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ROBUST LCMP BEAMFORMER WITH NEGATIVE LOADING

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Abstract—The adaptive linearly constrained minimum power (LCMP) beamformer can improve the robustness of the Capon beamformer. And quadratic constraints on the weighting vector of the LCMP beamformer can improve the robustness to pointing errors and to random perturbations in sensor parameters. But how to solve it and how to select the constraint parameters are its key problems. In this paper, the Lagrange multiplier method is proposed to solve the LCMP beamformer under quadratic inequality constraint (QIC). The problem of finding the optimal weight vector is solved, and the choice of the quadratic constraint parameter is analyzed and the selected bound is also given. Since the quadratic equality constraint (QEC) is stronger than the quadratic inequality constraint (QIC), the performance of the QECLCMP beamformer is more robust than that of the QICLCMP beamformer. Therefore, the QECLCMP beamformer is proposed and is solved effectively. Numerical examples attest the correctness and the efficiency of the proposed algorithms. And the results show that the QECLCMP beamformer has the advantage of overcoming the steering vector mismatch, namely the optimal negative loading has the preferable robustness.

1. INTRODUCTION

Beamforming is a ubiquitous task in array signal processing with applications, among others, in radar, sonar, acoustics, astronomy, seismology, communications, and medical imaging. Without loss

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of generality, herein we consider beamforming in array processing applications.

The traditional approach to the design of adaptive beamformers assumes that the desired signal components are not present in training data, and the robustness of beamformer is known to depend essentially on the availability of signal-free training data. However, in many important applications such as mobile communications, passive location, microphone array speech processing, medical imaging, and radio astronomy, the signal-free training data cells are unavailable. In such scenarios, the desired signal is always present in the training snapshots, and the adaptive beamforming methods become very sensitive to any violation of underlying assumptions on the environment, sources, or sensor array. In fact, the performances of the existing adaptive array algorithms are known to degrade substantially in the presence of even slight mismatches between the actual and presumed array responses to the desired signal [1–3]. Similar types of degradation can take place when the array response is known precisely but the training sample size is small, namely the mismatch between the actual and the estimated covariance matrix [4–6]. Therefore, robust approaches to adaptive beamforming appear to be of primary importance in these cases [7–31].

Li et al. propose a Capon beamforming approach with the norm inequality constraint (NICCB) to improve the robustness against array steering vector errors and noise [19]. The exact solution is given, and optimal loading level can be computed via the proposed method. But its efficiency is not as good as expectation according to analysis and simulation. Since the constraint parameter determines the approach's robustness, and how to select the constraint parameter is not discussed.

Quadratic inequality constraints (QIC) on the weight vector of LCMP beamformer can improve robustness to pointing errors and to random perturbations in sensor parameter [20]. The weights that minimize the output power subject to linear constraints and an inequality constraint on the norm of the weight vector have the same form as the optimum LCMP beamformer, with diagonal loading of the data covariance matrix. But the optimal loading level cannot be directly expressed as a function of the constraint in a closed form, and cannot be solved exactly. Hence, its application is restricted by finding of optimal weight vector. So that some numerically algorithms are proposed to implement the QICLCMP, such as Least Mean Squares (LMS) or Recursive Least Squares (RLS) [20], but the application's effect isn't good as expectation.

In this paper, we dedicate to the robust LCMP beamformer with weight norm constraint.

For the QICLCMP beamformer, the key problems are how to solve the optimal weighting vector and how to select the quadratic constraint parameter. Hence, the Lagrange multiplier method is proposed to solve the QICLCMP beamformer, and the problem of finding the optimal weight vector is solved. Furthermore, the choice of the quadratic constraint parameter is analyzed and the selected bound is given. Since the quadratic equality constraint (QEC) is stronger than the quadratic inequality constraint (QIC), the LCMP beamformer under quadratic equality constraint (QEC) has more ascendant robust performance than the QICLCMP beamformer. Therefore, the QECLCMP beamformer is proposed and is solved effectively. Numerical examples attest the correctness and the validity of the proposed algorithm, which show that the QECLCMP beamformer has the best performance to overcome the signal direction mismatch, namely the optimal negative loading has the preferable robustness.

This paper is organized as follows. First, the signal model and the LCMP beamformer are introduced. Second, the Lagrange multiplier method is proposed to solve the QICLCMP beamformer, particularly the choice of the quadratic constraint parameter and the selecting bound is discussed. Third, the QECLCMP beamformer is proposed and is solved effectively. Finally, the simulation analyses and the conclusion are given.

2. SIGNAL MODEL AND LCMP BEAMFORMER

Consider an array comprising N sensors. And let $\mathbf{x}(k)$ denote the $N\times1$ vector of signals received by the array. The vector $\mathbf{x}(k)$ is given by:

$$\mathbf{x}(k)=\sum_{i=1}^{D}\mathbf{v}_i s_i(k)+\mathbf{n}(k) \tag{1}$$

where $s_i(k)$ is the k-th signal sample transmitted by the i-th user, $\mathbf{v}_i$ the $N\times1$ complex array response vector of the i-th user, and $\mathbf{n}(k)$ the complex vector of ambient noise samples. In narrow-band beamforming, a complex weight is applied to the signal at each sensor and summed to form the beamformer output:

$$y(k)=\mathbf{w}^H\mathbf{x}(k) \tag{2}$$

where $(\cdot)^H$ denotes the conjugate transposition.

To account for the array steering vector errors, additional linear constraints, including point and derivative constraints, can be imposed to improve the robustness of the Capon beamformer [21–24]. And they are used to widen and flatten the main beam to provide robustness to mismatch in the array response vector and null constraint that are used to place explicit nulls, or zeros, in certain directions. Therefore, the LCMP beamformer is the generalization of the standard Capon beamformer, and the standard Capon beamformer is a special case of the general LCMP beamformer.

The weights of an LCMP beamformer are chosen to minimize the output power of the beamformer which subjects to a set of m linear constraints of the form $\mathbf{C}^H\mathbf{w} = \mathbf{f}$, where $\mathbf{C}$ is the $N \times m$ constraint matrix, and $\mathbf{f}$ is the $m \times 1$ vector of constraint values.

The LCMP optimization problem can be formulated as:

$$\begin{cases} \min\limits_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{\mathbf{x}} \mathbf{w} \\ s.t. \quad \mathbf{C}^H \mathbf{w} = \mathbf{f} \end{cases} \tag{3}$$

where $\mathbf{R}_{\mathbf{x}} = E\left\{\mathbf{x}(k)\,\mathbf{x}^H(k)\right\}$ is the data covariance matrix, and $E\{\cdot\}$ denotes the expectation operation. The final weight vector expression for the LCMP beamformer is given by:

$$\mathbf{w} = \mathbf{R}_{\mathbf{x}}^{-1}\mathbf{C}\left(\mathbf{C}^H\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{C}\right)^{-1}\mathbf{f} \tag{4}$$

Under the ideal condition, namely there isn't any error in the steering vector or the covariance matrix, the weight vector (4) is optimal, whereas, the performance will be degraded greatly, especially, when the presumed steering vector differs from the actual steering vector. For instance, when this case happens in practice, the beamformer pattern will suffer severe distortion. Hence, the robustness is the essential requirement for the beamformer in practice.

3. LCMP BEAMFORMER UNDER QUADRATIC INEQUALITY CONSTRAINT

The LCMP beamformer can experience significant performance degradation when there is a mismatch between the presumed and actual characteristics of the source or array. The goal of the QICLCMP beamformer is to impose an additional quadratic inequality constraint on the Euclidean norm of $\mathbf{w}$ for which the purpose is to improve the robustness to pointing errors and to random perturbations in sensor parameters. This requires incorporating a quadratic inequality constraint on $\mathbf{w}$ of the form:

$$\|\mathbf{w}\|^2 \leq \varepsilon \tag{5}$$

where $\|\cdot\|$ denotes the vector l_2 norm.

Consequently, the QICLCMP beamformer problem is formulated as follows [20]:

$$\begin{cases} \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R_x} \mathbf{w} \\ s.t. \quad \mathbf{C}^H \mathbf{w} = \mathbf{f} \\ \qquad \|\mathbf{w}\|^2 \leq \varepsilon \end{cases} \tag{6}$$

Since the beamformer is not solved efficiently, its application is restricted so far. In this paper, the Lagrange multiplier methodology is proposed to solve the QICLCMP beamformer as follows.

3.1. Solution to the QICLCMP Beamformer

Let S be the set defined by the constraints in the optimization problem (6), namely:

$$S = \left\{ \mathbf{w} | \, \mathbf{C}^H \mathbf{w} = \mathbf{f}, \quad \|\mathbf{w}\|^2 \leq \varepsilon \right\} \tag{7}$$

Define the function:

$$f_1(\mathbf{w}, \lambda, \boldsymbol{\mu}) = \mathbf{w}^H \mathbf{R_x} \mathbf{w} + \lambda \left(\|\mathbf{w}\|^2 - \varepsilon \right) + \boldsymbol{\mu}^H \left(\mathbf{f} - \mathbf{C}^H \mathbf{w} \right) + \left(\mathbf{f} - \mathbf{C}^H \mathbf{w} \right)^H \boldsymbol{\mu} \tag{8}$$

where λ is the real-valued Lagrange multiplier, and $\lambda \geq 0$ satisfies $\mathbf{R_x} + \lambda \mathbf{I} > 0$ so that $f_1(\mathbf{w}, \lambda, \boldsymbol{\mu})$ can be minimized with respect to $\mathbf{w}$, and $\boldsymbol{\mu}$ is the arbitrary Lagrange multiplier vector. Then:

$$f_1(\mathbf{w}, \lambda, \boldsymbol{\mu}) \leq \mathbf{w}^H \mathbf{R_x} \mathbf{w}, \quad \mathbf{w} \in S \tag{9}$$

with equality on the boundary of S.

Consider the condition:

$$\begin{aligned} &\left[\mathbf{R_x^{-1}} \mathbf{C} \left(\mathbf{C}^H \mathbf{R_x^{-1}} \mathbf{C} \right)^{-1} \mathbf{f} \right]^H \left[\mathbf{R_x^{-1}} \mathbf{C} \left(\mathbf{C}^H \mathbf{R_x^{-1}} \mathbf{C} \right)^{-1} \mathbf{f} \right] \\ &= \mathbf{f}^H \left(\mathbf{C}^H \mathbf{R_x^{-1}} \mathbf{C} \right)^{-1} \mathbf{C}^H \mathbf{R_x^{-2}} \mathbf{C} \left(\mathbf{C}^H \mathbf{R_x^{-1}} \mathbf{C} \right)^{-1} \mathbf{f} \leq \varepsilon. \end{aligned} \tag{10}$$

When the condition (10) is satisfied, the solution of LCMP beamformer (6) becomes:

$$\hat{\mathbf{w}} = \mathbf{R_x^{-1}} \mathbf{C} \left(\mathbf{C}^H \mathbf{R_x^{-1}} \mathbf{C} \right)^{-1} \mathbf{f} \tag{11}$$

And (11) satisfies the norm constraint of the QICLCMP beamformer. Therefore, it is also the solution to the QICLCMP beamformer. In this case, $\lambda = 0$ and the norm constraint in the QICLCMP beamformer is inactive.

Otherwise, we have the condition:

$$\varepsilon < \mathbf{f}^H \left(\mathbf{C}^H \mathbf{R_x^{-1}} \mathbf{C} \right)^{-1} \mathbf{C}^H \mathbf{R_x^{-2}} \mathbf{C} \left(\mathbf{C}^H \mathbf{R_x^{-1}} \mathbf{C} \right)^{-1} \mathbf{f} \stackrel{\Delta}{=} \varepsilon_0 \tag{12}$$

which is an upper bound on ε so the QICLCMP beamformer is different from the LCMP beamformer. To deal with this case, we can rewrite $f_1(\mathbf{w}, \lambda, \boldsymbol{\mu})$ as follows:

$$f_1(\mathbf{w}, \lambda, \boldsymbol{\mu}) = \left[\mathbf{w} - (\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\boldsymbol{\mu}\right]^H (\mathbf{R_x} + \lambda\mathbf{I}) \left[\mathbf{w} - (\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\boldsymbol{\mu}\right] - \boldsymbol{\mu}^H\mathbf{C}^H(\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\boldsymbol{\mu} - \lambda\varepsilon + \boldsymbol{\mu}^H\mathbf{f} + \mathbf{f}^H\boldsymbol{\mu} \tag{13}$$

Hence, the unconstrained minimizer of $f_1(\mathbf{w}, \lambda, \boldsymbol{\mu})$, for fixed λ and $\boldsymbol{\mu}$, is given by:

$$\hat{\mathbf{w}}_{\lambda,\boldsymbol{\mu}} = (\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\boldsymbol{\mu} \tag{14}$$

Clearly, we have:

$$f_2(\lambda, \boldsymbol{\mu}) \stackrel{\Delta}{=} f_1(\hat{\mathbf{w}}_{\lambda,\boldsymbol{\mu}}, \lambda, \boldsymbol{\mu}) = -\boldsymbol{\mu}^H\mathbf{C}^H(\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\boldsymbol{\mu} - \lambda\varepsilon + \boldsymbol{\mu}^H\mathbf{f} + \mathbf{f}^H\boldsymbol{\mu} \leq \mathbf{w}^H\mathbf{R_x}\mathbf{w}, \ \mathbf{w} \in S \tag{15}$$

The maximization of $f_2(\lambda, \boldsymbol{\mu})$ with respect to $\boldsymbol{\mu}$, is given by:

$$\frac{\partial f_2(\lambda, \boldsymbol{\mu})}{\partial \boldsymbol{\mu}} = -2\mathbf{C}^H(\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C} \cdot \boldsymbol{\mu} + 2\mathbf{f} \tag{16}$$

Hence, $\boldsymbol{\mu}$ is given by:

$$\hat{\boldsymbol{\mu}} = \left[\mathbf{C}^H(\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\right]^{-1}\mathbf{f} \tag{17}$$

Insert $\hat{\boldsymbol{\mu}}$ into $f_2(\lambda, \boldsymbol{\mu})$, and let:

$$f_3(\lambda) \stackrel{\Delta}{=} f_2(\lambda, \hat{\boldsymbol{\mu}}) = -\lambda\varepsilon + \mathbf{f}^H\left[\mathbf{C}^H(\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\right]^{-1}\mathbf{f} \tag{18}$$

For any matrix function $\mathbf{F}$ of λ, we have:

$$\frac{d\left(\mathbf{F}^{-1}\right)}{d\lambda} = -\mathbf{F}^{-1} \cdot \frac{d(\mathbf{F})}{d\lambda} \cdot \mathbf{F}^{-1} \tag{19}$$

The maximization of the function $f_3(\lambda)$ in (18) with respect to λ gives:

$$\varepsilon = \mathbf{f}^H\left[\mathbf{C}^H(\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\right]^{-1}\left[\mathbf{C}^H(\mathbf{R_x} + \lambda\mathbf{I})^{-2}\mathbf{C}\right] \left[\mathbf{C}^H(\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\right]^{-1}\mathbf{f} \tag{20}$$

Hence, the optimal Lagrange multiplier $\hat{\lambda}$ can be obtained efficiently via, for example, the Newton's method from the Equation (20) of λ.

Note that using $\hat{\boldsymbol{\mu}}$ in $\hat{\mathbf{w}}_{\lambda,\boldsymbol{\mu}}$ yields:

$$\hat{\mathbf{w}} = (\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\left[\mathbf{C}^H(\mathbf{R_x} + \lambda\mathbf{I})^{-1}\mathbf{C}\right]^{-1}\mathbf{f} \tag{21}$$

which satisfies the constraints of the QICLCMP beamformer, namely:

$$\mathbf{C}^H \hat{\mathbf{w}} = \mathbf{f} \tag{22}$$

And

$$\|\hat{\mathbf{w}}\|^2 = \varepsilon \tag{23}$$

Hence, $\hat{\mathbf{w}}$ belongs to the boundary of S. Therefore, $\hat{\mathbf{w}}$ is our sought solution to the LCMP optimization problem with a quadratic inequality constraint, which has the same form as the LCMP beamformer with a diagonal loading term $\lambda\mathbf{I}$ added to $\mathbf{R_x}$, namely, the QICLCMP beamformer also belongs to the class of diagonal loading approaches.

Based on the analysis above, we can see that if the Lagrange multiplier λ is obtained, the optimal weight vector for the QICLCMP beamformer will be solved. In order to obtain the Lagrange multiplier λ, we must solve the following equation via Newton's method, and let:

$$f(\lambda) \triangleq \varepsilon - \mathbf{f}^H \left[\mathbf{C}^H (\mathbf{R_x} + \lambda\mathbf{I})^{-1} \mathbf{C}\right]^{-1} \left[\mathbf{C}^H (\mathbf{R_x} + \lambda\mathbf{I})^{-2} \mathbf{C}\right] \left[\mathbf{C}^H (\mathbf{R_x} + \lambda\mathbf{I})^{-1} \mathbf{C}\right]^{-1} \mathbf{f} = 0 \tag{24}$$

Hence, the key problem of the QICLCMP beamformer is to find the optimal Lagrange multiplier by above Equation (24).

3.2. Solution to the Optimal Lagrange Multiplier

In order to solve Equation (24), we perform the eigenvalue decomposition (EVD) of the sample covariance matrix as follows:

$$\mathbf{R_x} = \mathbf{U} \cdot \mathbf{\Lambda} \cdot \mathbf{U}^H = \sum_{i=1}^{N} \lambda_i \mathbf{u}_i \mathbf{u}_i^H \tag{25}$$

where $\mathbf{\Lambda} = \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$ is a diagonal matrix; $\mathbf{U} = (\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_N)$ is Hermitian; λ_i $(i = 1, 2, \ldots, N)$ and $\mathbf{u}_i$ $(i = 1, 2, \ldots, N)$ are the eigenvalues and eigenvectors of $\mathbf{R_x}$; N is the total number of degrees-of-freedom. For the convenience of analysis, we assume that the eigenvalues/eigenvectors of $\mathbf{R_x}$ are sorted in descending order, i.e.,

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N \tag{26}$$

Therefore, we can have:

$$(\mathbf{R_x} + \lambda\mathbf{I})^{-1} = \mathbf{U} \cdot (\mathbf{\Lambda} + \lambda\mathbf{I})^{-1} \cdot \mathbf{U}^H = \sum_{i=1}^{N} \frac{\mathbf{u}_i \mathbf{u}_i^H}{\lambda_i + \lambda} \tag{27}$$

Then:

$$\begin{aligned}\mathbf{C}^H\left(\mathbf{R_x}+\lambda\mathbf{I}\right)^{-1}\mathbf{C} &= \left(\mathbf{C}^H\mathbf{U}\right)\cdot\left(\mathbf{\Lambda}+\lambda\mathbf{I}\right)^{-1}\cdot\left(\mathbf{C}^H\mathbf{U}\right)^H \\ &= \sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\lambda_i+\lambda} \qquad (28)\end{aligned}$$

$$\begin{aligned}\mathbf{C}^H\left(\mathbf{R_x}+\lambda\mathbf{I}\right)^{-2}\mathbf{C} &= \left(\mathbf{C}^H\mathbf{U}\right)\cdot\left(\mathbf{\Lambda}+\lambda\mathbf{I}\right)^{-2}\cdot\left(\mathbf{C}^H\mathbf{U}\right)^H \\ &= \sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\left(\lambda_i+\lambda\right)^2} \qquad (29)\end{aligned}$$

And let:

$$\gamma=\mathbf{f}^H\cdot\mathbf{C}^H\mathbf{U}\mathbf{U}^H\mathbf{C}\cdot\mathbf{f}=\mathbf{f}^H\cdot\left[\sum_{i=1}^{N}\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H\right]^{-1}\cdot\mathbf{f} \qquad (30)$$

Therefore, $f(\lambda)$ can be rewritten as follows:

$$\begin{aligned}f(\lambda)=\varepsilon-\mathbf{f}^H\cdot&\left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\lambda_i+\lambda}\right]^{-1}\cdot\left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\left(\lambda_i+\lambda\right)^2}\right] \\ &\cdot\left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\lambda_i+\lambda}\right]^{-1}\cdot\mathbf{f} \qquad (31)\end{aligned}$$

And, we have:

$$\begin{aligned}\varepsilon \;\le\; \mathbf{f}^H&\left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\lambda_1+\lambda}\right]^{-1}\left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\left(\lambda_N+\lambda\right)^2}\right] \\ &\left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\lambda_1+\lambda}\right]^{-1}\mathbf{f}=\frac{\left(\lambda_1+\lambda\right)^2}{\left(\lambda_N+\lambda\right)^2}\cdot\gamma \qquad (32)\end{aligned}$$

$$\begin{aligned}\varepsilon \;\ge\; \mathbf{f}^H&\left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\lambda_N+\lambda}\right]^{-1}\left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\left(\lambda_1+\lambda\right)^2}\right] \\ &\left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\lambda_N+\lambda}\right]^{-1}\mathbf{f}=\frac{\left(\lambda_N+\lambda\right)^2}{\left(\lambda_1+\lambda\right)^2}\cdot\gamma \qquad (33)\end{aligned}$$

Under the condition of $\varepsilon < \mathbf{f}^H\left(\mathbf{C}^H\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{C}\right)^{-1}\mathbf{C}^H\mathbf{R}_{\mathbf{x}}^{-2}\mathbf{C}$

$\left(\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\right)^{-1}\mathbf{f}$, we have:

$$\begin{aligned}\varepsilon &< \mathbf{f}^H \cdot \left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\lambda_i}\right]^{-1} \cdot \left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\lambda_i^2}\right] \\ &\quad \cdot \left[\sum_{i=1}^{N}\frac{\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H}{\lambda_i}\right]^{-1} \cdot \mathbf{f} \\ &\le \frac{\lambda_1^2}{\lambda_N^2} \cdot \mathbf{f}^H \cdot \left[\sum_{i=1}^{N}\left(\mathbf{C}^H\mathbf{u}_i\right)\left(\mathbf{C}^H\mathbf{u}_i\right)^H\right]^{-1} \cdot \mathbf{f} = \frac{\lambda_1^2}{\lambda_N^2} \cdot \gamma \end{aligned} \tag{34}$$

Alternately, the inequality relationship above can be expressed as:

$$\sqrt{\frac{\varepsilon}{\gamma}} \le \frac{\lambda_1 + \lambda}{\lambda_N + \lambda} \tag{35}$$

$$\sqrt{\frac{\varepsilon}{\gamma}} \ge \frac{\lambda_N + \lambda}{\lambda_1 + \lambda} \tag{36}$$

$$\sqrt{\frac{\varepsilon}{\gamma}} < \frac{\lambda_1}{\lambda_N} \tag{37}$$

Let $a \triangleq \sqrt{\varepsilon/\gamma}$. Then, we establish the bound of the Lagrange multiplier λ and its existence.

(1) If $a > 1$, then from (35) and (36), we can have:

$$a \le \frac{\lambda_1+\lambda}{\lambda_N+\lambda} \Rightarrow a\lambda_N + a\lambda \le \lambda_1 + \lambda \Rightarrow (a-1)\lambda \le \lambda_1 - a\lambda_N \Rightarrow \lambda \le \frac{\lambda_1 - a\lambda_N}{a-1} \tag{38}$$

$$a \ge \frac{\lambda_N+\lambda}{\lambda_1+\lambda} \Rightarrow a\lambda_1 + a\lambda \ge \lambda_N + \lambda \Rightarrow (a-1)\lambda \ge \lambda_N - a\lambda_1 \Rightarrow \lambda \ge \frac{\lambda_N - a\lambda_1}{a-1} \tag{39}$$

Since $\lambda \ge 0$, but $\lambda_N - a\lambda_1 < 0$, the bound of the Lagrange multiplier λ under $a > 1$ is given as follows:

$$\lambda_{\min}^{(1)} \triangleq 0 \le \lambda \le \frac{\lambda_1 - a\lambda_N}{a-1} \triangleq \lambda_{\max}^{(1)} \tag{40}$$

Then, we have:

$$f\left(\lambda_{\min}^{(1)}\right) = f(0) = \varepsilon - \mathbf{f}^H\left(\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\right)^{-1}\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-2}\mathbf{C}\left(\mathbf{C}^H\mathbf{R}_\mathbf{x}^{-1}\mathbf{C}\right)^{-1}\mathbf{f} < 0 \tag{41}$$

$$\begin{aligned} f\left(\lambda_{\max}^{(1)}\right) &= f(\lambda)\Big|_{\lambda=\lambda_{\max}^{(1)}} \ge \varepsilon - \frac{(\lambda_1+\lambda)^2}{(\lambda_N+\lambda)^2}\cdot\gamma\Bigg|_{\lambda=\lambda_{\max}^{(1)}} \\ &= \varepsilon - \frac{\left(\lambda_1+\lambda_{\max}^{(1)}\right)^2}{\left(\lambda_N+\lambda_{\max}^{(1)}\right)^2}\cdot\gamma = 0 \end{aligned} \tag{42}$$

Using the condition of $a < \lambda_1/\lambda_N$, when $1 < a < \lambda_1/\lambda_N$, there is a solution $\lambda \in \left[\lambda_{\min}^{(1)}, \lambda_{\max}^{(1)}\right]$ that satisfies $f(\lambda) = 0$.

(2) If $a < 1$, then from (35) and (36), we can have:

$$a \le \frac{\lambda_1+\lambda}{\lambda_N+\lambda} \Rightarrow a\lambda_N + a\lambda \le \lambda_1+\lambda \Rightarrow (1-a)\lambda \ge a\lambda_N - \lambda_1 \Rightarrow \lambda \ge \frac{a\lambda_N-\lambda_1}{1-a} \quad (43)$$

$$a \ge \frac{\lambda_N+\lambda}{\lambda_1+\lambda} \Rightarrow a\lambda_1 + a\lambda \ge \lambda_N+\lambda \Rightarrow (1-a)\lambda \le a\lambda_1 - \lambda_N \Rightarrow \lambda \le \frac{a\lambda_1-\lambda_N}{1-a} \quad (44)$$

Since $\lambda \ge 0$, but $a\lambda_N - \lambda_1 \le 0$, if $a\lambda_1 - \lambda_N \ge 0$, then $a \ge \lambda_N/\lambda_1$. Hence, when $\lambda_N/\lambda_1 \le a < 1$, the bound of the Lagrange multiplier λ is given as follows:

$$\lambda_{\min}^{(2)} \triangleq 0 \le \lambda \le \frac{a\lambda_1 - \lambda_N}{1-a} \triangleq \lambda_{\max}^{(2)} \quad (45)$$

Then, we have:

$$\begin{aligned} f\left(\lambda_{\max}^{(2)}\right) &= f(\lambda)\Big|_{\lambda=\lambda_{\max}^{(2)}} \le \varepsilon - \frac{(\lambda_N+\lambda)^2}{(\lambda_1+\lambda)^2}\cdot\gamma\Bigg|_{\lambda=\lambda_{\max}^{(2)}} \\ &= \varepsilon - \frac{\left(\lambda_N+\lambda_{\max}^{(2)}\right)^2}{\left(\lambda_1+\lambda_{\max}^{(2)}\right)^2}\cdot\gamma = 0 \end{aligned} \quad (46)$$

Hence, when $\lambda_N/\lambda_1 \le a < 1$, there isn't a solution $\lambda \in \left[\lambda_{\min}^{(2)}, \lambda_{\max}^{(2)}\right]$ which satisfies $f(\lambda) = 0$.

From the expressions of the QICLCMP beamformer and NICCB, we know that NICCB is the specialism of the QICLCMP beamformer, and the QICLCMP beamformer is the generalization of NICCB. For the case of NICCB, $f(\lambda)$ is monotonically increasing function of $\lambda \ge 0$ [17]. Hence, we can educe that $f(\lambda)$ of the QICLCMP beamformer is also a monotonically increasing function of $\lambda \ge 0$. Namely, if the Lagrange multiplier λ exists, it must be unique.

Based on the analysis above, we can conclude that when $1 < a < \lambda_1/\lambda_N$, there is a unique solution $\lambda \in \left[\lambda_{\min}^{(1)}, \lambda_{\max}^{(1)}\right]$ that satisfies $f(\lambda) = 0$.

4. QUADRATIC INEQUALITY CONSTRAINT PARAMETER SELECTION

Based on the analysis above, we can see that it is important to select the quadratic inequality constraint parameter ε for the QICLCMP beamformer. If the quadratic inequality constraint parameter ε is large, it is inactive. On the contrary, if the quadratic inequality constraint parameter ε is small, there isn't a solution to satisfy the QICLCMP beamformer.

We have analyzed that when $1 < a < \lambda_1/\lambda_N$, there is a unique solution $\lambda \in [\lambda_{\min}^{(1)}, \lambda_{\max}^{(1)}]$ which satisfies $f(\lambda) = 0$. Hence, we can have the selecting bound of the quadratic inequality constraint parameter ε as follows:

$$1 < \sqrt{\frac{\varepsilon}{\gamma}} < \frac{\lambda_1}{\lambda_N} \tag{47}$$

Namely:

$$\gamma < \varepsilon < \gamma \cdot \left(\frac{\lambda_1}{\lambda_N}\right)^2 \tag{48}$$

Under the condition of $\varepsilon < \varepsilon_0 = \mathbf{f}^H \left(\mathbf{C}^H \mathbf{R}_\mathbf{x}^{-1} \mathbf{C}\right)^{-1} \mathbf{C}^H \mathbf{R}_\mathbf{x}^{-2} \mathbf{C}$ $(\mathbf{C}^H \mathbf{R}_\mathbf{x}^{-1} \mathbf{C})^{-1} \mathbf{f}$, we can obtain:

$$\varepsilon_{\min} \overset{\Delta}{=} \gamma < \varepsilon < \min \left\{ \gamma \cdot \left(\frac{\lambda_1}{\lambda_N}\right)^2, \quad \varepsilon_0 \right\} \overset{\Delta}{=} \varepsilon_{\max} \tag{49}$$

If the quadratic inequality constraint parameter ε is out of the above bound, there is no solution to the QICLCMP beamformer. Hence, the quadratic inequality constraint parameter ε should be chosen in the interval defined by the inequalities above.

5. LCMP BEAMFORMER UNDER QUADRATIC EQUALITY CONSTRAINT

Based on the analysis above, we can see that the quadratic inequality constraint can enhance the robustness of LCMP beamformer. Since the inequality relationship has a wide range, the norm of the weight vector will vary in the relevant wide range. If the fluctuation of the weight vector norm is acute, the performance improvement will be weakened greatly. Because the quadratic equality constraint (QEC) is stronger than the quadratic inequality constraint (QIC), the QECLCMP beamformer will have more ascendant robust performance than the QICLCMP beamformer. Hence, the QECLCMP beamformer is proposed and is solved effectively.

The QECLCMP beamformer is to impose an additional quadratic equality constraint on the Euclidean norm of $\mathbf{w}$. Hence, The

QECLCMP beamformer problem is formulated as follows:

$$\begin{cases} \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{\mathbf{x}} \mathbf{w} \\ s.t. \quad \mathbf{C}^H \mathbf{w} = \mathbf{f} \\ \qquad \|\mathbf{w}\|^2 = \varepsilon \end{cases} \tag{50}$$

Comparing the QECLCMP with QICLCMP beamformer, we can educe the conclusions as follows. (I) The solution to the QICLCMP beamformer is obtained on the boundary of its constraint. Similarly, for the QECLCMP beamformer, the solution is also obtained on its constraint boundary. (II) However, the solving methods of the two beamformer (or the optimization problem) is different, such as the forenamed solution to the QICLCMP beamformer. The Lagrange multiplier of the QICLCMP beamformer is taken as a positive real-value only, but for the QECLCMP beamformer, the Lagrange multiplier is taken as an arbitrary real-value, that is to say, it will be not only the positive real-value, but also the negative real-value. Hence, if we analyze from the view of solving the optimization problem, the QECLCMP beamformer has two solutions to the optimal Lagrange multiplier, one is positive, the other is negative. Actually, the positive one is the solution to the QICLCMP beamformer. For the sake of distinguishing the otherness, the negative solution is interested to the QECLCMP beamformer. In order to solve the QECLCMP beamformer, we must make use of the discussed results of the QICLCMP beamformer. Since the manipulation of some inequality, such as the inequality lessening and enlarging are only right for the positive real-value when we solve the QICLCMP beamformer.

Similar to the QICLCMP beamformer, the solution to the QECLCMP beamformer can also be solved by the Lagrange multiplier methodology. And the optimal weight vector of the QECLCMP beamformer has the same form as the QICLCMP beamformer. The only difference between the QECLCMP and QICLCMP beamformer is the Lagrange multiplier $\breve{\lambda}$. For the QICLCMP beamformer, $\lambda \geq 0$, here $\breve{\lambda}$ is arbitrary real-value.

5.1. Solution to the Optimal Lagrange Multiplier

Although the solution to the QECLCMP beamformer has the same form as the QICLCMP beamformer, the bound of the Lagrange multiplier is different. In order to use the analyzed results of the QICLCMP beamformer to disscuss the QECLCMP beamformer, we replace the Lagrange multiplier with its absolute value, namely the bound of the Lagrange multiplier $\breve{\lambda}$ for the QECLCMP beamformer is given by:

$$\sqrt{\frac{\varepsilon}{\gamma}} \leq \frac{\lambda_1 + \left|\breve{\lambda}\right|}{\lambda_N + \left|\breve{\lambda}\right|} \tag{51}$$

$$\sqrt{\frac{\varepsilon}{\gamma}} \geq \frac{\lambda_N + \left|\breve{\lambda}\right|}{\lambda_1 + \left|\breve{\lambda}\right|} \tag{52}$$

(1) If $a > 1$, then from (51) and (52), we can have:

$$\frac{\lambda_N - a\lambda_1}{a-1} \leq \left|\breve{\lambda}\right| \leq \frac{\lambda_1 - a\lambda_N}{a-1} \tag{53}$$

If $\lambda_1 - a\lambda_N > 0$, then $a < \lambda_1/\lambda_N$. Since $\lambda_N - a\lambda_1 < 0$, but $\left|\breve{\lambda}\right| > 0$. Therefore, when $1 < a < \lambda_1/\lambda_N$, we can have:

$$\breve{\lambda}^{(1)}_{\min} \triangleq -\frac{\lambda_1 - a\lambda_N}{a-1} \leq \breve{\lambda} \leq \frac{\lambda_1 - a\lambda_N}{a-1} \triangleq \breve{\lambda}^{(1)}_{\max} \tag{54}$$

Since $\breve{\lambda}^{(1)}_{\max} > 0$, and $\breve{\lambda}^{(1)}_{\min} = -\breve{\lambda}^{(1)}_{\max} < 0$. Hence, when $1 < a < \lambda_1/\lambda_N$, the solution to the QECLCMP beamformer in the bound of $[0, \breve{\lambda}^{(1)}_{\max}]$ is the same as the QICLCMP beamformer, but the solution in the bound of $[\breve{\lambda}^{(1)}_{\min}, 0]$ is the true solution to the QECLCMP beamformer.

(2) If $a < 1$, then from (51) and (52), we can have:

$$\frac{a\lambda_N - \lambda_1}{1-a} \leq \left|\breve{\lambda}\right| \leq \frac{a\lambda_1 - \lambda_N}{1-a} \tag{55}$$

If $a\lambda_1 - \lambda_N > 0$, then $a > \lambda_N/\lambda_1$. Since $a\lambda_N - \lambda_1 < 0$, but $\left|\breve{\lambda}\right| > 0$. Therefore, when $\lambda_N/\lambda_1 < a < 1$, we can have:

$$\breve{\lambda}^{(2)}_{\min} \triangleq -\frac{a\lambda_1 - \lambda_N}{1-a} \leq \breve{\lambda} \leq \frac{a\lambda_1 - \lambda_N}{1-a} \triangleq \breve{\lambda}^{(2)}_{\max} \tag{56}$$

Since $\breve{\lambda}^{(2)}_{\max} > 0$, and $\breve{\lambda}^{(2)}_{\min} = -\breve{\lambda}^{(2)}_{\max} < 0$, with the analysis of the QICLCMP beamformer above, we can obtain that when $\lambda_N/\lambda_1 < a < 1$ there isn't a solution in the bound of $[0, \breve{\lambda}^{(2)}_{\max}]$ to the QECLCMP beamformer. But the solution in the bound of $[\breve{\lambda}^{(2)}_{\min}, 0]$ is the true solution to the QECLCMP beamformer.

Based on the analysis above, we can conclude as follows.

(I) When $1 < a < \lambda_1/\lambda_N$, the solution in the bound of $[\breve{\lambda}_{\min}^{(1)}, 0]$ is the true solution to the QECLCMP beamformer, and the quadratic equality constraint parameter ε should be chosen in the interval defined by $\gamma < \varepsilon < \min\{\gamma \cdot (\lambda_1/\lambda_N)^2, \varepsilon_0\}$.

(II) When $\lambda_N/\lambda_1 < a < 1$, the solution in the bound of $[\breve{\lambda}_{\min}^{(2)}, 0]$ is the true solution to the QECLCMP beamformer, and the quadratic equality constraint parameter ε should be chosen in the bound of $\gamma \cdot (\lambda_N/\lambda_1)^2 < \varepsilon < \min\{\gamma, \varepsilon_0\}$.

(III) The QECLCMP beamformer has the form of diagonal loading with negative loading level, while the QICLCMP beamformer has the form of diagonal loading with positive loading level.

5.2. The QECLCMP Beamforming Algorithm

Similar to the QICLCMP beamforming algorithm, we summarize the QICLCMP beamforming algorithm below.

Step 1) Compute the eigendecomposition of the data covariance matrix $\mathbf{R_x}$, obtain the eigenvalues/eigenvectors of $\mathbf{R_x}$. And compute the parameter γ by the definition (30).

Step 2) For the given constraint parameter ε, compute the parameter a by the definition $a = \sqrt{\varepsilon/\gamma}$. If the inequality relationship $1 < a < \lambda_1/\lambda_N$ is satisfied, solve the Equation (31), namely $f(\breve{\lambda}) = 0$ in the bound of $[-(\lambda_1 - a\lambda_N)/(a-1), 0]$ (i.e., $[\breve{\lambda}_{\min}^{(1)}, 0]$). So the optimal Lagrange multiplier will be obtained by the Newton's method. Else if $\lambda_N/\lambda_1 < a < 1$, solve the Equation (31), namely $f(\breve{\lambda}) = 0$ in the bound of $[-(a\lambda_1 - \lambda_N)/(1-a), 0]$, (i.e., $[\breve{\lambda}_{\min}^{(2)}, 0]$). Except for the two bounds above, there isn't the solution to equation $f(\breve{\lambda}) = 0$. And the constraint parameter should be modified, so that the solution condition can be satisfied.

Step 3) Use the optimal Lagrange multiplier obtained in Step 2 to get the optimal weight vector according as (21), or as follows:

$$\hat{\mathbf{w}} = \mathbf{U}\left(\mathbf{\Lambda} + \breve{\lambda}\mathbf{I}\right)^{-1}\mathbf{U}^H\mathbf{C}\left[\left(\mathbf{U}^H\mathbf{C}\right)^H\left(\mathbf{\Lambda} + \breve{\lambda}\mathbf{I}\right)^{-1}\mathbf{U}^H\mathbf{C}\right]^{-1}\mathbf{f} \quad (57)$$

where the inverse of the diagonal matrix $\mathbf{\Lambda} + \breve{\lambda}\mathbf{I}$ is easily computed, and the matrix $\mathbf{U}^H\mathbf{C}$ is also available from Step 1.

6. SIMULATION ANALYSIS

In order to validate the correctness and the efficiency of the proposed algorithms, we analyze as follows. In our simulations, we assume a uniform linear array with $N = 10$ omnidirectional sensors spaced half a wavelength apart. Through all examples, we assume that there is one desired and two interfering sources, namely, there is a signal from direction $0°$, and two equi-powered interferers are located at $-40°$ and $60°$ respectively. The Signal Noise Ratio (SNR) and the Interferer Noise Ratio (INR) of the array data are both -5 dB. Therein, the presumed signal direction is equal to $5°$ (i.e., there is a $5°$ look direction mismatch). In the simulation, the sample number is 1000.

For comparison, the benchmark LCMP algorithm that corresponds to the ideal case when the covariance matrix is estimated by the Maximum Likelihood Estimator (MLE) and the actual steering vector is used. This algorithm does not correspond to any real situation but is included in our simulations for the sake of comparison only, and is denoted by Ideal-LCMP. The other algorithms include: LCMP, variable loading recursive least square (VLRLS)-LCMP [20], QICLCMP, QECLCMP. For the QICLCMP and QECLCMP beamformer, the constraint parameter are selected as the median of the allowable bound.

6.1. Effectivity Analyzing

In order to attest the efficiency of the proposed algorithms, the beamformer patterns are compared particularly, and the beamformer output SNR versus sample number and angle mismatch are analyzed in detail.

The LCMP beamformer pattern is given in Fig. 1. Since the mismatch of the the signal direction exists, the mainlobe of the LCMP beamformer departs from the signal actual direction, but in the interferers directions, there are deep nulls. The pattern of the VLRLS-LCMP beamformer is close to that of the LCMP beamformer. The QICLCMP beamformer is almost the same as the LCMP beamformer, and the QECLCMP beamformer is the best of all. The direction mismatch is overcame commendably, and its pattern has the lowest sidelobe level and interferer nulls. Here, the QICLCMP beamformer uses the positive optimal loading level, the QECLCMP beamformer uses the negative optimal loading level. From the comparison, we can see that the QECLCMP beamformer has a better performance than the QICLCMP beamformer and the others.

The variation of the beamformer output SNR versus samples number is given in Fig. 2. Apparently, the SNRs of the QICLCMP and VLRLS-LCMP beamformer are almost closed to that of the

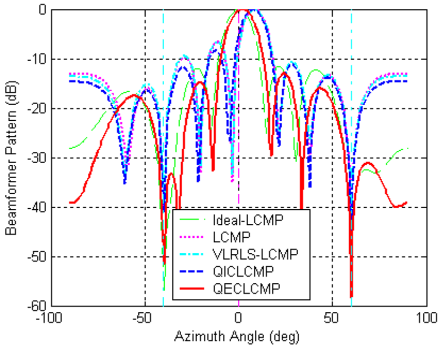

Figure 1. LCMP beamformer pattern comparison.

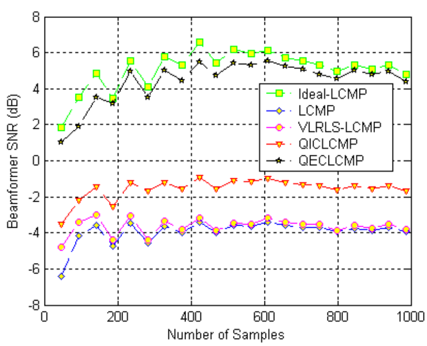

Figure 2. Output SNR versus samples number.

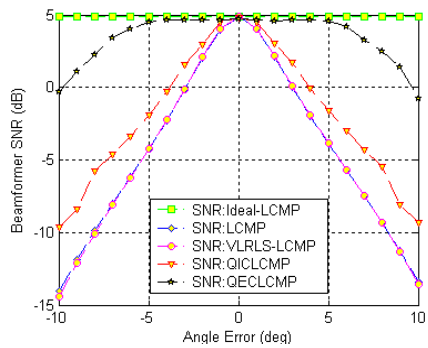

Figure 3. Output SNR versus angle mismatch.

LCMP beamformer, which are lower than that of the Ideal-LCMP beamformer. But the QECLCMP beamformer is the best of all, especially for the small number, it has the preferable performance. This is because that the QECLCMP beamformer not only has the precise pointing performance, but also has the lowest sidelobe level. Therefore, the quadratic equality constraint can improve the output SNR of the LCMP beamformer.

The variation of the LCMP beamformer output SNR versus signal direction mismatch or angle mismatch is given in Fig. 3. We can see that when the angle error is in the bound of $[-7°, 7°]$, the QECLCMP beamformer has a higher SNR than the QICLCMP, VLRLS-LCMP, LCMP beamformer. This is caused by the QECLCMP beamformer which not only has the precise pointing performance, but also has the lowest sidelobe level.

Based on the analysis above, we can see that the QECLCMP beamformer has the best robustness against the signal direction mismatch.

6.2. Correctness Analyzing

The QICLCMP and QECLCMP beamformers have the same form as the LCMP beamformer with diagonal loading. However, their key problems are to find their own optimal loading level or Lagrange multiplier. In order to show the impact of loading level on the LCMP beamformer under quadratic constraint (QCLCMP) and attest the correctness of the proposed algorithms, the simulation results are given as follows.

The variation of the output SNR versus diagonal loading level is given in Fig. 4. We can see that with the change of the loading level in the bound of $[\breve{\lambda}^{(1)}_{\min}, \breve{\lambda}^{(1)}_{\max}]$, the SNR of the QCLCMP beamformer varies accordingly. When the loading level is positive, QCLCMP is QICLCMP, whereas, when the loading level is negative, QCLCMP is QECLCMP. By comparison, we can see that the QECLCMP beamformer has higher SNR than the QICLCMP beamformer. For the optimal loading, namely when the loading level is equal to -4.23, the QECLCMP beamformer has the best pointing performance, and its SNR is the highest one. Hence, the loading level has a great impact on the SNR of the LCMP beamformer, and determines the performance improvement.

From the simulation results above, we can see that the loading level has a great impact on the performance of the LCMP beamformer,

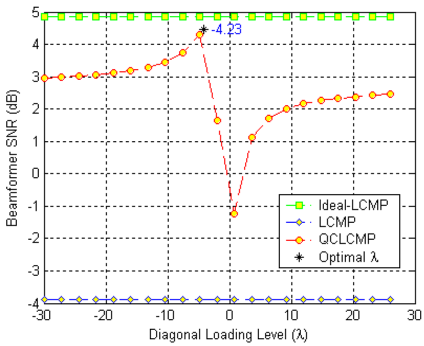

Figure 4. Output SNR versus loading level.

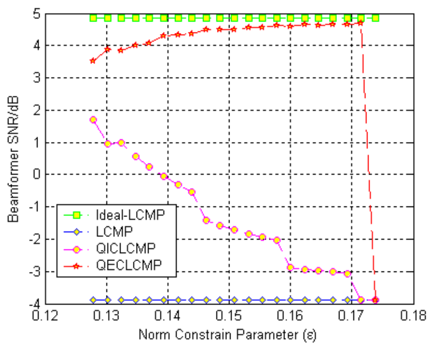

Figure 5. Output SNR versus constraint parameter.

and the QECLCMP beamformer has the best pointing performance, namely, the optimal negative loading is the best. This is also consistent to the theory analysis. For the robust beamformer with diagonal loading, the improvement is determined by the optimal loading level. When the loading level is optimal, the performance improvement will be the optimal. But for other values, the improvement will be little, or even worse.

6.3. Constraint Parameter Selection Analyzing

For the QCLCMP beamformer, there are two key problems, one is how to find the optimal loading level, and the other is how to select the norm constraint parameter. Although we have solved the two problems in theory, there is another key problem, namely, how to select the optimal norm constraint parameter. Therefore, the impact of norm constraint parameter on the QCLCMP beamformer is analyzed here particularly.

The variation of the output SNR versus norm constraint parameter is given in Fig. 5. We can see that with the change of the norm constraint parameter in the allowable bound of $(\varepsilon_{\min},\ \varepsilon_{\max})$, the SNR of the LCMP beamformer varies accordingly. The QICLCMP beamformer has a little higher SNR than that of the LCMP beamformer, and the QECLCMP beamformer has the highest SNR. And with the norm constraint parameter increasing, the SNR of the QECLCMP beamformer increases correspondingly, but the SNR of the QICLCMP beamformer is inclined to the SNR of the LCMP beamformer. When the norm constraint parameter is equal to the maximum, the constraint is inactive, and the three SNRs tend to the same value. Hence, the SNR is determined by the choice of the norm constraint parameter, especially for the QECLCMP beamformer.

According to the simulation results above, we can see that if the norm constraint parameter is selected in the allowable bound, the norm constraint parameter has a great impact on the performance of the QICLCMP and QECLCMP beamformer, especially for the QECLCMP beamformer. But the QECLCMP beamformer with the larger constraint parameter has the better pointing performance, namely, when the constraint parameter is selected as a larger value in its allowable bound, the optimal negative loading has the optimal improvement.

7. CONCLUSION

Since the quadratic constraints on the weight vector of the LCMP beamformer can improve the robustness to pointing errors and to random perturbations in sensor parameter, the Lagrange multiplier method is developed to solve the LCMP beamformer under quadratic constraint which includes the inequality and equality constraint. Therein, the exact Lagrange multiplier or loading level is obtained, and the optimal weight vector can be computed exactly. The choice of the quadratic constraint parameter is analyzed and the selecting bound is given. Above all, this method gives the efficient solution to find the optimal loading level for the QCLCMP beamformer. From the theory analysis and the simulations, the QECLCMP beamformer has the best performance to overcome the steering vector mismatch, namely the optimal negative loading has the preferable robustness.

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REFERENCES

1. Cox, H., "Resolving power and sensitivity to mismatch of optimum array processors," *J. Acoust. Soc. Amer.*, Vol. 54, No. 3, 771–785, Mar. 1973.
2. Feldman, D. D. and L. J. Griffiths, "A projection approach to robust adaptive beamforming," *IEEE Trans. Signal Processing*, Vol. 42, 867–876, Apr. 1994.
3. Wax, M. and Y. Anu, "Performance analysis of the minimum variance beamformer in the presence of steering vector errors," *IEEE Trans. Signal Processing*, Vol. 44, 938–947, Apr. 1996.
4. Hung, E. K. and R. M. Turner, "A fast beamforming algorithm for large arrays," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. 19, 598–607, Jul. 1983.
5. Carlson, B. D., "Covariance matrix estimation errors and diagonal loading in adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. 24, 397–401, Jul. 1988.
6. Wax, M. and Y. Anu, "Performance analysis of the minimum variance beamformer," *IEEE Trans. Signal Processing*, Vol. 44, 928–937, Apr. 1996.
7. Cox, H., R. M. Zeskind, and M. H. Owen, "Robust adaptive beamforming," *IEEE Trans. Acoust., Speech, Signal Processing*, Vol. 35, 1365–1376, Oct. 1987.

8. Gershman, A. B., "Robust adaptive beamforming in sensor arrays," *Int. J. Electron. Commun.*, Vol. 53, 305–314, Dec. 1999.
9. Robert, G. L. and P. B. Stephen, "Robust minimum variance beamforming," *IEEE Trans. Signal Processing*, Vol. 53, No. 5, 1684–1696, May 2005.
10. Vorobyov, S. A., A. B. Gershman, and Z.-Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem," *IEEE Trans. Signal Processing*, Vol. 51, No. 2, 313–324, Feb. 2003.
11. Elnashar, A., S. M. Elnoubi, and H. A. El-Mikati, "Further study on robust adaptive beamforming with optimum diagonal loading," *IEEE Trans. Antennas Propagation*, Vol. 54, No. 12, 3647–3658, Dec. 2006.
12. Chen, C. Y. and P. P. Vaidyanathan, "Quadratically constrained beamforming robust against direction-of-arrival mismatch," *IEEE Trans. Signal Processing*, Vol. 55, No. 8, 4139–4150, Aug. 2007.
13. Mutapcic, A., S.-J. Kim, and S. Boyd, "Beamforming with uncertain weights," *IEEE Signal Processing Letter*, Vol. 14, No. 5, 348–351, May 2007.
14. Gu, Y. J., Z. G. Shi, K. S. Chen, and Y. Li, "Robust adaptive beamforming for steering vector uncertainties based on equivalent DOAs method," *Progress In Electromagnetics Research*, Vol. 79, 277–290, 2008.
15. Gu, Y. J., Z. G. Shi, K. S. Chen, and Y. Li, "Robust adaptive beamforming for a class of Gaussian steering vector mismatch," *Progress In Electromagnetics Research*, Vol. 81, 315–328, 2008.
16. Li, Y., Y. J. Gu, Z.-G. Shi, and K. S. Chen, "Robust adaptive beamforming based on particle filter with noise unknown," *Progress In Electromagnetics Research*, Vol. 90, 151–169, 2009.
17. Li, J., P. Stotica, and Z. Wang, "On robust capon beamformer and diagonal loading," *IEEE Trans. Signal Processing*, Vol. 51, No. 7, 1702–1715, Jul. 2003.
18. Shahram, S., A. B. Gershman, Z. Luo, et al., "Robust adaptive beamforming for general-rank signal models," *IEEE Trans. Signal Processing*, Vol. 51, No. 9, 2257–2269, Sep. 2003.
19. Li, J., P. Stotica, and Z. Wang, "Doubly constrained robust capon beamformer," *IEEE Trans. Signal Processing*, Vol. 52, No. 9, 2407–2423, Sep. 2004.
20. Tian, Z., K. L. Bell, and H. L. Van Trees, "A recursive least squares implementation for LCMP beamforming under quadratic constraint," *IEEE Trans. Signal Processing*, Vol. 49, No. 6, 1365–

1376, Jun. 2001.

21. Frost III, O. L., "An algorithm for linearly constrained adaptive array processing," *Proc. IEEE*, Vol. 60, 926–935, Aug. 1972.
22. Er, M. H. and A. Cantoni, "Derivative constraints for broad-band element space antenna array processors," *IEEE Trans. Acoust., Speech, Signal Processing*, Vol. 31, 1378–1393, Dec. 1983.
23. Buckley, K. M. and L. J. Griffiths, "An adaptive generalized sidelobe canceller with derivative constraints," *IEEE Trans. Antennas Propagation*, Vol. 349, 311–319, Mar. 1986.
24. Zhang, S. and I. L. Thng, "Robust presteering derivative constraints for broadband antenna arrays," *IEEE Trans. Signal Processing*, Vol. 50, 1–10, Jan. 2002.
25. Byrne, D., M. O'Halloran, E. Jones, and M. Glavin, "Transmitter-grouping robust capon beamforming for breast cancer detection," *Progress In Electromagnetics Research*, Vol. 108, 401–416, 2010.
26. Umrani, A. W., Y. Guan, and F. A. Umrani, "Effect of steering error vector and angular power distributions on beamforming and transmit diversity systems in correlated fading channel," *Progress In Electromagnetics Research*, Vol. 105, 383–402, 2010.
27. O'Halloran, M., M. Glavin, and E. Jones, "Performance and robustness of a multistatic mist beamforming algorithm for breast cancer detection," *Progress In Electromagnetics Research*, Vol. 105, 403–424, 2010.
28. Mallipeddi, R., J. P. Lie, P. N. Suganthan, S. G. Razul, and C. M. S. See, "A differential evolution approach for robust adaptive beamforming based on joint estimation of look direction and array geometry," *Progress In Electromagnetics Research*, Vol. 119, 381–394, 2011.
29. Zaharis, Z. D. and T. V. Yioultsis, "A novel adaptive beamforming technique applied on linear antenna arrays using adaptive mutated boolean PSO," *Progress In Electromagnetics Research*, Vol. 117, 165–179, 2011.
30. Zaharis, Z. D., C. Skeberis, and T. D. Xenos, "Improved antenna array adaptive beamforming with low side lobe level using a novel adaptive invasive weed optimization method," *Progress In Electromagnetics Research*, Vol. 124, 137–150, 2012.
31. Zaharis, Z. D., K. A. Gotsis, and J. N. Sahalos, "Comparative study of neural network training applied to adaptive beamforming of antenna arrays," *Progress In Electromagnetics Research*, Vol. 126, 269–283, 2012.