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SCATTERING OF A RADIALLY ORIENTED HERTZ DIPOLE FIELD BY A PERFECT ELECTROMAGNETIC CONDUCTOR (PEMC) SPHERE

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Abstract—PEMC medium is a special type of metamaterial which generalizes the pre-existing concepts of perfect electric conductor (PEC) and perfect magnetic conductor (PMC). PEMC medium is described by a special parameter named as admittance which decides the nature of medium as PEC or PMC. Electromagnetic fields scattered by a PEMC sphere are investigated theoretically. A Hertz dipole as a source of excitation is considered. Co-polarized and cross-polarized components of the scattered fields are taken into consideration. A general solution of fields scattered by the PEMC sphere has been sought.

1. INTRODUCTION

The concept of perfect electromagnetic conductor (PEMC) was introduced a few years ago. Lindell and Sihvola proposed a generalization of the perfect electric conductor (PEC) and perfect magnetic conductor (PMC) as perfect electromagnetic conductor (PEMC) [1]. In order to solve the problems involving PEMC structures Lindell and Sihvola introduced the transformation method [2]. At the same time they worked for the realization of PEMC boundary [3]. By this realization, the PEMC surface was proved to be a perfect reflector

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for electromagnetic energy. Both these pioneers also discussed possible applications of this newly introduced concept [4] and the reflection and transmission from the PEMC interface [5]. In light of all this work it is accepted by the past & recent researchers that perfect electromagnetic conductors (PEMC) are metamaterials that possess special properties which are not usually observed in nature. Due to this reason, the field of study of perfect electromagnetic conductors (PEMC) has fascinated the interest of many scientists. In recent times, much theoretical research work has been done in the study of PEMC. Hussain discussed the comparison of PEMC with fractional duality [6] and fractional waveguide [7]. Previous methods for the calculation of fields in different regions and with different apparatus considerations were also implemented for PEMC material [8–11].

It is well known that PEC has following boundary conditions

$$\mathbf{n} \times \mathbf{E} = 0, \quad \mathbf{n} \cdot \mathbf{B} = 0 \tag{1}$$

while PMC boundary may be defined by the conditions

$$\mathbf{n} \times \mathbf{H} = 0, \quad \mathbf{n} \cdot \mathbf{D} = 0 \tag{2}$$

At the surface of a PEMC, boundary conditions are of more general form and can be read as

$$\mathbf{n} \times (\mathbf{H} + M\mathbf{E}) = 0, \quad \mathbf{n} \cdot (\mathbf{D} - M\mathbf{B}) = 0 \tag{3}$$

where M denotes the admittance of the PEMC boundary and $\mathbf{n}$ is the unit normal vector. It is obvious from above boundary conditions that PEMC corresponds to the PMC when $M = 0$, while it corresponds to the PEC for $M \to \pm\infty$.

In order to fulfill the boundary conditions, co-polarized as well as cross-polarized field components are required to represent the field and as a result PEMC is found as a non-reciprocal feature. When PEMC material is represented in differential form, it is found as the simplest probable medium [12]. It has been verified theoretically that a PEMC material acts as a perfect reflector of electromagnetic waves. The difference between PEMC and PMC or PEC is that the reflected wave has a cross-polarized component. This non-reciprocal effect of the PEMC medium has been established for the planar, cylindrical and spherical geometries. Scattering of electromagnetic field by different geometries of PEMC material and different conditions and situations has been studied time to time. Two main geometries discussed by most of the researchers in the topic of scattering are circular cylinder [13–22] and sphere [23–29]. We have studied scattering of electromagnetic waves from a PEMC sphere. In real sense as used in practical applications, there exist two sources of excitation which can be treated as the basic sources. One is the electric source and second the magnetic source. The simplest form of electric source is electric dipole and of magnetic source is magnetic dipole [30]. The source of excitation for the sphere in our case is a Hertz dipole. The study of scattering of electromagnetic radiation from a sphere has found its applications in many recent scientific fields such as biomedical applications involving (a) implantations inside the human head for hyperthermia or biotelemetry as well as excitation of the human brain by the neuron's current, (b) modeling of the interaction between different antennas and the human head in order to seek important information about the biological effects of electromagnetic radiation and (c) analysis of biological phenomena at the cell level of living organisms, e.g., investigations of biological cells involving a central spherical nucleus. The space communication is also concerned with the scattering of electromagnetic waves from different planets and other space probes and objects.

2. PEMC SPHERE ILLUMINATED BY HERTZ DIPOLE

Consider a PEMC sphere of radius a, which is a scattering body, in this case as shown in the Figure 1. If the radius of the PEMC sphere is considered very small as compared to the wavelength, the fields falling on the PEMC sphere can be approximated as plane wave. This approximation is known as Raleigh approximation.

In case of validation of Raleigh approximation, the fields scattered by the PEMC sphere can be approximated as the fields radiating from a point source with a dipole moment directed along the direction of incident field [23]. Let this sphere is located at the origin of the spherical coordinates system. The electromagnetic field incident on the sphere is those, of a Hertz dipole located at $P(r', \theta', \phi')$ in radial direction. It is assumed that the dipole is parallel to the axis of sphere. Our interest is to find scattered field at an arbitrary observation point $R(r, \theta, \phi)$. It is assumed the medium surrounding PEMC sphere is free space having constructive parameters μ_0 and ϵ_0.

The radiated field due to a dipole may be obtained by introducing the vector potentials, i.e., magnetic vector potential $\mathbf{A}$ for electric dipole and electric vector potential $\mathbf{F}$ for magnetic dipole [14].

The vector potentials and fields are related as [31]

$$\mathbf{B} = \nabla \times \mathbf{A}, \qquad \mathbf{E} = -j\omega \left[\mathbf{A} + \frac{1}{k^2}\nabla\nabla \cdot \mathbf{A}\right] \tag{4}$$

$$\mathbf{E} = -\frac{1}{\epsilon_0}\nabla \times \mathbf{F}, \quad \mathbf{H} = -j\omega \left[\mathbf{F} + \frac{1}{k^2}\nabla\nabla \cdot \mathbf{F}\right] \tag{5}$$

To find out the fields scattered by the sphere, the equation in terms of

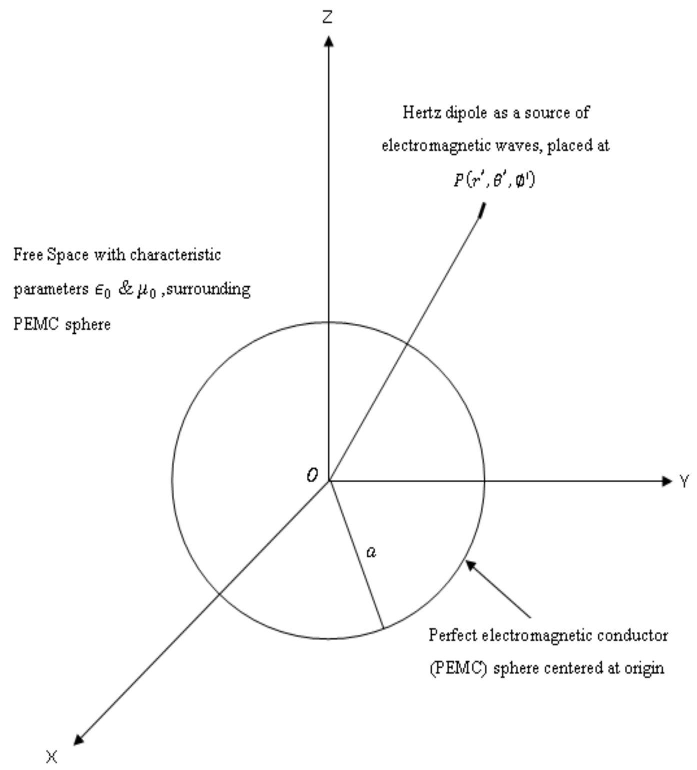

Figure 1. PEMC sphere as a scattering body illuminated by a hertz dipole which is in radial direction.

radial component of electric Hertz vector is as follows:

$$\left(\boldsymbol{\nabla}^2 + k^2\right) \pi_{er} = -\frac{J_r}{j\omega\epsilon r} \tag{6}$$

where π_{er} represents the radial component of electric Hertz vector. For a short Hertz dipole

$$J_r = I_0 dl\delta\left(\mathbf{r} - \mathbf{r}'\right) \tag{7}$$

The incident and scattered components of the fields are found in the form of appropriate products of Bessel/Hankel functions and Legendre polynomials and applying boundary conditions.

In case of Hertz dipole incident electric field can be found as under:

$$\mathbf{E} = \mathbf{E}_A + \mathbf{E}_F \tag{8}$$

where Electric field in terms of magnetic vector potential $\mathbf{A}$ can be written as

$$\mathbf{E}_A = -j\omega\mathbf{A} - j\frac{1}{\omega\mu_0\epsilon_0}\boldsymbol{\nabla}\left(\boldsymbol{\nabla}\cdot\mathbf{A}\right) \tag{9}$$

Magnetic field of the dipole can be written as

$$\mathbf{H} = \mathbf{H}_A + \mathbf{H}_F \tag{10}$$

where magnetic field in terms of magnetic vector potential $\mathbf{A}$ can be written as

$$\mathbf{H}_A = \frac{1}{\mu_0}\boldsymbol{\nabla}\times\mathbf{A} \tag{11}$$

Electric and magnetic fields in terms of their components are as follows

$$\mathbf{E}_A = E_r\hat{i}_r + E_\theta\hat{i}_\theta + E_\phi\hat{i}_\phi, \quad \mathbf{H}_A = H_r\hat{i}_r + H_\theta\hat{i}_\theta + H_\phi\hat{i}_\phi \tag{12}$$

For spherical coordinates

$$\begin{aligned}\boldsymbol{\nabla}\times\mathbf{A} &= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(A_\phi\sin\theta\right)-\frac{\partial A_\theta}{\partial\phi}\right]\hat{i}_r+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi}-\frac{\partial}{\partial r}\left(rA_\phi\right)\right]\hat{i}_\theta \\ &\quad +\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\theta\right)-\frac{\partial A_r}{\partial\theta}\right]\hat{i}_\phi \qquad (13)\\ \boldsymbol{\nabla}\mathbf{A} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2A_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi} \qquad (14)\\ \boldsymbol{\nabla}\varphi &= \frac{\partial\varphi}{\partial r}\hat{i}_r+\frac{1}{r}\frac{\partial\varphi}{\partial\theta}\hat{i}_\theta+\frac{1}{r\sin\theta}\frac{\partial\varphi}{\partial\phi}\hat{i}_\phi \qquad (15)\end{aligned}$$

For a short electric dipole located at (r', θ', ϕ') and directed along z-axis, magnetic vector potential is given by [23]

$$A_z = \frac{\mu_0 I_0 dl}{4\pi}\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tag{16}$$

Radial and theta components of magnetic vector potential are as [23]

$$\begin{aligned}A_r &= \frac{jk\mu_0 I_0 dl\cos\theta}{4\pi}h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right) \qquad (17)\\ A_\theta &= \frac{jk\mu_0 I_0 dl\sin\theta}{4\pi}h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right) \qquad (18)\end{aligned}$$

where

$$h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right) = \sum_{n=0}^{\infty}\left(2n+1\right)h_n^{(2)}\left(kr'\right)j_n\left(kr\right)P_n\left(\cos\gamma\right) \tag{19}$$

&

$$P_n(\cos\gamma)=\sum_{m=1}^{n}\sum_{n=0}^{\infty}\frac{2(n-m)!}{(n+m)!}P_n^m(\cos\theta)\,P_n^m\left(\cos\theta'\right)\cos m\left(\phi-\phi'\right) \quad (20)$$

In above equations, $h_n^{(2)}(\ldots)$ is Hankel function of second kind and $P_n^m(\ldots)$ is associated Legendre's polynomial.

3. ELECTRIC FIELD

3.1. Incident Electric Field Components

Using Equations (9), (12), (14) & (15) radial component of incident electric field E_r^i can be expressed as

$$E_r^i=-j\omega A_r-j\frac{1}{\omega\mu_0\epsilon_0}\frac{\partial}{\partial r}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2A_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right)\right] \quad (21)$$

Putting the values of A_r & A_θ from Equations (17) & (18) in above equation we get

$$\begin{aligned}E_r^i &= \frac{jk\mu_0I_0dl}{4\pi}\left[-j\omega\cos\theta h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\right.\\ &\quad -j\frac{1}{\omega\mu_0\epsilon_0}\frac{\partial}{\partial r}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cos\theta h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\right)\right.\\ &\quad \left.\left.+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin^2\theta h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\right)\right\}\right] \end{aligned} \quad (22)$$

Using Equations (19) & (20) in above equation, the radial component of incident electric field E_r^i becomes

$$\begin{aligned}E_r^i &= \frac{kI_0dl\cos\theta}{4\pi\omega r^2\epsilon_0}h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\left[r^2\omega^2\epsilon_0\mu_0\right.\\ &\quad \left.+\frac{r^2j_n''(kr)+R_n}{j_n(kr)}\left(4+\tan\theta\left(\frac{P_n^{m'}(\cos\theta)}{P_n^m(\cos\theta)}\right)\right)\right] \end{aligned} \quad (23)$$

where

$$R_n=rj_n'(kr)-j_n(kr) \quad (24)$$

In above equation, $j_n(kr)$ is Bessel's function of first king and $j_n'(kr)$ is its first derivative.

Again using Equations (9), (12), (14) & (15), however now for theta component of incident electric field E_θ^i

$$E_\theta^i=-j\omega A_\theta-j\frac{1}{\omega\mu_0\epsilon_0}\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2A_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right)\right] \quad (25)$$

Putting the values of A_r & A_θ from Equations (17) & (18) in above we get

$$\begin{aligned} E_\theta^i \;=\; & \frac{jk\mu_0 I_0 dl}{4\pi}\left[-j\omega\sin\theta h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\right. \\ & -j\frac{1}{\omega\mu_0\epsilon_0}\frac{1}{r}\frac{\partial}{\partial r}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cos\theta h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\right)\right. \\ & \left.\left.+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin^2\theta h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\right)\right\}\right] \end{aligned} \tag{26}$$

Using Equations (19) & (20) in above equation, the radial component of incident electric field E_r^i becomes

$$\begin{aligned} E_\theta^i \;=\; & \frac{k\mu_0 I_0 dl\sin\theta}{4\pi}h_0^{(2)}\left(k\left|r-r'\right|\right)\left[\left[\omega\mu_0+\frac{1}{r^2\omega\epsilon_0}\left\{\frac{1}{P_n\left(\cos\gamma\right)}\right.\right.\right. \\ & \left(\frac{\partial^2}{\partial\theta^2}P_n\left(\cos\gamma\right)\right)+\left(\frac{\partial}{\partial\theta}P_n\left(\cos\gamma\right)\right)\frac{\cot\theta}{P_n\left(\cos\gamma\right)} \\ & \left.\left.\left.+\left(3+\frac{T_n}{j_n\left(kr\right)}\right)\left(\left(\frac{\partial}{\partial\theta}P_n\left(\cos\gamma\right)\right)\frac{\cot\theta}{P_n\left(\cos\gamma\right)}-1\right)\right\}\right]\right] \end{aligned}$$

Above equation can also be written as

$$\begin{aligned} E_\theta^i \;=\; & \frac{k\mu_0 I_0 dl\sin\theta}{4\pi}h_0^{(2)}\left(k\left|r-r'\right|\right) \\ & \left[\left[\omega\mu_0+\frac{1}{r^2\omega\epsilon_0}\left\{\left(\frac{P_n^{m''}\left(\cos\theta\right)}{P_n^m\left(\cos\theta\right)}\right)+\left(\frac{P_n^{m'}\left(\cos\theta\right)}{P_n^m\left(\cos\theta\right)}\right)\cot\theta\right.\right.\right. \\ & \left.\left.\left.+\left(3+\frac{T_n}{j_n\left(kr\right)}\right)\left(\left(\frac{P_n^{m'}\left(\cos\theta\right)}{P_n^m\left(\cos\theta\right)}\right)\cot\theta-1\right)\right\}\right]\right] \end{aligned}$$

where

$$T_n = rj_n'\left(kr\right)+j_n\left(kr\right) \tag{27}$$

3.2. Scattered Electric Field Components

Let us suppose that electric hertz vector in radial direction π_{er}^s can be written as

$$\pi_{er}^s = \frac{I_0 dl}{j\omega\epsilon r'}\sum_{n=0}^{\infty}a_n h_n^{(2)}\left(k_1 r\right)P_n\left(\cos\gamma\right) \tag{28}$$

Now scattered radial component of electric field can be expressed as

$$E_r^s = \left(\frac{\partial^2}{\partial r^2}+k^2\right)r\pi_{er}^s \tag{29}$$

Using equation given below

$$\left(\frac{d^2}{dr^2}+k^2-\frac{n\,(n+1)}{r^2}\right)r\left[Z_n\,(kr)\right]=0 \tag{30}$$

where $Z_n(kr)$ is any spherical Bessel function, E_r^s can be written as

$$E_r^s = \frac{n\,(n+1)}{r}\pi_{er}^s \tag{31}$$

$$E_r^s = \frac{I_0 dl}{j\omega\epsilon r r'}\sum_{n=0}^{\infty} n(n+1)a_n h_n^{(2)}(k_1 r)P_n(\cos\gamma) \tag{32}$$

Let us suppose that magnetic hertz vector in radial direction π_{mr}^s can be expressed as

$$\pi_{mr}^s=\sum_{n=0}^{\infty} b_n h_n^{(2)}\,(k_1 r)P_n\,(\cos\gamma) \tag{33}$$

Now

$$E_\theta^s=-\frac{1}{\sin\theta}j\omega\mu\frac{\partial}{\partial\phi}\pi_{mr}^s+\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(r\pi_{er}^s\right) \tag{34}$$

With Equation (33), first part of RHS of Equation (34) is

$$\frac{1}{\sin\theta}j\omega\mu\frac{\partial}{\partial\phi}\pi_{mr}^s=\frac{1}{\sin\theta}j\omega\mu\frac{\partial}{\partial\phi}\left[\sum_{n=0}^{\infty} b_n h_n^{(2)}\,(k_1 r)P_n\,(\cos\gamma)\right] \tag{35}$$

Using Equation (20) and applying derivative, Equation (35) takes the form

$$=\frac{1}{\sin\theta}j\omega\mu\sum_{n=0}^{\infty} b_n h_n^{(2)}\,(k_1 r)\left(\frac{\partial}{\partial\phi}P_n\,(\cos\gamma)\right) \tag{36}$$

The second part of Equation (34), with the help of Equation (28), can be written as

$$\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(r\pi_{er}^s\right)=\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left(r\frac{I_0 dl}{j\omega\epsilon r'}\sum_{n=0}^{\infty} a_n h_n^{(2)}\,(k_1 r)P_n\,(\cos\gamma)\right) \tag{37}$$

Using Equation (20) and applying derivative, Equation (37) takes the form

$$=\frac{1}{r}\frac{I_0 dl}{j\omega\epsilon r'}\sum_{n=0}^{\infty} a_n\left(\frac{\partial}{\partial\theta}P_n\,(\cos\gamma)\right)\left(h_n^{(2)}\,(k_1 r)+r h_n^{(2)'}\,(k_1 r)\right) \tag{38}$$

After substituting the values from Equations (36) & (38) and simplifying, theta component of scattered electric field E_θ^s becomes

$$E_\theta^s = \sum_{n=0}^{\infty}\sum_{m=1}^{n} h_n^{(2)}(k_1 r) P_n(\cos\gamma) \left[\frac{jm\omega\mu\tan m(\phi-\phi')}{\sin\theta} b_n + \frac{I_0 dl}{j\omega\epsilon r'}\left(\frac{P_n^{m'}(\cos\theta)}{P_n^m(\cos\theta)}\right) Q_n a_n\right] \tag{39}$$

where

$$Q_n = \frac{1}{r} + \frac{h_n^{(2)'}(k_1 r)}{h_n^{(2)}(k_1 r)} \tag{40}$$

4. MAGNETIC FIELD

4.1. Incident Magnetic Field Components

Using Equations (11), (12) & (13) we get the radial component of the incident magnetic field H_r^i

$$H_r^i = \frac{1}{\mu_0 r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right] \tag{41}$$

As field is assumed to be along z-axis so radial component can be written in simplified form

$$H_r^i = -\frac{1}{\mu_0 r\sin\theta}\frac{\partial A_\theta}{\partial\phi} \tag{42}$$

Using A_θ from Equation (18) in Equation (42) above we get

$$H_r^i = -\frac{1}{r\mu_0\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{jk\mu_0 I_0 dl\sin\theta}{4\pi} h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\right] \tag{43}$$

Using Equations (19) & (20) in above equation and then simplifying

$$H_r^i = \frac{jmkI_0 dl}{4\pi r}\sum_{n=0}^{\infty}\sum_{m=1}^{n} h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\tan m\left(\phi-\phi'\right) \tag{44}$$

H_r^i can also be expressed as

$$H_r^i = \left(\frac{\partial^2}{\partial r^2} + k^2\right) r\pi_{mr}^i \tag{45}$$

π_{mr}^i denotes the incident radial component of magnetic Hertz vector. Using Equation (30)

$$H_r^i = \frac{n(n+1)}{r}\pi_{mr}^i \tag{46}$$

Comparing Equations (44) & (46) π^i_{mr} can be expressed as

$$\pi^i_{mr} = jmkI_0dl \sum_{n=0}^{\infty}\sum_{m=1}^{n} \frac{\tan m\left(\phi-\phi'\right)}{4\pi n\left(n+1\right)} h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right) \tag{47}$$

Using Equations (11), (12) & (13) we get the theta component of the incident magnetic field H^i_θ

$$H^i_\theta = \frac{1}{\mu_0 r\sin\theta}\frac{\partial A_r}{\partial\phi} \tag{48}$$

Using Equation (17) in above

$$H^i_\theta = \frac{1}{\mu_0 r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{jk\mu_0 I_0 dl\cos\theta}{4\pi}h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\right] \tag{49}$$

Putting values from Equations (19) & (20) in above equation, we get theta component of incident magnetic field H^i_θ

$$H^i_\theta = -\frac{jmkI_0dl\cot\theta}{4\pi r}\sum_{n=0}^{\infty}\sum_{m=1}^{n} h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)\tan m\left(\phi-\phi'\right) \tag{50}$$

4.2. Scattered Magnetic Field Components

Using Maxwell's equations and the values of E^s_r & E^s_θ from Equations (32) & (39) we get radial and theta components of scattered magnetic field, i.e., H^s_r & H^s_θ as follows

$$H^s_r = -\frac{I_0dl}{\eta\omega\epsilon rr'}\sum_{n=0}^{\infty} n\left(n+1\right)a_n h_n^{(2)}\left(k_1 r\right)P_n\left(\cos\gamma\right) \tag{51}$$

&

$$\begin{aligned} H^s_\theta \;=\; & -\frac{1}{\eta}\sum_{n=0}^{\infty} h_n^{(2)}\left(k_1 r\right)\left[\frac{1}{\sin\theta}\left(\frac{\partial}{\partial\phi}P_n\left(\cos\gamma\right)\right)\omega\mu b_n \right.\\ & \left. +\frac{I_0dl}{\omega\epsilon r'}\left(\frac{\partial}{\partial\theta}P_n\left(\cos\gamma\right)\right)Q_n a_n\right] \end{aligned} \tag{52}$$

Above equation can also be written as

$$\begin{aligned} H^s_\theta \;=\; & -\frac{1}{\eta}\sum_{n=0}^{\infty}\sum_{m=1}^{n} h_n^{(2)}\left(k_1 r\right)P_n\left(\cos\gamma\right)\left[-\frac{m\omega\mu}{\sin\theta}\tan m\left(\phi-\phi'\right)b_n \right.\\ & \left. +\frac{I_0dl}{\omega\epsilon r'}\left(\frac{P_n^{m'}\left(\cos\theta\right)}{P_n^{m}\left(\cos\theta\right)}\right)Q_n a_n\right] \end{aligned} \tag{53}$$

where

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

5. SCATTERING BY PEMC SPHERE

The tangential field components have to satisfy the boundary condition at the PEMC sphere surface, i.e., $r = a$

$$H_\theta^i + ME_\theta^i + H_\theta^s + ME_\theta^s = 0 \tag{54}$$

and the boundary condition for the radial component is

$$\epsilon_0 E_r^i - M\mu_0 H_r^i + \epsilon_0 E_r^s - M\mu_0 H_r^s = 0 \tag{55}$$

Applying the above conditions to the theta and radial components of the fields, we obtain a system of linear equations, from these equations we find that the expansion coefficients a_n and b_n are given by

$$\begin{aligned} a_n \;=\; & -\left[\frac{1}{1+jM\eta}\right]\frac{jk\epsilon r'}{4\pi a\epsilon_0}\sum_{n=0}^{\infty}\sum_{m=1}^{n}\frac{h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)}{n\left(n+1\right)h_n^{(2)}\left(k_1a\right)P_n\left(\cos\gamma\right)} \\ & \left[\cos\theta\left\{a^2\omega^2\epsilon_0\mu_0+\frac{a^2j_n''\left(ka\right)+R_n}{j_n\left(ka\right)}\left(4+\tan\theta\frac{P_n^{m'}\left(\cos\theta\right)}{P_n^m\left(\cos\theta\right)}\right)\right\}\right. \\ & \left.-jMa\omega\mu_0 m\tan m\left(\phi-\phi'\right)\right] \end{aligned} \tag{56}$$

$$\begin{aligned} b_n \;=\; & \left[\frac{1}{1+jM\eta}\right]\sum_{n=0}^{\infty}\sum_{m=1}^{n}\frac{jkI_0dl\sin\theta h_0^{(2)}\left(k\left|\mathbf{r}-\mathbf{r}'\right|\right)}{4\pi\omega\mu h_n^{(2)}\left(k_1a\right)P_n\left(\cos\gamma\right)}\left[\frac{Q_n}{n\left(n+1\right)}\left[\cos\theta\right.\right. \\ & \left\{a\omega\mu_0+\frac{a^2j_n''\left(kr\right)+R_n}{a\omega\epsilon_0 j_n\left(ka\right)}\left(4+\tan\theta\frac{P_n^{m'}\left(\cos\theta\right)}{P_n^m\left(\cos\theta\right)}\right)\right\} \\ & \left.-Mm\tan m\left(\phi-\phi'\right)\right]+\frac{\eta}{m\tan m\left(\phi-\phi'\right)}\left[\frac{\cot\theta}{a}m\tan m\left(\phi-\phi'\right)\right. \\ & +jM\mu_0\sin\theta\left[\omega\mu_0+\frac{1}{a^2\omega\epsilon_0}\left\{\frac{P_n^{m''}\left(\cos\theta\right)}{P_n^m\left(\cos\theta\right)}+\left(4+\frac{T_n}{j_n\left(ka\right)}\right)\right.\right. \\ & \left.\left.\left.\left.\frac{P_n^{m'}\left(\cos\theta\right)}{P_n^m\left(\cos\theta\right)}\cot\theta-3-\frac{T_n}{j_n\left(ka\right)}\right\}\right]\right]\right] \end{aligned} \tag{57}$$

By introducing the values of these constants a_n & b_n in Equations (32), (39), and (53), scattered electric and magnetic fields can be found. As the illuminating Hertz dipole is oriented in radial direction so the radial components of the scattered electric and magnetic fields are considered to be the co-polarized components of the fields and theta components of the two fields are considered to be the cross-polarized components.

6. RESULTS & DISCUSSION

The equations for co-polarized components as well as cross-polarized components of the electric and magnetic fields are implemented in terms of Bessel, Hankel and Legendre function along with the derivatives of these functions. This is done by programming in MATHEMATICA. Built-in functions for spherical Bessel and Hankel functions and Legendre function are available in the software. In order to check the functionality of the MATHEMATICA on the system, some graphs from [14] were plotted and thus the program was verified.

Following figures show the variation of co-polarized and cross-polarized components of the electric field scattered from the PEMC sphere for different values of admittance parameter. Three values of admittance are taken for obtaining the results. When $M\eta = 0$ the PEMC sphere acts as PMC sphere and when $M\eta = \infty$, sphere acts as a PEC sphere. Radius of the sphere is taken to be $0.15 \times 2\pi$ and the position of the Hertz dipole illuminating the sphere is taken as $5.05 \times 2\pi$. ϕ is varied from 0 to 2π whereas ϕ' is taken as constant, i.e., $\phi' = \frac{\pi}{3}$. I_0 & dl both are taken to be unity. The series is taken from 0 to 5.

It is clear from Figures 2 & 3 that the co-polarized and cross-polarized components of the electric field vary approximately inversely to each other. However on the other hand Figure 4 shows that the behavior of the two components of the scattered electric field became

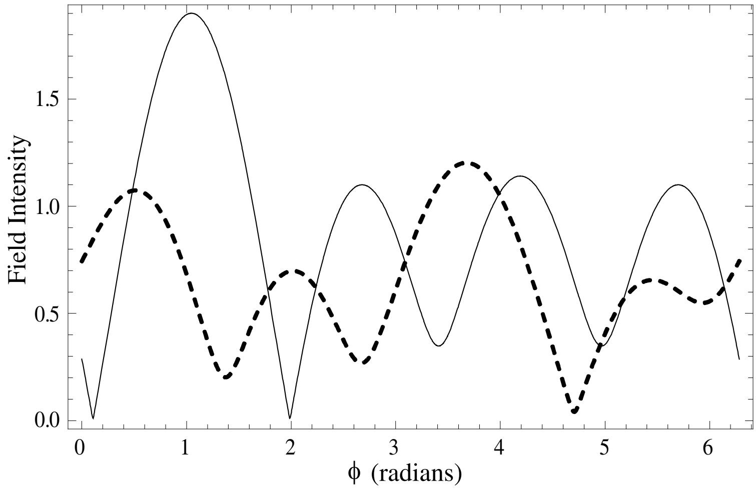

Figure 2. Variation of co & cross-polarized components of scattered electric field. (when $M\eta = 0$, $\omega = 3 \times 10^8$ & $\theta = \theta' = \frac{\pi}{3}$).

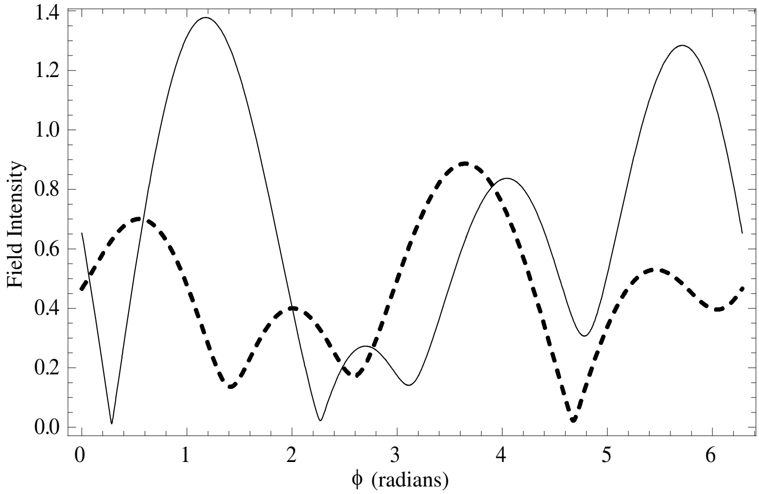

Figure 3. Variation of co & cross-polarized components of scattered electric field, when $M\eta = 1$, $\omega = 3 \times 10^8$ & $\theta = \theta' = \frac{\pi}{3}$).

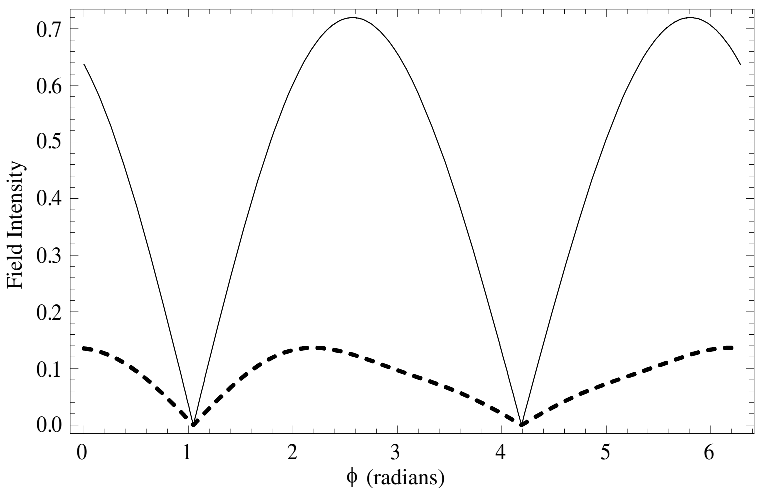

Figure 4. Variation of co & cross-polarized components of scattered electric field, (when $M\eta = \infty$, $\omega = 3 \times 10^8$ & $\theta = \theta' = \frac{\pi}{3}$).

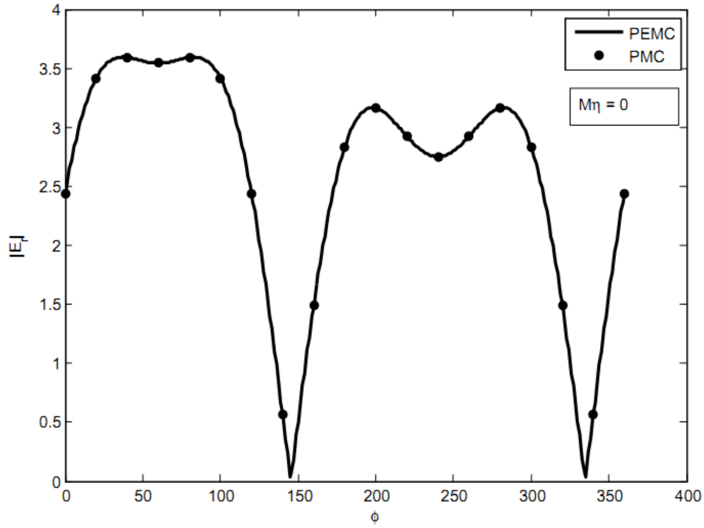

Figure 5. Comparison of co-polarized components of scattered electric field for PEMC and PMC sphere.

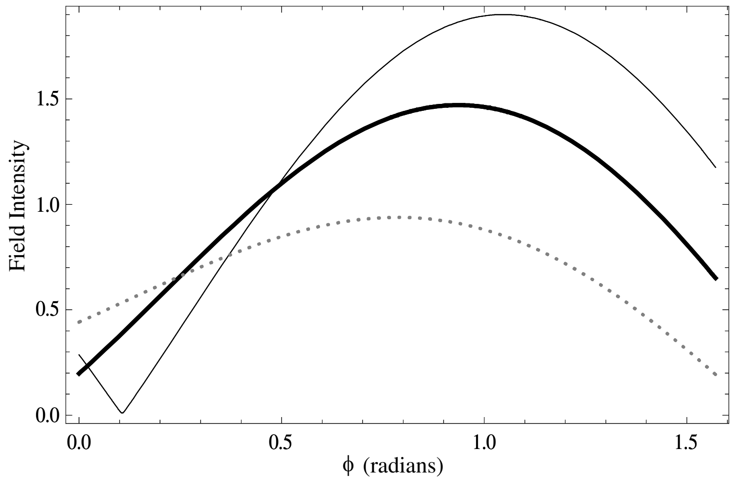

Figure 6. Variation of co-polarized component of scattered electric field.

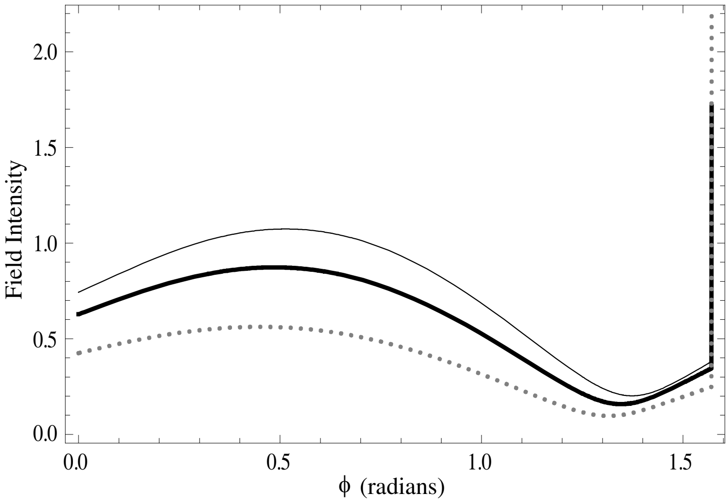

Figure 7. Variation of cross-polarized component of scattered electric field.

similar to each other. So it can be concluded that for small values of M the co & cross-polarized component of the scattered electric field behave in such a way so as to cancel each other's effect and for large value of M these components behave in such a way so as to add up and give a uniformly varying resultant field. Also by the increase in the value of M, the maximum peaks of scattered field shift to a smaller value.

It can also be noted from these plots that the maximum peak value of co-polarized component is greater than that of the cross-polarized component for most of the values of angle of observation. Figure 5 shows that for $M\eta = 0$ the behavior of PEMC sphere exactly matches to that of PMC sphere. In Figures 6 and 7, it can be observed that the field intensity in case of co-polarized component as well as cross-polarized component of the electric field decreases as the value of $M\eta$ increases.

In above Figures 2, 3 and 4

— Co-Polarized Component of Scattered Electric Field.

--- Cross-Polarized Component of Scattered Electric Field.

In above Figures 6 & 7 (for $M\eta = \text{TanV}$)

— When $V = 0°$.

— When $V = 15°$.

... When $V = 90°$.

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