for a radar system that consists of the following components. The transmitter is the same as in a classical MIMO chaotic waveform radar; the transmit antenna emit the bandlimited and chaotic logistic signal. However, the receiver does not consist of a matched filter and high-rate A/D converter but rather only a low-rate A/D converter that operates not at the Nyquist rate but at a rate proportional to the target sparsity (see Figure 2).

By CS theory, we can construct a $N_T N_I \times N_I$ basis matrix $\mathbf{\Psi}_j$ for the j-th receive antenna as

$$\mathbf{\Psi}_j = \text{diag}\left(\mathbf{A}^T(\theta)\,\text{b}_j(\theta), \ldots, \mathbf{A}^T(\theta)\,\text{b}_j(\theta)\right) \tag{12}$$

Considering the discrete azimuth grid, it is assumed that N_θ is the resolution of the angle, N_I is an odd number, we have linear projections of the received signal at the j-th antenna as

$$x_j(k) = \mathbf{\Phi}_j \mathbf{\Psi}_j \boldsymbol{\eta} + \mathbf{N}_j(k) \tag{13}$$

where $1 \leq k \leq N_s$ and N_s is the snapshot number. The measurement matrix $\mathbf{\Phi}_j$ is incoherent with the basis matrix $\mathbf{\Psi}_j$ (the signal steering matrix of a discrete-angle). Placing the output of N_R receive antennas, i.e., $x_1, x_2, \ldots, x_{N_R}$, in measurement vector $\mathbf{X}$ one have

$$\mathbf{X} = \mathbf{\Phi}\mathbf{\Psi}\boldsymbol{\eta} + \mathbf{N} \tag{14}$$

where $\mathbf{X} = [x_1(1), \ldots, x_1(N_s), x_2(1), \ldots, x_{N_R}(N_s)]^T$ is the $N_R N_s \times 1$ virtual data vector associated with the CS-MIMO chaotic waveform radar.

$$\mathbf{\Phi} = \text{diag}(\mathbf{\Phi}_1,\ \mathbf{\Phi}_2, \ldots, \mathbf{\Phi}_{N_R}) \tag{15}$$

is a $N_R N_s \times N_T N_R N_I$ diagonal matrix of the discrete-time waveform. $\mathbf{\Phi}_j = \mathbf{S} \otimes \mathbf{I}$, where

$$\mathbf{I} = \left[e^{-i2\pi N_I/2N_I}, e^{i2\pi(N_I-1)/2N_I}, \ldots, e^{i2\pi N_I/2N_I}\right] \tag{16}$$

represents the interpolation.

$$\mathbf{\Psi} = \left[\mathbf{\Psi}_1^T, \mathbf{\Psi}_2^T, \ldots, \mathbf{\Psi}_{N_R}^T\right]^T \tag{17}$$

is a $N_T N_R N_I \times N_\theta N_I$ basis matrix.

$$\mathbf{N} = \left[\mathbf{N}_1^T, \mathbf{N}_2^T, \ldots, \mathbf{N}_{N_R}^T\right]^T \tag{18}$$

is a $N_R N_s \times 1$ matrix of the noise term.

3.3. Sparse Recovery

The received data is complex valued, while the standard sparse optimization algorithms have been developed for real valued signals. We break the signal into its real and imaginary parts as follows

$$\begin{aligned} \mathbf{Y} &= \begin{bmatrix} \Re(\mathbf{X}) \\ \Im(\mathbf{X}) \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} \Re(\mathbf{\Phi\Psi}) & -\Im(\mathbf{\Phi\Psi}) \\ \Im(\mathbf{\Phi\Psi}) & \Re(\mathbf{\Phi\Psi}) \end{bmatrix}, \\ \boldsymbol{\alpha} &= \begin{bmatrix} \Re(\boldsymbol{\eta}) \\ \Im(\boldsymbol{\eta}) \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} \Re(\mathbf{N}) \\ \Im(\mathbf{N}) \end{bmatrix} \end{aligned} \tag{19}$$

where $\Re(\cdot)$ and $\Im(\cdot)$ indicate the real and imaginary part of the complex number, respectively. Equation (14) becomes

$$\mathbf{Y} = \mathrm{T}\boldsymbol{\alpha} + \mathbf{Z} \tag{20}$$

Our goal is to solve for the vector $\boldsymbol{\eta}$ in Equation (14). Solving for $\boldsymbol{\eta}$ in Equation (14) can be viewed as an ordinary inverse problem. Sparse optimization strategies such as convex relaxation, nonconvex (often gradient based) local optimization or greedy search strategies are used in practice. In this paper, we apply the GP method [15] (as discussed in Section 4 below) in the monostatic MIMO noise radar to reconstruct the sparse target scene vector $\boldsymbol{\eta}$ in Equation (14).

4. GP METHOD FOR SPARSE RECOVERY

In this section, let Γ^j be a set containing the indices of the elements selected up to and including iteration j. Using this index set as a subscript enables the matrix $\mathbf{T}_{\Gamma^j}$ to be a submatrix of $\mathbf{T}$ containing only those columns of $\mathbf{T}$ with indices in Γ^j. The same convention is used for vectors. For example, $\boldsymbol{\eta}_{\Gamma^j}$ is a subvector of $\boldsymbol{\eta}$ containing only those elements of $\boldsymbol{\eta}$ with indices in Γ^j. In addition, we use the superscript j in the subscript Γ^j of $\boldsymbol{\eta}_{\Gamma^j}$ to label the j-th iteration However, we sometimes resort to using superscript j of $\boldsymbol{\eta}^j$ to label the j-th iteration. The same convention is used for vectors. We have inner products between vectors with angled brackets (e.g., $\langle \boldsymbol{\eta}, \boldsymbol{\eta} \rangle = \boldsymbol{\eta}^T \boldsymbol{\eta}$).

The GP algorithm can be summarized as follows [15]:

1) Initialize $\mathbf{r}^0 = \mathbf{Y}$, $\boldsymbol{\eta}^0 = 0$ and $\Gamma^0 = \emptyset$;

2) For $j = 1$; $j \triangleq j + 1$ until the stopping criterion is met:

a) $\gamma^j = \mathbf{T}^T \mathbf{r}^{j-1}$;
b) $k^j = \arg\max_k \left|\gamma_k^j\right|$;
c) $\Gamma^j = \Gamma^{j-1} \cup k^j$;
d) $\mathbf{d}_{\Gamma^j} = \mathbf{T}^T \mathbf{r}^{j-1}$;
e) $\boldsymbol{\delta}_{\Gamma^j} = \left\langle \mathbf{r}^{j-1}, \mathbf{T}\mathbf{d}_{\Gamma^j} \right\rangle / \|\mathbf{T}\mathbf{d}_{\Gamma^j}\|_2^2$;
f) $\boldsymbol{\eta}_{\Gamma^j} = \boldsymbol{\eta}_{\Gamma^{j-1}} + \boldsymbol{\delta}_{\Gamma^j} \mathbf{d}_{\Gamma^j}$;
g) $\mathbf{r}^j = \mathbf{r}^{j-1} - \mathbf{T}_{k^j} \boldsymbol{\delta}_{\Gamma^j} \mathbf{d}_{\Gamma^j}$

3) Output $\mathbf{r}^j$ and $\boldsymbol{\eta}^j$.

The problem addressed in this section is to solve the Equation (20) by using GP algorithm and SVD method, determine the number of targets in the received signals and estimate the azimuth angles of targets at Low-SNR. We propose the singular value decomposition (SVD) of the complex matrix $\mathbf{X}\mathbf{X}^H$ (where $\mathbf{X}^H$ is the conjugate transpose of $\mathbf{X}$)

$$\mathbf{X}\mathbf{X}^H = \mathbf{U}_X \Lambda_X \mathbf{V}_X^H \tag{21}$$

where $\mathbf{U}_X$ and $\mathbf{V}_X$ denote the $N_R N_s \times N_R N_s$ complex unitary matrixes, and Λ_X is a diagonal matrix, with nonnegative diagonal elements in decreasing order

$$\Lambda_X = \operatorname{diag}\left(\lambda_1, \ldots, \lambda_{N_R}, \ldots, \lambda_{N_R N_s}\right) \tag{22}$$

The vector $\boldsymbol{\upsilon}$ is defined by

$$\boldsymbol{\upsilon} = \left[\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_{N_R}}, 0, \ldots, 0\right]^T \tag{23}$$

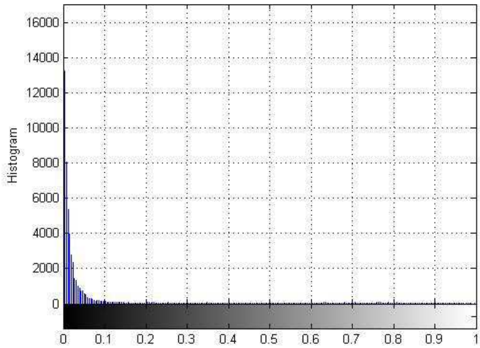

Figure 3. Histogram of the off-diagonal elements of the Gramm matrix.

from which we can define

$$\mathbf{X}_{\upsilon} = \mathbf{U}_X \boldsymbol{\upsilon} \tag{24}$$

Solving for $\boldsymbol{\alpha}_{\upsilon}$ in (20) can be viewed as the problem

$$\min_{\alpha_{\upsilon}} \|\boldsymbol{\alpha}_{\upsilon}\|_1, \quad \text{subject to} \quad \|\mathbf{Y}_{\upsilon} - \mathbf{T}\boldsymbol{\alpha}_{\upsilon}\|_2^2 < \varepsilon \tag{25}$$

where

$$\mathbf{Y}_{\upsilon} = \begin{bmatrix} \Re(\mathbf{X}_{\upsilon}) \\ \Im(\mathbf{X}_{\upsilon}) \end{bmatrix} \tag{26}$$

The GP algorithm is proposed to solve (25) for the estimation of the azimuth parameter θ and vector $\boldsymbol{\alpha}_{\upsilon}$. Then, we reshape the $N_{\theta}N_I \times 1$ vector $\boldsymbol{\alpha}_{\upsilon}$ into a $N_I \times N_{\theta}$ matrix $\mathbf{V}_{\alpha}$, and treat the columns of $\mathbf{V}_{\alpha}$ as vectors, obtaining a vector $\boldsymbol{\nu}_{\alpha}$ of the maximum element from each column vector.

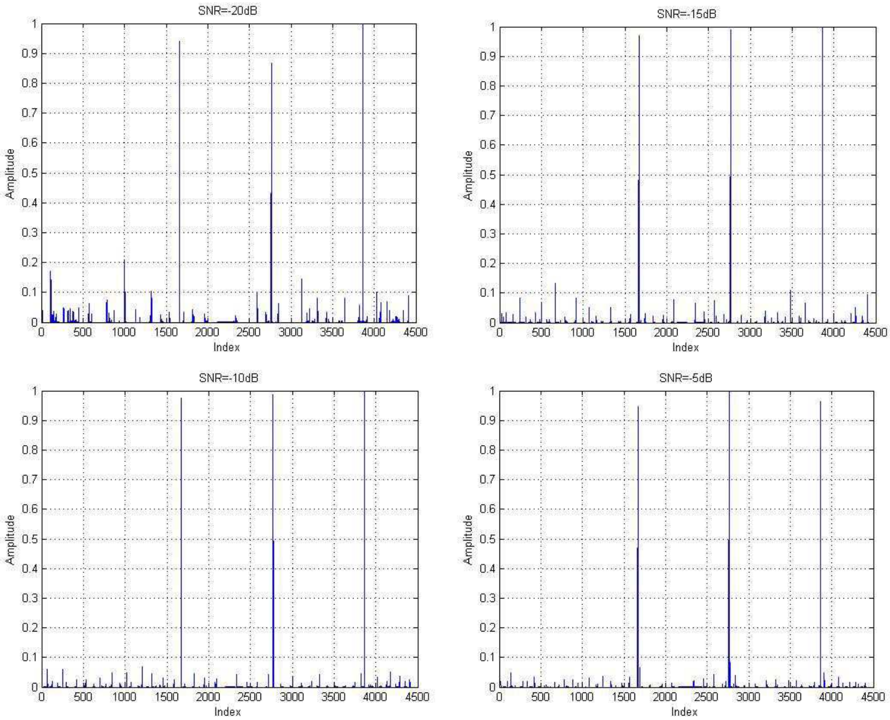

Figure 4. Estimation of the vector $\boldsymbol{\alpha}_{\upsilon}$.

5. SIMULATION RESULTS

In this section, the simulation is carried out to illustrate the correctness and the performance of the proposed method. A MIMO monostatic radar system, with uniform linear array (ULA) in which the half-wavelength spacing between adjacent antennas is used both for transmitting and for receiving. The transmitted waveforms are chaotic coded signals. The transmitted pulse width is 55 μs, and the pulse repeat period is 500 μs.

5.1. Coherence of Sensing Matrix

The sensing matrix $\mathbf{\Phi\Psi}$ was designed by making the Gram matrix as close to the identity matrix as possible. In the case, it was experimentally shown that the obtained sensing matrix leads to a better the sensing matrix and thus to improved CS reconstruction performance [13]. The histogram of the off-diagonal elements of the

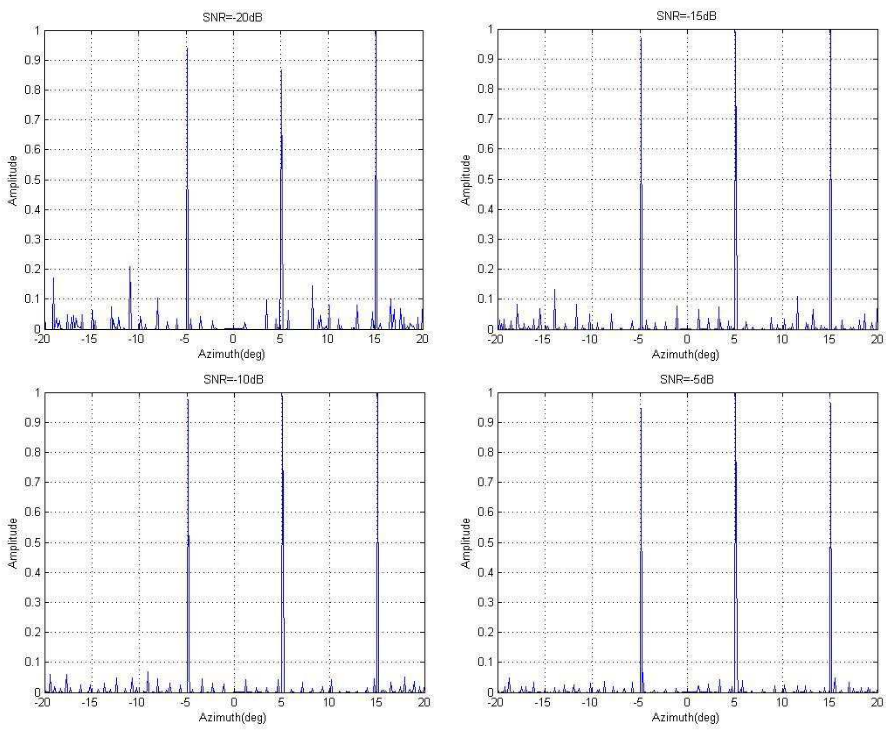

Figure 5. Estimation of the vector $\boldsymbol{\nu}_\alpha$.

Gramm matrix $(\mathbf{\Phi\Psi})^H \mathbf{\Phi\Psi}$ is shown in Figure 3. From Figure 3, one can see that the proposed chaotic coded signal has an ideal Gram matrix close to the identity matrix.

5.2. Estimation Using the Proposed Method

As parameter values N_T, N_I and N_s are increased, the reconstruction accuracy as well as the SNR advantage increases also according to Equation (14) and CS theory. The number N_R of receive antennas is greater than the number of targets. Here, $N_T = 100$, $N_R = 14$ and $N_I = 11$ are considered. The number of snapshots is $N_s = 70$ during the simulation. $\Delta_a = 0.1°$, azimuth angle of minimum resolution, is used during the simulation experiments. Assume that three targets locate at $\theta_1 = -5.0°$, $\theta_2 = 5.0°$ and $\theta_3 = 15.0°$ with the same complex amplitudes $\boldsymbol{\eta} = 1$, $K = 3$. The estimated result for azimuth angles θ, vector $\boldsymbol{\alpha}_v$, and vector $\boldsymbol{\nu}_\alpha$ are shown in Figure 4 and Figure 5. From Figure 4 and Figure 5, one can see that the azimuth angles of the

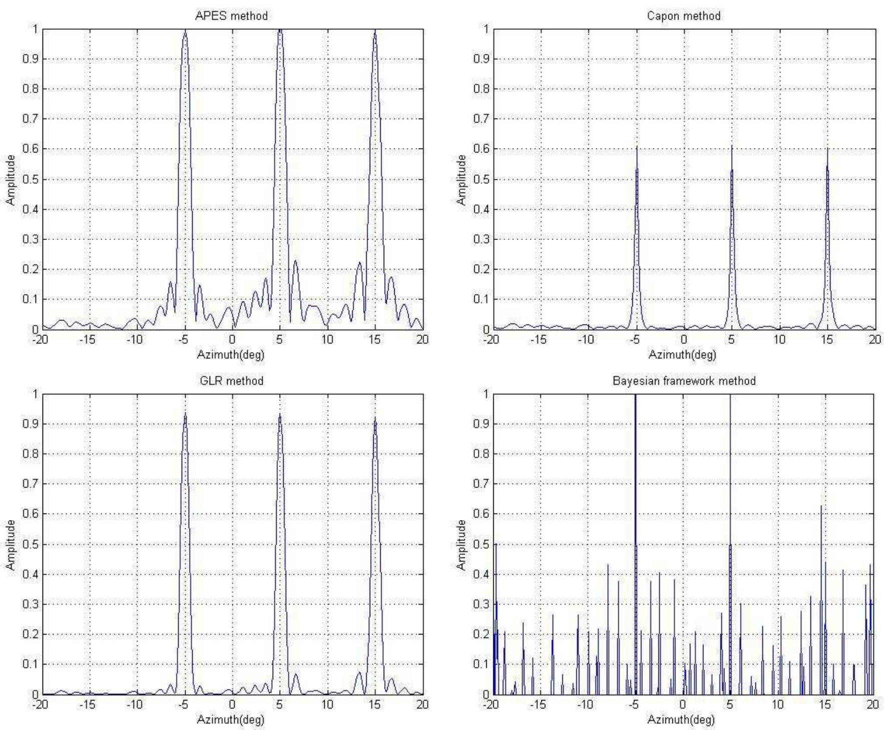

Figure 6. Estimated results with $N_s = 256$.

targets can accurately be estimated by the proposed method at low enough SNR ($-25\,\text{dB} \sim -5\,\text{dB}$).

5.3. Estimation Using the Existing Methods

$N_T = 100$ and $N_R = 12$, are considered $\Delta_a = 0.1°$, azimuth angle of minimum resolution, is used during the simulation experiments. Assume that three targets locate at $\theta_1 = -5.0°$, $\theta_2 = 5.0°$ and $\theta_3 = 15.0°$ with the same complex amplitudes $\boldsymbol{\eta} = 1$, SNR $= -20\,\text{dB}$, $K = 3$. Figure 6 and Figure 7 show the reconstructed target scene using APES method, Capon method, GLR method [6], and Bayesian framework method (i.e., Bayes-ics in CS-based monostatic noise MIMO radar [13, 14]).

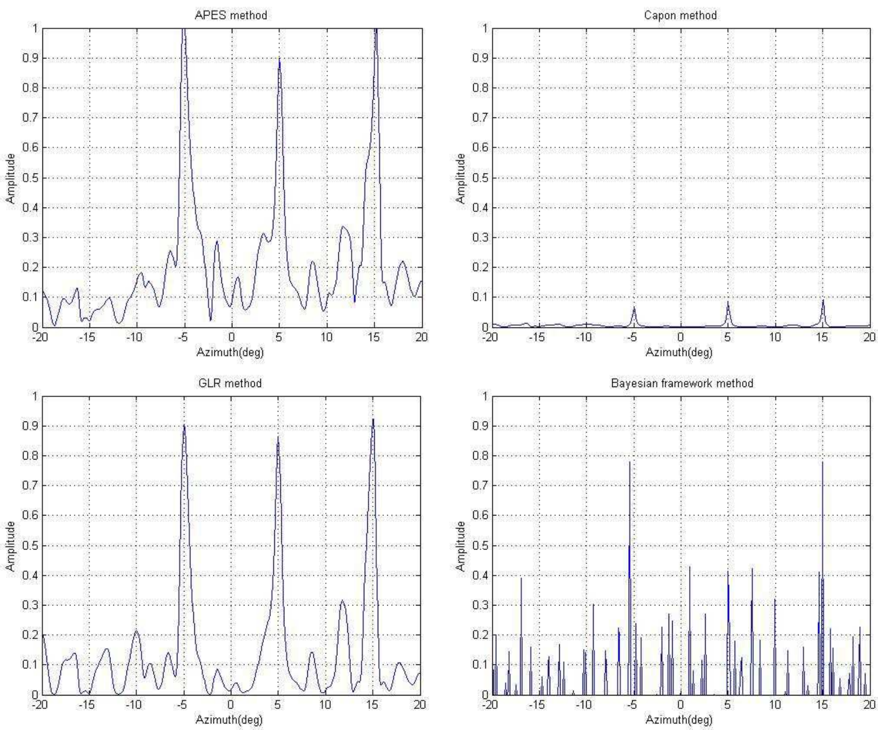

Figure 7. Estimated results with $N_s = 110$.

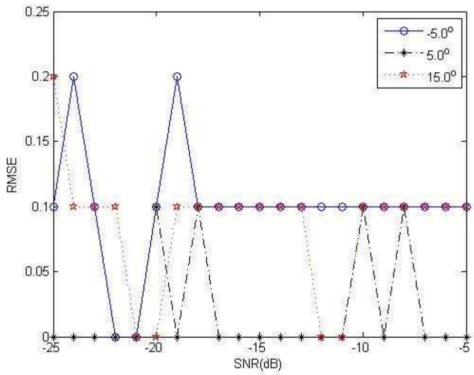

Figure 8. RMSE of azimuth angles versus SNR.

5.4. Robustness of the Proposed Method

$N_T = 100$, $N_R = 14$ and $N_I = 11$ are considered. The number of snapshots is $N_s = 70$ during the simulation. $\Delta_a = 0.1°$, azimuth angle of minimum resolution, is used during the simulation experiments. Assume that three targets locate at $\theta_1 = -5.0°$, $\theta_2 = 5.0°$ and $\theta_3 = 15.0°$ with the same complex amplitudes $\boldsymbol{\eta} = 1$, $K = 3$, and the number of Monte-Carlo trials is 10000. The root mean square error (RMSE) of the azimuth angles versus SNR are shown in Figure 8. It can be seen from Figure 8, the proposed method has low RMSE for azimuth angles estimation.

6. CONCLUSION

In this paper, we investigate sparsity-exploiting algorithm for the estimation of targets in the azimuth domain at Low-SNR. A new waveform design technique that exploits the chaotic behavior of nonlinear dynamical systems to generate a quasi-orthogonal set is developed. The chaotic coded waveforms are proposed to construct measurement operator incoherent with sparse bases and noise. Therefore, noise is suppressed using the SVD method. Sparse bases are constructed using interpolation method to improve the robustness of azimuth estimation. One can solve for the sparse vector by GP algorithm with many fewer samples than some existing methods, i.e., the APES method, Capon method GLR method, and Bayesian framework method. The proposed method is superior to these existing methods at Low-SNR.

ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China under grant 61071163 and in part by the Research and Innovation Project for College Graduates of Jiangsu Province under grant CXLX11_0197.

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