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SPATIAL POWER SPECTRUM OF MULTIPLE SCATTERED ORDINARY AND EXTRAORDINARY WAVES IN MAGNETIZED PLASMA WITH ELECTRON DENSITY FLUCTUATIONS

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Abstract—Features of the spatial power spectrum (SPS) of multiple scattered ordinary and extraordinary waves in randomly inhomogeneous magnetized plasma are investigated using the smooth perturbation method taking into account diffraction effects. Second order statistical moments are derived for arbitrary correlation function of electron density fluctuations at oblique illumination of magnetized plasma by mono-directed incident radiation. Numerical calculations have been carried out for anisotropic Gaussian correlation function taking into account anisotropy factor and angle of inclination of prolate irregularities with respect to the external magnetic field. It was shown that SPS has a double-peaked shape. External magnetic field narrows SPS for ordinary wave and the gap arises in the direction of prolate irregularities. For extraordinary wave the gap increases with a distance passing by the wave in anisotropic magnetized plasma, the width broadens and maximum slightly displaced.

1. INTRODUCTION

Peculiarities of electromagnetic waves propagation in randomly statistically isotropic media have been intensively studied [1, 2]. However in many cases irregularities are anisotropic. Particularly,

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molecules of thermotropic liquid crystals havying characteristic spatial scale about 20 angstrom the ratio of longitudinal and transverse sizes equals $4 \div 8$ [3]. Chloroplasts of algae and plants are ellipsoidal in shape with a diameter $1 \div 5\,\mu\text{m}$ and length of $1 \div 10\,\mu\text{m}$ [3]. Polymer macromolecules are oriented in a liquid flow with velocity gradient [4, 5]. Random plasma inhomogeneities in the Earth's ionosphere are aligned along geomagnetic field [6].

The features of the spatial power spectrum (SPS) of scattered radiation in magnetized anisotropic plasma in the complex geometrical optics approximation using the perturbation method have been investigated in [7–10]. Evolution of the angular distribution of ray intensity at light propagation in random medium with prolate irregularities has been considered in [11]. Numerical simulation has been carried out by Monte-Carlo method. It was shown that strongly pronounced dip exists in the angular spectrum of a single-scattered radiation along the direction of prolate inhomogeneities caused due to permittivity fluctuations. The same peculiarities have been revealed analytically in the SPS of a multiple scattered radiation at oblique illumination of medium with prolate inhomogeneities by mono-directed incident radiation using the smooth perturbation method: the SPS has a double-peaked shape, the location of its maximum weakly varies and width substantially broadens with increasing distance passing by electromagnetic waves in a randomly-inhomogeneous medium [12].

This paper is devoted to the analytical and numerical investigations of a double-peak shape SPS of multiple scattered ordinary and extraordinary waves in anisotropic inhomogeneous magnetized plasma. Diffraction effects are taken into account using the smooth perturbation method. Numerical calculations are carried out for anisotropic Gaussian correlation function of electron density fluctuations applying experimental data.

2. FORMULATION OF THE PROBLEM

Initial is the vector wave equation for the electric field $\mathbf{E}$:

$$\left(\frac{\partial^2}{\partial x_i \partial x_j} - \Delta \delta_{ij} - k_0^2 \varepsilon_{ij}(\mathbf{r}) \right) \mathbf{E_j}(\mathbf{r}) = 0. \tag{1}$$

If an incident electromagnetic wave propagates along z axis and the unit vector τ of an external magnetic field lies in the yoz coordinate plane ($\mathbf{k_0} \parallel z$, $\mathbf{H_0} \in yoz$-principle plane), components of second-rank tensor ε_{ij} of collisionless magnetized plasma have the following form [13]:

$$\varepsilon_{xx}=1-\frac{v}{1-u},\ \ \varepsilon_{yy}=1-\frac{v\left(1-u\sin^2\alpha\right)}{1-u},\ \ \varepsilon_{zz}=1-\frac{v\left(1-u\cos^2\alpha\right)}{1-u},$$
$$\varepsilon_{xy}=-\varepsilon_{yx}=i\frac{v\sqrt{u}\cos\alpha}{1-u},\quad \varepsilon_{yz}=\varepsilon_{zy}=\frac{uv\sin\alpha\cos\alpha}{1-u},\tag{2}$$
$$\varepsilon_{xz}=-\varepsilon_{zx}=-i\frac{v\sqrt{u}\sin\alpha}{1-u},$$

where α is the angle between $\mathbf{k_0}$ and $\mathbf{H_0}$ vectors; $\varepsilon_{xy}=i\tilde{\varepsilon}_{xy}$, $\varepsilon_{xz}=-i\tilde{\varepsilon}_{xz}$, $u=(eH_0/mc\omega)^2$, $v=\omega_p^2/\omega^2$ are the magneto-ionic parameters; $\omega_p=(4\pi Ne^2/m)^{1/2}$ is the plasma frequency; $\Omega_H=eH_0/mc$ is the electron gyrofrequency.

Dielectric permittivity of turbulent magnetized plasma is a random function of spatial coordinates $\varepsilon_{ij}(\mathbf{r})=\varepsilon_{ij}^{(0)}+\varepsilon_{ij}^{(1)}(\mathbf{r})$, $|\varepsilon_{ij}^{(1)}(\mathbf{r})|\ll 1$. First component represents zero-order approximation, second one takes into account electron density fluctuations; complex phase fluctuations are of the order $\varphi_1\sim\varepsilon_{ij}^{(1)}$, $\varphi_2\sim\varepsilon_{ij}^{(1)2}$. Wave field we introduce as $E_j(\mathbf{r})=E_{0j}\exp(\varphi_0+\varphi_1+\varphi_2+\ldots)$, $\varphi_0=ik_\perp y+ik_0z$ $(k_\perp\ll k_0)$.

In zero-order approximation we have wave equation

$$\left[\frac{\partial^2\varphi_0}{\partial x_i\partial x_j}+\frac{\partial\varphi_0}{\partial x_i}\frac{\partial\varphi_0}{\partial x_j}+\left(k_\perp^2+k_0^2\right)\delta_{ij}-k_0^2\varepsilon_{ij}^{(0)}\right]E_{0j}=0\tag{3}$$

containing the set of three algebraic equations for E_{0j} $(j=x,y,z)$ regular field components:

$$\begin{aligned}\left(\mu^2+1-\varepsilon_{xx}^{(0)}\right)E_{0x}-i\tilde{\varepsilon}_{xy}^{(0)}E_{0y}+i\tilde{\varepsilon}_{xz}^{(0)}E_{0z}&=0,\\ i\tilde{\varepsilon}_{xy}^{(0)}E_{0x}+\left(1-\varepsilon_{yy}^{(0)}\right)E_{0y}-\left(\mu+\varepsilon_{yz}^{(0)}\right)E_{0z}&=0,\\ i\tilde{\varepsilon}_{xz}^{(0)}E_{0x}+\left(\mu+\varepsilon_{yz}^{(0)}\right)E_{0y}-\left(\mu^2-\varepsilon_{zz}^{(0)}\right)E_{0z}&=0.\end{aligned}\tag{4}$$

Solution of determinant allows us to calculate parameter $\mu=k_\perp/k_0$.

Taking into account inequalities characterizing the smooth perturbation method [1, 2, 14]:

$$\left|\frac{\partial\varphi_1}{\partial z}\right|\ll k_0|\varphi_1|,\quad \left|\frac{\partial^2\varphi_1}{\partial z^2}\right|\ll k_0\left|\frac{\partial\varphi_1}{\partial z}\right|,$$
$$\left|\frac{\partial\varphi_2}{\partial z}\right|\ll k_0|\varphi_2|,\quad \left|\frac{\partial^2\varphi_2}{\partial z^2}\right|\ll k_0\left|\frac{\partial\varphi_2}{\partial z}\right|,$$

in the first approximation we obtain:

$$\left[\frac{\partial^2\varphi_1}{\partial x_i\partial x_j}+\frac{\partial\varphi_0}{\partial x_i}\frac{\partial\varphi_1}{\partial x_j}+\frac{\partial\varphi_1}{\partial x_i}\frac{\partial\varphi_0}{\partial x_j}-\delta_{ij}\left(\Delta_\perp+2ik_\perp\frac{\partial\varphi_1}{\partial y}+2ik_0\frac{\partial\varphi_1}{\partial z}\right)\right.$$
$$\left.-k_0^2\varepsilon_{ij}^{(0)}\right]E_{0j}=0. \tag{5}$$

where $\Delta_\perp=(\partial^2\varphi_1/\partial x^2)+(\partial^2\varphi_1/\partial y^2)$ is the transversal Laplasian.

Two-dimensional Fourier transformation for the phase fluctuations is

$$\varphi_1(x,y,z)=\int\limits_{-\infty}^{\infty}dk_x\int\limits_{-\infty}^{\infty}dk_y\psi(k_x,k_y,z)\exp(ik_xx+ik_yy),$$

For $j=x,\,y,\,z$ components from Equation (5) we obtain set of stochastic differential equations:

$$\frac{\partial\psi}{\partial z}+\frac{i}{k_x\frac{E_{0z}}{E_{0x}}-2k_0}\left[k_x(k_y+k_\perp)\frac{E_{0y}}{E_{0x}}+k_xk_0\frac{E_{0z}}{E_{0x}}-k_y(k_y+2k_\perp)\right]\psi$$
$$=-i\frac{k_0^2}{k_x\frac{E_{0z}}{E_{0x}}-2k_0}\cdot\left(\varepsilon_{xx}^{(1)}+i\tilde{\varepsilon}_{xy}^{(1)}\frac{E_{0y}}{E_{0x}}-i\tilde{\varepsilon}_{xz}^{(1)}\frac{E_{0z}}{E_{0x}}\right), \tag{6}$$

$$\frac{\partial\psi}{\partial z}+\frac{i}{(k_y+k_\perp)\frac{E_{0z}}{E_{0x}}-2k_0\frac{E_{0y}}{E_{0x}}}\left[k_x(k_y+k_\perp)+k_0k_y\frac{E_{0z}}{E_{0x}}-k_x^2\frac{E_{0y}}{E_{0x}}\right]\psi$$
$$=-i\frac{k_0^2}{(k_y+k_\perp)\frac{E_{0z}}{E_{0x}}-2k_0\frac{E_{0y}}{E_{0x}}}\cdot\left(-i\tilde{\varepsilon}_{xy}^{(1)}+\varepsilon_{yy}^{(1)}\frac{E_{0y}}{E_{0x}}+\varepsilon_{yz}^{(1)}\frac{E_{0z}}{E_{0x}}\right), \tag{7}$$

$$\frac{\partial\psi}{\partial z}+\frac{i}{k_x+(k_y+k_\perp)\frac{E_{0y}}{E_{0x}}}\left[k_0k_x+k_0k_y\frac{E_{0y}}{E_{0x}}-\left(k_x^2+k_y^2+2k_\perp k_y\right)\frac{E_{0z}}{E_{0x}}\right]\psi$$
$$=-i\frac{k_0^2}{k_x+(k_y+k_\perp)\frac{E_{0y}}{E_{0x}}}\cdot\left(i\tilde{\varepsilon}_{xz}^{(1)}+\varepsilon_{yz}^{(1)}\frac{E_{0y}}{E_{0x}}+\varepsilon_{zz}^{(1)}\frac{E_{0z}}{E_{0x}}\right). \tag{8}$$

Mean electric field components are determined by well-known formulae: $(E_{0y}/E_{0x})=iP_j$, $(E_{0z}/E_{0x})=i\Gamma_j$, where polarization coefficients are [13]:

$$P_j=\frac{2\sqrt{u}(1-v)\cos\alpha}{u\sin^2\alpha\pm\sqrt{u^2\sin^4\alpha+4u(1-v)^2\cos^2\alpha}},$$
$$\Gamma_j=-\frac{v\sqrt{u}\sin\alpha+P_juv\sin\alpha\cos\alpha}{1-u-v+uv\cos^2\alpha}, \tag{9}$$

minus sign and index $j=1$ correspond to the extraordinary wave, plus sign and index $j=2$ to the ordinary wave. Ordinary and extraordinary waves in magnetized plasma generally are elliptically polarized.

Turbulence of the ionospheric plasma is caused by electron density fluctuations, which are random functions of the spatial coordinates: $v(\mathbf{r}) = v_0[1 + n_1(\mathbf{r})]$. Substituting this expression in (2) we can easily restore regular and fluctuating components of magnetized plasma permittivity tensor.

Transverse correlation function of a scattered field has the following form $W_{EE^*}(\rho) = \langle E(\mathbf{r})E^*(\mathbf{r} + \rho)\rangle$. Taking into account that the observation points are spaced apart at a small distances $\rho = \{\rho_x, \rho_y\}$, we have [12]

$$W_{EE^*}(\rho, k_\perp) = E_0^2 \exp(-i\rho_y k_\perp) \exp\left[\frac{1}{2}\left(\left\langle\varphi_1^2(\mathbf{r})\right\rangle + \left\langle\varphi_1^{2*}(\mathbf{r}+\rho)\right\rangle\right)\right] \exp(2\langle\varphi_2\rangle) \cdot \exp\left[\langle\varphi_1(\mathbf{r})\varphi_1^*(\mathbf{r} + \rho)\rangle\right], \quad (10)$$

where E_0^2 is the intensity of an incident radiation.

SPS of scattered field in case of incident plane wave $W(k, k_\perp)$ is easily calculated by Fourier transform of the transversal correlation function of a scattered field [2]

$$W(k, k_\perp) = \int_{-\infty}^{\infty} d\rho_y W_{EE^*}(\rho_y, k_\perp) \exp(ik\rho_y). \quad (11)$$

On the other hand, when the angular spectrum of an incident wave has a finite width and its maximum is directed along z axis, SPS of scattered radiation is given by the expression [12]:

$$I(k) = \int_{-\infty}^{\infty} dk_\perp W(k, k_\perp) \exp(-k_\perp^2 \beta^2), \quad (12)$$

where β characterizes the dispersal of an incident radiation, and k is a transverse component of the wave vector of scattered field evaluating by Fourier transform from the correlation function [1, 2].

2.1. Second Order Statistical Moments of the Phase Fluctuations

Two-dimensional spectral component of the phase fluctuation of scattered electromagnetic field (6) along the direction perpendicular to the principle plane ($j = x$) in the first approximation satisfies stochastic differential equation:

$$\frac{\partial\psi}{dz} + \frac{id_1 - d_2}{\Gamma_j k_x + 2ik_0}\psi(k_x, k_y, z) = -\frac{k_0^2}{\Gamma_j k_x + 2ik_0}\left\{\varepsilon_{xx}^{(1)}(k_x,k_y,z) - \left[P_j \tilde{\varepsilon}_{xy}^{(1)}(k_x,k_y,z) - \Gamma_j \tilde{\varepsilon}_{xz}^{(1)}(k_x,k_y,z)\right]\right\}, \quad (13)$$

where: $d_1 = k_x(k_y + k_\perp)P_j + k_0 k_x \Gamma_j$, $d_2 = k_y(k_y + 2k_\perp)$. The solution of the Equation (13) with the boundary condition $\psi(k_x,\, k_y,\, z = 0) = 0$ is:

$$\psi(k_x,k_y,z) = -\frac{k_0^2}{\Gamma_j k_x + 2ik_0}\int\limits_{-\infty}^{\infty} dz' \left\{\varepsilon_{xx}^{(1)}(k_x,k_y,z')\right.$$
$$\left. -\left[P_j\tilde{\varepsilon}_{xy}^{(1)}(k_x,k_y,z')-\Gamma_j\tilde{\varepsilon}_{xz}^{(1)}(k_x,k_y,z')\right]\right\}\cdot\exp\left[-\frac{d_2-id_1}{\Gamma_j k_x+2ik_0}(L-z')\right], \quad (14)$$

where L is a distance traveling by the wave in a turbulent magnetized plasma. Using Equation (14), taking into account that

$$\left\langle T_{\alpha\beta}^*(\kappa,z')T_{\gamma\delta}^*(\kappa',z'')\right\rangle = W'_{\alpha\beta,\gamma\delta}(\kappa,z'-z'')\delta(\kappa+\kappa'),$$
$$\left\langle T_{\alpha\beta}(\kappa,z')T_{\gamma\delta}^*(\kappa',z'')\right\rangle = W_{\alpha\beta,\gamma\delta}(\kappa,z'-z'')\delta(\kappa-\kappa'),$$

where $W_{\alpha\beta,\gamma\delta}\,(\kappa,\, z' - z'')$ is 2D spatial spectrum of electron density fluctuations; indices denote the product of fluctuating components of second rank tensor (2); δ is Dirac delta function, changing the variables $z' - z'' = \rho_z$, $z' + z'' = 2\,\eta$. We obtain second order statistical moments of scattered electromagnetic waves:

$$\left\langle\varphi_1^2(\mathbf{r})\right\rangle = \frac{\pi k_0^2}{2}T\int\limits_{-\infty}^{\infty} dk_x\int\limits_{-\infty}^{\infty} dk_y\frac{G_1+iG_2}{G_1^2+G_2^2}\,[V_{xx,xx}+V_{xy,xy}+V_{xz,xz}$$
$$+2(V_{xx,xz}-V_{xx,xy}-V_{xy,xz})]\cdot\{1-\exp[(G_1-iG_2)L]\}\,. \quad (15)$$

$$\left\langle\varphi_1^{*2}(\mathbf{r}+\rho)\right\rangle = \frac{\pi k_0^2}{2}\int\limits_{-\infty}^{\infty} dk_x\int\limits_{-\infty}^{\infty} dk_y\frac{G_1-iG_2}{G_1^2+G_2^2}\left[V'_{xx,xx}+V'_{xy,xy}+V'_{xz,xz}\right.$$
$$\left.+2(V'_{xx,xz}-V'_{xx,xy}-V'_{xy,xz})\right]\cdot\{1-\exp[(G_1+iG_2)L]\}\,, \quad (16)$$

$$\langle\varphi_1(\mathbf{r})\varphi_1^*(\mathbf{r}+\rho)\rangle = \frac{\pi k_0^2 L}{2}\int\limits_{-\infty}^{\infty} dk_x\int\limits_{-\infty}^{\infty} dk_y\left[V''_{xx,xx}+V''_{xy,xy}+V''_{xz,xz}\right.$$
$$\left.+2(V''_{xx,xz}-V''_{xx,xy}-V''_{xy,xz})\right]\cdot\exp(-ik_x\rho_x-ik_y\rho_y), \quad (17)$$

where correlation functions are: $V_{\alpha\beta,\gamma\delta} \equiv V_{\alpha\beta,\gamma\delta}(k_x,k_y,iG_3-G_4)$, $V'_{\alpha\beta,\gamma\delta}\equiv V'_{\alpha\beta,\gamma\delta}(k_x,k_y,-iG_3-G_4)$, $V''_{\alpha\beta,\gamma\delta}\equiv V''_{\alpha\beta,\gamma\delta}(k_x,k_y,-\frac{d_1\Gamma_j k_x+2d_2k_0}{4k_0^2})$

$$G_1=\frac{1}{k_0^2}(\Gamma_j k_\perp - P_j k_0)k_x k_y, \quad G_2=\frac{1}{2k_0^2}\left[\Gamma_j(P_j k_\perp+\Gamma_j k_0)k_x^2+2k_0k_y^2\right],$$

$$G_3=\frac{1}{4k_0^2}\left(2\Gamma_j k_0^2+2P_j k_0 k_\perp-\Gamma_j k_y^2\right)k_x, \quad G_4=\frac{1}{4k_0^2}\left(P_j\Gamma_j k_x^2+4k_0k_\perp\right)k_y.$$

In the absence of an external magnetic field the expressions (15)–(17) are simplified and coincide with [12]. Contrary to the ray-(optics) approximation, these correlation functions containing parameter k_y^2/k_0^2 take into account diffraction effects. These effects become essential if $L \gg (l/\lambda)$ is substantially big, l is characteristic spatial scale of electron density fluctuations, λ-wavelength.

Using Equation (1), phase fluctuation in second order approximation satisfies differential equation

$$\left[\left(\frac{\partial^2\varphi_2}{\partial x_i \partial x_j}+\frac{\partial\varphi_0}{\partial x_i}\frac{\partial\varphi_2}{\partial x_j}+\frac{\partial\varphi_2}{\partial x_i}\frac{\partial\varphi_0}{\partial x_j}\right)-\delta_{ij}\left(\frac{\partial^2\varphi_2}{\partial x^2}+\frac{\partial^2\varphi_2}{\partial y^2}+2ik_\perp\frac{\partial\varphi_2}{\partial y}\right.\right.$$
$$\left.\left.+2ik_0\frac{\partial\varphi_2}{\partial z}\right)\right]E_{0j}=\left\{-\frac{\partial\varphi_1}{\partial x_i}\frac{\partial\varphi_1}{\partial x_j}+\delta_{ij}\left[\left(\frac{\partial\varphi_1}{\partial x}\right)^2+\left(\frac{\partial\varphi_1}{\partial y}\right)^2\right]\right\}E_{0j}. \quad (18)$$

From Equation (18) for $j = x$ component we obtain stochastic differential equation:

$$iP_j\frac{\partial^2\varphi_2}{\partial x\partial y}+i\Gamma_j\frac{\partial^2\varphi_2}{\partial x\partial z}-(k_\perp P_j+k_0\Gamma_j)\frac{\partial\varphi_2}{\partial x}-\frac{\partial^2\varphi_2}{\partial y^2}-2ik_\perp\frac{\partial\varphi_2}{\partial y}-2ik_0\frac{\partial\varphi_2}{\partial z}$$
$$=-iP_j\frac{\partial\varphi_1}{\partial x}\frac{\partial\varphi_1}{\partial y}+\left(\frac{\partial\varphi_1}{\partial y}\right)^2, \quad (19)$$

Using Fourier transformation to the Equation (19) we have

$$\mathrm{Re}\langle\varphi_2(\mathbf{r})\rangle=\mathrm{Re}\frac{\pi k_0}{4}\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y v\frac{1}{G_1^2+G_2^2}\left\{L(A_1+iB_1)+\frac{1}{G_1^2+G_2^2}\right.$$
$$\left.[(A_1G_1-B_1G_2)+i(B_1G_1+A_1G_2)]-\frac{1}{G_1^2+G_2^2}(A_2+iB_2)\exp(G_1L)\right\}$$
$$[V_{xx,xx}+V_{xy,xy}+V_{xz,xz}+2(V_{xx,xz}-V_{xx,xy}-V_{xy,xz})], \quad (20)$$

where: $A_1 = -(P_jG_1k_xk_y + G_2k_y^2)$, $A_2 = (A_1G_1 - B_1G_2)\cos(G_2L) + (B_1G_1 + A_1G_2)\sin(G_2L)$, $B_1 = G_1k_y^2 - P_jG_2k_xk_y$, $B_2 = (B_1G_1 + A_1G_2)\cos(G_2L) - (A_1G_1 - B_1G_2)\sin(G_2L)$. In the absence of an external magnetic field we obtain [12].

3. NUMERICAL CALCULATIONS

SPS of scattered radiation caused by electron density fluctuations analytically and numerically will be investigated for anisotropic Gaussian correlation function having in the principle yoz plane following form

$$W_n(k_x,k_y,k_z)=\sigma_n^2\frac{l_\perp^2 l_\parallel}{8\pi^{3/2}}\exp\left(-\frac{k_x^2 l_\perp^2}{4}-p_1\frac{k_y^2 l_\parallel^2}{4}-p_2\frac{k_z^2 l_\parallel^2}{4}-p_3 k_y k_z l_\parallel^2\right), \qquad (21)$$

where: $p_1 = (\sin^2\gamma_0+\chi^2\cos^2\gamma_0)^{-1}[1+(1-\chi^2)^2\sin^2\gamma_0\cos^2\gamma_0/\chi^2]$, $p_2 = (\sin^2\gamma_0+\chi^2\cos^2\gamma_0)/\chi^2$, $p_3 = (1-\chi^2)\sin\gamma_0\cos\gamma_0/2\chi^2$, σ_n^2 is variance of electron density fluctuations. This function contains anisotropy factor of irregularities $\chi = l_\parallel/l_\perp$ (ratio of longitudinal and transverse linear scales of plasma irregularities) and inclination angle of prolate irregularities with respect to the external magnetic field γ_0.

Substituting Equation (21) into (17) we have:

$$\langle\varphi_1(\mathbf{r})\varphi_1^*(\mathbf{r}+\rho)\rangle =$$
$$\frac{\sigma_n^2\Omega}{16\sqrt{\pi}}\frac{T^3 k_0 L}{\chi^2}\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}ds\exp\left[-\frac{T^2}{4}\left(p_2 m_3^2 x^4+b_2 x^2+b_3\right)\right]\exp(-i\xi x-i\eta s), \qquad (22)$$

where: $b_2=\frac{1}{\chi^2}+2p_2 m_3 m_4+4p_3 m_3 s$, $b_3=\frac{1}{4}p_2 s^4+(p_2\mu+2p_3)s^3+(p_1+p_2\mu^2+4p_3\mu)s^2$, $\Omega = \frac{v_0^2}{(1-u_0)^2}[1+u_0-2\sqrt{u_0}(\sin\alpha-\cos\alpha+\sqrt{u_0}\sin\alpha\cos\alpha]$, $s = \frac{k_y}{k_0}$, $x = \frac{k_x}{k_0}$, $\eta = k_0\rho_y$, $\xi = k_0\rho_x$ (ρ_x — distance between observation points spaced apart along direction perpendicular to the principle plane), $m_3 = \frac{1}{4}[(s+\mu)P_j+\Gamma_j]\Gamma_j$, $m_4 = \frac{1}{2}(s^2+2s\mu)$.

Applying the saddle point method ($T = k_0 l_\parallel \gg 1$) Equation (22) is reduced to the expression:

$$\langle\varphi_1(\mathbf{r})\varphi_1^*(\mathbf{r}+\rho)\rangle = \frac{\Omega T B_0}{2\sqrt{\pi}\chi}\int_{-\infty}^{\infty}ds\frac{1}{\sqrt{b_2}}$$
$$\exp\left(-\frac{T^2}{2}\left[\frac{1}{4}p_2 s^4+(p_2\mu+2p_3)s^3+\left(p_1+p_2\mu^2+4p_3\mu\right)s^2\right]\right)\cdot\exp(-i\eta s).$$

Similar calculations can be carried out using Equations (15) and (16) having quite bulky forms. As a result from Equation (10) for normalized correlation function of scattered radiation caused by electron density fluctuations in turbulent anisotropic magnetized plasma, we obtain:

$$\frac{W_{EE^*}(\eta,\mu)}{E_0^2} = \exp(-i\eta\mu)\exp\left\{-\sqrt{\frac{\pi}{2}}\frac{\Omega T B_0}{\chi}\int_{-\infty}^{\infty} ds\frac{1}{\sqrt{b_1}}\right.$$

$$\exp\left[-\frac{s^2T^2}{4}\left(p_1+p_2\mu^2+4p_3\mu\right)\right]+\frac{1}{2\sqrt{\pi}}\frac{\Omega T B_0}{\chi}\cdot\int_{-\infty}^{\infty} ds\frac{1}{\sqrt{b_2}}\exp$$

$$\left.\left(-\frac{T^2}{2}\left[\frac{1}{4}p_2s^4+(p_2\mu+2p_3)s^3+\left(p_1+p_2\mu^2+4p_3\mu\right)s^2\right]\right)\exp(-i\eta s)\right\}. \quad (23)$$

where: $b_1=\frac{1}{4\chi^2}-\frac{1}{4}p_2(m_1^2-2m_2\mu s)+p_3m_2s$, $m_1=\frac{1}{4}(2\Gamma_j+2P_j\mu-\Gamma_js^2)$, $m_2=\frac{1}{4}P_j\Gamma_j$, $B_0=\sigma_n^2\frac{\sqrt{\pi}}{4}\frac{Tk_0L}{\chi}$.

This expression containes: angle α between wave propagation and external magnetic field, parameter of anisotropy χ, angle of inclination γ_0 of prolate irregularities with respect to the external magnetic field, non-dimensional plasma parameters v_0 and u_0, distance between observation points in the principle plane ρ_y, distance L passing by the electromagnetic wave in turbulent magnetized plasma.

Numerical calculation are carried out for 3 MHz incident electromagnetic wave, $v_0 = 0.28$, $u_0 = 0.22$, $\alpha = 20°$, $\mu = 0.06$; irregularities are streached along external magnetic field, $\gamma_0 = 0°$. Figure 1 illustrates dependence of real part of the normalized correlation function of scattered ordinary wave in magnetized plasma (solid line) and non-magnetized plasma (dotted line) with electron density fluctuations ($\chi = 180$) versus nondimensional parameter η at

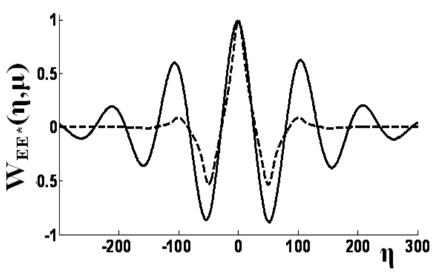

Figure 1. Dependence of real part of the normalized corralation function of scattered field $W_{EE^*}(\eta,\mu)/E_0^2$ versus non-dimensional parameter η at: $T = 4000$, $B_0 = 1.88$, $\chi = 180$. Solid line corresponds to the ordinary wave in magnetized plasma, dotted line — non-magnetized plasma.

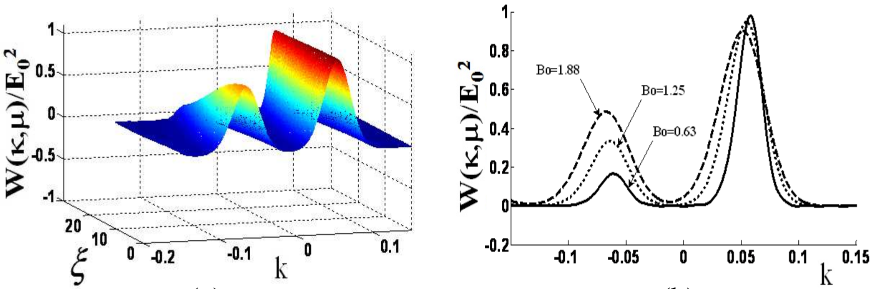

Figure 2. Dependence of SPS of scattered electromagnetic waves versus non-dimensional parameter k in non-magnetized plasma, (a) 3D picture of SPS at $\chi = 190$, $B_0 = 1.5$; $\xi = k_0\rho_x$, (b) profile of SPS for different parameter B_0.

$T = 4000$ and $B_0 = 1.88$. The curves are normalized on their maximum values, have a symmetrical form; maxima are located at $\eta = 0$. With increasing distance between observation points correlation function fast decreases in non-magnetized inhomogeneous plasma, while in magnetized plasma amplitude of this function slowly oscillates and attenuates with increasing parameter η.

Figure 2(a) gives 3D picture of the dependence SPS of scattered electromagnetic wave versus both parameters and k for non-magnetized plasma at $\chi = 190$, $B_0 = 1.5$, $T = 4000$. Values of SPS are plotted in Figure 2(b) as a function of parameter k for various values of the numerical distance B_0. The curves are normalized on their maximum values. Increasing distance passing by electromagnetic wave in anisotropic non-magnetized turbulent plasma, at fixed parameter of anisotropy $\chi = 170$, $T = 4000$, amplitude of small hump increases, while amplitude of big hump decreases. Maxima of both humps weakly shift. These results are in a good agreement with the results for 3D prolate inhomogeneities calculated by statistical Monte-Carlo method [11]. Numerical analyses also show that that neglecting diffraction effects, i.e., neglecting the term k_y^2/k_0^2 in the argument of 2D spectrum of the correlation functions of electron density fluctuations, "double-humping" effect in SPS disappears.

Figure 3 represents the dependence of SPS versus non-dimensional parameter k in turbulent non-magnetized (solid line) and magnetized plasma (dotted line) respectively, at fixed parameters $\chi = 180$ and $B_0 = 1.6$. Calculations show that a dip of the curve caused by electron density fluctuations getting much more pronounced in non-magnetized plasma than in the presence of an external magnetic field. Increasing

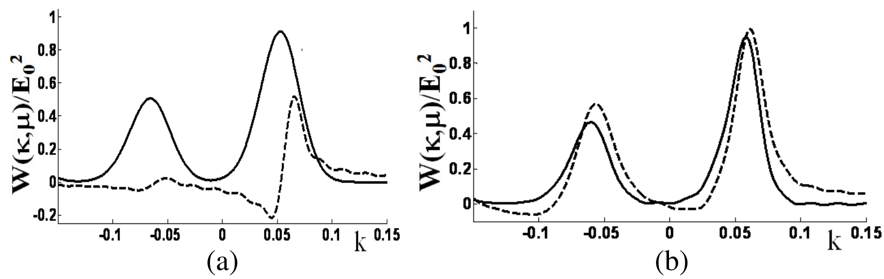

Figure 3. Dependence of SPS versus non-dimensional parameter k, (a) solid line corresponds to the non-magnetized plasma, dotted line — ordinary wave in magnetized plasma, $\chi = 180$, (b) ordinary wave in magnetized plasma, solid line $\chi = 150$, dotted line $\chi = 160$.

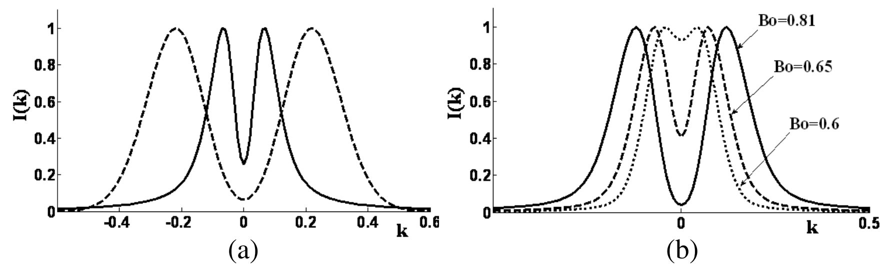

Figure 4. Dependence of SPS (12) versus non-dimensional k parameter for a finite width of an original mode $\beta = 0$. Figure 4(a) represents SPS of ordinary wave at: $\chi = 100$, $T = 4500$, $B_0 = 0.8$; solid line — in magnetized plasma, dotted line — in non-magnetized plasma. Figure 4(b) corresponds to the SPS of scattered extraordinary wave in the anisotropic magnetized plasma for different parameters B_0 at fixed values $\chi = 450$, $T = 6500$.

parameter of anisotropy χ amplitude of SPS increases and weakly displaced (Figure 3(b)).

Figure 4 describes dependence of SPS versus parameter k at $\beta = 10$; for ordinary wave: $\chi = 100$, $T = 4500$, $B_0 = 0.8$; for extraordinary wave: $\chi = 450$, $T = 6500$, $B_0 = 0.6$. SPS has double-hump shape. Figure 4(a) shows that external magnetic field leads to narrowing of SPS of the ordinary wave, maximum displaced and a dip decreases. Maxima for ordinary wave are spaced symmetrically with respect to $k = 0$; in non-magnetized plasma maxima are located at $k = \pm 0.22$, in magnetized plasma at $k = \pm 0.07$. Dips in SPS for both cases arise at $k = 0$; $I(k) = 0.06$ and 0.26 in non-magnetized and magnetized anisotropic inhomogeneous plasma, respectively. Figure 4(b) describes SPS of the extraordinary wave in anisotropic turbulent magnetized plasma. Calculations ahow that increasing distance travelling by the wave in plasma a dip become more pronounced in the direction $k = 0$ and maxima are displaced. The depth of a gup increases in proportion distance covered by ordinary wave in magnetized plasma: $B_0 = 0.6$, $I(k) = 0.93$; $B_0 = 0.65$, $I(k) = 0.41$; $B_0 = 0.81$, $I(k) = 0.04$; maxima are located at: $B_0 = 0.6$, $k = \pm 0.04$; $B_0 = 0.65$, $k = \pm 0.07$; $B_0 = 0.81$, $k = \pm 0.12$. All maxima are located symmetrically with respect to $k = 0$.

Physical reason of peculiarities of the spatial spectrum of scattered radiation in random medium has been considered in [11]. At single scattering of electromagnetic waves on smooth strongly extended irregularities spectrum of scattered radiation has one narrow maximum in the direction of an incident wave, second maximum — in the direction of specular reflection with respect to a plane passing through z axis, along which irregularities mainly are prolate. If an incident wave propagates along z axis these maxima combine [1] and broaden at multiple scattering of electromagnetic wave oblique incident with respect to z axis. If an incident wave propagates along z axis, single scattered field has a broadened spectrum with maximum in zero (along z axis). Therefore inclined components arise. New maxima arise under specular angles and the two-humped spectrum is gradually formed at multiple scattering of radiation in random medium. This effect cannot be obtained by smooth perturbation method [1, 2]. If an incident wave has a broadened spectrum with maximum in zero, increasing distance travelling by wave in random medium, two-humped spectrum arises [11]. This effect is revealed also in anisotropic turbulent magnetized plasma using smooth perturbation method.

Statistical characteristics of SPS of multiple scattered ordinary and extraordinary waves in anisotropic randomly inhomogeneous magnetized plasma with prolate irregularities caused by electron density fluctuations are considered for the first time.

4. CONCLUSION

Features of SPS of multiple scattered ordinary and extraordinary waves in anisotropic magnetized plasma with electron density fluctuations have been studied analytically and numerically. Investigation has shown that strongly pronounced dip appears in SPS of scattered electromagnetic waves along prolate irregularities at oblique illumination of magnetized plasma by mono-directed incident radiation. SPS has a "double-hump" shape. External magnetic field narrows the width of SPS for the ordinary wave weakly. SPS for the extraordinary wave broadens and its maximum is weakly displaced in proportion to a distance passing by the wave in magnetized inhomogeneous plasma.

The obtained results will have applications at observations of electromagnetic waves propagation in the upper atmosphere and remote sensing.

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