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GAIN-ASSISTED NEGATIVE REFRACTIVE INDEX IN A QUANTUM COHERENT MEDIUM

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Abstract—A new scheme for overcoming losses with incoherent optical gain in a quantum-coherent left-handed atomic vapor is suggested. In order to obtain low-loss, lossless or active left-handed media (LHM), a pump field, which aims at realizing population inversion of atomic levels, is introduced into a four-level atomic system. Both analytical and numerical results are given to illustrate that such an atomic vapor can exhibit intriguing electric and magnetic responses required for achieving simultaneously negative permittivity and permeability (and hence a gain-assisted quantum-coherent negative refractive index would emerge). The quantum-coherent left-handed atomic vapor presented here could have four fascinating characteristics: *i*) three-dimensionally isotropic negative refractive index, *ii*) double-negative atomic medium at visible and infrared wavelengths, *iii*) high-gain optical amplification, and *iv*) tunable negative refractive index based on quantum coherent control. Such a *three-dimensionally isotropic gain medium* with negative refractive index *at visible and infrared frequencies* would have a potential application in design of new quantum optical and photonic devices, including superlenses for perfect imaging and subwavelength focusing.

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1. INTRODUCTION

The left-handed media (LHM) with simultaneously negative electric permittivity and magnetic permeability [1–3] can exhibit many intriguing optical and electromagnetic properties [4], such as unusual photon tunnelling effect [5], negative Goos-Hänchen shift [6], amplification of evanescent waves [7] (and hence subwavelength focusing [7, 8]). One of the most promising scenarios for obtaining LHM is the artificial composite metamaterials [1–3], which can have various potential applications, including perfect lens [7, 8], subwavelength cavity [9, 10], metamaterial cloak [11], and high-efficiency wireless energy transmission [12]. The left-handed media that were prepared successfully in earlier experiments are actually *anisotropic* in nature [13–15]. Although there have been some new techniques to realize isotropic metamaterials [16–18], yet fabrication of ideally isotropic and homogeneous negatively refracting materials is believed to be still a challenging issue [19]. Obviously, the impact on the research of artificial electromagnetic LHM would be enormous if a three-dimensionally isotropic and homogeneous material of simultaneously negative permittivity and permeability can be realized in optical frequency ranges by using new schemes, e.g., quantum optical approaches. In the literature, there have been some scenarios suggested for realizing the negative refractive indices based on electromagnetically induced transparency (EIT) and photonic resonance in atomic vapors [20–25]. However, all these media exhibit quite large absorption to light fields. With the development of artificial material technology, the reduction of losses in metamaterials (and other relevant artificial composite structures) becomes an increasingly important issue that needs to be resolved [26]. Quite recently, some new schemes for compensating losses in metamaterials have been suggested by Soukoulis et al. who have developed both theoretical (computational) and experimental work on the loss compensation (or light amplification) in metamaterial structure units (e.g., split ring resonator and its array) with four-level layer underneath gain and InGaAs single-quantum-well gain [27–30]. In addition to the mechanisms of coupling to gain substrates [27–30], some scenarios for designing low-loss and almost zero-absorption media of negative refractive index have also been proposed by means of metamaterial analog of EIT [31] and incoherent manipulation of population in atoms [32]. Here, we shall suggest an alternative way to compensate the loss in a quantum-coherent left-handed atomic vapor (and hence to achieve a large optical gain in the negative refractive index) by introducing a pump field to realize the population inversion of a proper

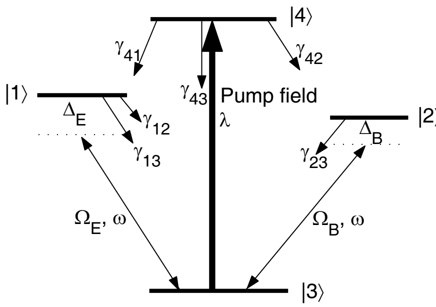

Figure 1. The schematic diagram of a four-level system driven by the electric and the magnetic fields of a weak probe light at angular frequency ω (the atom-light interactions are characterized by the electric and magnetic Rabi frequencies Ω_{E}, Ω_{B}). The $|3\rangle$-$|4\rangle$ transition of the quantum-coherent system is pumped by a strong laser beam (at the pumping rate λ), and can lead to high-gain optical amplification for the probe field. The electric-dipole allowed transition $|3\rangle$-$|1\rangle$ and the magnetic-dipole allowed transition $|3\rangle$-$|2\rangle$ can give rise to simultaneously negative permittivity and permeability of the atomic vapor.

multilevel atomic system [33], which can give rise to intriguing electric and magnetic responses for achieving the simultaneously negative permittivity and permeability. Such a kind of negative refracting media that can overcome the losses with gain can be referred to as "gain LHM".

In this paper, as an illustrative example, the chosen multilevel system is a four-level atomic system (see Fig. 1 for its schematic diagram), which can be found in, e.g., alkali-metal atoms. The presented quantum-coherent left-handed atomic vapor that can exhibit high gain for overcoming the losses has four advantages (they can also be viewed as four fascinating characteristics): i) the produced LHM is *three-dimensionally isotropic*, and can have a potential application to superlens for perfect imaging, ii) the present left-handed atomic vapor would have a negative refractive index *at visible and infrared wavelengths* (compared with the negative refractive index of artificial metamaterials at microwave frequencies), iii) such a quantum-coherent medium can be a low-loss, or loss-free and active optical negative-index material, and iv) the negative refractive index can be quantum-coherently manipulated.

In the sections that follow, we shall consider the equation of motion of the density matrix elements of the atomic system interacting with both the applied light field and the incoherent pump field, analyze the behavior of the electric- and magnetic-dipole allowed transitions, and then present the solution to the equation of the density matrix of the present atomic system for obtaining the explicit expressions for the microscopic electric and magnetic polarizabilities of the atomic system. An illustrative example with numerical results of the simultaneously negative permittivity and permeability in certain frequency bands will be given. We will show that a negative refractive index accompanied by the incoherent pumping gain for overcoming the losses would be exhibited in such a multilevel atomic vapor.

2. OPTICAL RESPONSES OF THE QUANTUM-COHERENT MEDIUM

We shall in this section present the physical mechanism for achieving the double-negative vapor medium composed of this multilevel system (see Fig. 1) that can exhibit the required electromagnetic responses leading to the simultaneously negative permittivity and permeability. For the first, let us consider the optical behavior of the four-level system driven by both the incoherent pump field and the electromagnetic fields of a probe light field. The $|3\rangle$-$|1\rangle$ electric-dipole allowed transition and the $|3\rangle$-$|2\rangle$ magnetic-dipole allowed transition of the system are excited simultaneously by the electric field and the magnetic field, respectively, of the weak probe light field. In order to achieve a gain LHM, we suppose that the atoms are pumped into their upper level $|4\rangle$ from the ground level $|3\rangle$ at a pumping rate λ [33]. We will find some proper values of the pumping rate λ for our scheme to compensate the loss of the quantum-coherent material when it becomes a negatively refracting medium in certain frequency bands. Since the electric- and magnetic-dipole allowed transitions of atoms can be excited by visible and infrared light, the refractive index of the atomic vapor medium can display its negative refractive index behavior in such high-frequency bands.

According to the quantum mechanical Schrödinger equation, the equation of motion of the density matrix of the present atomic system is given by

$$
\begin{aligned}
\dot{\rho}_{11} &= -\left(\gamma_{13}+\gamma_{12}\right)\rho_{11}+\gamma_{41}\rho_{44}-\frac{\mathrm{i}}{2}\left(\Omega_{\mathrm{E}}^{*}\rho_{13}-\Omega_{\mathrm{E}}\rho_{31}\right), \\
\dot{\rho}_{22} &= \gamma_{12}\rho_{11}-\gamma_{23}\rho_{22}+\gamma_{42}\rho_{44}-\frac{\mathrm{i}}{2}\left(\Omega_{\mathrm{B}}^{*}\rho_{23}-\Omega_{\mathrm{B}}\rho_{32}\right), \\
\dot{\rho}_{33} &= -\lambda\rho_{33}+\gamma_{13}\rho_{11}+\gamma_{23}\rho_{22}+\gamma_{43}\rho_{44} \\
&\quad +\frac{\mathrm{i}}{2}\left(\Omega_{\mathrm{E}}^{*}\rho_{13}+\Omega_{\mathrm{B}}^{*}\rho_{23}\right)-\frac{\mathrm{i}}{2}\left(\Omega_{\mathrm{E}}\rho_{31}+\Omega_{\mathrm{B}}\rho_{32}\right),
\end{aligned}
$$

$$\dot{\rho}_{32} = -\left(\frac{\gamma_{23}+\lambda}{2}+\gamma_{\rm ph}-{\rm i}\Delta_{\rm B}\right)\rho_{32}+\frac{\rm i}{2}\left[\Omega_{\rm E}^{*}\rho_{12}+\Omega_{\rm B}^{*}\left(\rho_{22}-\rho_{33}\right)\right],$$
$$\dot{\rho}_{12} = -\left[\frac{\gamma_{13}+\gamma_{12}+\gamma_{23}}{2}+{\rm i}\left(\Delta_{\rm E}-\Delta_{\rm B}\right)\right]\rho_{12}-\frac{\rm i}{2}\left(\Omega_{\rm B}^{*}\rho_{13}-\Omega_{\rm E}\rho_{32}\right),$$
$$\dot{\rho}_{13} = -\left(\frac{\gamma_{13}+\gamma_{12}+\lambda}{2}+{\rm i}\Delta_{\rm E}\right)\rho_{13}-\frac{\rm i}{2}\left[\Omega_{\rm B}\rho_{12}+\Omega_{\rm E}\left(\rho_{11}-\rho_{33}\right)\right],$$
$$\dot{\rho}_{44} = \lambda\rho_{33}-\left(\gamma_{41}+\gamma_{42}+\gamma_{43}\right)\rho_{44}. \quad (1)$$

Here, the electric and magnetic Rabi frequencies of the weak probe light are defined as $\Omega_{\rm E}=\wp_{13}\mathcal{E}/\hbar$ and $\Omega_{\rm B}=m_{23}\mathcal{B}/\hbar$, respectively, where $\mathcal{E}$ and $\mathcal{B}$ represent the field envelopes (slowly-varying amplitudes). The parameters γ_{12}, γ_{13}, γ_{42}, γ_{43} and γ_{23}, γ_{41} denote the electric- and magnetic-dipole induced spontaneous emission decay rates and $\gamma_{\rm ph}$ the collisional dephasing rate. As the probe field is weak while the pump field is strong (i.e., the electric and magnetic Rabi frequencies $\Omega_{\rm E}, \Omega_{\rm B}$ are less than the pumping rate λ), the zeroth-order steady populations $\rho_{11}^{(0)}$, $\rho_{22}^{(0)}$, $\rho_{33}^{(0)}$ and $\rho_{44}^{(0)}$ of the four-level system can be obtained when the electric and magnetic Rabi frequencies $\Omega_{\rm E}$ and $\Omega_{\rm B}$ are neglected in Eq. (1), namely, the diagonal density matrix elements are governed by

$$\begin{aligned}-\left(\gamma_{13}+\gamma_{12}\right)\rho_{11}^{(0)}+\gamma_{41}\rho_{44}^{(0)} &=0,\\ \gamma_{12}\rho_{11}^{(0)}-\gamma_{23}\rho_{22}^{(0)}+\gamma_{42}\rho_{44}^{(0)} &=0,\\ -\lambda\rho_{33}^{(0)}+\gamma_{13}\rho_{11}^{(0)}+\gamma_{23}\rho_{22}^{(0)}+\gamma_{43}\rho_{44}^{(0)} &=0,\\ \lambda\rho_{33}^{(0)}-\left(\gamma_{41}+\gamma_{42}+\gamma_{43}\right)\rho_{44}^{(0)} &=0,\end{aligned} \quad (2)$$

where the last formula is an identity, since it can be obtained with the help of the first three formulas in (2). It follows from Eq. (2) that the steady population of the upper level, $|1\rangle$, of the electric-dipole transition is

$$\rho_{11}^{(0)}=\frac{1}{1+\frac{\gamma_{12}}{\gamma_{23}}+\frac{\gamma_{42}(\gamma_{13}+\gamma_{12})}{\gamma_{23}\gamma_{41}}+\frac{(\gamma_{41}+\gamma_{42}+\gamma_{43})(\gamma_{13}+\gamma_{12})}{\lambda\gamma_{41}}+\frac{\gamma_{13}+\gamma_{12}}{\gamma_{41}}}. \quad (3)$$

Then from Eq. (2), one can show that the populations of the other three levels can be expressed in terms of $\rho_{11}^{(0)}$, i.e.,

$$\begin{aligned}\rho_{22}^{(0)} &= \left[\frac{\gamma_{12}}{\gamma_{23}}+\frac{\gamma_{42}\left(\gamma_{13}+\gamma_{12}\right)}{\gamma_{23}\gamma_{41}}\right]\rho_{11}^{(0)},\\ \rho_{33}^{(0)} &= \frac{\left(\gamma_{41}+\gamma_{42}+\gamma_{43}\right)\left(\gamma_{13}+\gamma_{12}\right)}{\lambda\gamma_{41}}\rho_{11}^{(0)},\\ \rho_{44}^{(0)} &= \frac{\gamma_{13}+\gamma_{12}}{\gamma_{41}}\rho_{11}^{(0)}.\end{aligned} \quad (4)$$

Now the weak light field is introduced, i.e., the terms in Eq. (1) associated with the electric and magnetic Rabi frequencies $\Omega_{\mathrm{E}}, \Omega_{\mathrm{B}}$ are taken into account. Substitution of (4) into the fourth to sixth formulae in Eq. (1) yields

$$\rho_{12} = \frac{\frac{\Omega_{\mathrm{E}}\Omega_{\mathrm{B}}^{*}}{4}\left(\frac{\rho_{33}^{(0)}-\rho_{11}^{(0)}}{\frac{\gamma_{13}+\gamma_{12}+\lambda}{2}+\mathrm{i}\Delta_{\mathrm{E}}} - \frac{\rho_{22}^{(0)}-\rho_{33}^{(0)}}{\frac{\gamma_{23}+\lambda}{2}+\gamma_{\mathrm{ph}}-\mathrm{i}\Delta_{\mathrm{B}}}\right)}{\left[\frac{\gamma_{13}+\gamma_{12}+\gamma_{23}}{2}+\mathrm{i}\left(\Delta_{\mathrm{E}}-\Delta_{\mathrm{B}}\right)\right]+\frac{1}{4}\Upsilon}, \tag{5}$$

where the parameter Υ in the denominator is given by

$$\Upsilon = \frac{\Omega_{\mathrm{B}}^{*}\Omega_{\mathrm{B}}}{\frac{\gamma_{13}+\gamma_{12}+\lambda}{2}+\mathrm{i}\Delta_{\mathrm{E}}} + \frac{\Omega_{\mathrm{E}}^{*}\Omega_{\mathrm{E}}}{\frac{\gamma_{23}+\lambda}{2}+\gamma_{\mathrm{ph}}-\mathrm{i}\Delta_{\mathrm{B}}}, \tag{6}$$

and the off-diagonal density matrix elements relevant to the electric- and magnetic-dipole allowed transitions are of the form

$$\begin{aligned}\rho_{32} &= \frac{\frac{\mathrm{i}}{2}\Omega_{\mathrm{E}}^{*}\rho_{12}+\frac{\mathrm{i}}{2}\Omega_{\mathrm{B}}^{*}\left(\rho_{22}^{(0)}-\rho_{33}^{(0)}\right)}{\frac{\gamma_{23}+\lambda}{2}+\gamma_{\mathrm{ph}}-\mathrm{i}\Delta_{\mathrm{B}}}, \\ \rho_{13} &= \frac{-\frac{\mathrm{i}}{2}\Omega_{\mathrm{B}}\rho_{12}+\frac{\mathrm{i}}{2}\Omega_{\mathrm{E}}\left(\rho_{33}^{(0)}-\rho_{11}^{(0)}\right)}{\frac{\gamma_{13}+\gamma_{12}+\lambda}{2}+\mathrm{i}\Delta_{\mathrm{E}}}.\end{aligned} \tag{7}$$

The microscopic electric and magnetic polarizabilities of the atoms are defined as $\beta_{\mathrm{e}} = 2\wp_{31}\rho_{13}/(\varepsilon_0\mathcal{E})$, $\beta_{\mathrm{m}} = 2\mu_0 m_{32}\rho_{23}/\mathcal{B}$. With the help of the expressions for the electric and magnetic Rabi frequencies (Ω_{E} and Ω_{B}), the electric and magnetic polarizabilities can be rewritten as $\beta_{\mathrm{e}} = (2|\wp_{31}|^2/\varepsilon_0\hbar)\rho_{13}/\Omega_{\mathrm{E}}$ and $\beta_{\mathrm{m}} = (2\mu_0|m_{32}|^2/\hbar)\rho_{23}/\Omega_{\mathrm{B}}$, respectively. Thus, we have the explicit expressions for the electric and magnetic polarizabilities of the coherent atoms

$$\begin{aligned}\beta_{\mathrm{e}} &= \frac{2|\wp_{31}|^2}{\varepsilon_0\hbar}\left[\frac{\frac{\mathrm{i}}{2}\left(\rho_{33}^{(0)}-\rho_{11}^{(0)}\right)-\frac{\mathrm{i}}{2}\frac{\Omega_{\mathrm{B}}}{\Omega_{\mathrm{E}}}\rho_{12}}{\frac{\gamma_{13}+\gamma_{12}+\lambda}{2}+\mathrm{i}\Delta_{\mathrm{E}}}\right], \\ \beta_{\mathrm{m}} &= \frac{2\mu_0|m_{32}|^2}{\hbar}\left[\frac{-\frac{\mathrm{i}}{2}\left(\rho_{22}^{(0)}-\rho_{33}^{(0)}\right)-\frac{\mathrm{i}}{2}\frac{\Omega_{\mathrm{E}}}{\Omega_{\mathrm{B}}}\rho_{21}}{\frac{\gamma_{23}+\lambda}{2}+\gamma_{\mathrm{ph}}+\mathrm{i}\Delta_{\mathrm{B}}}\right].\end{aligned} \tag{8}$$

In order to realize the negative permittivity and the negative permeability, the chosen vapor should be dense, so that one should consider the local field effect, i.e., one must distinguish between the applied macroscopic fields and the microscopic local fields that act upon the atoms in the vapor when addressing how the properties of the atomic transitions between the energy levels are related to the electric and magnetic susceptibilities [34, 35]. According to the Clausius-Mossotti relation that accounts for the contribution of all the other neighboring atoms to the polarization (local field correction), the relative electric permittivity and magnetic permeability of the present quantum-coherent atomic medium are given by $\varepsilon_r = 1 + N\beta_e/(1 - N\beta_e/3)$ [34] and $\mu_r = 1 + N\beta_m/(1 - N\beta_m/3)$ [35], respectively. Here, N denotes the atomic concentration (total number of atoms per unit volume) of the vapor.

In the next section, we will give an illustrative example to demonstrate how the gain-assisted negative refractive index can emerge under certain conditions (relevant to proper atomic and optical parameters of the present system).

3. AN ILLUSTRATIVE EXAMPLE OF GAIN-ASSISTED NEGATIVE REFRACTIVE INDEX

We shall in this numerical example present the behavior of dispersion of the simultaneously negative permittivity and permeability in such a gain-assisted left-handed medium. We choose the electric- and magnetic-dipole moments for the present system as $\wp_{31} = 1.0 \times 10^{-29}$ C·m and $m_{32} = 6.6 \times 10^{-23}$ C·m^{2}s^{-1}, respectively. We shall adopt typical atomic parameters of energy level and optical parameters of vapor medium (i.e., typical in the order of magnitude) for alkali metallic atoms that have been used in references on quantum coherence (atomic phase coherence) [20, 23, 24, 36–38]. The typical values for the decay rates due to spontaneous emission are $\gamma_{12} = 0.9 \times 10^7\,\mathrm{s}^{-1}$, $\gamma_{13} = 0.7 \times 10^7\,\mathrm{s}^{-1}$, $\gamma_{41} = 0.5 \times 10^7\,\mathrm{s}^{-1}$, $\gamma_{42} = 0.4 \times 10^7\,\mathrm{s}^{-1}$, $\gamma_{43} = 0.8 \times 10^7\,\mathrm{s}^{-1}$, $\gamma_{23} = 0.08 \times 10^7\,\mathrm{s}^{-1}$, and the collisional dephasing rate $\gamma_{ph} = 0.6 \times 10^6\,\mathrm{s}^{-1}$. The electric and magnetic Rabi frequencies of the weak probe field are chosen as $\Omega_E = 5.0 \times 10^5\,\mathrm{s}^{-1}$ and $\Omega_B = 1.1 \times 10^4\,\mathrm{s}^{-1}$, respectively, and the atomic concentration is $N = 3.0 \times 10^{24}$ Atoms m^{-3}. In the dimensionless variable Δ/Γ in Figs. 2 and 3, Δ is the frequency detuning, Δ_E, of the probe electric field driving the electric-dipole allowed transition $|3\rangle$-$|1\rangle$, and the normalized frequency is $\Gamma = \gamma_{12} + \gamma_{13}$ (i.e., the decay rate of level $|1\rangle$). The frequency detuning, Δ_B, of the probe electric field driving the $|3\rangle$-$|2\rangle$ transition is $\Delta - \omega_{12}$, where we choose $\omega_{12} = 1.0 \times 10^8\,\mathrm{s}^{-1}$. The dispersion behavior of the relative permittivity ε_r and the relative permeability μ_r is plotted in Fig. 2 and the relative refractive index n_r and the relative impedance η_r in Fig. 3. Here, the pumping rate is $\lambda = 5.7 \times 10^7\,\mathrm{s}^{-1} = 3.5\,\Gamma$.

It follows from Fig. 2 that the simultaneously negative permittivity and permeability occur in the frequency detuning range $[4.5\Gamma, 11.0\Gamma]$:

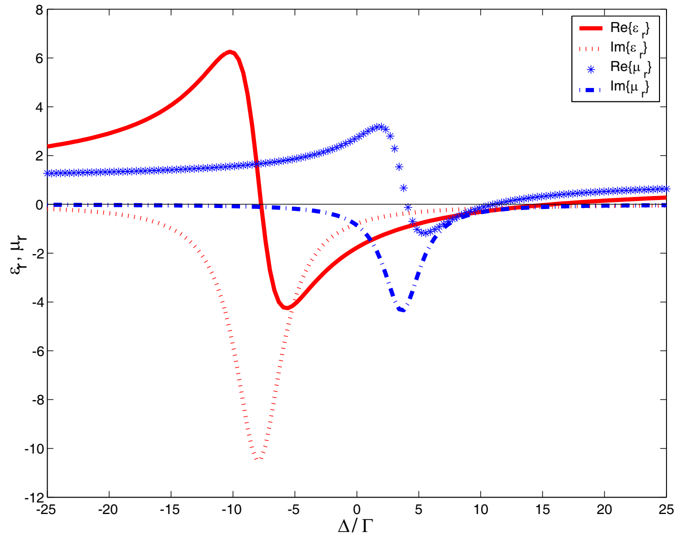

Figure 2. The dispersion characteristics of the real and imaginary parts of the relative electric permittivity ε_{r} and the relative magnetic permeability μ_{r}. The simultaneously negative permittivity and permeability emerge in the frequency detuning range $[4.5\Gamma, 11.0\Gamma]$, and thus the atomic vapor becomes a gain LHM, since the imaginary parts of both ε_{r} and μ_{r} in this frequency band are negative.

specifically, the real parts of the permittivity ε_{r} and the permeability μ_{r} range from -1.0 to -0.5 and from -1.3 to 0, respectively. In this frequency band, the real part of the relative refractive index is negative, which is shown in Fig. 3. Note that the present quantum-coherent medium is a single-negative material in some other frequency ranges shown in Fig. 2, and the imaginary parts of both the permittivity ε_{r} and the permeability μ_{r} are quite negative (i.e., high-gain optical amplification in the probe field due to strong field pumping action). One can see from Fig. 3 that the relative refractive index has a negative real part in the frequency detuning range $[1.0\Gamma, 12.5\Gamma]$, which is broader than the double-negative frequency detuning range $[4.5\Gamma, 11.0\Gamma]$ in Fig. 2. Although it is a single-negative medium in the frequency detuning ranges $[1.0\Gamma, 4.5\Gamma]$ and $[11.0\Gamma, 12.5\Gamma]$, yet the large gain (characterized by the negative imaginary parts of both the permittivity ε_{r} and the permeability μ_{r}) can also make the refractive index become negative.

It can be seen from expression (8) for the atomic electric and magnetic polarizability that the present scheme can lead to a tunable negative refractive index by the pump field. The dependence of

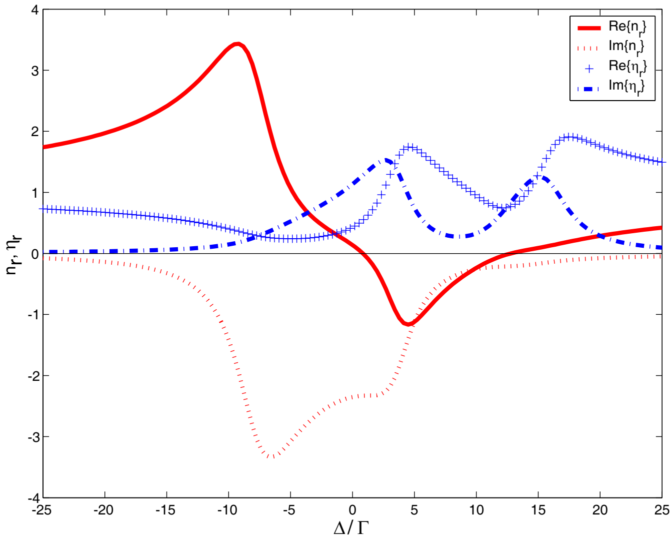

Figure 3. The dispersion characteristics of the real and imaginary parts of the relative refractive index n_{r} and the relative impedance η_{r}. There are some bands, where both the real and the imaginary parts of the refractive index are negative.

the optical response on the pumping rate λ in the present gain-assisted quantum-coherent medium also deserves consideration. We have shown in Figs. 4 and 5 the three-dimensional behavior of ε_{r}, μ_{r}, n_{r} and η_{r} when both the probe frequency detuning Δ and the pumping rate λ vary. It follows from Fig. 4 that both the permittivity and the permeability have their negative real parts in certain ranges of both the frequency detuning Δ and the pumping rate λ, and the imaginary parts of the permittivity and the permeability can also be negative. The real part of the refractive index plotted in Fig. 5 is negative in certain bands, where the permittivity and the permeability also have negative real parts (see Fig. 4). As is known, in the literature, the coherent control in atomic vapors gives rise to many intriguing quantum optical applications such as electromagnetically induced focusing (EIF) [36], all-optical switches [37] as well as single-photon turnstile device [38]. We expect that the controllable coherent manipulation in the negative index of refraction of the atomic vapors would also find new applications to quantum optical and photonic devices, e.g., tunable subwavelength compact cavity resonator and three-dimensional focusing devices and subdiffraction (or subwavelength) imaging devices [39–41] that aim at controlling light

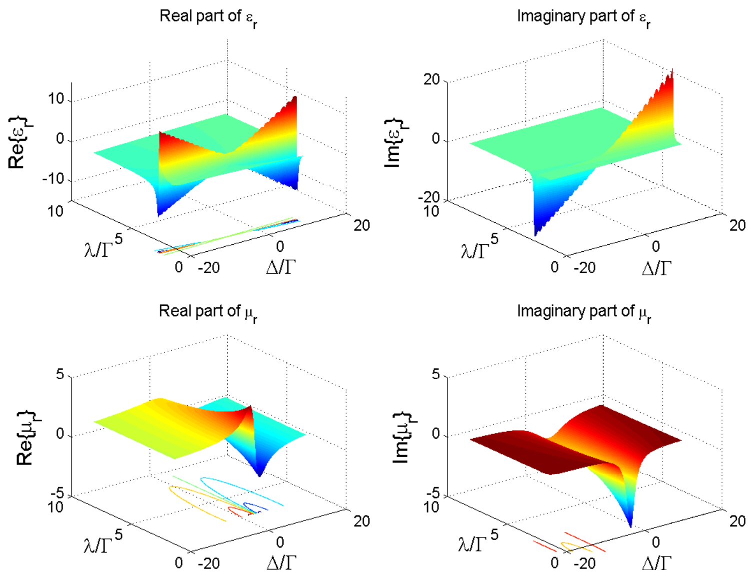

Figure 4. The relative permittivity and permeability versus the normalized frequency detuning Δ/Γ and the normalized pumping rate λ/Γ. There are some frequency bands, in which both the permittivity and the permeability have negative real parts. The imaginary parts of the permittivity and the permeability can also be negative in these bands.

waves by taking full advantage of the negative indices of the media.

To close this section, we point out that the effect of motion of light atoms in vapor at room temperature would influence (or modify) the dispersion characteristics of the refractive index, e.g., shifting the negative-index bands, or even drastically changing the profile of dispersion curves. For heavy atoms, however, such an effect is small, and can be ignored. This can be interpreted as follows: The transition frequency ω_{ij} in the present atomic system will be corrected due to Doppler effect by a numerical factor $1 + \mathbf{k} \cdot \mathbf{v}/\omega_{ij}$ (or approximately $1 \pm v/c$) [42]. The order of magnitude of v/c at room temperature for heavy alkali metallic atoms (e.g., Rb and Cs) is $10^{-7} \sim 10^{-6}$. If the transition frequency ω_{ij} is of the order $10^{13} \sim 10^{14}\,\mathrm{s}^{-1}$, then the Doppler correction of the transition frequency is of the order $10^6 \sim 10^8\,\mathrm{s}^{-1}$. In our numerical example, the negative-index frequency detuning band width is about $10^8\,\mathrm{s}^{-1}$. Thus, the effect of atom motion is small or negligibly small. But for the large Doppler shift correction of the transition frequency (e.g., the Doppler shift is larger than $10^8\,\mathrm{s}^{-1}$

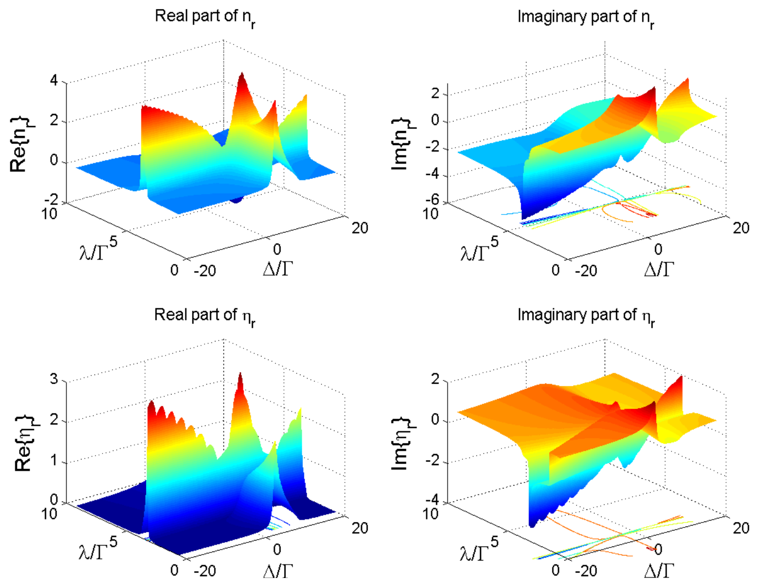

Figure 5. The relative refractive index $n_{\rm r}$ and the relative impedance $\eta_{\rm r}$ versus the normalized frequency detuning Δ/Γ and the normalized pumping rate λ/Γ. There are some frequency bands, in which the refractive index has its negative real part, and the imaginary part can also be negative in these bands (high-gain optical amplification).

for light atoms), the effect of atom motion needs to be taken into account.

4. CONCLUDING REMARKS

We have proposed a new scheme to realize the simultaneously negative permittivity and permeability (and hence the negative refractive index) by means of gain-assisted quantum coherence in the atomic transition processes of a multilevel coherent atomic system. Expressions for the electric permittivity and the magnetic permeability of the atomic vapor have been given in the present paper, and numerical results have been presented to show a left-handed atomic medium that can exhibit the high-gain optical amplification could be realized. This may be viewed as a new route to low-loss and lossless negative refracting materials. In addition to the characteristic of active loss compensation, the most remarkable feature of the present scheme is that the negative values of the permittivity and permeability of such an *isotropic* and *homogeneous* left-handed medium (with atomic-scale microscopic structure units) can be tuned by adjusting an incoherent pump field, and such a scenario for achieving a *tunable* negative refractive index would therefore have some potentially important applications in the development of new technologies in quantum optics and photonics, including new devices such as tunable subwavelength cavity resonator and superlenses for perfect imaging and subwavelength focusing [39–41]. We expect that such an atomic vapor that exhibits a three-dimensionally isotropic *gain-assisted quantum-coherent negative refractive index* would be realized experimentally in the near future.

It should also be pointed out that a semiconductor-quantum-dot (SQD) solid consisting of such a multilevel system would also exhibit such a behavior of negative refractive index (the present model is also applicable to SQD systems). Since the energy level structures in quantum dots ("artificial atoms") can be controllably manipulated by a variety of routes, such as chemical, photonic and optical techniques, the expected energy-level systems that can possibly lead to the negative indices ($\varepsilon_{\rm r}$, $\mu_{\rm r}$ and $n_{\rm r}$) of an SQD medium could in principle be designed and fabricated. Therefore, the work on the negative refractive index in an SQD solid medium should also be developed (based on the present scheme) both theoretically and experimentally.

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