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A MULTI-OBJECTIVE MEMETIC OPTIMIZATION APPROACH TO THE CIRCULAR ANTENNA ARRAY DESIGN PROBLEM

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Abstract—The paper provides a novel approach to the design of non-uniform planar circular antenna arrays for achieving maximal side lobe level suppression and directivity. The current excitation amplitudes and phase perturbations of the array elements are determined using an Adaptive Memetic algorithm resulting from a synergy of Differential Evolution (DE) and Learning Automata that is able to significantly outperform existing state-of-the-art approaches to the design problem. Moreover, existing literature considers the design problem as a single-objective optimization task that is formulated as a linear sum of all the performance metrics. Due to the conflicting nature of the various design objectives, improvements in a certain design measure causes deterioration of the other measures. Following this observation, the single-objective design problem is reformulated as a constrained multi-objective optimization task. The proposed memetic algorithm is extended to the multi-objective framework to generate a set of non-dominated solutions from which the best compromising solution is selected employing a fuzzy membership based approach. An instantiation of the design problem clearly depicts that the multi-objective approach provides simultaneous side lobe level suppression and directivity maximization in comparison to the single-objective scenario.

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1. INTRODUCTION

Circular antenna array design is a growing avenue of research in electromagnetics. They find interesting applications in radio direction finding, sonar, radar and space navigation and other systems and are advantageous over other array geometries [1, 2]. Although substantial research work has been conducted for the design of linear arrays, optimal selection of array parameters of circular antenna arrays for achieving superior performance remained as an open research problem. The main design problem in the present context is to determine the optimal geometric and electrical configurations for the circular array that provides maximum directivity, reduced side-lobe level and other suitable criteria. With the rapid growth of wireless and long-distance communications, antenna arrays have witnessed growing applications to meet the demands of directive radiation patterns and substantial power gain [3–5].

Recently researchers have concentrated on the usage of meta-heuristic algorithms for the array design problem due to the inherent complexity of the search space involved in the optimization process. Traditional techniques involving derivatives often get trapped in local optima and fail to obtain promising results. Panduro et al. [6] first applied population based search algorithms in the circular antenna array design problem. They employed the popular Genetic Algorithm (GA) to ensure maximal side lobe level reduction along with a specific beam width. Later Shihab et al. [7] used Particle Swarm Optimization (PSO) for similar purpose. Experimental results indicated that PSO was able to outperform GA in a statistically significant manner. Later Panduro et al. [8] provided a comparative performance analysis of GA, PSO and Differential Evolution (DE) to the above problem. The main objective of his work was to determine the current excitations and phase perturbations of the circular antenna array to provide maximum directivity and reduced side lobe level. PSO and DE provided comparable results but were able to outperform GA by a large margin. Recently Invasive Weed Optimization has occupied a special place in various antenna design problems. In [9] the authors propose a modified IWO algorithm (referred to as MIWO in this paper) and applied it to the circular antenna design problem to obtain promising results. Other significant research works in this field include [10, 11]. In this article we describe the problem of determining optimal current excitation amplitudes and phase perturbations of the array elements to ensure maximum directivity and minimum side lobe level.

Recently Memetic Algorithms (MA) [25] have earned popularity in solving complex numerical optimization problems. The class of algorithms fall in a broad category of population based meta-heuristics that incorporate strategies for individual learning. The word "meme" was first introduced by Dawkins [12]. It is involved in the context of cultural evolution and it performs the same role as the "gene" in the evolution process. MAs utilize the problem domain knowledge through interaction with the members of the population to culturally improve the quality of the trial solutions in the evolutionary setting [13–15].

The paper proposes a MA, called LA-DE which employs Differential Evolution (DE) [16–19] as the global search tool and Learning Automata [20–22] for local refinement. Recently Differential Evolution (DE) has emerged as one of the most powerful EA [36] and has successfully been applied to various electromagnetic problems [33]. It is a simple yet efficient population based search algorithm. However, it still suffers from the problems of premature convergence or stagnation. The performance of the heuristic used in DE is governed by three control parameters — the population size NP, the scale factor F and the Crossover Ratio Cr [24]. The parameter F controls the evolution process of a particular member of the population while parameter Cr controls the degree of diversity introduced in the population and is highly sensitive to problem selection. The main inspiration behind this work is outlined next. The basic idea is that a population member with good fitness should search the local neighborhood, whereas a poor performing member should be made to participate in the global search. Thus the scaling factor F should decrease for fitter individuals in comparison to the others in order to ensure fitter genes to participate in local search and the remaining ones to participate in global search. With variation in the generation count the extent of global and local exploration should also vary and must be adapted for better performance. Thus it is intuitively understandable that proper variation of the scaling factor with both fitness and generation would improve the performance of the DE algorithm. In the proposed method we have employed Learning Automata to choose appropriate values for the control parameter F from the meme pool for each population member during successive generations. Proper balancing between the exploration and exploitation capabilities of the members is crucial and must be maintained dynamically as evolution continues and hence the choice of Learning Automata over any predetermined control rules for the scaling factor.

In existing literature the array design problem is formulated as the optimization of a single objective function which is taken as the linear (or weighted) sum of the various performance metrics of the array. However, the output in this case is a single optimal solution which may not always provide optimal values of all the metrics simultaneously. Determining the optimal choice of weights to produce a non-dominated solution is also a cumbersome process. Further, in most real life scenarios such single objective functions often consist of conflicting components and hence it is preferable to generate a set of optimal solutions instead of a single one and employ a knowledgeable decision maker to select the appropriate choice. As a result, research paradigm for real world optimization scenarios are gradually shifting to Multi-objective Optimization (MO) [31, 32]. MO provides a set of non-dominated solutions which constitute the Pareto-front [26–30] from which it is suitable to extract an optimal solution. The second part of our article extends the proposed memetic algorithm to a multi-objective framework. The design problem is formulated as a constrained multi-objective optimization process to generate a number of non-dominated solutions to the design problem.

The major contribution of this paper lies in the integration of two recent research directions in the field of evolutionary computing namely, Memetic Algorithms and Multi-objective Optimization. Simulated results demonstrate that the proposed memetic algorithm is able to outperform several state-of-the-art evolutionary algorithms applied before in this research area for the single-objective optimization problem. In the next part of the article we propose a multi-objective memetic framework to the aforesaid problem and establish the superiority of our approach in comparison to the single-objective one through extensive experimental results. To the best of our knowledge, such a multi-objective approach to the mentioned design problem has not been reported before and this work may be considered as the humble beginning to the multi-objective optimization of the circular antenna array design problem.

2. THE DESIGN PROBLEM

2.1. The Single-objective Approach

The array factor of a circular antenna array of N antenna elements placed on a circle of radius r in the x-y plane (Fig. 1) is [6–11]:

$$AF(\Phi)=\sum_{i=1}^{N} I_n \exp\left(jkr\left(\cos\left(\Phi-\Phi_{ang}^n\right)-\cos\left(\Phi_0-\Phi_{ang}^n\right)\right)+\beta_n\right) \quad (1)$$

where, $\Phi_{ang}^n = 2\pi(n-1)/N$ is the angular position of the nth element in the x-y plane, $kr = Nd$ where, k is the wave-number, d is the angular spacing between elements, and r is the radius of the circle defined by the antenna array, Φ_0 is the direction of maximum radiation, Φ is the angle of incidence of the plane wave, I_n is the current excitation and β_n is the phase excitation of the nth element.

Here our main objective is to suppress side-lobes and minimize beam width by varying the current and phase excitations of the antenna elements. For a symmetrical excitation of the circular antenna array,

$$\begin{aligned} I_{n/2+1}\angle\beta_{n/2+1} &= conj\left(I_1\angle\beta_1\right) \\ I_{n/2+2}\angle\beta_{n/2+2} &= conj\left(I_2\angle\beta_2\right) \\ &\dots \\ I_n\angle\beta_n &= conj\left(I_n\angle\beta_n\right) \end{aligned}$$

Hence we model the objective function as:

$$f = \left|AR(\varphi_{sll}, \vec{I}, \vec{\beta}, \varphi_0)\right| \Big/ \left|AR\left(\varphi_{\max}, \vec{I}, \vec{\beta}, \varphi_0\right)\right| + 1/DIR\left(\varphi_0, \vec{I}, \vec{\beta}\right) + \left|\varphi_0 - \varphi_{des}\right| \tag{2}$$

where φ_{sll} is the angle at which maximum sidelobe level is attained, φ_{des} is the desired maxima and $DIR(\varphi_0, \vec{I}, \vec{\beta})$ is the directivity of the array at the direction indicated by Φ_0. The range of variation of normalized amplitude excitation is [0, 1]. The range of phase excitation is [−180°, 180°]. The element spacing should lie between 0.5λ and λ to prevent mutual coupling and grating lobes.

The first component attempts to suppress the sidelobes. Nowadays directivity has become a very useful figure of merit for comparing array patterns. The second component attempts to

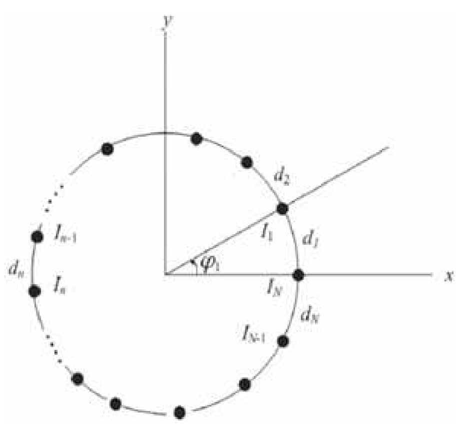

Figure 1. Non-uniform circular antenna array with N isotropic radiators.

maximize directivity of the array pattern. The third component serves basically as a constraint and strives to drive the maxima of the array pattern close to the desired maxima.

2.2. Extension to Multi-objective Framework

As we shall show later in Section 5 the single-objective function f is composed of conflicting terms. Hence the various terms of f in (2) are decomposed to form a multi-objective optimization task. The first two components of f are treated as separate objective functions to be optimized simultaneously in the multi-objective framework. The third component is treated as a constraint. The idea behind this approach lies in the fact that one is interested to find out the best compromise solution among minimum side lobe level and maximum directivity while maintaining the maxima of the array pattern close to the desired maxima. One should not consider trade-off of the final solution with respect to the maxima of the array pattern. Thus the multi-objective optimization task may be specified as:

$$\begin{aligned}
\text{Optimize} : f_1 &= \left|AR\left(\varphi_{sll}, \vec{I}, \vec{\beta}, \varphi_0\right)\right| \Big/ \left|AR\left(\varphi_{\max}, \vec{I}, \vec{\beta}, \varphi_0\right)\right| \\
f_2 &= 1 \Big/ DIR\left(\varphi_0, \vec{I}, \vec{\beta}\right) \\
\text{subject to} : |\varphi_0 - \varphi_{des}| &\leq \epsilon
\end{aligned} \tag{3}$$

where ϵ is a small positive constant (taken as 5° in our case).

3. LA-DE FOR SINGLE-OBJECTIVE OPTIMIZATION

The proposed algorithm employs a synergy of Differential Evolution and Learning Automata to realize a Memetic Algorithm for achieving superior performance in global optimization problems. It maintains a meme pool for parameter F in order to select the control parameters for individual members of the population in every generation. The state transition probability matrix controls the meme selection process. The row indices of the matrix, corresponding to the states of the stochastic automata, denote the members ranked in order of decreasing fitness value, while the column indices represent the actions performed by the automata at a particular state corresponding to uniform quantized values of the control parameter in the range [0, 2]. The state transition matrix and the associated update and selection rules constitute the Learning Automata, which learns according to the reward/penalty responses from its environment, DE. The basic algorithm is outlined in the following sections.

3.1. Initialization

The algorithm employs a population of NP D-dimensional parameter vectors representing the candidate solutions initialized uniformly and randomly over the entire search space, constrained by the prescribed minimum and maximum bounds.

The state transition probability matrix is initialized with equal values of 0.05 for 20 quantized levels of the parameter F. This is in accordance with the principle of unavailability of *a priori* information about the environment and assuming all actions to be equally likely at the initial stage.

3.2. Adaptive Selection of Memes

We employ Fitness proportionate selection, also known as Roulette-Wheel selection, for the selection of potentially useful memes. For state S_i, the selection of F_j is such that the cumulative probability of selection of $F = F_1$ through F_{j-1} is greater than r (a random number in the range [0, 1]), i.e.,

$$\sum_{m=1}^{j-1} p_{S_i,m} < r \le \sum_{m=j}^{20} p_{S_i,m}. \tag{4}$$

This selection mechanism ensures that although fitter memes would enjoy much higher probability of selection, yet the memes with poorer fitness do manage to survive and may contribute some components as evolution continues, and thus the diversity of the meme population can be maintained.

3.3. Differential Evolution

In our implementation we use the "DE/current-to-best/1" strategy for performing the mutation operation, where a donor vector $\vec{V}_{i,G}$ corresponding to each population member $\vec{X}_{i,G}$ in the current generation is created according to the following rule:

$$\vec{V}_{i,G} = \vec{X}_{i,G} + F \cdot \left(\vec{X}_{best,G} - \vec{X}_{i,G}\right) + F \cdot \left(\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G}\right). \tag{5}$$

The indices r_1^i and r_2^i are mutually exclusive integers, different from the base index i, randomly chosen from the range [1, NP], and $\vec{X}_{best,G}$ is the vector with the best fitness in the population at generation G. The parameter F is obtained by selection from the meme pool.

The trial vector is generated using Binomial Crossover where the components from the donor are inherited according to the following rule

$$u_{j,i,G} = \begin{cases} v_{j,i,G} & \text{if } rand_{i,j}(0,1) \leq Cr \text{ or } j = j_{rand} \\ x_{j,i,G} & \text{otherwise} \end{cases}. \tag{6}$$

where $rand_{i,j}(0,1) \in [0,1]$ is a uniformly distributed random number lying between 0 and 1 and is instantiated independently for each j-th component of the i-th vector. $j_{rand} \in [1, 2, \ldots, D]$ is a randomly chosen index, which ensures that $\vec{U}_{i,G}$ gets at least one component from $\vec{V}_{i,G}$.

This is followed by the selection step where if the new trial vector yields an equal or lower value of the objective function, it replaces the corresponding target vector in the next generation; otherwise the target is retained in the population.

3.4. Update of State Transition Probability Matrix

The state transition probabilities (Fig. 2) are updated according to Linear Reinforcement Scheme. If the fitness of the trial vector increases for the i-th state and choice of F_i from the meme pool then the action probability $p_{S_{i,j}}(t)$ is increased and all other components are decreased as shown:

$$\begin{cases} p_{S_{i,j}}(t+1) = (1-a) \cdot_{S_{i,j}}(t) \quad \forall j \neq F_i \\ p_{S_{i,F_i}}(t+1) = p_{S_{i,F_i}}(t) + a \cdot \left(1 - p_{S_{i,F_i}}(t)\right) \end{cases} \tag{7}$$

where the parameter $a \in [0,1]$ is associated with the reward/penalty response. Otherwise, in the case of a penalty input, $p_{S_{i,j}}(t)$ is decreased and all other components are increased as follow:

$$\begin{cases} p_{S_{i,j}}(t+1) = \frac{a}{r-1} + (1-a) \cdot_{S_{i,j}}(t) \quad \forall j \neq F_i \\ p_{S_{i,F_i}}(t+1) = (1-a) \cdot_{S_{i,F_i}}(t) \end{cases} \tag{8}$$

This process is repeated for all the population members.

3.5. State Assignment

The population members are now ranked in decreasing order of fitness and assigned corresponding states.

The Sections 3.2–3.5 are repeated till maximum number of iterations is reached. The algorithm is outlined in Table 1.

4. LA-DEMO: THE MULTIOBJECTIVE EXTENSION

The two major goals in multi-objective optimization may be stated as follows:

(a) to find solutions as close to the Pareto front as possible.

Meme Pool	F_1	F_2	…	F_{20}
State 1 (S_1)	p_{S_1,F_1}	p_{S_1,F_2}	…	$p_{S_1,F_{20}}$
State 2 (S_2)	p_{S_2,F_1}	p_{S_2,F_2}	…	$p_{S_2,F_{20}}$
…	…	…	…	
State *NP* (S_{NP})	p_{S_{Np},F_1}	p_{S_{Np},F_2}	…	$p_{S_{Np},F_{20}}$

Figure 2. The state transition probability matrix governing the adaptive selection of memes from the meme pool.

Table 1. LA-DE for single-objective optimization.

1. *Initialize* the random population *P* of *NP* individuals and evaluate the members.
   Initialize state transition probability matrixof Learning Automata.
2. *While* stopping criterion is not reached, *do*
   - 2.1. *For* each individual $P_i\ \forall\ i=1,\ldots, NP$ *do*
     - 2.1.1. Select F_i from $\{F_1, F_2,\ldots, F_{20}\}$ by *Roulette-Wheel Selection* from the meme pool.
     - 2.1.2. Perform *Mutation* of the original individual.
     - 2.1.3. Generate candidate through *Crossover* on the mutated individual.
     - 2.1.4. *Selection*: *If* the candidate is fitter than the parent,the candidate is selected.
       else the candidateis discarded.
     - 2.1.5 *If* candidate is selected, update state transition probability matrixof Learning Automata based on rewardscheme.
       else update state transition probability matrix based on penalty scheme.
   - 2.2. Evaluate new population and assign states to the members in accordance to their fitness.
   - 2.3. Increase the generation count.

(b) to find solutions as diverse as possible in the non-dominated front.

In order to apply DE to multi-objective optimization the selection of potential candidates has to be changed. In single-objective optimization, selection is based simply on the fitness value of the individual members. However, in multi-objective optimization the selection criterion requires certain modifications. In this article, we employ the selection procedure outlined in DEMO/parent [26]. The candidate replaces the parent only if it dominates the parent. If the parent dominates the candidate, it is discarded. Otherwise, if the candidate and parent are non-dominated with respect to each other, the candidate is added to the population. If the population size exceeds the maximum value, truncation is achieved by sorting the individuals with non-dominated sorting and then evaluating members of the same front with the crowding distance metric [26].

We extend our proposed adaptive memetic algorithm LA-DE to the multi-objective framework by incorporating DEMO's selection criteria. In comparison to LA-DE, LA-DEMO updates the state transition probability matrix based on the penalty scheme *iff* the candidate is discarded. The reward scheme is used if the candidate replaces the parent or it is added to the population before truncation. Another core feature of the LA-DEMO algorithm is the state assignment process. In LA-DE the state assignment is based on the relative fitness values of the individuals. However, in LA-DEMO members are assigned states based on their rank. In case two individuals belong to the same front (i.e., they have the same rank) their crowding distances are compared. A large average crowding distance will ensure better diversity of the population. Hence for two individuals with the same rank, the one with a greater crowding distance is assigned a higher fitness. The resultant algorithm, named LA-DEMO is used to generate a set of non-dominated solutions constituting the Pareto front. In absence of any decision-maker, we use a fuzzy membership based method as described in [34]. The i-th objective function is represented by a membership function μ_i where,

$$\mu_i = \begin{cases} 1 & f_i \le f_i^{\min} \\ \frac{f_i^{\max} - f_i}{f_i^{\max} - f_i^{\min}} & f_i^{\min} \le f_i \le f_i^{\max} \\ 0 & f_i \ge f_i^{\max} \end{cases} \tag{9}$$

$f_i^{\max}$ and $f_i^{\min}$ are the maximum and minimum values of the i-th objective function among all non-dominated solutions respectively.

For each non-dominated solution j, the normalized membership function μ^j can be evaluated as:

$$\mu^j = \frac{\sum_{i=1}^{N_1} \mu_i^j}{\sum_{k=1}^{N_2} \sum_{i=1}^{N_1} \mu_i^k} \tag{10}$$

where N_1 is the number of objective functions and N_2 is the number of non-dominated solutions. The best compromise is the one having the highest value of μ^j. The algorithm is briefly outlined in Table 2.

5. SIMULATED RESULTS

In order to illustrate the efficiency of our approach to the antenna array design problem we compare our results with state-of-the-art methodologies, namely PSO, DE and MIWO. These algorithms have been previously applied to various electromagnetic problems including the circular antenna array design scenario and have been successful in providing superior results [7–11]. The parameter settings for the

Table 2. LA-DE for multi-objective optimization.

1. *Initialize* the random population P of NP individuals and evaluate the members.
   Initialize state transition probability matrix o f Learning Automata
2. *While* stopping criterion is not reached, *do*
   2.1 For each individual $P_i\ \forall\ i=1,\ldots, NP$ *do*
      2.1.1. Select F_i from $\{F_1, F_2,\ldots, F_{20}\}$ by *Roulette-Wheel Selection* from the meme pool.
      2.1.2. Perform *Mutation* of the original individual.
      2.1.3. Generate candidate through *Crossover* on the mutated individual.
      2.1.4. *Selection: If* the candidate dominatesthe parent, t he candidate is selected.
         else if the parent dominates the candidate, the candidateis discarded.
         else the candidate is added to the population
      2.1.5 *If* candidate is discarded, update state transition probability matrix o f Learning Automata based on penaltyscheme.
         else update state transition probability matrix based on rewardscheme.
   2.2 If population size is more than NP then truncate it.
   2.3 Evaluate new population and assign states to the members in accordance to their rank and crowding distance.
   2.4 Increase the generation count.

Table 3. Brief description of the parameter configurations of the competitor algorithms.

PSO	Value	DE/LA-DE	Value	MIWO	Value
Population size	50	Population size	50	Initial population	10
C_1	2	Scaling Factor	0.5	Max population	50
C_2	2	Crossover ratio	0.9	Max No. of seeds	4
Inertia weight	0.9–0.4	Reward/penalty rate, a	0.01	Min No. of seeds	0
				Std. dev. for seed dispersal	0.8–0.001

various competitor algorithms are briefly outlined in Table 3. They have been determined following the guidelines suggested in [7–11]. For LA-DE the learning rate was set equal to 0.01 after a set of tuning experiments. We consider the design problem for a 20 element array.

In order to handle the problem constraint for LA-DEMO we used the method outlined in [35] as follows:

(a) Any feasible solution is preferred to any infeasible solution;
(b) Between two feasible solutions, the one with better fitness is preferred;
(c) Between two infeasible solutions, the one with a smaller constraint violation is preferred.

To tackle the constraints presented the algorithm was initialized with a population of around 200 particles with randomly initialized positional coordinates. Out of these, 50 fittest particles were selected, space coordinates of which obeyed the constraints imposed by (3). During the evolution phase sorting was accomplished based on both fitness and constraint violation. Thus, only those members were promoted to the next generation which satisfied the constraints besides having greater fitness value.

The first section considers the performance of the various algorithms with respect to the two design objectives: side lobe level and directivity. The results are tabulated in Table 4.

Two important aspects are reflected in the above results. Although LA-DE provides statistically superior results with respect to the competitor algorithms (statistical significance was measured using unpaired t-tests [23]), the performance improvement is mainly reflected in the domain of side lobe level suppression. This is mainly because of the fact that the single-objective function was formulated as a linear sum of the design objectives and consequently the side lobe level component enjoyed a higher weightage in comparison to directivity. Scaling of the directivity component may provide better results but determining the relative scaling factors for the various components is again a cumbersome process. The final results therefore seem to be biased to side lobe level suppression. An interesting fact may also be pointed out in this context. For a uniform circular antenna array with 20 elements the side lobe level and directivity are -6.06 dB and 11.72 dB respectively. Thus the main motive for shifting to the non-

Table 4. Mean and Standard Deviation (within parenthesis) results obtained after 30 independent runs.

Algorithms	Side lobe level	Directivity
PSO	−9.81 (2.11)	11.61 (0.32)
DE	−10.94 (2.73)	11.71(0.24)
MIWO	−11.18 (2.84)	11.67 (0.29)
LA-DE	−17.67 (2.45)	11.87 (0.21)
LA-DEMO	−14.475 (1.24)	13.04 (0.10)

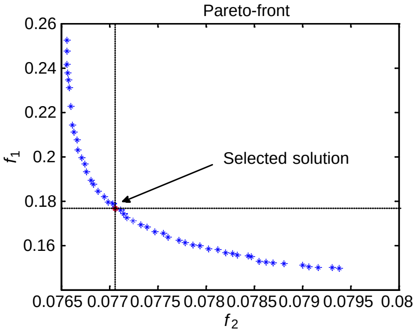

Figure 3. Optimal Pareto-front obtained by LA-DEMO.

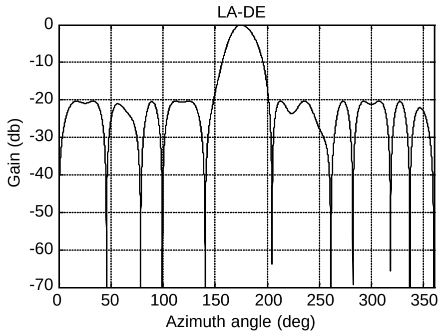

Figure 4. Best array pattern obtained by LA-DE.

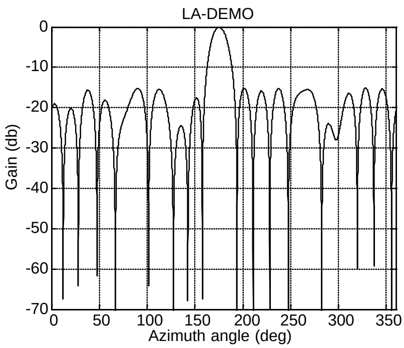

Figure 5. Best array pattern obtained by LA-DEMO.

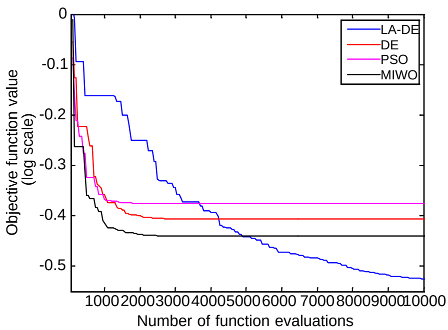

Figure 6. Median convergence characteristics.

uniform array case is somewhat lost, provided the fact that PSO, DE and MIWO are unable to provide directivity measures that outperform the uniform array case and LA-DE enjoys only minor improvements in directivity measure with respect to the uniform array.

We further investigate the scope of improvement through the multi-objective framework by extending our LA-DE algorithm. The average compromise solution obtained justifies our claim. Significant improvements are observed in both the design objectives in comparison to the uniform array. The optimal Pareto-front obtained is shown in Fig. 3. The nature of the front clearly depicts the conflicting nature of the design objectives and serves as the major motivation for the application of multi-objective optimization in the concerned problem.

The best array patterns obtained by LA-DE and LA-DEMO have been shown in Figs. 4 and 5 respectively. The current excitation

Table 5. Current excitation amplitudes and phase perturbations obtained by LA-DE and LA-DEMO (Best result among 30 independent runs).

Algorithms	Normalized I_n	Phase Perturbations β_n
LA-DE	0.0652, 0.5157, 0.6565, 0.6057, 0.3001, 0.2693, 0.5691 0.6459 0.5648, 0.1738	-46.7409, -54.5813, -12.7593, 84.8221, -63.5517, -129.9736, 106.7660, -102.2548, -10.1277, 59.9692
LA-DEMO	0.0139, 0.7266, 0.7612, 0.6873, 0.6415, 0.2981, 0.5826, 0.6162, 0.7216, 0.1732	34.0229, -39.9371, -25.0957, 71.2258, -39.4931, -128.1003, 121.1700, -100.9375, -23.8889, 140.8587

amplitudes and phase perturbations obtained by LA-DE and LA-DEMO for the circular array under consideration have been presented in Table 5.

Finally, the robustness of the LA-DE approach is presented through the convergence graph in Fig. 6. We used number of function evaluations as a measure to compare the convergence speed. It is observed that although MIWO enjoys the highest convergence speed, yet PSO, DE as well as MIWO often get entrapped in the local optima. In contrast, LA-DE continues to exhibit converging property throughout the entire evolution phase and is able to provide better solutions in comparison to the other competitor algorithms. The lower convergence speed may be attributed to the fact that LADE searches the problem space thoroughly in the initial stage and is able to avoid local optima. This prevents stagnation of LADE unlike the competitor algorithms even after 3000 function evaluations.

6. CONCLUSIONS

In this article we addressed a challenging problem in computational electromagnetics, namely determination of current excitations and phase perturbations of non-uniform circular antenna array for achieving minimal side lobe level and maximum directivity. Circular antenna arrays have recently gained much attention due to its several applications in the communication domain [37, 38]. A novel memetic algorithm was proposed and its efficiency in comparison to state-of-the-art approaches was established through extensive experimental simulations. The design problem was again reformulated as a constrained multi objective optimization task resulting in a host of non-dominated solutions from which the best compromise solution was selected using a fuzzy membership based function. Simultaneous performance improvements were observed in both the performance metrics of the array.

Future research work will investigate the application of other multi objective evolutionary algorithms to the design problem and efficient ways to extract the compromise solution from the non-dominated Pareto front.

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