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NONSYNCHRONOUS NONCOMMENSURATE IMPEDANCE TRANSFORMERS

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Abstract—Nonsynchronous noncommensurate impedance transformers consist of a combination of two types of transmission lines: transmission lines with a characteristic impedance equal to the impedance of the source, and transmission lines with a characteristic impedance equal to the load. The practical advantage of such transformers is that they can be constructed using sections of transmission lines with a limited variety of characteristic impedances. These transformers also provide comparatively compact size in applications where a wide transformation ratio is required. This paper presents the data which allows to estimate the achievable total electrical length and in-band reflection coefficient for transformers consisting of up to twelve transmission line sections in the range of transformation ratios $r = 1.5$ to 10 and bandwidth ratios $\chi = 2$ to 20. This data is obtained using wave transmission matrix approach and experimentally verified by synthesizing a 12-section nonsynchronous noncommensurate impedance transformer. The measured characteristics of the transformer are compared to the characteristics of a conventional tapered line transformer.

1. INTRODUCTION

Fast-growing performance requirements demanded in industrial and scientific applications continuously challenge the standard approaches to the design of impedance transformers. Implementation of traditional designs based on distributed components leads to a large circuit size, which is highly undesirable in practice. The transformers based on lumped elements are compact, but suffer from low quality-factor at high frequencies. *Nonsynchronous noncommensurate* (NN) altering transmission line impedance transformers are one of the promising

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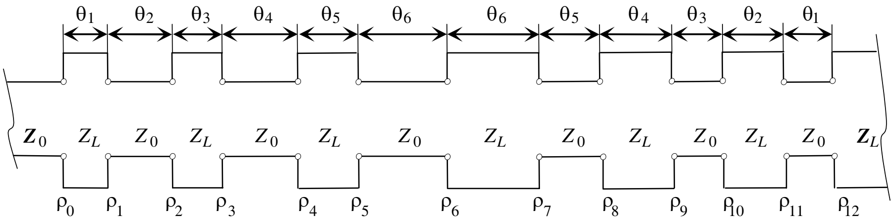

Figure 1. A schematic view of a 12-section NN impedance transformer.

types of transformers for a broadband matching. The schematic view of a 12-section NN transformer is shown in Fig. 1 as an example. The NN impedance transformer consists of transmission line sections of different lengths (*noncommensurate*) with the same characteristic impedances as the impedances of the source Z_0 and the load Z_L which should be matched.

Every two sections of transmission lines in the transformer introduce one zero in the spectrum of the reflection coefficient. The word *nonsynchronous* in the name of the impedance transformer means that the impedance ratio between the steps can be equal or exceed the output-to-input transformation ratio r. In the literature, NN impedance transformers are also sometimes referred as stepped transformers of second class [1].

Compared to the traditional transformers based on distributed components, NN impedance transformers exhibit a number of advantages: they are more compact than the standard transformers based on quarter-wave sections [1, 2]; they are less sensitive to fabrication errors than the compact transformers based on coupled lines [3]; and finally, their fabrication process is often compatible with the fabrication process of the transmission lines employed in the system. For example, if one part of the system with the output impedance of $50\,\Omega$ should be connected to another part with the input impedance of $300\,\Omega$, this can be achieved by NN impedance transformer containing the same types of transmission line sections having mentioned ($300\,\Omega$, and $50\,\Omega$) characteristic impedances. The practical advantage of such transformers is that they can be constructed using sections of transmission lines with a limited variety of characteristic impedances. This is useful in systems based on standardized transmission lines, like, for example, off-the-shelf coaxial cables, where the available range of characteristic impedances is limited. A custom design would be required for a coaxial line with a nonstandard characteristic impedance [4], not to mention tapered coaxial lines. In the example described above, assuming that 300 Ω and 50 Ω lines exist in the system, it would be relatively easy to construct NN impedance transformer using only those two types of lines, since they readily available. The alternative way is to use a quarter-wave transformer with a characteristic impedance of 122.5 Ω, which is not readily available. In addition, NN impedance transformers can be constructed using transmission lines with different cross sections, for example, combining coaxial and twin lead lines.

Even though limiting the variety of characteristic impedances to two brings convenience in practice, in theory, even better matching characteristics could be achieved by extending the variety of characteristic impedances, but this is left out of the scope of this work.

In this paper, the general analysis of NN impedance transformers for resistive loads is presented. The dependences of the reflection coefficient and total electrical length on frequency bandwidth are given for the range of transformation ratios suitable for most practical applications. The presented data is for NN impedance transformers consisting of up to twelve transmission line sections, and, according to the authors' knowledge, is the most complete analysis data available in the literature. More number of sections has not been considered here because this would usually lead to a large circuit size, and in most cases, has a limited practical use.

The presented data is useful for design engineers, allowing to choose the required number of sections for the transformer, based on the design specification such as the transformation ratio, operating frequency band, and required reflection coefficient. The presented data also allows to estimate the resulting electrical length associated with the chosen transformer configuration. The design process can then be followed by a synthesis of the chosen transformer configuration using the design data given in the literature for 2-, 4-, 6-sections [5, 6], 8-sections [7], 10-sections [8], and for 12-section impedance transformers given in Section 4 of this paper.

2. ANALYSIS OF NN IMPEDANCE TRANSFORMERS USING WAVE TRANSMISSION MATRIX FORMULATION

The NN impedance transformers can be described by the wave transmission matrices (T-matrices) [9] taking into account multiple reflections from the impedance steps. The transformer is assumed to be lossless, reciprocal, and antimetric. Lengths of the sections, θ_n ($n = 1, 2, \ldots, N$), are symmetric with regard to the transformer center, $\theta_n = \theta_{N-(n-1)}\,(n = 1, 2, \ldots, N/2)$, leading to the symmetry of the partial reflection coefficients from the impedance steps with regard to the transformer centre: $\rho_{n-1} = \rho_{N-(n-1)}\,(n = 1, 2, \ldots, N/2)$ (refer to the schematic of the 12-section NN impedance transformer in Fig. 1). Moreover, the electrical lengths of the sections are related to each other linearly $\theta_n = \nu_n\theta_1\,(n = 2, 3, \ldots, N)$ where θ_1 is the electrical length of the first section at the center frequency f_0 of the assumed matching bandwidth, ν_n is the n-th component of the N-dimensional vector defining the electrical lengths of the remaining sections $\nu = (1, \nu_2, \nu_3, \ldots, \nu_N)$.

NN impedance transformer in Fig. 1 consists of 12 altering sections with the electrical lengths $\theta_1 = \theta_{12}$, $\theta_2 = \theta_{11}$, $\theta_3 = \theta_{10}$, $\theta_4 = \theta_9$, $\theta_5 = \theta_8$, $\theta_6 = \theta_7$, and having the same characteristic impedances as the impedance of the source Z_0 and the load Z_L.

For analysis, the impedance transformer is split into components (two-port networks) consisting of two transmission lines with different characteristic impedances forming impedance step, and the entire impedance transformer is treated as series connected two-port networks. The first such a two-port network is obtained by shifting the input reference plane of the impedance transformer for a half an electrical length, $\theta_1/2$, in the direction of the source. All the following two-port networks consequently consist of two transmission lines with length $\theta_1/2$ and $\theta_n - \theta_1/2\ (n = 2, 3, \ldots, N)$. Such a two-port network is shown in Fig. 2, where V_1^+, V_2^- are the voltage amplitudes for the waves entering port 1 and port 2, and V_1^-, V_2^+ are the voltage amplitudes for the waves traveling out of port 1 and port 2, respectively.

For the chosen direction of propagation, the relation between the

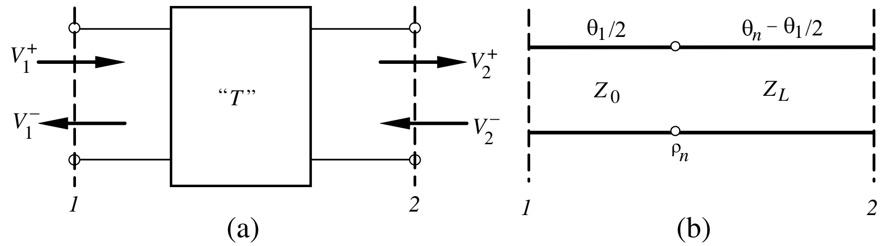

Figure 2. (a) NN impedance transformer is represented as a series connection of two-port networks, (b) containing an impedance step formed by two transmission lines with different characteristic impedances.

voltage amplitudes at port 1 and port 2 can be written as

$$\begin{bmatrix} V_1^+ \\ V_1^- \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_2^+ \\ V_2^- \end{bmatrix}, \tag{1}$$

leading to the following relationship between the T parameters and S parameters:

$$[T] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{s_{21}} & -\frac{s_{22}}{s_{21}} \\ \frac{s_{11}}{s_{21}} & S_{12} - \frac{s_{22}s_{11}}{s_{21}} \end{bmatrix}. \tag{2}$$

The T-matrix for the step, formed by two transmission lines n and $(n+1)$ of different characteristic impedances with lengths $\theta_1/2$ and $\theta_{n+1} - \theta_1/2$, is expressed as:

$$[T]_n = \frac{1}{1-\sqrt{1-\rho_n^2}} \begin{bmatrix} e^{j\theta_{n+1}} & \rho_n e^{j(\theta_{n+1}-\theta_1)} \\ \rho_n e^{j(\theta_{n+1}-\theta_1)} & e^{-j\theta_{n+1}} \end{bmatrix}. \tag{3}$$

The impedance transformer has more impedance steps, $(N+1)$, than the number of sections, N, therefore, the last two-port network is formed by two transmission lines of length $\theta_1/2$.

The advantage of such a T parameter representation over the conventional S parameter representation is that the resulting matrix of the overall NN impedance transformer can now be easily found by multiplying the T-matrices of the individual two-ports:

$$[T]_{\text{tot}} = \prod_{n=0}^{N} [T]_n. \tag{4}$$

Since NN impedance transformer is assumed to be lossless, reciprocal, and antimetric, this leads to the following relations between the elements of the total T-matrix: $T_{11} = T_{22}^*$, $|T_{11}|^2 = 1 + |T_{12}|^2$, $T_{12} = T_{21} \cdot \text{Im}T_{12} = \text{Im}T_{21} = 0$.

The synthesis of the NN impedance transformers with Chebyshev characteristic is performed solving a mini-max problem for the magnitude of the total reflection coefficient

$$\min|\Gamma|_{\max}, \Gamma|_{\max} = \max|\Gamma(\theta_1, \nu)|, \quad \theta_1 \in [\theta_1(f_1), \theta_1(f_2)], \tag{5}$$

where $\theta_1\,(f_1)$ and $\theta_1\,(f_2)$ are the electrical lengths of the first section θ_1 at the lowest frequency f_1 and the highest frequency f_2 of the assumed matching bandwidth and

$$|\Gamma|_{\max} = \frac{|T_{21\text{tot}}|_{\max}}{\sqrt{1 + |T_{21\text{tot}}|^2_{\max}}} \tag{6}$$

where $T_{21\text{tot}}$ is the corresponding parameter of the total wave transmission matrix for the NN impedance transformer from Equation (4) according to (1).

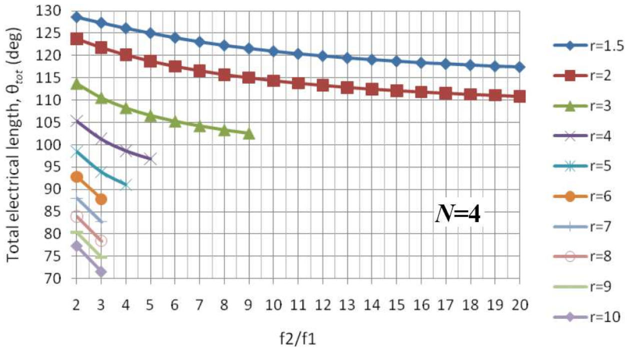

Figure 3. Total electrical length of 4-section NN impedance transformer. Here N is the number of sections.

3. PROPERTIES OF NN IMPEDANCE TRANSFORMERS

The wave transmission matrix formulation described above has been used for analysis of NN impedance transformers. The total electrical lengths θ_{tot} (which is a sum of all sections in Fig. 1) of up to 12-section NN impedance transformer, and achievable magnitude of the reflection coefficients $|\Gamma|$ are shown in Fig. 3 through Fig. 12. The data are presented for the transformers providing bandwidth $\chi = 2$ to 20, the transformation ratio $r = 1.5$ to 10, and the maximum tolerated magnitude of the reflection coefficient $|\Gamma| \leq -7\,\text{dB}$. The transformation ratio r is defined as a ratio between the load impedance and the source impedance: $r = Z_L/Z_0$ (refer to Fig. 1). Bandwidth ratio χ is defined as the ratio between the highest frequency f_2 and the lowest frequency f_1 of operation: $\chi = f_2/f_1$.

The total electrical length of a 2-section impedance transformer is not presented graphically, but can be calculated analytically [10]:

$$\theta_{\text{tot}} = 2 \cdot \text{atan}\left(\frac{1}{\sqrt{r + {}^{1}\!/_{r} + 1}}\right). \tag{7}$$

Using this equation, it is easy to show that the total length of a 2-section impedance transformer for $r = 2$ is equal to approximately 56° which is considerably shorter than the traditional quarter-wave transformer.

The behavior of the reflection coefficient for 2-section NN impedance transformer is very similar to the single section quarter-wave transformer [2], and therefore, is not considered here.

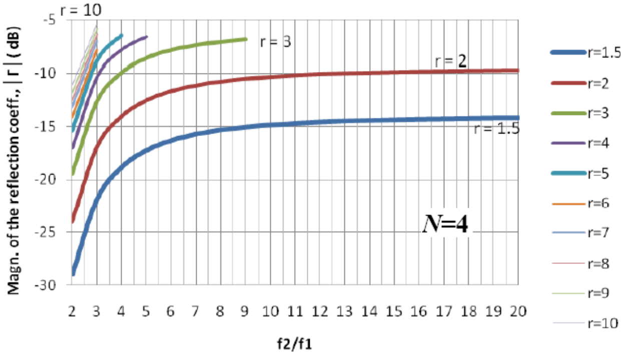

Figure 4. Magnitude of the reflection coefficient for 4-section NN impedance transformer.

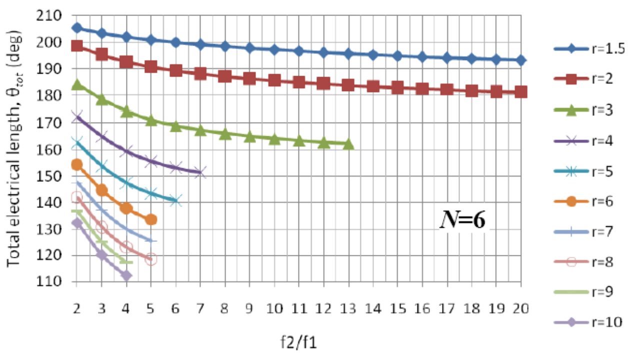

Figure 5. Total electrical length of 6-section NN impedance transformer.

It is interesting to note that the length of NN impedance transformers (refer to Figs. 3, 5, 7, 9, 11) decreases while increasing the transformation ratios or bandwidth. However, this is also accompanied by degradation of reflection coefficient (refer to Figs. 4, 6, 8, 10, 12). The degradation of the reflection coefficient, in turn, can be compensated by increasing the number of employed transmission line sections.

As one can see from the data presented in Fig. 3, the total electrical length of the 4-section NN impedance transformer with a transformation ratio to $r = 1.5$ to 10 and bandwidth ratio $\chi = 2$ to 20 varies in the range from 71° to 128°.

The magnitude of the reflection coefficient $|\Gamma|$ deteriorates while increasing the transformation ratio r and bandwidth ratio χ (refer to Fig. 3), as expected.

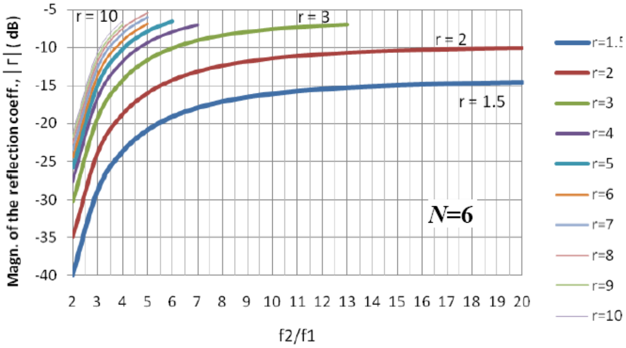

Figure 6. Magnitude of the reflection coefficient for 6-section NN impedance transformer.

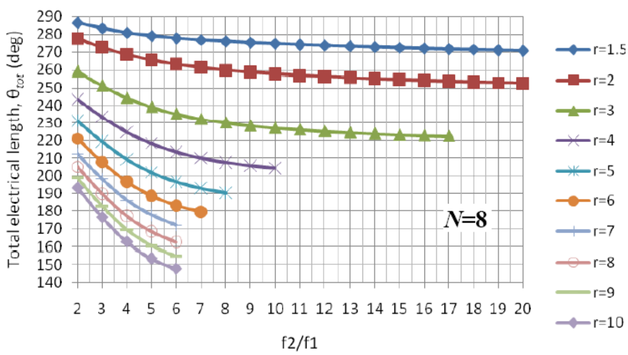

Figure 7. Total electrical length of 8-section NN impedance transformer.

If the reflection coefficient of the 4-section NN impedance transformer does not fulfill the required impedance transformer specification, a 6-section impedance transformer can be implemented. Naturally, the impedance transformer total electrical length will increase while increasing the number of sections. The total electrical length of the 6-section NN impedance transformer varies in the range from 112° to 205° (refer to Fig. 5).

Data in Fig. 6 shows the variation of the reflection coefficient $|\Gamma|$ for the 6-section NN impedance transformer.

Data in Figs. 7 and 8 allows to estimate the total electrical length and magnitude of the reflection coefficient for 8-section NN impedance transformer. The electrical length of the 8-section NN impedance

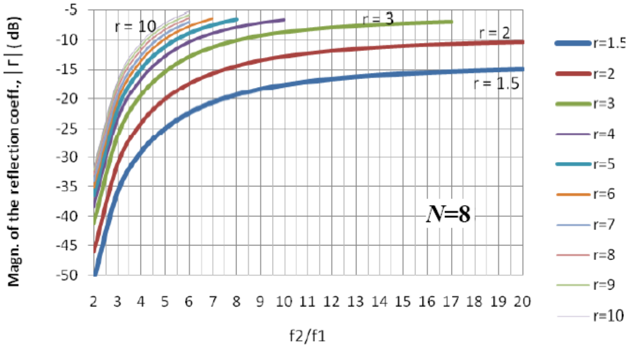

Figure 8. Magnitude of the reflection coefficient for 8-section NN impedance transformer.

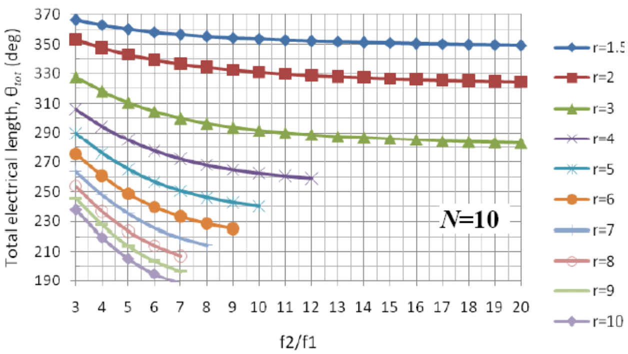

Figure 9. Total electrical length of 10-section NN impedance transformer.

transformer varies in the range from 148° to 287°.

The total electrical length and the magnitude of the reflection coefficient for 10-section NN impedance transformer are shown in Figs. 9 and 10, respectively.

The electrical length of the 10-section NN impedance transformer varies in the range from 188° to 367°.

From Figs. 11 and 12, the total electrical length and magnitude of the reflection coefficient of a 12-section NN impedance transformer can be determined. The electrical length of the 12-section NN impedance transformer varies in the range from 229° to 429°.

The analysis of the obtained design data allows to draw the following conclusions regarding the properties of NN impedance transformers:

- NN impedance transformers with a fixed transformation ratio r and bandwidth ratio χ exhibit decrease in the magnitude of the reflection coefficient while increasing the number of sections N, as expected.
- NN impedance transformers with a fixed number of sections N and bandwidth χ become shorter while increasing the transformation ratio r. At the same time, the magnitude of the reflection coefficient degrades.
- NN impedance transformers with a fixed transformation ratio r and number of sections N, become shorter while increasing the bandwidth ratio χ, which also leads to deterioration of the reflection coefficient.

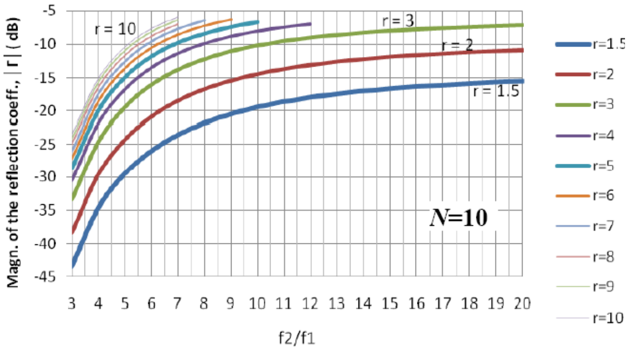

Figure 10. Magnitude of the reflection coefficient for 10-section NN impedance transformer.

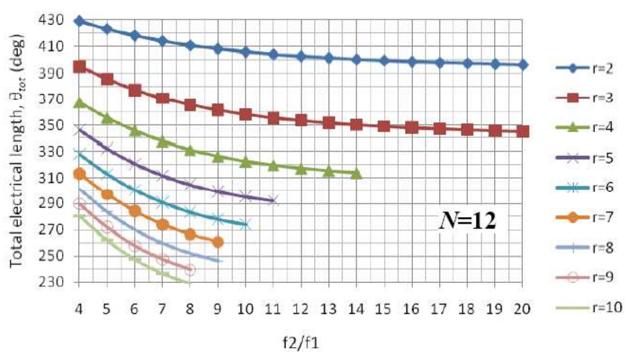

Figure 11. Total electrical length of 12-section NN impedance transformer.

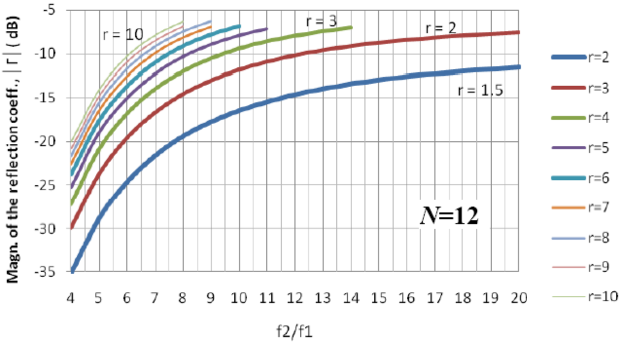

Figure 12. Magnitude of the reflection coefficient for 12-section NN impedance transformer.

4. SYNTHESIS OF 12-SECTION NN IMPEDANCE TRANSFORMER

In order to obtain the design data for the synthesis of the 12-section NN impedance transformers, (5) has been solved numerically for the range of the transformation ratios r and bandwidth ratios χ. Tables 1 through 9 represent the design data for the synthesis of 12-section NN impedance transformer with bandwidth ratios $\chi = 4$ to 20, the transformation ratios $r = 2$ to 10, and the maximum tolerated reflection coefficient magnitude $|\Gamma| \leq -10$ dB, extending the range of ratios r given in [11].

The given range of bandwidth ratios and transformation ratios has been chosen for practical reasons. $\chi > 20$ and $r > 10$ would lead to poor matching. In most practical cases, this would require implementation of a larger transformer with number of sections of more than twelve. Choosing $\chi < 4$ would lead to a very low level of reflection coefficient and in most practical cases it would be reasonable to use a shorter transformer (with a number of sections of less than twelve) [5–8].

The second column in the tables gives the maximum achieved amplitude of the in-band reflection coefficient $|\Gamma|$. The following columns give the electrical lengths of the sections (θ_1 through θ_{12}) and the total electrical length θ_{tot} of the transformer.

As one could expect, the reflection coefficient magnitude $|\Gamma|$ deteriorates while increasing the transformation ratio r and bandwidth ratio χ.

The electrical length of the first and twelfth sections, third and tenth sections, and the fifth and eighth sections of the 12-section NN

Table 1. Design data for 12-section NN impedance transformer and transformation ratio $r = 2$.

χ	$\|\Gamma\|$, dB	$\theta_1=\theta_{12}$,deg	$\theta_2=\theta_{11}$,deg	$\theta_3=\theta_{10}$,deg	$\theta_4=\theta_9$,deg	$\theta_5=\theta_8$,deg	$\theta_6=\theta_7$,deg	θ_{total}, deg
4.0	-35.12	4.50	70.29	13.43	57.85	26.38	41.91	428.72
5.0	-28.76	6.27	65.93	15.51	55.17	27.41	41.13	422.85
6.0	-24.59	8.05	61.96	17.38	52.85	28.27	40.46	417.94
7.0	-21.60	9.80	58.34	19.13	50.74	29.05	39.87	413.83
8.0	-19.37	11.48	55.11	20.74	48.84	29.76	39.34	410.51
9.0	-17.73	12.95	52.44	22.11	47.28	30.32	38.90	407.98
10.0	-16.43	14.30	50.02	23.40	45.77	30.89	38.47	405.69
11.0	-15.44	15.49	48.00	24.47	44.55	31.31	38.11	403.85
12.0	-14.61	16.58	46.23	25.46	43.44	31.71	37.79	402.4
13.0	-13.95	17.50	44.75	26.30	42.51	32.04	37.52	401.24
14.0	-13.39	18.34	43.41	27.05	41.67	32.33	37.25	400.11
15.0	-12.95	19.06	42.32	27.69	40.97	32.58	37.04	399.32
16.0	-12.55	19.72	41.30	28.27	40.32	32.79	36.83	398.44
17.0	-12.25	20.26	40.46	28.75	39.77	32.96	36.65	397.70
18.0	-11.97	20.76	39.74	29.19	39.30	33.13	36.50	397.22
19.0	-11.75	21.19	39.10	29.56	38.88	33.25	36.36	396.68
20.0	-11.53	21.61	38.48	29.90	38.50	33.36	36.23	396.15

Table 2. Design data for 12-section NN impedance transformer and transformation ratio $r = 3$.

χ	$\|\Gamma\|$, dB	$\theta_1=\theta_{12}$, deg	$\theta_2=\theta_{11}$,deg	$\theta_3=\theta_{10}$,deg	$\theta_4=\theta_9$,deg	$\theta_5=\theta_8$, deg	$\theta_6=\theta_7$,deg	θ_{total}, deg
4.0	-29.89	4.25	66.87	12.19	53.22	23.48	37.52	395.03
5.0	-23.76	5.88	61.81	14.02	50.15	24.28	36.53	385.33
6.0	-19.59	7.56	57.16	15.73	47.45	25.00	35.71	377.20
7.0	-16.69	9.16	53.10	17.28	45.12	25.63	35.01	370.60
8.0	-14.57	10.67	49.65	18.66	43.15	26.19	34.43	365.49
9.0	-13.02	12.00	46.73	19.90	41.46	26.68	33.92	361.37
10.0	-11.82	13.20	44.25	21.00	39.96	27.11	33.48	357.98
11.0	-10.93	14.22	42.22	21.91	38.75	27.45	33.10	355.30
12.0	-10.19	15.15	40.48	22.75	37.68	27.78	32.77	353.21
13.0	-9.60	15.96	39.01	23.46	36.79	28.03	32.50	351.47

impedance transformer increases with increase of bandwidth ratio χ and decrease of transformation ratio r. The electrical length of the second and eleventh sections, fourth and ninth sections, and sixth and seventh sections decrease with increase of a bandwidth ratio χ

Table 3. Design data for 12-section NN impedance transformer and transformation ratio $r = 4$.

χ	$\|\Gamma\|$,dB	θ_1=θ_{12},deg	θ_2=θ_{11},deg	θ_3=θ_{10},deg	θ_4=θ_9,deg	θ_5=θ_8,deg	θ_6=θ_7,deg	θ_{total}, deg
4.0	-27.18	3.95	63.97	11.05	49.51	21.10	34.05	367.24
5.0	-20.93	5.49	58.39	12.74	46.20	21.82	33.02	355.31
6.0	-16.82	7.07	53.34	14.33	43.37	22.48	32.18	345.54
7.0	-14.00	8.56	48.99	15.75	40.94	23.04	31.45	337.47
8.0	-12.01	9.90	45.38	17.00	38.90	23.52	30.84	331.08
9.0	-10.55	11.10	42.44	18.08	37.22	23.93	30.33	326.18
10.0	-9.42	12.20	39.93	19.07	35.74	24.31	29.89	322.27

Table 4. Design data for 12-section NN impedance transformer and transformation ratio $r = 5$.

χ	$\|\Gamma\|$, dB	θ_1=θ_{12},deg	θ_2=θ_{11},deg	θ_3=θ_{10},deg	θ_4=θ_9, deg	θ_5=θ_8, deg	θ_6=θ_7, deg	θ_{total}, deg
4.0	-25.22	3.70	61.73	10.15	46.67	19.29	31.45	345.99
5.0	-18.97	5.16	55.65	11.73	43.17	19.95	30.37	332.04
6.0	-15.02	6.59	50.36	13.14	40.26	20.50	29.49	320.66
7.0	-12.29	7.96	45.85	14.46	37.79	21.01	28.77	311.68
8.0	-10.34	9.23	42.11	15.65	35.72	21.48	28.16	304.68
9.0	-8.95	10.35	39.11	16.66	34.01	21.88	27.65	299.32

Table 5. Design data for 12-section NN impedance transformer and transformation ratio $r = 6$.

χ	$\|\Gamma\|$, dB	θ_1=θ_{12}, deg	θ_2=θ_{11}, deg	θ_3=θ_{10}, deg	θ_4=θ_9,deg	θ_5=θ_8,deg	θ_6=θ_7, deg	θ_{total}, deg
4.0	-23.75	3.48	59.74	9.40	44.29	17.82	29.32	328.07
5.0	-17.60	4.83	53.42	10.86	40.72	18.43	28.25	313.00
6.0	-13.65	6.20	47.83	12.21	37.72	18.96	27.37	300.56
7.0	-10.99	7.48	43.20	13.44	35.24	19.44	26.65	290.91
8.0	-9.11	8.68	39.40	14.56	33.17	19.87	26.05	283.45

and transformation ratio r. At the same time, the total electrical length of the impedance transformer becomes shorter with increase of a bandwidth ratio χ and transformation ratio r.

5. DESIGN EXAMPLE

In order to validate the data obtained for the design of NN impedance transformers, an impedance transformer with a transformation ratio

Table 6. Design data for 12-section NN impedance transformer and transformation ratio $r = 7$.

χ	$\lvert\Gamma\rvert$, dB	$\theta_1=\theta_{12}$,deg	$\theta_2=\theta_{11}$,deg	$\theta_3=\theta_{10}$,deg	$\theta_4=\theta_9$, deg	$\theta_5=\theta_8$, deg	$\theta_6=\theta_7$, deg	θ_{total}, deg
4.0	-22.57	3.28	58.10	8.75	42.33	16.60	27.59	313.29
5.0	-16.48	4.56	51.47	10.14	38.69	17.18	26.53	297.14
6.0	-12.58	5.85	45.76	11.41	35.68	17.69	25.65	284.09
7.0	-9.96	7.08	41.01	12.60	33.18	18.16	24.95	273.94

Table 7. Design data for 12-section NN impedance transformer and transformation ratio $r = 8$.

χ	$\lvert\Gamma\rvert$, dB	$\theta_1=\theta_{12}$, deg	$\theta_2=\theta_{11}$, deg	$\theta_3=\theta_{10}$,deg	$\theta_4=\theta_9$,deg	$\theta_5=\theta_8$,deg	$\theta_6=\theta_7$,deg	θ_{total}, deg
4.0	-21.66	3.10	56.72	8.22	40.72	15.61	26.17	301.07
5.0	-15.56	4.32	49.76	9.53	36.95	16.14	25.07	283.55
6.0	-11.68	5.57	43.85	10.77	33.89	16.65	24.22	269.89
7.0	-9.17	6.72	39.14	11.88	31.46	17.09	23.54	259.66

Table 8. Design data for 12-section NN impedance transformer and transformation ratio $r = 9$.

χ	$\lvert\Gamma\rvert$, dB	$\theta_1=\theta_{12}$, deg	$\theta_2=\theta_{11}$,deg	$\theta_3=\theta_{10}$, deg	$\theta_4=\theta_9$, deg	$\theta_5=\theta_8$, deg	$\theta_6=\theta_7$, deg	θ_{total}, deg
4.0	-20.80	2.95	55.36	7.76	39.22	14.74	24.91	289.87
5.0	-14.79	4.11	48.31	9.00	35.48	15.26	23.84	271.99
6.0	-10.91	5.32	42.20	10.22	32.38	15.76	23.00	257.78
7.0	-8.50	6.40	37.54	11.26	29.98	16.18	22.33	247.36

Table 9. Design data for 12-section NN impedance transformer and transformation ratio $r = 10$.

χ	$\lvert\Gamma\rvert$, dB	$\theta_1=\theta_{12}$, deg	$\theta_2=\theta_{11}$, deg	$\theta_3=\theta_{10}$,deg	$\theta_4=\theta_9$, deg	$\theta_5=\theta_8$, deg	$\theta_6=\theta_7$, deg	θ_{total}, deg
4.0	-20.04	2.83	54.18	7.37	37.95	14.02	23.85	280.37
5.0	-14.11	3.93	47.00	8.56	34.21	14.51	22.80	262.01
6.0	-10.32	5.08	40.87	9.71	31.12	15.00	21.97	247.50
7.0	-7.95	6.12	36.13	10.73	28.70	15.40	21.30	236.76

$r = 4$ ($Z_L = 50\,\Omega$, $Z_0 = 12.5\,\Omega$), bandwidth ratio $\chi = 5$ ($f_1 = 0.45\,\text{GHz}$, $f_2 = 2.25\,\text{GHz}$), and a magnitude of the reflection coefficient better than $|\Gamma| \leq -20\,\text{dB}$ has been synthesized based on the design data in Table 3. The calculated S-parameters of the impedance transformer using expression (4) are shown in Fig. 13.

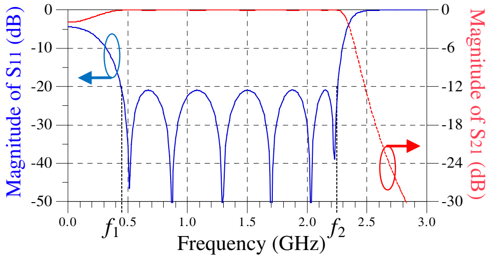

Figure 13. Magnitudes of S_{11} and S_{21} for the synthesized NN impedance transformer in Fig. 1.

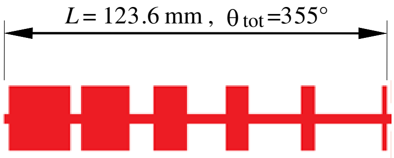

Figure 14. Layout of the synthesized microstrip 12-section NN impedance transformer.

The response of the NN impedance transformer exhibits six minima in the spectrum of the reflection coefficient. The same number of minima could be achieved cascading six quarter-wave sections at the expense of a longer matching circuit.

For simplicity reasons, the 12-section NN impedance transformer has been realized on low-cost microstrip technology. The transformer is fabricated on a substrate with a thickness $h = 1.524$ mm, relative dielectric constant $\varepsilon_r = 3.55$, dielectric loss tangent $\tan\delta = 0.002$, and conductor thickness $t = 0.035$ mm. The layout of the microstrip 12-section NN impedance transformer is shown in Fig. 14.

The widths of 50 Ω and 12.5 Ω microstrip lines are 3.35 mm and 20.85 mm respectively. The physical lengths of the NN impedance transformer sections are listed in Table 10. The sections of the impedance transformer in Fig. 14 are numbered from the left to the right.

As one can see from the data in Table 10, the microstrip realization of the transformer is not exactly symmetrical. This is due to the fact that the propagation properties of the 50 Ω and 12.5 Ω microstrip transmission lines are slightly different and the same electrical length leads to slightly different physical length of those two lines.

Table 10. The dimensions of the microstrip NN impedance transformer in Fig. 14 using RO4003 substrate.

Section #	1	2	3	4	5	6	7	8	9	10	11	12
Length,mm	1.63	19.89	3.46	15.77	7.72	11.1	12.26	7.47	16.99	4.36	21.2	1.77

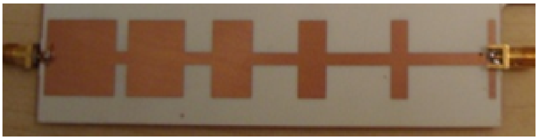

Figure 15. A photo of the fabricated 12-section NN impedance transformer.

6. MEASUREMENT RESULTS

The photo of the fabricated NN impedance transformer is shown in Fig. 15. It has been characterized using a Vector Network Analyzer. The scattering parameters of the impedance transformer could not be obtained directly due to the fact that the circuit has the input impedance $Z_0 = 12.5\,\Omega$ and the output impedance $Z_L = 50\,\Omega$, while the Network Analyzer utilizes standard $50\,\Omega$ input and output ports.

Therefore, the renormalization and de-embedding [12] of the $50\,\Omega$ coaxial connector at the input port of the transformer has been performed in order to obtain the actual S-parameters of the structure. For de-embedding, the coaxial connector and coaxial-to-microstrip line transition have been modeled using an FDTD full-wave simulator. The obtained scattering parameters of the SMA connector and the transition have been subtracted from the measured data.

Simulated and measured S-parameters of the impedance transformer are shown in Fig. 16. Measurements show that the magnitude of the reflection coefficient deteriorates at high frequencies. One minimum is not visible compared to MoM simulations of the structure. This is most likely caused by the inaccuracy of the fixture model in the de-embedding process and the fabrication errors. The measured maximum of in-band reflection reaches the level of approximately -15 dB while the expected level was below -20 dB. For this -15 dB level the measured operating frequency range of the transformer is from 0.33 GHz to 1.97 GHz. The magnitude of the transmission coefficient is better than -0.72 dB up to 1.97 GHz. The measured and simulated with MoM characteristics in Fig. 16 are shifted to lower frequencies in compari-

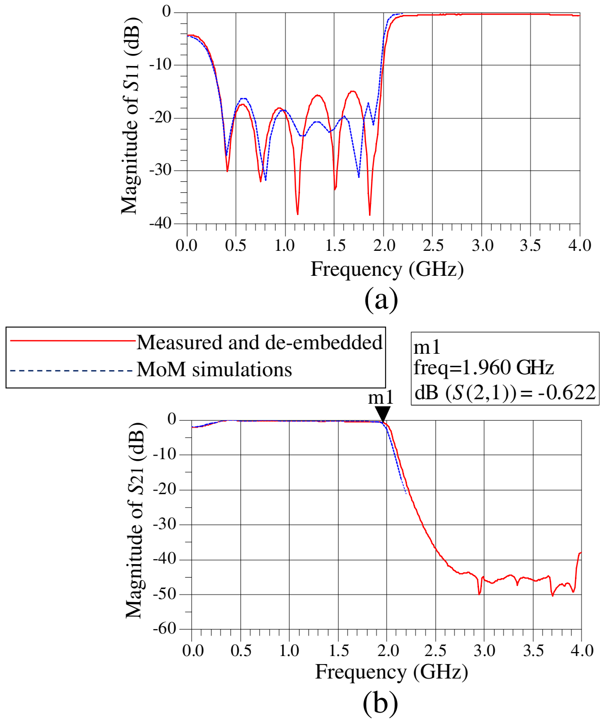

Figure 16. S-parameters of the NN impedance transformer in Fig. 15. (a) S_{11}. (b) S_{21}.

son to the calculated data in Fig. 13. This is due to the fact that the model for the impedance transformer presented in Section 2 is developed for a general case and does not depend on realization technology (microstrip, coaxial, etc.).

Therefore, even though multiple reflections from impedance steps are taken into account, there will always be discontinuities associated with the realization technology. And even within the same technology these discontinuities can be different depending on implemented materials and dimensions of the transmission lines. For example, in microstrip technology the same $12\,\Omega$-to-$50\,\Omega$ step will form different discontinuities if using substrates with different permittivity or thickness, due to different resulting width of the transmission line conductors for such impedances. This, however, can be taken into account using freely and commercially available simulation tools containing models for transmission line components, or even using a full-wave simulator, as it shown in Fig. 16.

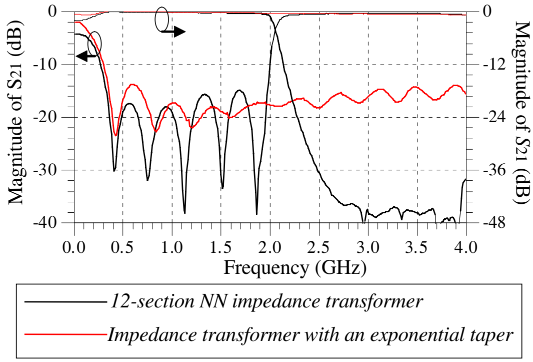

Figure 17. Measured S-parameters of the tapered and NN impedance transformers.

7. COMPARISON WITH A TAPERED LINE

In order to evaluate the performance of the synthesized NN transformer in comparison to the traditional designs, the 12-section NN impedance transformer and a tapered line impedance transformer have been fabricated on the same substrate. The length of the tapered transformer has been chosen such that both transformers have the same frequency of the first minimum, as it is shown in Fig. 17. As a result, both transformers had approximately the same matching characteristics in the frequency range from 0.33 GHz to 1.97 GHz.

The resulting total length of the NN transformer is 123.6 mm. The total length of the fabricated tapered line transformer is 215 mm. The length of the tapered transformer is considerably longer even though both transformers exhibit similar matching in the given frequency range.

It should also be noted that the magnitude of the reflection coefficient for the tapered line does not decrease (improves) continuously at high frequencies, as it was expected. This indicates that the tapered transformer becomes very sensitive to fabrication errors at high frequencies, and a special care should be taken when using tapered line transformer for ultra-wideband applications.

It can be concluded that the NN impedance transformer is more attractive in practice since it provides approximately the same matching level as the tapered impedance transformer in the specified frequency range, but has shorter length. A further miniaturization can be achieved by meandering the high impedance transmission line sections of the transformer as it is demonstrated in [11].

8. CONCLUSIONS

The distinct feature of NN impedance transformers is that their length becomes shorter as the transformation ratio increases. This feature makes the transformer attractive for applications, where a wide operating band and high transformation ratios are required, for example, an output matching of wideband power amplifiers. The transformer consists of a combination of high- and low-impedance transmission lines, which allows for further miniaturization. For example, in microstrip realization, further miniaturization can be achieved by meandering high impedance transmission line sections. In addition, the fabrication is simple, and does not require via holes, air bridges, neither etching of the ground plane.

Due to a lowpass behavior of the impedance transformer the structure can be used for matching power amplifiers simultaneously providing effective suppression of high order harmonics exhibited by the amplifier.

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