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APPLYING EFFECTIVE MEDIUM THEORY IN CHARACTERIZING DIELECTRIC CONSTANT OF SOLIDS

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Abstract—We present a simple approach to measure the dielectric constant of solid materials. In this approach, the powder for the solid under investigation is mixed with the oil at a specific volume fraction. By measuring the oil and the mixture, the permittivity of the inclusion, i.e., the solid, can be accurately derived from the Maxwell-Garnett effective medium theory. With this method, the strict requirements for the solid shape and surface flatness in the conventional measuring configurations can be waved off, and meanwhile the broadband permittivity can be obtained. The demonstrations on alumina, glucose, and pearl show this approach is valid and robust.

1. INTRODUCTION

The characterization of dielectric constants or permittivity of materials at radio and microwave frequencies has been intensely studied and widely applied in science and industry [1–4]. Several approaches have been well developed in terms of the transmission line and the resonator configuration. For instance, the material under investigation is filled into a section of transmission line, like coaxials and waveguides, or a resonant cavity, and then S-parameters for the transmission line/resonator are measured, and lastly the permittivity is obtained from the S-parameters [2, 5]. The resonant cavity method, leading to very accurate values of permittivity, works well on the single frequency. In contrast, the transmission line method enables either narrowband or broadband measurements, depending on the type of transmission line.

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However, a common problem has been identified for both methods to measure a solid sample, for which the sample needs to be machined and polished to fit precisely and tightly into the internal shape/cavity of the transmission line/resonator. Even in the most tightly fitting situation, the air gap between the sample and the conducting walls exists, which can reduce the measuring precision to some extent [6]. This difficulty becomes more severe when one is dealing with fragile solids.

In electromagnetic materials characterization, it is well known that the Maxwell-Garnett (MG) effective medium theory predicts the bulk effective permittivity of a composite in terms of the permittivities of the inclusion and host materials [7, 8]. The prediction can be quite accurate especially for the cases of low volume fractions of the inclusion inside the composites, such as the system of photonic crystals and functional fluids [9–12]. In the long wavelength regime, the electromagnetic properties of such composites are similar to homogeneous materials with the permittivity that can be tuned readily via the volume fraction of the inclusions, and can give rise to the realization of controllable dielectrics or even present the extraordinary electromagnetic properties [13, 14].

In this paper, we introduce a simple method to measure the permittivity of solids based on the MG theory. Instead of working on a proper-shaped solid specimen, we use the powder sample of the solids and mix it with silicone oil to form the composite fluid at certain volume fraction. The dielectric constant of the composite fluid and the oil are characterized by using a coaxial probe that is a convenient and standard way to measure the liquids through the immersion. According to the MG theory, the permittivity of the inclusion, i.e., the solid, can be simply derived, given the volume fraction, permittivity of the mixture and the oil. From this method, one can easily see that an advantage is no concerns/requirements about the solid shape and surface flatness in measurements, since we are dealing with the powder. Another advantage is this approach can precisely characterize the permittivity of materials that are naturally in powder form.

2. METHOD

The principle of measuring dielectric constant in microwave frequencies is to insert the material into the wave space and then to measure the resultant reflection/transmission or the frequency shifting of the original resonant circuit and finally to extract the permittivity from these spectra. The well-established configurations typically include transmission line and resonant cavity, which provide a convenient and fast test of dielectric constant and are applied broadly in general labs. Different materials are tested with different types (shapes) of transmission lines (resonant cavities). For instance, the coaxial probe, an open-ended coaxial, is usually used to measure a liquid through simple immersion where gap-free contact between the probe and the liquid is guaranteed. However, when solids are directly measured with either of the two configurations, their shapes/surfaces must match the inserted space in the microwave circuit, as aforementioned.

In order to liftoff the requirement in characterizing solids and enable the measurement on dielectric particle individuals, we can turn to the effective medium theory. The MG effective medium formula has the expression

$$\left(\frac{\varepsilon_m - \varepsilon_h}{\varepsilon_m + 2\varepsilon_h}\right) = \delta_i \left(\frac{\varepsilon_i - \varepsilon_h}{\varepsilon_i + 2\varepsilon_h}\right), \tag{1}$$

where ε_m, ε_h, and ε_i are the permittivities of the mixture, host medium, and inclusions, respectively, and δ_i is the volume fraction of the inclusions [7, 8]. This formula has been regarded as valid to calculate the dielectric constant of the composite materials at lower volume fractions. On the other hand, if we look at the expression from the right hand side to the left, we can derive ε_i provided that ε_m, ε_h, and δ_i are measurable easily and precisely in experiments, which provides us a way to evaluate dielectric constant.

We employ silicon oil as the host medium, and mix the powder into the oil. The volume fraction is determined through weighting the powder and the oil and then calculating their respective volumes, V_{powder} and V_{oil}, according to their densities which are readily measured in the lab or tabulated in the literature. During the calculation ($\delta_i = V_{powder}/(V_{powder} + V_{oil})$), an assumption is that the composite volume is the sum of the component volumes, which requires that the powder is not soluble in the silicone oil. Other liquids can also be used, provided that they don't dissolve the powder and are not volatile, either. Measuring the dielectric constants of the oil and the mixture is performed with the coaxial probe (Agilent 85070E), which is a good choice to measure the permittivity for liquids. Furthermore, the coaxial probe enables us to perform broadband measurements because its fundamental waveguiding mode has no cutoff frequency. In experiments, the silicone oil was measured first according to the standard procedure, and then its permittivity, 3.0, was used as one of custom dielectric standards (another two being air and distilled water) to calibrate the probe for the mixture measurement.

To avoid the moisture effect, all the powders under investigation are dried carefully before making the mixture. During the measurement as illustrated in the inset of Fig. 1, the mixture is

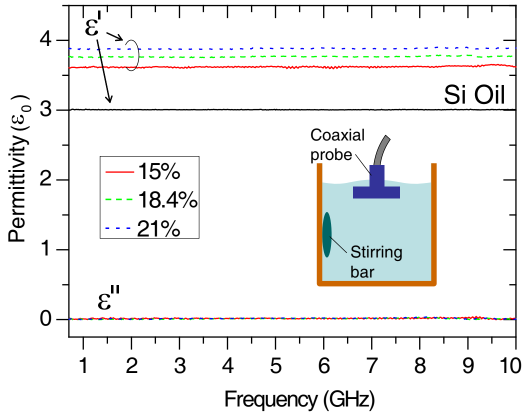

Figure 1. The permittivity of the mixture, alumina-silicone oil, of volume fraction 15%, 18.4%, and 21%. For comparison, the permittivity of the silicone oil is also plotted. ε' and ε'' denote the real and imaginary parts of the permittivity, respectively. The inset is the schematic illustration of the measurement.

contained in a plexiglass box, and the coaxial probe with a large flange of diameter 19 mm is immersed into the suspension with its other end connected to a microwave network analyzer. In order to have rigorous mixing and prevent any sedimentation during the measurement, we stir the mixture with a magnetic stirring bar on the side wall during the measurement. The bar is kept away from the probe and its peripheries, so that it doesn't interfere the microwave measurements.

3. RESULTS AND DISCUSSIONS

First, we use commercially available alumina powder to prove our method. The purity of Al_2O_3 powder used for our experiment is 99.8%. Its SEM image is shown in the inset of Fig. 2, indicating the particle size smaller than 10 μm. We make three volume fractions, 15%, 18.4%, and 21%, of mixtures for the alumina powder, and measure their corresponding dielectric constants. Fig. 1 shows the results at three volume fractions in the frequency range from 0.7 GHz to 10 GHz, and it is seen that the real part ε' is non-dispersive and increases with δ_i while the imaginary part ε'' is nearly zero.

From the results in Fig. 1, we calculate the permittivities of alumina using Eq. (1), and plot them in Fig. 2. We notice that the three volume fractions of measurements display the consistent results with an average value of the permittivity 9.60 + 0.20i for the commercially

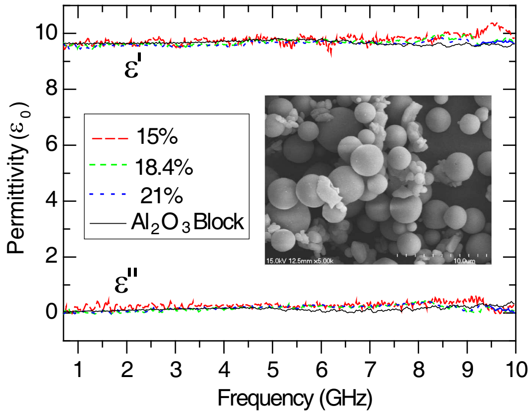

Figure 2. The permittivity of alumina measured from the mixture of volume fraction 15%, 18.4%, 21% and the block sample (solid line). ε' and ε'' denote the real and parts of the permittivity, respectively. The inset is the SEM image of alumina powder.

available alumina. This indicates the feasibility of our method to measure the dielectric constant of solid samples.

In order to find the accuracy of our method, we have measured the permittivity of alumina solid block using the conventional method. The alumina block is $6 \times 6 \times 5\,\text{cm}^3$ in size and one of the surfaces has been polished elaborately with surface flatness of $\pm 10\,\mu\text{m}$ to make a firm contact with the measuring probe. The result obtained from directly pressing the probe against the alumina solid agrees very well with the results from the powder test, as shown in Fig. 2 (solid line), which reveals that our approach is convincing and as accurate as conventional measurements. We also notice that the typical value of the real part of permittivity of Al_2O_3 in the literature [5] is 9.5–10, which is in good agreement with our results, while the imaginary part of the permittivity is < 0.01 which is smaller than ours (the larger imaginary part in our study is due to the inherent limitation of the Agilent dielectric probe in measuring ε).

It is known that glucose is an important energy provider in biological bodies. The concentration of glucose in the body fluid need often be monitored and sensed in bio-engineering. Therefore, it is expected that the permittivity of the glucose solid might be helpful to some sensing situations or data analysis [15]. However, it is hard to crystallize a big block of glucose with centimeter size matching the microwave wavelength. In contrast, the glucose is more usual in powder form. By our method, the permittivity value can be determined quickly and accurately from its powder form. In experiment, we purchase the glucose crystal (G8270 powder from the Sigma-Aldrich Chemical Co) and mix it with the silicone oil. As illustrated in the SEM image (the inset of Fig. 3), the glucose powder has an irregular shaped particles of size less than $\sim 250\,\mu\text{m}$. We have performed two volume fractions

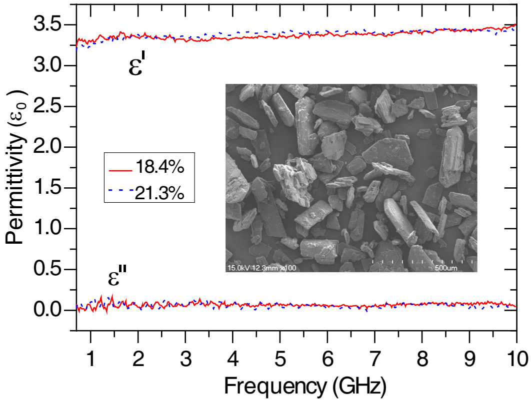

Figure 3. The permittivity of glucose measured from the mixture of volume fraction 18.4%, 21.3%. ε' and ε'' denote the real and imaginary parts of the permittivity, respectively. The inset is the SEM image of glucose sample.

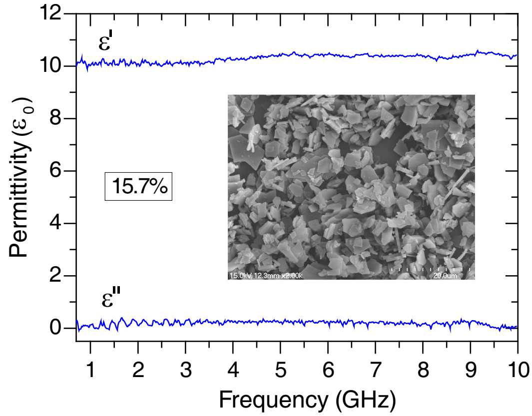

Figure 4. The permittivity of pearl measured from the mixture of volume fraction 15.7%. ε' and ε'' denote the real and imaginary parts of the permittivity, respectively. The inset is the SEM image of pearl powder.

of measurements in Fig. 3 and show that the average permittivity of glucose is $3.38 + 0.06\text{i}$.

Our method is especially beneficial to measure the permittivity of solids which are fragile and can not be machined into definite shapes as required for the conventional methods. The pearl is regarded as an example in this kind of materials. In our test, we grind the fresh water pearls into the powder, as shown in the SEM picture in the inset of Fig. 4. Then, we mix the pearl powder with the silicone oil at the volume fraction 15.7%. Our measurement in Fig. 4 shows the average permittivity of pearl is $10.30+0.19\text{i}$, which is consistent with the value of calcium carbonate, the main ingredient of the pearl.

Because the MG theory is valid in the low volume fraction where the host can be recognized to be continuum phase, we keep the volume fraction considerably lower than 50% in our study. We empirically find that the value around 18% works best for our measuring conditions, taking into account the mixture homogeneity and the stirring conditions. Moreover, in our results, the frequency band ranges 0.7 to 10 GHz, where the upper limit frequency is one order of magnitude larger than the lower limit, and still can be extended to be broader in principle provided that the grain size in the powder meets the long wavelength approximation for the MG formula. Thus, our approach also enables the broadband measurement. Although the demonstrated samples are not dispersive or dissipative, this method is believed able to cope with the dispersive and dissipative materials, based on the validity of the MG theory.

In this method, the accuracy of the permittivity of the powder will be influenced by the uncertainties in measuring ε_m, ε_h, and δ_i. Within these three error sources, the first two are considered to be major, as δ_i can be determined quite precisely relying on the state-of-art instruments. In order to minimize the error in permittivity measurement, we took the silicone oil as one of dielectric standards for the probe calibration. So the uncertainty in ε_i comes mainly from ε_m and is estimated to be $\pm 0.05|\varepsilon_i|$ for both real and imaginary parts, according to the used Agilent probe.

4. CONCLUSIONS

In conclusion, we present a simple approach to measure the permittivity of solid materials, based on the MG theory. In our method, the powder of the solids under investigation is mixed with silicone oil to form the composite fluid at certain volume fractions. From the measured permittivity of the oil and the composite, we can simply derive the permittivity of the inclusion, i.e., the solid, according to the MG formula. The proof demonstrations are performed on several materials including alumina, glucose, and pearl, and the results show the validity and the robustness of our method. Comparing with the conventional measuring configurations, our advantage is no requirements for solid shape and surface flatness in measurements and the broadband availability. Recently, the system of colloidal suspensions has paved a new way for realizing tunable metamaterials and exotic electromagnetic properties like negative refraction [14]. Here our method shows such system is also providing an alternative platform for implementing the material characterization. Finally, the capacity of our directly addressing the powder particles makes it very important to characterize the electromagnetic properties of some nanomaterials, such as ferroelectric nanoparticles and so on [16, 17].

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REFERENCES

1. Chen, L. F., C. K. Ong, C. P. Neo, V. V. Varadan, and V. K. Varadan, *Microwave Electronics: Measurement and Materials Characterization*, John Wiley & Sons, Chichester, 2004.
2. Stuchly, M. A. and S. S. Stuchly, "Coaxial line reflection methods for measuring dielectric properties of biological substances at radio and microwave frequencies — A review," *IEEE Transactions on Instrumentation and Measurement*, Vol. 29, 176–183, 1980.
3. Adous, M., P. Queffelec, and L. Laguerre, "Coaxial/cylindrical transition line for broadband permittivity measurement of civil engineering materials," *Meas. Sci. Technol.*, Vol. 17, 2241–2246, 2006.
4. Skierucha, W. and A. Wilczek, "A FDR sensor for measuring

complex soil dielectric permittivity in the 10–500 MHz frequency range," *Sensors*, Vol. 10, 3314–3329, 2010.

5. Pozar, D. M., *Microwave Engineering*, Ch. 3, John Wiley & Sons, Hoboken, 1998.
6. Baker-Jarvis, J., "Transmission/reflection and short-circuit line methods," NIST Technical Note 1341, 1990.
7. Maxwell-Garnett, J. C., "Colours in metal glasses and in metallic films," *Phil. Trans. R. Soc. London*, Vol. 203, 385–420, 1904.
8. Cai, W. and V. M. Shalaev, *Optical Metamaterials: Fundamentals and Applications*, Ch. 2, Springer, New York, 2010.
9. Joannopoulos, J. D., S. G. Johnson, J. N. Winn, and R. D. Meade, *Photonic Crystals: Molding the Flow of Light*, Princeton University Press, New Jersey, 2008.
10. Wen, W., X. Huang, S. Yang, K. Lu, and P. Sheng, "The giant electrorheological effect in suspensions of nanoparticles," *Nature Materials*, Vol. 2, 727–730, 2003.
11. Philip, J., P. D. Shima, and B. Raj, "Nanofluid with tunable thermal properties," *Appl. Phys. Lett.*, Vol. 92, 043108, 2006.
12. Shima, P. D. and J. Philip, "Tuning of thermal conductivity and rheology of nanofluids using an external stimulus," *J. Phys. Chem. C*, Vol. 115, 20097–20104, 2011.
13. Hu, X., C. T. Chan, J. Zi, M. Li, and K. M. Ho, "Diamagnetic response of metallic photonic crystals at infrared and visible frequencies," *Phys. Rev. Lett.*, Vol. 96, 223901, 2006.
14. Gao, Y., J. P. Huang, Y. M. Liu, L. Gao, K. W. Yu, and X. Zhang, "Optical negative refraction in ferrofluids with magnetocontrollablility," *Phys. Rev. Lett.*, Vol. 104, 034501, 2010.
15. Tao, H., et al., "Metamaterials on paper as a sensing platform," *Advanced Materials*, Vol. 23, 3197–3201, 2011.
16. Wada, S., H. Yasuno, T. Hoshina, S. M. Nam, H. Kakemoto, and T. Tsurumi, "Preparation of nm-sized barium titanate fine particles and their powder dielectric properties," *Jpn. J. Appl. Phys.*, Vol. 42, 6188–6195, 2003.
17. Hung, D. S., P. C. Chiang, C. W. Lee, C. S. Ho, S. H. Chieng, and Y. D. Yao, "Observation of effective permittivity of water-dispersible FePt nanoparticles at microwave frequencies," *IEEE Transactions on Magnetics*, Vol. 43, 879–881, 2007.