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A MODIFIED HAIR-PIN RESONATOR FOR THE DESIGN OF COMPACT BANDPASS FILTER WITH SUPPRESSION OF EVEN HARMONICS

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Abstract—A new design consideration is explored for a hair-pin resonator. A grounding via at the mid-point of the resonator acts as a perturbation to split the resonant frequencies. The via also suppresses even harmonics of the fundamental. The principle operation of the hair-pin resonator with a via is analyzed and verified by measurement. It is shown that such a hair-pin resonator can be made more compact using stepped impedance line. A compact 4-pole bandpass filter using the modified compact hair-pin resonator with a via is demonstrated. Simulation and measured results showed good agreement.

1. INTRODUCTION

Many applications, such as mobile stations, require compact and light weight electronics; compact designs are also important for IC technology. For RF applications, these have led to the research on compact RF band pass filters. One way of making such filters is to use dual mode resonators, since dual mode resonator can replace 2 single mode resonators.

Many RF filters using distributed elements, such as ring resonators, also exhibit passband response at harmonics of the fundamental (centre frequency of the filter). It is sometimes necessary to suppress some or all harmonic responses. Several papers have been published on compact filters employing single [1–4] and dual modes [5–8]. On the other hand, several papers have also been published on harmonic suppression [9–15]. In [12], the authors combine dual mode and harmonic suppression. The suppression is quite low; about 9 dB

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at the first harmonic and about 6 dB at the second harmonic. Other dual mode resonators [13–15] suppress only the first harmonic, but the suppression is better. Only [12, 14, 15] illustrate filter design with harmonic suppression.

The objective of this paper is to report on a hair-pin microstrip resonator which has a dual mode with suppression of all even harmonics of the fundamental. The dual mode allows the design of compact hair-pin resonator filters. The principle of operation and further size reduction are illustrated by calculated expression, simulation and measurement. The circuit analysis employed here is an approximate; the resonator and filter designs have to be fine tuned by simulation. The expressions from the circuit analysis serve as guidelines for the variation of the parameters in the simulation. Finally, the simulated and measured response of a 4-pole filter employing the reduced size resonator is presented. The simulated and measured results are in reasonable agreement. The measured results exhibit good suppression of even harmonics of the fundamental.

2. PRINCIPLE OF OPERATION

A half wavelength microstrip transmission line open circuited at both ends and bent into a U shape is called a hair-pin resonator. This resonator can resonate at frequencies at which its length, ℓ, is an integer multiple of half wavelength, λ, that is at frequency $f = nv_p/(2\ell)$, where v_p is the phase velocity in the microstrip line and n is an integer.

Consider now the mid point A of a lossless hair-pin resonator excites from two sides as shown in Fig. 1.

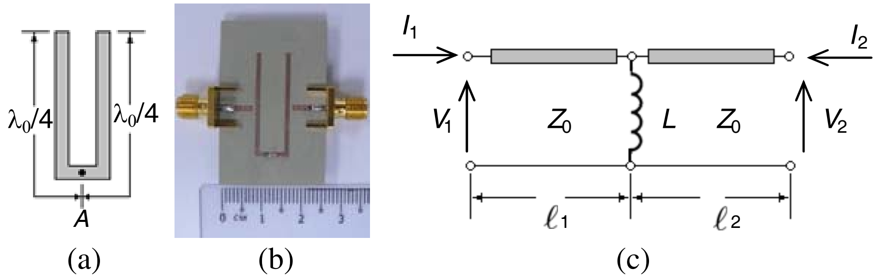


Figure 1. (a) Hair-pin resonator with a via at the centre. λ_0 is the wavelength of the fundamental resonant frequency, f_0 of the resonator without a via. (b) Photograph of the resonator with weak coupling. (c) Equivalent circuit of a hair-pin resonator with a via. ℓ_1 and ℓ_2 are the lengths of the two arms measured from the grounding via.

At the fundamental and its odd harmonics, there is a voltage minimum. A perfect ground at point A has no effect on the fundamental and its odd harmonics. On the other hand, point A has a voltage maximum at the even harmonics of the fundamental. Hence, a perfect ground will terminate such harmonics.

A perfect ground does not exist in practice. Grounding through a via has a small inductance, L, which acts as perturbation; this splits the resonance of the fundamental and its odd harmonics. The imperfect ground does not eliminate the even harmonics, but rather suppresses them. The equivalent circuit is shown in Fig. 1(c). Consider the circuit as a two-port network, the currents at each port is given by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{1}$$

Non-zero values of the voltages, V_1 and V_2, are for open circuit boundary conditions, and $I_1 = I_2 = 0$ can occur at the resonance if the determinant of the above Y-matrix above is zero [16],

$$Y_{11}Y_{22} - Y_{21}Y_{12} = 0. \tag{2}$$

As the structure is cascaded, it is algebraically easier to analyze using $ABCD$ parameters. By substituting $Y_{11} = D/B$, $Y_{22} = A/B$, and $Y_{12} = Y_{21} = -1/B$ (reciprocal network) into (2), this gives the resonance condition of

$$AD = 1. \tag{3}$$

By matrix multiplication,

$$A = \cos\beta(\ell_1 + \ell_2) + \frac{Z_0}{\omega L} \sin\beta\ell_1 \cos\beta\ell_2 \tag{4}$$

$$D = \cos\beta(\ell_1 + \ell_2) + \frac{Z_0}{\omega L} \cos\beta\ell_1 \sin\beta\ell_2 \tag{5}$$

where β is the propagation constant, and ω is $2\pi f$.

3. HAIR-PIN RESONATORS WITH EQUAL AND UNEQUAL ARM LENGTHS

3.1. Hair-pin Resonator with Equal Arm Lengths and a Via at the Center

The lengths of the arms are $\lambda_0/4$ each at the frequency, f_0, where λ_0 is the guided wavelength at the center frequency of the passband. When the network is symmetric, then $A = D$. Hence from (3),

$$A = \pm 1 \tag{6}$$

Considering $\ell_1 = \ell_2 = \ell$ in (6),

$$\cos 2\beta\ell + \frac{Z_0}{2\omega L}\sin 2\beta\ell = \pm 1 \tag{7}$$

The right-hand side of (7) equals -1 when $\beta\ell = \pi/2$, and the frequency for β is f_0 when $\ell = \lambda_0/4$.

The solution of (7) is very close to f_0, when the right-hand side equals $+1$, as the perturbation is small. By including the first correction, it is given approximately by

$$f \approx f_0\left(1 - \frac{8f_0L}{Z_0}\right) \tag{8}$$

Thus, one of the resonant frequencies is the same as the resonant frequency of the hair-pin resonator without a via, while the other is not. This is also supported by considering the odd and even modes for the symmetric structure of the resonator. The odd modes produce a zero voltage at the central point, A; the resonant frequency is not affected by the perturbation at this point. On the other hand, the even modes produce a voltage at this point and so the resonant frequency is affected.

For a band pass filter with resonators tuned to the same frequency, f_C, which is also the centre frequency of the filter, the coupling coefficient between two single mode resonators is given in [17] as

$$k = \frac{f_1 - f_2}{f_C} \tag{9}$$

where f_1 and f_2 are the resonant frequencies of the two coupled resonators. If the dual mode hair-pin resonator replaces two coupled single mode resonators, one must have

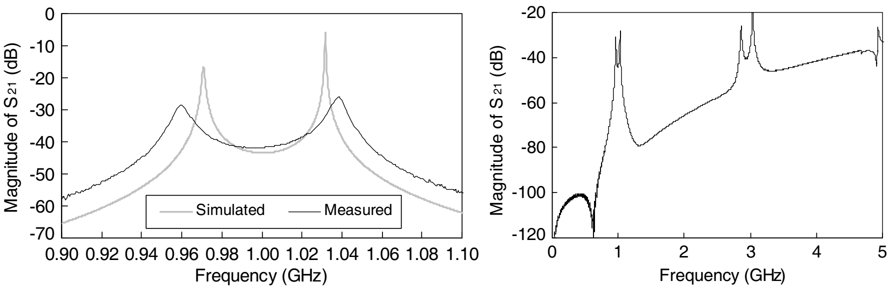
$$k = \frac{\left(8f_0^2L\right)/Z_0}{f_C}. \tag{10}$$

As f_C is close to f_0,

$$k \approx \frac{8f_CL}{Z_0} \quad \text{and} \quad f_0 \approx f_C\left(1 + \frac{4f_CL}{Z_0}\right). \tag{11}$$

We can determine the required values of f_0 and L for a chosen value of Z_0 given the values of k and f_C from bandpass filter design. The physical length of the quarter wavelength lines is determined by f_0.

In this paper, a via is implemented on RT Duroid 6010.2 with a substrate thickness of 1.27 mm. To implement the via, we use a copper wire with a diameter of 0.52 mm. The inductance of the via is extracted by a Sonnet [18] simulation with two half wavelength lines



(a) (b)

Figure 2. (a) Simulated and measured variation of the magnitude of S_{21} using the frequency for equal arm length hair-pin resonator with a via. The arm length is 29 mm. The centre frequency, f_C is 1.0 GHz. (b) Simulation broadband response of S_{21}.

on both sides of the via and a port at the end of each half wavelength. The inductance is given by

$$L = \mathrm{Im}(Z_{11})/\omega \tag{12}$$

where $\mathrm{Im}\,(Z_{11})$ refers to the imaginary part of Z_{11}. Over a range of frequencies around 1.0 GHz, the value of the inductance is determined to be approximately 0.39 nH.

Figure 2 shows the simulated [18] and measured magnitude of S_{21} with the frequency for the hairpin resonator with a via. The centre frequency is 1 GHz and the characteristic impedance, Z_0 is 53.68 Ω. For $L = 0.39$ nH, the difference between the two resonant frequencies is calculated as $8f_0^2L/Z_0 = 0.06$ GHz. The calculated two resonant frequencies are 0.968 GHz and 1.030 GHz, respectively. From simulation, the difference between the resonant frequencies is also 0.06 GHz. The measured value of 0.08 GHz is larger, as we are using a soldered copper wire as a via, which can have a higher inductance. Fig. 2(b) shows even harmonics suppression in its broadband response. We note that the split frequencies of the hair-pin resonator with a via can be used to measure the inductance of the via, just as it has been done using ring resonators with vias [19].

3.2. Hair-pin Resonator with Unequal Arm Lengths

The separation between the resonant frequencies may be increased using hairpin resonators with unequal arm lengths. Fig. 3(a) shows the resonator with one arm of length $\lambda_0/4 + x$ and the other arm of length $\lambda_0/4 - x$. Then by substituting (4) and (5) into (3) will give

$$\left(\frac{Z_0}{2\omega L}\right)^2 \sin 2\beta\ell_1 \sin 2\beta\ell_2 + \left(\frac{Z_0}{2\omega L}\right) \sin 2\beta(\ell_1+\ell_2) - \sin^2\beta(\ell_1+\ell_2) = 0 \quad (13)$$

For small x compared to λ_0, consider the solutions to lie close to the frequency f_0, the approximated solution is

$$f = f_0(1+\delta) \quad (14)$$

$$\delta = -\frac{4f_0L}{Z_0}\left(1 \pm \sqrt{1 + \left(\frac{Z_0}{f_0L}\frac{x}{\lambda_0}\right)^2}\right) \quad (15)$$

As a result, none of the solutions is f_0. The parameter x allows further tuning of the resonator. It can be seen that if $x = 0$, (15) reduces to the solutions for equal arm lengths.

Figure 3(b) shows the variation of the simulated [18] and measured magnitude of S_{21} with frequency for the unequal arm hair-pin resonator with a via. The centre frequency is at 1 GHz, the characteristic impedance of the microstrip line, Z_0, is 53.68 Ω and the value of inductor, L, is 0.39 nH. The calculated upper resonant frequency is 1.0456 GHz and the lower resonant frequency is 0.952 GHz. The calculated separation value between the two resonant frequencies is 0.0936 GHz. The simulated result gives a higher resonant frequency of

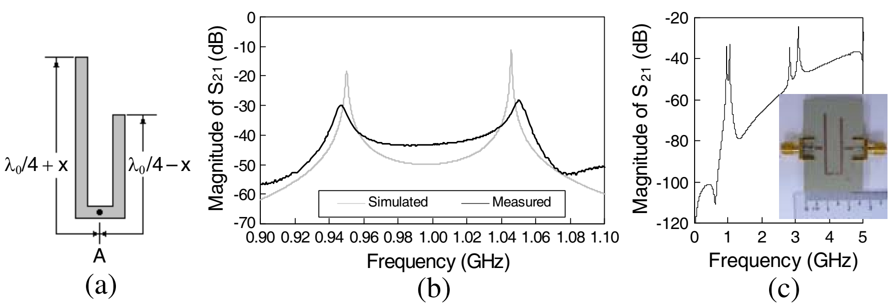
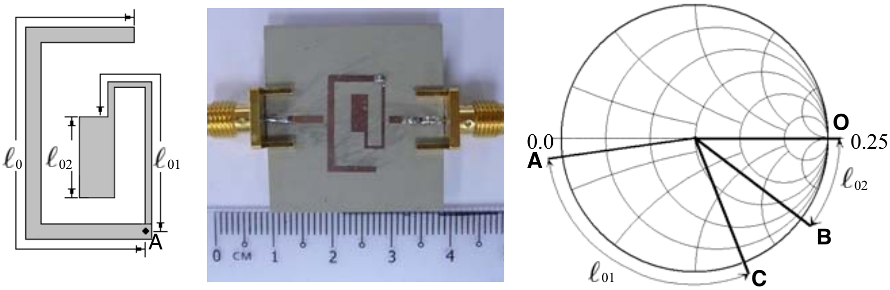


Figure 3. (a) Hair-pin resonator with a via and unequal arm lengths, $\lambda_0/4 + x$ and $\lambda_0/4 - x$. λ_0 is the wavelength corresponding to the fundamental resonant frequency, f_0 of the equal arm length resonator without a via. (b) Simulated and measured variation of the magnitude of S_{21} using the frequency for an unequal arm length hair-pin resonator with a via. The arm lengths measured from the via are $\lambda_0/4 + x = 30$ mm and $\lambda_0/4 - x = 28$ mm. The center frequency is 1.0 GHz. (c) Simulated broadband response of S_{21}. Inset is photograph of the resonator with weak coupling.



(a) (b) (c)

Figure 4. (a) Proposed compact resonator. (b) Photograph of the proposed compact resonator with weak coupling. (c) Smith chart illustrating the reduction in the length of the resonator.

1.046 GHz and a lower resonant frequency of 0.951 GHz; the separation between the frequencies is 0.095 GHz. The measured result gives a higher resonant frequency of 1.051 GHz and a lower resonant frequency of 0.948 GHz; here the separation between the resonant frequencies is 0.103 GHz. The broadband response of S_{21} in Fig. 3(c) shows even harmonic suppression.

4. MAKING A MORE COMPACT RESONATOR

The unequal arm length resonator can be made more compact if the length of the smaller arm can be reduced and the longer arm is folded around. This can be done by using lines of two different characteristic impedances to replace the shorter arm. The structure is shown in Figs. 4(a) and 4(b).

The principle is best explained by the use of Smith Impedance Chart shown in Fig. 4(c). The normalized wavelength, λ_0, corresponds to the fundamental resonant frequency, f_0 of the equal arm length resonator without a via. Open circuit is represented by the point O on the Smith Impedance chart. The normalized impedance of the open circuited shorter arm is $\ell_0 = \ell_0/4 - x$, and the characteristic impedance Z_0 is represented by point A; its electrical length is OA. Instead of this, we use an open circuited line of characteristic impedance Z_{02} less than Z_{01}. Its electrical length is OB, while B represents its normalized impedance. On renormalization with respect to Z_{01}, point B is moved to point C because the impedance normalized to Z_{01} is $(Z_{02}/Z_{01} < 1)$ times the impedance at B normalized to Z_{02}. A line of characteristic impedance Z_{01} and the electrical length CA are then required to achieve the same impedance as the single line arm; the

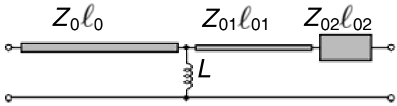


Figure 5. Equivalent circuit of the proposed compact resonator.

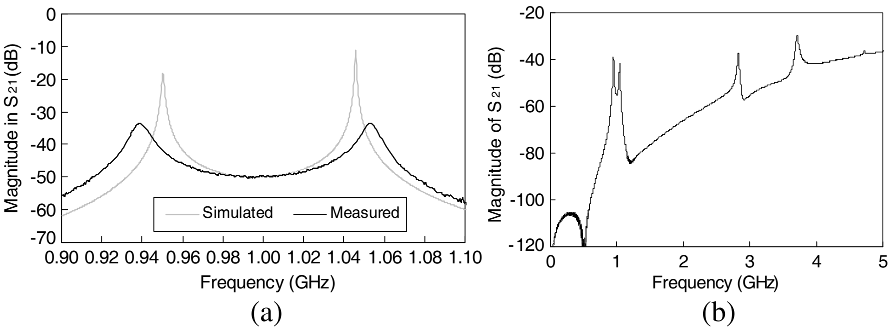


Figure 6. (a) Simulated and measured variation of the magnitude of S_{21} with the frequency for the weakly coupled compact resonator. (b) Simulated broadband response of S_{21}.

saving in length is BC. Another way of looking at this is to recall that a step change in characteristic impedance is equivalent to a transformer.

Although the explanation above is correct, the lengths obtained this way will not give the same frequency separation; this is because the $ABCD$ parameters of the line pair and a single line are different. The equivalent circuit for the improved resonator structure is shown in Fig. 5. The following is the simplified equation from the $ABCD$ parameters for the equivalent circuit to obtain the two resonant frequencies.

$$\left(\frac{1}{Z_0}\tan\beta_0\ell_0-\frac{1}{\omega L}\right)\left(1-\frac{Z_{01}}{Z_{02}}\tan\beta_{01}\ell_{01}\tan\beta_{02}\ell_{02}\right)+\frac{1}{Z_{01}}\tan\beta_{01}\ell_{01}+\frac{1}{Z_{02}}\tan\beta_{02}\ell_{02}=0 \tag{16}$$

where ℓ_0 is $\lambda_0/4+x$. β is the propagation constant of each microstrip line.

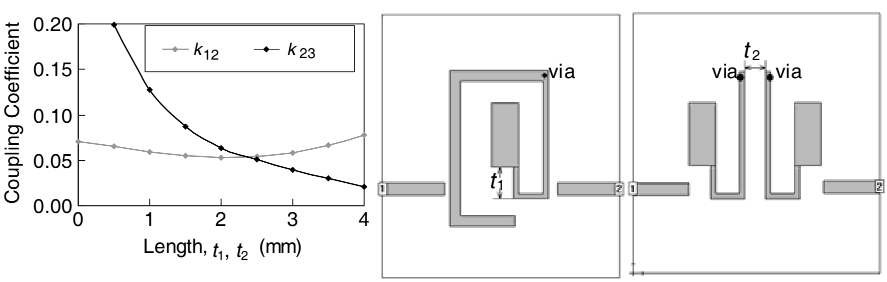
Figure 4(b) shows the folded compact resonator, whose lengths have been adjusted to achieve a simulated frequency separation of 0.0954 GHz in Fig. 6. The high impedance line has a characteristic impedance value Z_{01} of 70.44 Ω; its length, ℓ_{01}, is 15.85 mm. The low impedance line has a characteristic impedance Z_{02} of 32.32 Ω with ℓ_{02} equal to 6.00 mm.

The calculated resonant frequencies from (16) are 0.9340 GHz and 1.0360 GHz; their separation is 0.1020 GHz. The simulated higher and lower resonant frequencies are 1.0458 GHz and 0.9504 GHz, respectively; their separation is 0.0954 GHz. The measured upper and lower resonant frequencies are 1.055 GHz and 0.9406 GHz, respectively, with a separation of 0.1144 GHz, are plotted in Fig. 6(a) as a comparison to for comparison with the simulated frequencies. Fig. 6(b) shows the even harmonics suppression in the broadband response of the weakly coupled resonator.

5. DEMONSTRATION OF A FOUR POLE BANDPASS FILTER USING THE IMPROVED COMPACT RESONATOR

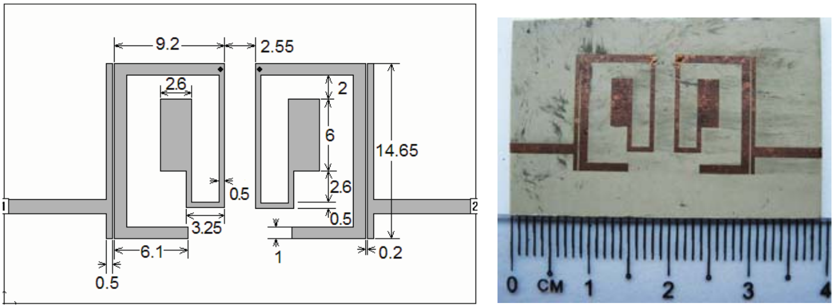
A 4-pole bandpass filter was designed using the proposed compact resonator as discussed in Section 4. The filter has a centre frequency of 1.0 GHz, a passband ripple of 0.5 dB and a fractional bandwidth of 8.5%. Given the filter specification, the required coupling coefficients and external quality factor computed using lowpass prototype g-parameter [17] are $k_{12} = k_{34} = 0.0602$, $k_{23} = 0.0506$ and $Q_{e1} = 19.65$, respectively. For this filter, two identical resonators are used, because k_{12} and k_{34} are the same. The filter is designed and fabricated on a RT/Duroid 6010.2 substrate with a thickness of 1.27 mm and a dielectric constant of 10.2.

Knowing the center frequency of the passband is 1.0 GHz and the coupling coefficient value of k_{12}, the two resonant frequencies calculated from coupling coefficient formula (9) are 0.970 GHz and 1.030 GHz.



(a) (b) (c)

Figure 7. (a) Design curve for the coupling coefficients, k_{12} and k_{23}. The layout of the resonators to obtain the design curve for (b) the coupling coefficient value of k_{12} varies with length t_1 and (c) the coupling coefficient value of k_{23} varies with length t_2.



(a) (b)

Figure 8. (a) Layout of the 4-pole bandpass filter designed using the proposed compact resonator. (b) Photograph of the fabricated 4-pole bandpass filter.

These resonant frequencies are then substituted separately into (16) to obtain the length of ℓ_0 and ℓ_{01}. The calculated ℓ_0 and ℓ_{01} are 16.95 mm and 27.8 mm, respectively. The parameters of the stepped impedance line: Z_{01}, Z_{02}, and ℓ_{02} are the same as in Section 4. They are 70.44 Ω, 32.32 Ω and 6.00 mm, respectively. Another way to obtain the coupling coefficient value of k_{12} is to use the filter design curve in Fig. 7(a). This design curve is obtained by varying the length t_2 of the resonators as shown in Fig. 7(b). Then the coupling coefficient of k_{23} is determined by the separation of the two stepped impedance resonators (layout as shown in Fig. 7(c)), t_2, and the design curve for the coupling coefficient value of k_{23} [17] is plotted in Fig. 7(a). The narrow gap coupling of the input and output signal feed structures are then used to obtain the required bandwidth of the passband.

The layout of the complete finalized 4-pole bandpass filter designed using the proposed compact resonator is shown in Fig. 8(a). The whole fabricated filter in Fig. 8(b) covers an area of 39.35 mm × 25.00 mm. Fig. 9(a) shows the simulated and measured results of the 4-pole bandpass filter. The measured bandpass has a center frequency of 1.0 GHz with a 3-dB fractional bandwidth of 9.1%. The measured in-band has an insertion loss is 2.54 dB and a return loss about 15 dB. The insertion loss is attributed to the copper and dielectric losses. The difference between the simulated and the measured filter's bandwidth is due to the tolerance in fabrication. The broadband response of this filter is shown in Fig. 9(b) as evident; the filter exhibits even harmonics suppression.

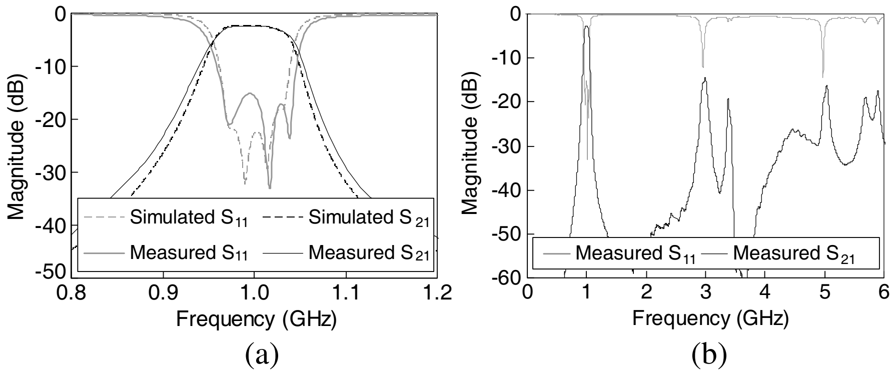


Figure 9. (a) Simulated and measured results of the 4-pole bandpass filter. (b) Measured broadband response of the 4-pole band pass filter.

6. CONCLUSION

A new design consideration has been proposed for a hairpin resonator. With a grounding via at the mid-point of the resonator, the hair-pin resonator can have two resonant frequencies at the fundamental mode of the conventional hair-pin resonator. The new hairpin resonator was investigated using $ABCD$ parameter for its resonant frequencies. To make the resonator more compact, one side of the uniform line on the hair-pin resonator is replaced with a stepped impedance line. Analytical results showed good agreement with simulated and measured ones. A four-pole bandpass filter was designed using the modified compact hair-pin resonator and measured to prove the design theories. This filter is more compact than those filters reported earlier, and it suppresses all even harmonics. The higher order even harmonics show reduced response.

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