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NONLINEAR SUBPROFILE SPACE FOR RADAR HRRP RECOGNITION

D. Zhou*, X. Shen, and Y. Liu

School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China

Abstract—In this paper, a novel approach, namely nonlinear subprofile space (NSS), is proposed for radar target recognition using high-resolution range profile (HRRP). First, the HRRP samples are mapped into a high-dimensional feature space using nonlinear mapping. Second, the nonlinear features, namely nonlinear subprofiles, are extracted by nonlinear discriminant analysis. Then, for each class, the nonlinear subprofile space is formed using all the training nonlinear subprofiles of class. Finally, the minimum hyperplane distance classifier (MHDC) is used for classification. The aim of NSS method is to represent the feature area of target using nonlinear subprofile space, and effectively measure the distance between the test HRRP and feature area via minimum hyperplane distance (MHD) metric. The experimental results of measured data show that the proposed method has better performance of recognition than KPCA and KFDA.

1. INTRODUCTION

With the advent of wideband radar, it is easy task to obtain the high-resolution range profile (HRRP). It contains the target structure signatures, such as target size, scatterer distribution, etc., which is useful for target recognition. Therefore, more and more researchers have paid close attention to radar target recognition using HRRP. Li and Yang directly used HRRPs as feature vectors for target recognition [1]. Zyweck and Bogner applied the HRRP to recognize the commercial aircraft [2]. Eom and Chellappa studied a hierarchical model for radar HRRP recognition [3]. Kim et al. presented some

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* Corresponding author: Daiying Zhou (dyzhou2011@163.com).

invariant features of HRRP [4]. Nelson et al. proposed a new feature selection method for HRRP [5]. Wong applied the features in frequency domain for non-cooperative target recognition [6]. Du et al. studied the two-distribution compounded statistical model for radar HRRP target recognition [7]. However, these methods often fail to gain good target recognition when HRRPS are subject to complex nonlinear variations due to change of target-aspect and noise' effect, for they are linear method in nature. Thus, kernel function is introduced to deal with the nonlinear problems, such as kernel-based methods. Chen et al. proposed kernel principal component analysis (KPCA) [8]. Mika et al. studied kernel-based Fisher discriminant analysis (KFDA) by regularizing the within-class scatter matrix [9]. Fu and Yang presented a kernel method via multiclass synthetical discriminant analysis [10]. These methods do not still discuss how to represent the feature area of target and how to measure the distance between the test HRRP sample and feature area, which is very important to target recognition.

In this paper, a novel radar HRRP target recognition method, namely nonlinear subprofile space (NSS), is proposed. This method achieves good recognition performance by effectively representing the feature area of target using nonlinear subprofile space and measuring the distance between the test HRRP sample and feature area using minimum hyperplane distance metric. The experiments on measured data of three airplanes are simulated to verify the effectiveness of the proposed method.

The rest of this paper is organized as follows. In Section 2, we discuss nonlinear subprofile space. Section 3 proposed minimum hyperplane distance classifier. Section 4 presents an experimental study. Section 5 draws conclusions.

2. NONLINEAR SUBPROFILE SPACE

Let $\mathbf{x}_{ij}$ represents the jth training HRRP of ith class, where $i = 1, 2, \ldots, g$, $j = 1, 2, \ldots, N_i$, $N = N_1 + N_2 + \ldots + N_g$, and g is the number of target classes, N_i is the number of training samples for ith class, and N is the number of total training samples.

A nonlinear function ϕ is used to map $\mathbf{x}_{ij}$ into a high-dimensional feature space F as follow

$$R^n : \mathbf{x}_{ij} \rightarrow F : \phi(\mathbf{x}_{ij}) \tag{1}$$

where the dimensionality of feature space F is n'. Here n' may be any value or infinite. The between-class scatter matrix $\mathbf{S}_B$ and within-class scatter matrix $\mathbf{S}_W$ in feature space F can be calculated as follows

$$\mathbf{S}_B=\sum_{i=1}^{g}\left(\frac{1}{N_i}\sum_{j=1}^{N_i}\phi(\mathbf{x}_{ij})-\frac{1}{N}\sum_{i=1}^{g}\sum_{j=1}^{N_i}\phi(\mathbf{x}_{ij})\right)\cdot\left(\frac{1}{N_i}\sum_{j=1}^{N_i}\phi\left(\mathbf{x}_{ij}\right)-\frac{1}{N}\sum_{i=1}^{g}\sum_{j=1}^{N_i}\phi\left(\mathbf{x}_{ij}\right)\right)^T \tag{2}$$

$$\mathbf{S}_W=\sum_{i=1}^{g}\sum_{j=1}^{N_i}\left(\phi(\mathbf{x}_{ij})-\frac{1}{N_i}\sum_{j=1}^{N_i}\phi(\mathbf{x}_{ij})\right)\cdot\left(\phi(\mathbf{x}_{ij})-\frac{1}{N_i}\sum_{j=1}^{N_i}\phi(\mathbf{x}_{ij})\right)^T \tag{3}$$

where T represents the matrix transposition. Then, the columns of transformation matrix $\mathbf{H}$ are solved by maximizing Fisher ratio as follow

$$\boldsymbol{\alpha}_m=\underset{\{\boldsymbol{\alpha}\}}{\arg\max}\left\{\frac{\boldsymbol{\alpha}^T\mathbf{S}_B\boldsymbol{\alpha}}{\boldsymbol{\alpha}^T\mathbf{S}_\mathbf{w}\boldsymbol{\alpha}}\right\}\quad m=1,2,\ldots,(g-1) \tag{4}$$

where $\boldsymbol{\alpha}_m$ is a n'-dimensional column vector of matrix $\mathbf{H}$, i.e.,

$$\mathbf{H}=[\boldsymbol{\alpha}_1\ \boldsymbol{\alpha}_2\ \ldots\ \boldsymbol{\alpha}_{g-1}] \tag{5}$$

Because the explicit expression of nonlinear map $\phi(\cdot)$ is not defined, we can not directly get the orthogonal vectors $\boldsymbol{\alpha}_1\ \boldsymbol{\alpha}_2,\ldots,\boldsymbol{\alpha}_{g-1}$ from Equation (4). Kernel trick is introduced to solve this problem. Let

$$\boldsymbol{\alpha}=\sum_{i=1}^{g}\sum_{j=1}^{N_i}w_{ij}\phi(\mathbf{x}_{ij}) \tag{6}$$

and

$$k(\mathbf{x}_{ij},\mathbf{x}_{lk})=\phi(\mathbf{x}_{ij})^T\phi(\mathbf{x}_{lk}) \tag{7}$$

where w_{ij} is a coefficient; $\mathbf{x}_{ij}$ and $\mathbf{x}_{lk}$ are HRRP sample vectors. Substituting Equation (6) and Equation (7) into Equation (4), simplifying, we get

$$\mathbf{w}_m=\underset{\{\mathbf{w}\}}{\arg\max}\left\{\frac{\mathbf{w}^TP_B\mathbf{w}}{\mathbf{w}^TP_\mathbf{w}\mathbf{w}}\right\}\quad m=1,2,\ldots,(g-1) \tag{8}$$

where

$$\mathbf{w}=\begin{bmatrix}w_{11} & w_{12} & \ldots & w_{gN_g}\end{bmatrix}^T \tag{9}$$

$$\mathbf{P}_B=\sum_{l=1}^{g}(\mathbf{r}_l-\mathbf{q})(\mathbf{r}_l-\mathbf{q})^T \tag{10}$$

$$\mathbf{P}_W = \sum_{l=1}^{g}\sum_{k=1}^{N_l}(\mathbf{k}_{ijlk} - \mathbf{r}_l)(\mathbf{k}_{ijlk} - \mathbf{r}_l)^T \tag{11}$$

where

$$(\mathbf{r}_l)_{ij} = \frac{1}{N_l}\sum_{k=1}^{N_l} k(\mathbf{x}_{ij}, \mathbf{x}_{lk}) \quad l=1,2,\ldots,g; \quad k=1,2,\ldots,N_l \tag{12}$$

$$\mathbf{q}_{ij} = \frac{1}{N}\sum_{l=1}^{g}\sum_{k=1}^{N_l} k(\mathbf{x}_{ij}, \mathbf{x}_{lk}) \tag{13}$$

$$(\mathbf{k}_{ijlk})_{ij} = k(\mathbf{x}_{ij}, \mathbf{x}_{lk}) \tag{14}$$

Take the vector derivative of Equation (8) with respect to $\mathbf{w}$ and set resultant equation to zero. This generates the generalized eigenvector equation as follow

$$\mathbf{P}_W^{-1}\mathbf{P}_B\mathbf{w} = \lambda\mathbf{w} \tag{15}$$

where λ is the eigenvalue and $\mathbf{w}$ the eigenvector corresponding to λ. Thus, $\mathbf{w}_m$ is eigenvector corresponding to nonzero eigenvalue in Equation (15). After getting $\mathbf{w}_m$, substituting $\mathbf{w}_m$ into Equation (6), it follows that

$$\boldsymbol{\alpha}_m = [\phi(\mathbf{x}_{11})\ \phi(\mathbf{x}_{12}\ \ldots\ \phi(\mathbf{x}_{gN_g})] \cdot \mathbf{w}_m \tag{16}$$

After getting $\mathbf{H}$, projecting $\phi(\mathbf{x}_{ij})$ into nonlinear transformation space $\mathbf{H}$ in F, i.e.,

$$\mathbf{y}_{ij} = \mathbf{H}^T\phi(\mathbf{x}_{ij}) \tag{17}$$

substituting Equation (16) into Equation (17), then simplifying, we get

$$\mathbf{y}_{ij} = \begin{bmatrix} \sum_{l=1}^{g}\sum_{k=1}^{N_l} w_{1lk}k(\mathbf{x}_{lk}, \mathbf{x}_{ij}) \\ \sum_{l=1}^{g}\sum_{k=1}^{N_l} w_{2lk}k(\mathbf{x}_{lk}, \mathbf{x}_{ij}) \\ \vdots \\ \sum_{l=1}^{g}\sum_{k=1}^{N_l} w_{(g-1)lk}k(\mathbf{x}_{lk}, \mathbf{x}_{ij}) \end{bmatrix} \tag{18}$$

where w_{mlk} is the (lk)th element of vector $\mathbf{w}_m\,(l = 1,2,\ldots,g;\ k = 1,2,\ldots,N_l)$, and $\mathbf{y}_{ij}$ is $(g-1)$-dimensional vector, namely nonlinear subprofile of HRRP sample $\mathbf{x}_{ij}$. All of training nonlinear subprofiles for each class is used to form the subprofile space as follow.

$$\mathbf{O}_i = [\mathbf{y}_{i1}\ \mathbf{y}_{i2}\ \ldots\ \mathbf{y}_{iN_i}] \quad i = 1,2,\ldots,g \tag{19}$$

where $\mathbf{O}_i$ is named as nonlinear subprofile space (NSS) of ith class. From Equation (19), it is seen that the nonlinear subprofile space is spaned by all the training nonlinear feature vectors (subprofiles), thus it represents the feature area of target.

3. MINIMUM HYPERPLANE DISTANCE CLASSIFIER

Because the dimensionality of $\mathbf{O}_i$ is $(g-1)$, any $(g-1)$ nonlinear subprofiles from NSS $\mathbf{O}_i$ can form a hyperplane as follow

$$\begin{aligned} \mathbf{O}_{i,r} = [\mathbf{y}_{ij_1}\ \mathbf{y}_{ij_2}\ \ldots\ \mathbf{y}_{ij_{(g-1)}}] \quad & j_1, j_2, \ldots, j_{(g-1)} = 1, 2, \ldots, N_i \\ j_1 \neq j_2 \neq \ldots \neq j_{(g-1)} \quad & r = 1, 2, \ldots, C_{N_i}^{g-1} \end{aligned} \tag{20}$$

where $C_{N_i}^{g-1}$ is the number of hyperplanes in $\mathbf{O}_i$. The distance between the nonlinear subprofile $\mathbf{y}_x$ and the hyperplane $\mathbf{O}_{i,r}$ is defined as

$$d\left(\mathbf{y}_x, \mathbf{O}_{i,r}\right) = \|\mathbf{y}_x - \mathbf{v}_r\| \tag{21}$$

where $||\cdot||$ is the vector norm, and $\mathbf{v}_r$ is the projection of subprofile $\mathbf{y}_x$ on the hyperplane $\mathbf{O}_{i,r}$. $\mathbf{v}_r$ is given as follow

$$\mathbf{v}_r = \mathbf{Y}_r(\mathbf{Y}_r^T\mathbf{Y}_r)^{-1}\mathbf{Y}_r^T\mathbf{y}_x \tag{22}$$

where

$$\begin{aligned} \mathbf{Y}_r = [\mathbf{y}_{ij_{r1}}\ \mathbf{y}_{ij_{r2}}\ \ldots\ \mathbf{y}_{ij_{r(g-1)}}] \quad & j_{r1}, j_{r2}, \ldots, j_{r(g-1)} = 1, 2, \ldots, N_i \\ j_{r1} \neq j_{r2} \neq \ldots \neq j_{r(g-1)} & \end{aligned} \tag{23}$$

Thus, we can get the distance between $\mathbf{y}_x$ and the NSS $\mathbf{O}_i$

$$d(\mathbf{y}_x, \mathbf{O}_i) = \min_{\{r\}}\{d(\mathbf{y}_x, \mathbf{O}_{i,r})\} \tag{24}$$

Let $\mathbf{y}_t$ denotes the nonlinear subprofile of test HRRP sample $\mathbf{x}_t$, which is solved according to Equation (18). If

$$d(\mathbf{y}_t, \mathbf{O}_k) = \min_{\{i\}}\{d(\mathbf{y}_t, \mathbf{O}_i)\} \tag{25}$$

then the test HRRP sample $\mathbf{x}_t$ belongs to kth class.

4. EXPERIMENTAL RESULTS

To show effectiveness of the proposed method, the experiments are performed on HRRP samples measured from three airplanes, i.e., An-26, Jiang and Yark-42. 260 HRRPs obtained continuously of each airplane are adopted. For each airplane, half of all HRRPs are used as training data and the rest are used as testing data. Before running experiments, each HRRP is preprocessed by energy normalization. Two kernels are used, i.e., Gaussian kernel

$$k(\mathbf{x}_{ij}, \mathbf{x}_{lk}) = e^{-\frac{\left\|\mathbf{x}_{ij} - \mathbf{x}_{lk}\right\|^2}{2\sigma^2}} \tag{26}$$

and polynomial kernel

$$k(\mathbf{x}_{ij}, \mathbf{x}_{lk}) = (1 + \mathbf{x}_{ij} \cdot \mathbf{x}_{lk})^d \tag{27}$$

For subsequent analysis, we denote NSS method as NSS_G when the Gaussian kernel is used, and as NSS_P when the polynomial kernel is adopted. The parameters are empirically set as $\sigma = 1.34$ for Gaussian kernel and $d = 1$ for polynomial kernel.

4.1. A Comparison between Nonlinear Subprofile Space and Linear Subprofile Space

Table 1 shows the recognition results using nonlinear subprofile space (NSS) and linear subprofile space (LSS). The linear subprofile space is formed by LDA [3]. Then, the training subprofiles of NSS and LSS for each class are averaged as library template vectors. The nearest-neighbor classifier is used for classification.

From Table 1, it is seen that the average recognition rate for NSS is higher than that for LSS, for either the Gaussian kernel or the polynomial kernel. The average recognition rates for NSS (Gaussian kernel), NSS (Polynomial kernel), and LSS are 91%, 90% and 88%, respectively. The reason is that the nonlinear subprofile space can represent the nonlinear variations of HRRP samples by nonlinear kernel mapping.

Table 1. The confusion matrices and average recognition rates (ACR) for NSS and LSS (%).

	NSS (Gaussian kernel)			NSS (Polynomial kernel)			LSS		
	An-26	Jiang	Yak-42	An-26	Jiang	Yak- 42	An-26	Jiang	Yak-42
An-26	86	2	4	85	3	5	84	5	7
Jiang	5	92	2	5	92	2	5	88	2
Yak-42	9	6	94	10	5	93	11	7	91
ACR (%)	91			90			88		

Table 2. The confusion matrices and average recognition rates (ACR) for MHDC and MDC (%) (Gaussian kernel).

	MHDC			MDC		
	An-26	Jiang	Yak-42	An-26	Jiang	Yak-42
An-26	89	2	2	87	4	4
Jiang	3	95	1	3	90	2
Yak-42	8	3	97	10	6	94
ACR (%)	94			90		

4.2. A Comparison between Minimum Hyperplane Distance Classifier and Minimum Distance Classifier

After the nonlinear subprofile space of each class is formed based on training data, minimum hyperplane distance classifier (MHDC) and minimum distance classifier (MDC) are used for classification. The results are illustrated in Table 2. The MDC is given as follow

$$d_{MDC}\left(\mathbf{y}_t, \mathbf{O}_i\right) = \min_{\{j\}}\{\|\mathbf{y}_t - \mathbf{y}_{ij}\|\} \quad j = 1, 2, \ldots, N_i \tag{28}$$

where $d_{MDC}\left(\mathbf{y}_t, \mathbf{O}_i\right)$ is the distance between the test sample $\mathbf{x}_t$ and NSS $\mathbf{O}_i$. Then the test HRRP $\mathbf{x}_t$ belongs to kth class, $k = \arg\min_{\{i\}}\{d_{MDC}(\mathbf{y}_t, \mathbf{O}_i)\}$.

It is seen from Table 2 that recognition performance for MHDC is better than that for MDC. In Table 2, using Gaussian kernel, the average recognition rates for MHDC and MDC are 94% and 90%, respectively. This is because traditional MDC directly uses the Euclidean distance between subprofiles to measure the distance between the test HRRP sample and NSS. It is possible to increase the misclassification rates when there is the overlap between two different NSS. But MHDC applies the minimum hyperplane distance to measure the distance between the test HRRP sample and NSS, and is insensitive to the overlap between different NSS, thus it is more effective for classification than MDC.

4.3. Different Number of Samples

We choose 100, 140, 180, 220, and 260 HRRP samples for each class, and adopts the NSS_G method, NSS_P method, and directly using range profile (DRP) method [1] to identify these sample set. The results is shown in Fig. 1.

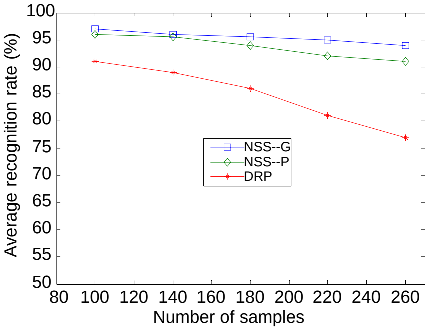

Figure 1. The average recognition rates of NSS_G, NSS_P, and DRP versus number of samples.

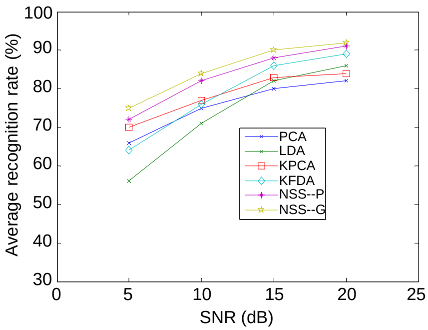

Figure 2. The average recognition rates of PCA, LDA, KPCA, KFDA, NSS_G and NSS_P versus SNR.

From Fig. 1, it can be observed that the average recognition rate of DRP method obviously decreases when the number of samples is increased, while that of NSS_G method and NSS_P method degrades little. The main reason is that the HRRP is sensitive to change of target-aspect, and the increase for number of samples is equivalent to larger variations of target-aspect, thus recognition rate degrades with the increase for the number of samples. But for the NSS method proposed in this paper, nonlinear features are used to effectively reduce the effect of the nonlinear variations of HRRP samples due to change of target-aspect, and the recognition performance of NSS method remains stable when the number of samples changes.

4.4. Recognition Performance Comparison

we also apply PCA [11], LDA [12], KPCA [8] and KFDA [9]. The experiments are simulated for SNR = 5 dB, 10 dB, 15 dB, and 20 dB. The Fig. 2 shows the average recognition rates of six methods versus SNR.

As can be seen, the average recognition rates of KPCA, KFDA, NSS_G, and NSS_P are significantly better than those of PCA and LDA. At SNR = 15 dB, the average recognition rates of KPCA, KFDA, NSS_G, and NSS_P are 81%, 85%, 90% and 89%, respectively; while those of PCA and LDA are 77% and 79%, respectively. It demonstrates that kernel function is effective for solving nonlinear problem. It is also observed that the average recognition rate of NSS method (NSS_G or NSS_P) is greater than that of other four methods (PCA, LDA, KPCA and KFDA) when SNR $\geqslant$ 5 dB. The reason is that NSS method takes into account the nonlinear feature area of target and the distance metric between test sample and feature area, thus improves recognition performance.

5. CONCLUSIONS

This paper proposes a novel radar target recognition method, namely nonlinear subprofile space (NSS). The NSS method has two prominent characteristics. First, nonlinear subprofile space is used to represent the feature area of target. Second, the minimum hyperplane distance is introduced to measure the distance between test sample and feature area, which further improves the classification performance. Experimental results for the measured data show that:

(1) The performance of nonlinear subprofile space is better than that of linear subprofile space;
(2) The classification performance for minimum hyperplane distance classifier is higher than that of traditional minimum distance classifier;
(3) The recognition rate of NSS method is insensitive to change of target-aspect;
(4) The recognition rate of NSS method is higher than that of KPCA, KFDA, PCA and LDA, when SNR $\geqslant 5\,\text{dB}$.

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REFERENCES

1. Li, H. J. and S. H. Yang, "Using range profiles as feature vectors to identify aerospace objects," *IEEE Trans. Antennas and Propagation,* Vol. 41, No. 3, 261–268, Mar. 1993.
2. Zyweck, A. and R. E. Bogner, "Radar target classification of commercial aircraft," *IEEE Trans. Aerospace Electronic Systems,* Vol. 32, No. 2, 598–606, Apr. 1996.
3. Eom, K. B. and R. Chellappa, "Noncooperative target classification using hierarchical modeling of high-range resolution radar signatures," *IEEE Trans. Signal Processing,* Vol. 45, No. 9, 2318–2326, Sep. 1997.

4. Kim, K. T., D. K. Seo, and H. T. Kim, "Efficient radar target recognition using the MUSIC algorithm and invariant feature," *IEEE Trans. Antennas and Propagation,* Vol. 50, No. 3, 325–337, Mar. 2002.
5. Nelson, D. E., J. A. Starzyk, and D. D. Ensley, "Iterated wavelet transformation and discrimination for HRR radar target recognition," *IEEE Trans. System Man and Cybernetics-part: System and Humans,* Vol. 33, No. 1, 52–57, Jan. 2003.
6. Wong, S. K., "Non-cooperative target recognition in the frequency domain," *IEE Proc. Radar Sonar Navig,* Vol. 151, No. 2, 77–84, Apr. 2004.
7. Du, L., H. W. Liu, Z. Bao, and J. Y. Zhang, "A two-distribution compounded statistical model for radar HRRP target recognition," *IEEE Trans. Signal Processing,* Vol. 54, No. 6, 2226–2238, Jun. 2006.
8. Chen, B., H. W. Liu, and Z. Bao, "PCA and kernel PCA for radar high range resolution profiles recognition," *2005 IEEE International Radar Conference*, 9–12, Arlington, VA, USA, May 2005.
9. Mika, S., G. Ratsch, J. Weston, B. Scholkopf, and K. R. Muler, "Fisher discriminant analysis with kernels," *IEEE International Workshop on Neural Networks for Signal Processing,* Berlin, Aug. 1999.
10. Fu, J. S. and W. L. Yang, "KFD-based multiclass synthetical discriminant analysis for radar HRRP recognition," *Journal of Electromagnetic Waves and Applications*, Vol. 26, Nos. 2–3, 169–178, 2012.
11. Novak, L. M. and G. J. Owirka, "Radar target recognition using an eigen-image approach," *IEEE National Radar Conference*, Atlanta, USA, Mar. 29–31, 1994.
12. Duda, R., R. Hart, and D. Stock, *Pattern Classification,* Wiley, 2002.