Progress In Electromagnetics Research B, Vol. 41, 397–417, 2012

GYROTROPIC-NIHILITY IN FERRITE-SEMICONDUCTOR COMPOSITE IN FARADAY GEOMETRY

V. R. Tuz[1, 2, *], O. D. Batrakov[2], and Y. Zheng[2]

[1]Institute of Radio Astronomy of National Academy of Sciences of Ukraine, 4, Krasnoznamennaya St., Kharkiv 61002, Ukraine

[2]School of Radio Physics, Karazin Kharkiv National University, 4, Svobody Square, Kharkiv 61022, Ukraine

Abstract—The reflection, transmission spectra and the polarization transformation of linearly polarized waves in the ferrite-semiconductor multilayer structure are considered. In the long-wavelength limit, the effective medium theory is applied to describe the studied structure as a uniaxial anisotropic homogeneous medium defined by the effective permittivity and effective permeability tensors. The investigations are carried out in the frequency band where the real parts of the diagonal elements of both the effective permittivity and permeability tensors are close to zero. In this frequency band the studied structure is referred to a gyrotropic-nihility medium. An enhancement of polarization rotation, impedance matching, backward propagation are revealed.

1. INTRODUCTION

The main purpose of the effective medium theory (EMT) is to determine the effective parameters of a complex structure for its given composition, shape and properties of the constitutive elements. These parameters are permittivity, permeability, conductivity or complex index of refraction. The identification of composite properties via effective parameters of a homogeneous medium requires averaging of microscopic electromagnetic fields and microscopic (local) polarization/magnetization which satisfy microscopic Maxwell's equations and the continuity equation. The averaging results in the material equations and in relations for material parameters entering these equations [1, 2]. In the scientific literature the identification

Received 26 April 2012, Accepted 21 June 2012, Scheduled 24 June 2012

* Corresponding author: Vladimir R. Tuz (tvr@rian.kharkov.ua).

procedure of a multicomponent system via a uniform continuous medium with some effective parameters is called homogenization [3].

From the viewpoint of the EMT, the homogenization procedure of a composite medium is reduced to a derivation of Maxwell's equations for the averaged fields and their spectral components inside the composite. In particular, such an approach is also applicable to the composite media realized in the form of multilayer periodic systems if the structure's period is much less than the length of wave propagating inside the structure. In the simplest case of isotropic dielectric constituents, the EMT gives us a description of a multilayer periodic structure as a uniaxial anisotropic homogeneous medium defined by the effective permittivity tensor [1, 2]. This effective permittivity is expressed by the actual material parameters and thickness of layers and does not depend on the number of periods inside the system. Further, in [4–7], the EMT is expanded to the general case of periodic systems with anisotropic layers including magnetic ones. Evidently, in the latter case, the homogenization procedure requires the definition of both effective permittivity and effective permeability tensors.

It has been also found that, under a special structure configuration and in a certain frequency band, the real parts of both effective permittivity and effective permeability of the homogenized composite structure can simultaneously acquire negative values. In general, the idea of electromagnetic complex materials which simultaneously have negative real parts of both permittivity and permeability (referred to double-negative media) is realized in metamaterials made of periodically arranged arrays of metallic rings/rods or metallic split ring resonators. A straightforward homogenization procedure is applied to describe the complicate system as a layer of the double-negative uniform medium, and then such double-negative layers can be combined with slabs of conventional dielectric double-positive medium to form some waveguide system. The main problem in such structure realization is the fact that the double-negative layers are described using the homogenization procedure applied to the metamaterials composed of metallic elements which have strong resonant characteristics. Such an approach can result in some fallibility in the EMT framework, which lead to failures in the attempts to obtain the expected optical properties of the system [8, 9].

On the other hand, the systems based on a combination of materials with naturally occurring negative permittivity and negative permeability can also be realized [10–17]. It is possible due to the internal resonant characteristics of some natural materials. As an example, ferrites have negative permeability in the microwave band near to the frequency of ferromagnetic resonance (the low-frequency magnetic resonance) [18], and conducting materials have negative permittivity below the plasma frequency (plasmalike effect of the metallic mesostructures and semiconductors) [19]. A remarkable feature of these natural materials is the dependence of their electromagnetic properties on the temperature or electric and magnetic fields which allows one to control the conditions of the wave propagation through such systems effectively.

In the case of materials in which only one of the two parameters permittivity and permeability has negative real part, not both, the entire structure is constructed by periodically arranging in the system such mu-negative ($\varepsilon' > 0$, $\mu' < 0$) and epsilon-negative ($\varepsilon' < 0$, $\mu' > 0$) layers, provided that these layers are optically thin [20]. As an example of such materials, ferrites (mu-negative) and semiconductors (epsilon-negative) can be considered. If the layers of the structure are optically thin, the EMT can be applied without any restrictions. It results in the consideration of the periodic system as a homogeneous gyrotropic medium described by tensors of the effective permittivity and the effective permeability which possess certain dispersion characteristics.

Typically, the normal wave incidence is considered and the transversal (the Cotton-Mouton or Voigt geometry) or longitudinal (the Faraday geometry) magneto-optic configuration of the biased external static magnetic field is chosen to study electromagnetic properties of ferrite-semiconductor structures [11–16]. In the transversal geometry, the electromagnetic wave can be presented as the *TE* and *TM* waves and in the longitudinal one, as the right-handed and left-handed circularly polarized waves. In either case these modes are uncoupled ones and the solution of the electromagnetic wave propagation problem is described via the 2×2 transfer matrix formulation. In the general case of oblique wave incidence, the use of the full 4×4 transfer matrix formulation is required [21, 22].

In the present paper the EMT is applied to predict optical properties of the ferrite-semiconductor multilayer structure in the Faraday geometry in both cases of normal and oblique incidence of the exciting wave. The long-wave approximation is applied to transform the rigorous solution of the Cauchy problem related to the tangential field components. Since the double-negative conditions in such a structure have been studied before both theoretically [10–16] and experimentally [17], our main interest in the present paper is to study the optical response of the system in the frequency band where the real parts of both permittivity and permeability simultaneously undergo the transition from negative values to positive ones. So, in this frequency band, the structure under study can be referred to a class of nihility media. Such media have many interesting characteristics, such as complete transmission, impedance matching, backward propagation which are the subjects of this research. In particular, all these effects are of special interest in the transformation optics [23, 24].

2. PROBLEM STATEMENT

A stack of N identical double-layer slabs (unit cells) which are arranged periodically along the z axis is investigated (Fig. 1). Each unit cell is composed of ferrite (with constitutive parameters ε_1, $\hat{\mu}_1$) and semiconductor (with constitutive parameters $\hat{\varepsilon}_2$, μ_2) layers with thicknesses d_1 and d_2, respectively. The structure's period is $L = d_1 + d_2$, and in the x and y directions the system is infinite. An external static magnetic field $\vec{M}$ is directed along the z axis (Faraday geometry). Generally, in such a configuration, the studied structure can be referred to gyromagnetic-gyroelectric one with magnetoplasma properties.

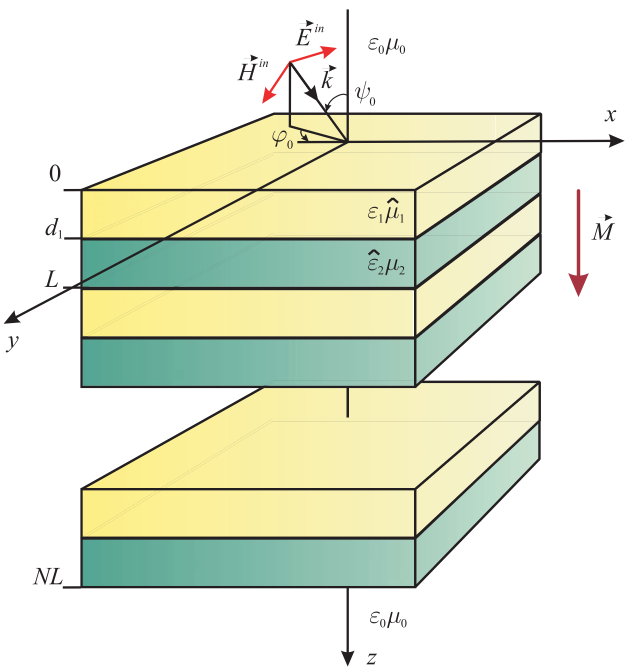


Figure 1. A periodic stack of one-dimensional double-layer ferrite-semiconductor structure under the action of an external static magnetic field.

The input $z \leq 0$ and output $z \geq NL$ half-spaces are homogeneous, isotropic and have constitutive parameters ε_0 and μ_0. Suppose that the incident field is a plane monochromatic wave of frequency ω and its direction of propagation in the region $z \leq 0$ is determined by angles ψ_0 and φ_0 relative to the z axis and x axis, respectively. Time dependence is assumed to be $\exp(-i\omega t)$ throughout the paper.

We use common expressions for constitutive parameters of normally magnetized ferrite and semiconductor layers with taking into account the losses. For ferrite layers the permittivity and permeability are defined in the form [18, 25]:

$$\varepsilon_1 = \varepsilon_f, \quad \hat{\mu}_1 = \begin{pmatrix} \mu_1^T & -i\alpha & 0 \\ i\alpha & \mu_1^T & 0 \\ 0 & 0 & \mu_1^L \end{pmatrix}, \tag{1}$$

where $\mu_1^T = 1 + \chi' + i\chi''$, $\chi' = \omega_0\omega_m[\omega_0^2 - \omega^2(1-b^2)]D^{-1}$, $\chi'' = \omega\omega_m b[\omega_0^2+\omega^2(1+b^2)]D^{-1}$, $\alpha = \Omega'+i\Omega''$, $\Omega' = \omega\omega_m[\omega_0^2-\omega^2(1+b^2)]D^{-1}$, $\Omega'' = 2\omega^2\omega_0\omega_m bD^{-1}$, $D = [\omega_0^2 - \omega^2(1+b^2)]^2 + 4\omega_0^2\omega^2 b^2$, $\mu_1^L = 1$, ω_0 is the Larmor frequency and b is a dimensionless damping constant.

For semiconductor layers the permittivity and permeability are defined as follows [19]:

$$\hat{\varepsilon}_2 = \begin{pmatrix} \varepsilon_2^T & -i\beta & 0 \\ i\beta & \varepsilon_2^T & 0 \\ 0 & 0 & \varepsilon_2^L \end{pmatrix}, \quad \mu_2 = \mu_s, \tag{2}$$

where $\varepsilon_2^T = \varepsilon_0\left[1 - \omega_p^2(\omega+i\nu)[\omega((\omega+i\nu)^2 - \omega_c^2)]^{-1}\right]$, $\varepsilon_2^L = \varepsilon_0\left[1 - \omega_p^2[\omega(\omega+i\nu)]^{-1}\right]$, $\beta = \varepsilon_0\omega_p^2\omega_c[\omega((\omega+i\nu)^2 - \omega_c^2)]^{-1}$, ε_0 is the part of permittivity attributed to the lattice, ω_p the plasma frequency, ω_c the cyclotron frequency, and ν the electron collision frequency in plasma.

For our calculation we use the same values of parameters of ferrite and semiconductor as in theoretical work [13] to maintain continuity. The frequency dependences of the permeability and permittivity calculated using Equations (1), (2) and parameters of [13] are presented in Fig. 2. Note that the values of $\mathrm{Im}(\mu_1^T)$, $\mathrm{Im}(\alpha)$ and $\mathrm{Im}(\varepsilon_2^T)$, $\mathrm{Im}(\beta)$ are so close to each other that the curves of their frequency dependences coincide in the figures.

Nevertheless we want to draw an attention to the experimental work [17] where the optical response of the ferrite-semiconductor periodic structure embedded into the hollow rectangular waveguide is measured in the millimeter waveband. The studied structure is composed of ferrite (brand 1SCH4) and InSb semiconductor layers which are under an action of the external magnetic field in the

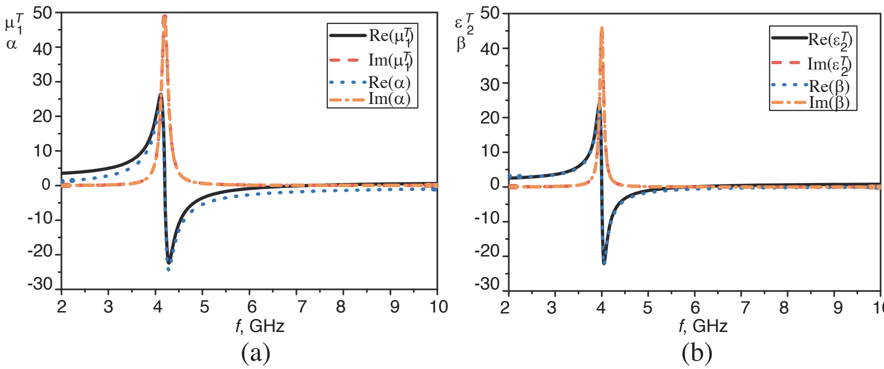


Figure 2. Frequency dependences of the (a) permeability and permittivity of ferrite and (b) semiconductor layers, respectively. We use parameters for these materials in the microwave region from [13]. For the ferrite layers, under saturation magnetization of 2000 G, parameters are: $\omega_0/2\pi = 4.2\,\text{GHz}$, $\omega_m/2\pi = 8.2\,\text{GHz}$, $b = 0.02$, $\varepsilon_f = 5.5$. For the semiconductor layers, parameters are: $\omega_p/2\pi = 4.5\,\text{GHz}$, $\omega_c/2\pi = 4.0\,\text{GHz}$, $\nu/2\pi = 0.05\,\text{GHz}$, $\varepsilon_0 = 1.0$, $\mu_s = 1.0$.

transversal magneto-optic configuration. In this study one can find the parameters of materials and strength of the external magnetic field under which the double-negative conditions are satisfied.

3. EFFECTIVE MEDIUM THEORY

In the long-wavelength limit, when the characteristic dimensions of the structure (d_1, d_2, L) are significantly smaller than the wavelength in the corresponding layer ($d_1 \ll \lambda$, $d_2 \ll \lambda$, $L \ll \lambda$), the interactions of electromagnetic waves with a periodic gyromagnetic-gyroelectric structure can be described analytically using the effective medium theory (EMT). From the EMT viewpoint, the periodic structure is represented approximately as an anisotropic (gyrotropic) uniform medium whose optical axis is directed along the structure periodicity, and this medium is described with some effective permittivity and permeability tensors $\hat{\varepsilon}_e$ and $\hat{\mu}_e$. By this means, the investigation of the wave interaction with an inhomogeneous periodic structure is reduced to the solution of the boundary-value problem of conjugations of an equivalent homogeneous anisotropic layer with surrounding spaces.

Let us consider a unit cell of the studied structure. It is made of two layers $0 < z < d_1$ and $d_1 < z < L$ of dissimilar materials whose constitutive relations are as follows:

$$\left.\begin{array}{l}\vec{D}=\varepsilon_1\vec{E}\\ \vec{B}=\hat{\mu}_1\vec{H}\end{array}\right\}\ 0<z<d_1,\quad \left.\begin{array}{l}\vec{D}=\hat{\varepsilon}_2\vec{E}\\ \vec{B}=\mu_2\vec{H}\end{array}\right\}\ d_1<z<L. \tag{3}$$

In general form, in Cartesian coordinates, the system of Maxwell's equations for each layer has a form

$$\begin{aligned} ik_yH_z-\partial_zH_y&=-ik_0\left(\hat{\varepsilon}_j\vec{E}\right)_x, & ik_yE_z-\partial_zE_y&=ik_0\left(\hat{\mu}_j\vec{H}\right)_x,\\ \partial_zH_x-ik_xH_z&=-ik_0\left(\hat{\varepsilon}_j\vec{E}\right)_y, & \partial_zE_x-ik_xE_z&=ik_0\left(\hat{\mu}_j\vec{H}\right)_y,\\ ik_xH_y-ik_yH_x&=-ik_0\left(\hat{\varepsilon}_j\vec{E}\right)_z, & ik_xE_y-ik_yE_x&=ik_0\left(\hat{\mu}_j\vec{H}\right)_z, \end{aligned} \tag{4}$$

where $\partial_z=\partial/\partial z$, $k_x=k_0\sin\psi_0\cos\varphi_0$, $k_y=k_0\sin\psi_0\sin\varphi_0$, $k_0=\omega/c$ is the free-space wavenumber, $j=1,2$, $\hat{\varepsilon}_1$, and $\hat{\mu}_2$ are the tensors with ε_1 and μ_2 on their main diagonal and zeros elsewhere, respectively ($\hat{\varepsilon}_1=\varepsilon_1\hat{I}$, $\hat{\mu}_2=\mu_2\hat{I}$, $\hat{I}$ is the identity tensor). From six components of the electromagnetic field $\vec{E}$ and $\vec{H}$, only four are independent. Thus the components E_z and H_z can be eliminated from the system (4) and derived a set of four first-order linear differential equations related to the transversal field components inside a layer of the structure [21, 22]. For the ferrite ($0<z<d_1$) and semiconductor ($d_1<z<L$) layers these systems, respectively, are:

$$\partial_z\begin{pmatrix}E_x\\E_y\\H_x\\H_y\end{pmatrix}=ik_0\begin{pmatrix}0&0&k_xk_y/k_0^2\varepsilon_1+i\alpha&\mu_1^T-k_x^2/k_0^2\varepsilon_1\\0&0&-\mu_1^T+k_y^2/k_0^2\varepsilon_1&-k_xk_y/k_0^2\varepsilon_1+i\alpha\\-k_xk_y/k_0^2\mu_1^L&-\varepsilon_1+k_x^2/k_0^2\mu_1^L&0&0\\\varepsilon_1-k_y^2/k_0^2\mu_1^L&k_xk_y/k_0^2\mu_1^L&0&0\end{pmatrix}\begin{pmatrix}E_x\\E_y\\H_x\\H_y\end{pmatrix}, \tag{5}$$

$$\partial_z\begin{pmatrix}E_x\\E_y\\H_x\\H_y\end{pmatrix}=ik_0\begin{pmatrix}0&0&k_xk_y/k_0^2\varepsilon_2^L&\mu_2-k_x^2/k_0^2\varepsilon_2^L\\0&0&-\mu_2+k_y^2/k_0^2\varepsilon_2^L&-k_xk_y/k_0^2\varepsilon_2^L\\-k_xk_y/k_0^2\mu_2-i\beta&-\varepsilon_2^T+k_x^2/k_0^2\mu_2&0&0\\\varepsilon_2^T-k_y^2/k_0^2\mu_2&k_xk_y/k_0^2\mu_2-i\beta&0&0\end{pmatrix}\begin{pmatrix}E_x\\E_y\\H_x\\H_y\end{pmatrix}. \tag{6}$$

The sets of Equations (5) and (6) can be abbreviated by using a matrix formulation:

$$\partial_z\vec{\Psi}(z)=ik_0\mathbf{A}(z)\vec{\Psi}(z),\qquad 0<z<L. \tag{7}$$

In this equation, $\vec{\Psi} = \{E_x, E_y, H_x, H_y\}^T$ is a four-component column vector (here upper index T is the matrix transpose operator), while the 4×4 matrix function $\mathbf{A}(z)$ is piecewise uniform as

$$\mathbf{A}(z) = \begin{cases} \mathbf{A}_1, & 0 < z < d_1, \\ \mathbf{A}_2, & d_1 < z < L, \end{cases} \tag{8}$$

where the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ correspond to Equations (5) and (6), respectively.

Since the vector $\vec{\Psi}$ is known in the plane $z = 0$, the Equation (7) is related to the Cauchy problem [26] whose solution is straightforward, because the matrix $\mathbf{A}(z)$ is piecewise uniform. Thus, the field components referred to boundaries of the double-layer period of the structure are related as

$$\begin{aligned} \vec{\Psi}(L) &= \mathbf{M}_2\vec{\Psi}(d_1) = \mathbf{M}_2\mathbf{M}_1\vec{\Psi}(0) = \mathfrak{M}\vec{\Psi}(0) \\ &= \exp[ik_0\mathbf{A}_2 d_2]\exp[ik_0\mathbf{A}_1 d_1]\vec{\Psi}(0), \end{aligned} \tag{9}$$

where $\mathbf{M}_j$ and $\mathfrak{M}$ are the transfer matrices of the corresponding layer and the period, respectively.

Suppose that γ_j is the eigenvalue of the corresponding matrix $\mathbf{A}_j$ ($\det[\mathbf{A}_j - \gamma_j\mathbf{I}] = 0$), $j = 1, 2$ and $\mathbf{I}$ is the 4×4 identity matrix. When $|\gamma_j|d_j \ll 1$ (i.e., both layers in the period are electrically thin), the next long-wave approximations can be used [27]

$$\exp[ik_0\mathbf{A}_2 d_2]\exp[ik_0\mathbf{A}_1 d_1] \simeq \mathbf{I} + ik_0\mathbf{A}_1 d_1 + ik_0\mathbf{A}_2 d_2. \tag{10}$$

Let us now consider a single layer of effective permittivity $\hat{\varepsilon}_e$, effective permeability $\hat{\mu}_e$ and thickness L. Quantity $\mathbf{A}_e$ can be defined in a way similar to (5), (6):

$$\partial_z \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix} = ik_0 \begin{pmatrix} 0 & 0 & k_x k_y/k_0^2\varepsilon_e^L + i\alpha_e & \mu_e^T - k_x^2/k_0^2\varepsilon_e^L \\ 0 & 0 & -\mu_e^T + k_y^2/k_0^2\varepsilon_e^L & -k_x k_y/k_0^2\varepsilon_e^L + i\alpha_e \\ -k_x k_y/k_0^2\mu_e^L - i\beta_e & -\varepsilon_e^T + k_x^2/k_0^2\mu_e^L & 0 & 0 \\ \varepsilon_e^T - k_y^2/k_0^2\mu_e^L & k_x k_y/k_0^2\mu_e^L - i\beta_e & 0 & 0 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix}, \tag{11}$$

and (9):

$$\vec{\Psi}(L) = \mathbf{M}_e\vec{\Psi}(0) = \mathfrak{M}_e\vec{\Psi}(0) = \exp[ik_0\mathbf{A}_e L]\vec{\Psi}(0). \tag{12}$$

Provided that γ_e is the eigenvalue of the matrix $\mathbf{A}_e$ ($\det[\mathbf{A}_e - \gamma_e\mathbf{I}] = 0$) and $|\gamma_e|L \ll 1$ (i.e., the entire composite layer is electrically thin as well), the next approximation follows

$$\exp[ik_0\mathbf{A}_eL] \simeq \mathbf{I} + ik_0\mathbf{A}_eL. \tag{13}$$

Equations (10) and (13) permit us to establish the following equivalence between bilayer and single layer:

$$\mathbf{A}_e = f_1\mathbf{A}_1 + f_2\mathbf{A}_2, \tag{14}$$

where $f_j = d_j/L$.

In the case when the directions of both wave propagation and static magnetic field are coincident ($k_x = k_y = 0$), the following simple expressions for the effective constitutive parameters of the homogenized medium can be obtained:

$$\begin{aligned} \mu_e^T &= f_1\mu_1^T + f_2\mu_2, \\ \varepsilon_e^T &= f_1\varepsilon_1 + f_2\varepsilon_2^T, \\ \alpha_e &= f_1\alpha, \\ \beta_e &= f_2\beta. \end{aligned} \tag{15}$$

The effective constitutive parameters calculated according to the formulae (15) are given in Fig. 3. The whole frequency range can be divided into three specific bands where parameters of the tensors

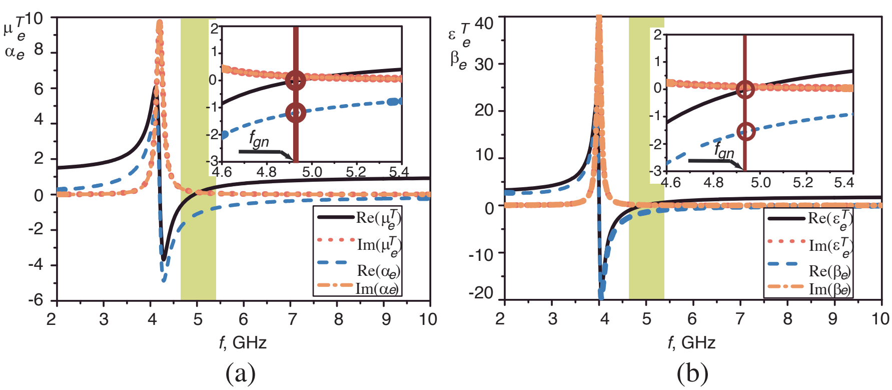


Figure 3. Frequency dependences of the (a) effective permeability and (b) effective permittivity of the homogenized ferrite-semiconductor medium. Parameters of the ferrite and semiconductor layers are the same as in Fig. 2; $d_1 = 0.05$ mm, $d_2 = 0.2$ mm. The circles mark the situation when $\mathrm{Re}(\mu_e^T)$ and $\mathrm{Re}(\varepsilon_e^T)$ are close to zero while $\mathrm{Re}(\alpha_e) \neq 0$, $\mathrm{Re}(\beta_e) \neq 0$ and losses in the ferrite and semiconductor layers are small.

$\hat{\mu}_e$ and $\hat{\varepsilon}_e$ acquire different properties. In the first band, located between 2 GHz and 3 GHz, μ_e^T, ε_e^T, α_e and β_e have positive values of their real parts and small imaginary parts. In the second band, between 3 GHz and 4.5 GHz, the real parts of parameters vary from positive values to negative ones as the frequency increases. These transitions occur at the frequencies of the ferromagnetic resonance of ferrite ($f_{fr} = 4.2$ GHz) and the cyclotron resonance of semiconductor ($f_{pr} = 4.0$ GHz), respectively. In this band the medium losses are very significant. Finally, in the third frequency band, located from 4.5 GHz to 5.5 GHz, the real parts of parameters have a transition from negative to positive values while their imaginary parts are small. The latter band is given in the insets of Fig. 3 on a larger scale. One can see that there is a frequency $f_{gn} \approx 4.94$ GHz where μ_e^T and ε_e^T simultaneously reach zero. It is significant that, by special adjusting ferrite and semiconductor type, external static magnetic field strength and thicknesses of layers, it is possible to obtain the condition when μ_e^T and ε_e^T acquire zero at the same frequency. Exactly this situation is marked in the insets of Fig. 3 with the circles. Note that at this frequency, the parameters α_e and β_e are far from zero and the medium losses are small.

The formulation of the eigenvalue problem of the matrix $\mathbf{A}_e$ ($\det[\mathbf{A}_e - \gamma_e \mathbf{I}] = 0$), whose coefficients are defined as (15), gives us the characteristic equation

$$\gamma^4 - 2\gamma^2(\varepsilon_e^T \mu_e^T + \alpha_e \beta_e) + (\varepsilon_e^T \mu_e^T)^2 - (\mu_e^T \beta_e)^2 - (\varepsilon_e^T \alpha_e)^2 = 0, \quad (16)$$

whose solution is

$$\gamma_e^{\pm} = k_0 \sqrt{(\varepsilon_e^T \pm \beta_e)(\mu_e^T \pm \alpha_e)} = k_0 \sqrt{\varepsilon^{\pm} \mu^{\pm}} = k_0 n^{\pm}. \quad (17)$$

Here the sign '±' defines two different eigenvalues. It is well known that in an unbounded gyrotropic medium these eigenvalues are related to the right circularly polarized (RCP, γ^+) and the left circularly polarized (LCP, γ^-) eigenwaves [28].

Since the medium losses are taken into account, the real and imaginary parts of the complex index of refraction $n^{\pm} = (n^{\pm})' + i(n^{\pm})''$ are defined from the solution of the equation

$$\{(n^{\pm})' + i(n^{\pm})''\}^2 = \{(\varepsilon^{\pm})' + i(\varepsilon^{\pm})''\}\{(\mu^{\pm})' + i(\mu^{\pm})''\}, \quad (18)$$

which, with using (17), is reduced to the next system of equations [16] (here and further the indexes '±' and T are omitted)

$$\begin{aligned} (n')^2 - (n'')^2 &= \varepsilon' \mu' - \varepsilon'' \mu'' \\ &= (\varepsilon_e' \pm \beta_e')(\mu_e' \pm \alpha_e') - (\varepsilon_e'' \pm \beta_e'')(\mu_e'' \pm \alpha_e''), \quad (19) \\ 2n'n'' &= \varepsilon'' \mu' + \varepsilon' \mu'' \\ &= (\varepsilon_e'' \pm \beta_e'')(\mu_e' \pm \alpha_e') + (\varepsilon_e' \pm \beta_e')(\mu_e'' \pm \alpha_e''). \quad (20) \end{aligned}$$

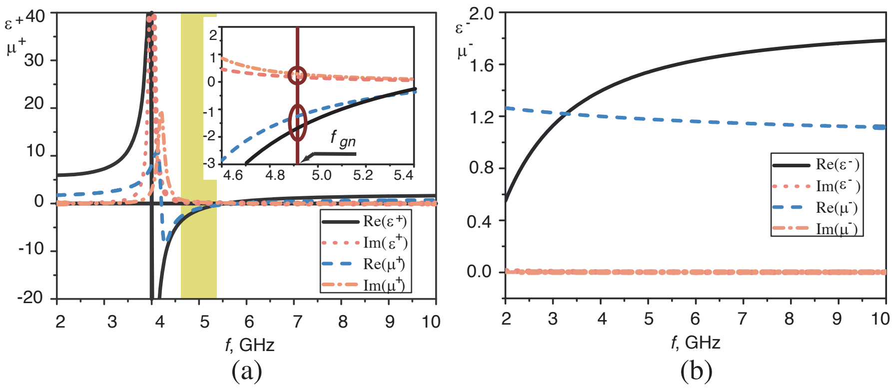


Figure 4. Frequency dependences of the material parameters of the equivalent gyrotropic medium for the (a) RCP and (b) LCP eigenwaves. Parameters of the ferrite and semiconductor layers are the same as in Fig. 2; $d_1 = 0.05\,\text{mm}$, $d_2 = 0.2\,\text{mm}$.

Here the signs '+' and '−' are related to the RCP and LCP eigenwaves, respectively.

Since the conditions $\varepsilon'' > 0$ and $\mu'' > 0$ are valid for the passive media, the imaginary part of the index of refraction must be positive value ($n'' > 0$) to ensure the damping of the wave as it propagates. Thus, according to the Equation (20), in the frequency band where the real parts of both the permittivity and the permeability are negative ($\varepsilon' < 0$, $\mu' < 0$) the real part of the index of refraction is negative as well ($n' < 0$). From (19) it is obvious that in the case of negligible losses, the double-negative condition can appear when $|\alpha'_e| > |\mu'_e|$ and $|\beta'_e| > |\varepsilon'_e|$ (see also, Ch. IV of [29]). In particular, in the frequency range 4.5–5.5 GHz, there are $\alpha'_e < 0$ and $\beta'_e < 0$, and the double-negative condition is satisfied for the RCP eigenwave $(\mu^+)' < 0$, $(\varepsilon^+)' < 0$ which is shown in Fig. 4(a). We should note here that, generally, the negative index of refraction can appear even in the case when only one of permittivity and permeability has negative real part, i.e., $\varepsilon'\mu' < 0$. Thus, from the equation $\varepsilon''\mu' + \varepsilon'\mu'' = 0$, the index of refraction becomes negative when $\mu' > 0$, $\varepsilon' < -\mu'\varepsilon''/\mu''$ and $\varepsilon' > 0$, $\mu' < -\varepsilon'\mu''/\varepsilon''$ [16].

Especially interesting situation appears if μ'_e and ε'_e are close to zero and the medium losses are small. In this case there is $|\varepsilon^\pm\mu^\pm| \approx |\alpha'_e\beta'_e|$, and the propagation constants become as:

$$\gamma_e = -\gamma_e^+ = \gamma_e^- \approx k_0\sqrt{|\alpha'_e\beta'_e|}. \tag{21}$$

Thus, the propagation constants of the RCP (γ_e^+) and LCP (γ_e^-) waves are equal in the magnitude but opposite in sign to each other, and the backward propagation appears for the RCP wave while for the LCP wave it is forward one. Recall that the backward wave is the wave in which the direction of the Poynting vector is opposite to that of its phase velocity [30]. The similar peculiarity of the RCP and LCP waves propagation occurs also in the chiral-nihility media [31–33], so in the analogy with them, the condition (21) can be related to the gyrotropic-nihility media [34]. The frequency band, at which the gyrotropic-nihility condition is satisfied for the RCP wave, is shown in Fig. 4. The gyrotropic-nihility frequency f_{gn} is marked in the inset of this figure with the red line. From our calculations it follows that at the frequency $f_{gn} = 4.94\,\text{GHz}$ the permeability and the permittivity related to the RCP and LCP eigenwaves, respectively, are $\mu^+ = -1.18 + i0.29$, $\varepsilon^+ = -1.58 + i0.19$ and $\mu^- = 1.18 + i0.002$, $\varepsilon^- = 1.54 + i0.002$. Evidently, that in the case of oblique wave incidence ($\psi_0 \neq 0$), the similar gyrotropic-nihility conditions can be derived for the elliptically polarized waves, but they are not presented here due to their cumbersomeness.

4. REFLECTED AND TRANSMITTED FIELDS. POLARIZATION TRANSFORMATION

In the general case, since the values of real and imaginary parts of the indexes of refraction $n^\pm$ and, therefore, the propagation constants $\gamma^\pm$ are different for the RCP and LCP eigenwaves, the incident linearly polarized wave propagating through an unbounded gyrotropic medium becomes elliptically polarized. The angle of the specific Faraday rotation (the rotation of the plane of polarization of the wave per unit length of the sample) is determined by the real parts of the propagation constants of the circularly polarized eigenwaves as [18]:

$$\mathcal{F} = \frac{1}{2}\left[(\gamma^-)' - (\gamma^+)'\right] = \frac{k_0}{2}\left[(n^-)' + (n^+)'\right]. \tag{22}$$

The peculiarity of the Faraday rotation in the relation to the structure under study is the fact that there is a frequency band where the propagation constants γ^+ and γ^- have opposite signs (i.e., $n^+ < 0$ and $n^- > 0$). As discussed above, in this case, the RCP eigenwave undergoes the backward propagation while the LCP eigenwave is the forward one. Therefore, in this band, the specific Faraday rotation is defined by the expression:

$$\mathcal{F} = \frac{1}{2}\left[(\gamma^-)' + |(\gamma^+)'|\right] = \frac{k_0}{2}\left[(n^-)' + |(n^+)'|\right], \tag{23}$$

from which it follows that the angle of the polarization plane rotation increases in comparison with the convenient double-positive medium where $n^+ > 0$ and $n^- > 0$ [16]. Remarkably, at the frequency where the gyrotropic-nihility condition (21) is satisfied, the specific Faraday rotation becomes directly proportional to the modulus of the real part of the propagation constant (21), i.e.,

$$\mathcal{F} = |(\gamma)'| = \sqrt{|\alpha_e' \beta_e'|}. \tag{24}$$

It is anticipated that if the frequency of the electromagnetic wave which incidents on a finite layer of such composite medium is chosen to be nearly the frequency of the gyrotropic-nihility condition f_{gn}, the transmitted and reflected fields will acquire some unusual properties. In order to demonstrate this, in the long-wavelength limit, the reflection and transmission coefficients can equivalently be defined using the rigorous solutions (9) or the approximate solution (12) of the Equation (7) because these solutions give the same result. So, the equation which defines the relation of the tangential components of the fields at the structure input and output has the form:

$$\vec{\Psi}(0) = (\mathfrak{M}^N)^{-1} \vec{\Psi}(NL) = (\mathfrak{M}_e^N)^{-1} \vec{\Psi}(NL) = \mathfrak{T} \vec{\Psi}(NL). \tag{25}$$

The field vector at the structure input is made up of two parts that consist of the incident and reflected wave contributions:

$$\vec{\Psi}(0) = \vec{\Psi}_{in} + \vec{\Psi}_{ref}. \tag{26}$$

The field at the structure output is matched only a single transmitted wavefield:

$$\vec{\Psi}(NL) = \vec{\Psi}_{tr}. \tag{27}$$

On the other hand, the incident, reflected and transmitted fields can be written as follows:

$$\begin{gathered} \vec{E}^{in}(\vec{r}) = \vec{E}_0^{in} \exp(i\vec{k}^{in} \cdot \vec{r}), \quad \vec{E}^{ref}(\vec{r}) = \vec{E}_0^{ref} \exp(i\vec{k}^{ref} \cdot \vec{r}), \\ \vec{E}^{tr}(\vec{r}) = \vec{E}_0^{tr} \exp(i\vec{k}^{tr} \cdot \vec{r}). \end{gathered} \tag{28}$$

The fields (28) can be represented in terms of the linearly polarized waves. If we assume that the incident field is either p-polarized or s-polarized, the reflected and transmitted fields will have the components of both types since two polarizations of the linearly polarized wave propagating in the gyrotropic medium are coupled due to the Faraday rotation, i.e., generally, the reflected and transmitted fields become elliptically polarized. So, we define the co-polarized reflection and transmission coefficients by the expressions $R^{vv} = B^v/A^v$ and $T^{vv} = C^v/A^v$, and the cross-polarized ones as $R^{vv'} = B^{v'}/A^v$ and $T^{vv'} = C^{v'}/A^v$, where A^v, B^v and C^v $(v = p,\ s)$ are the amplitudes of the incident, reflected and transmitted fields which are introduced in the Equation (28). In the case of normal wave incidence $(\psi_0 = 0)$

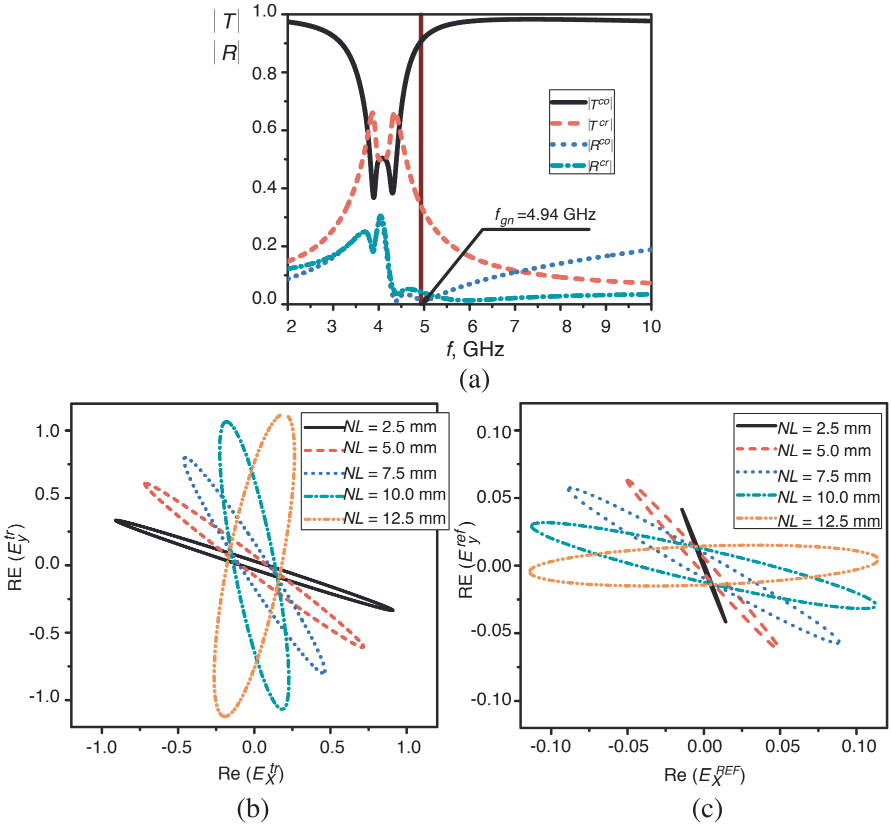
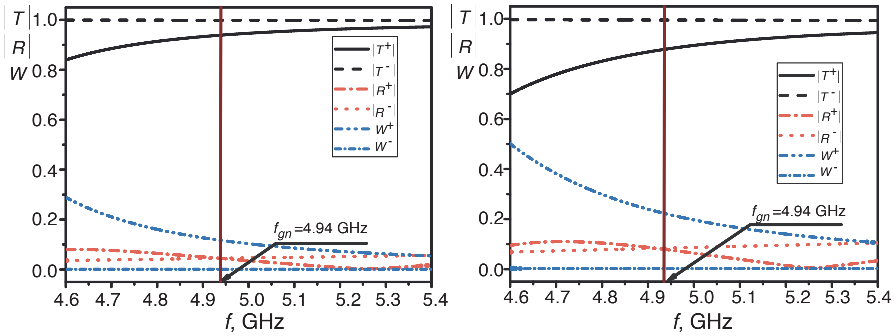


Figure 5. (a) The transmission, reflection spectra of the co-polarized and cross-polarized waves, and the polarization ellipses of the (b) transmitted and (c) reflected fields. Parameters of the ferrite and semiconductor layers are the same as in Fig. 2; $d_1 = 0.05\,\text{mm}$, $d_2 = 0.2\,\text{mm}$; (a) $NL = 2.5\,\text{mm}$; (b), (c) $f = 4.94\,\text{GHz}$.

there is not any difference in the two linear polarizations, therefore $T^{pp} = T^{ss} = T^{co}$, $R^{pp} = R^{ss} = R^{co}$ and $T^{ps} = T^{sp} = T^{cr}$, $R^{ps} = R^{sp} = R^{cr}$ for the co-polarized and cross-polarized waves, respectively, and one can represent them as a superposition of the RCP and LCP waves. The complete expressions evaluated via elements of the transfer matrix $\mathfrak{T}$ which allow us to calculate the reflection and transmission coefficients are given in [22]. The results obtained using these expressions in the case of normal wave incidence are summarized in Figs. 5–7.

Thus, from Fig. 5 one can conclude that the drastic changes in the



(a) (b)

Figure 6. The transmission, reflection spectra and absorption coefficient ($W^{\pm} = 1 - |R^{\pm}|^2 - |T^{\pm}|^2$) of the circularly polarized waves. Parameters of the ferrite and semiconductor layers are the same as in Fig. 2; $d_1 = 0.05\,\mathrm{mm}$, $d_2 = 0.2\,\mathrm{mm}$; (a) $NL = 2.5\,\mathrm{mm}$; (b) $NL = 5\,\mathrm{mm}$.

transmission and reflection characteristics occur in that three frequency bands mentioned earlier. In particular, in the frequency band from 3 GHz to 4.5 GHz, due to the significant level of medium losses nearly the frequencies of the ferromagnetic and cyclotron resonances, there is strong damping of the RCP wave. Directly, at the frequency of 4 GHz, both the transmitted and reflected fields become circularly polarized, since the RCP wave is completely suppressed, and, in the linear polarization space, there are $|R^{co}| = |R^{cr}|$ and $|T^{co}| = |T^{cr}|$ (Fig. 5). From the frequency of 4.5 GHz, the double-negative condition begins to be satisfied, and the reflected and transmitted fields become to be elliptically polarized. The angle of the polarization ellipse rotation is defined with the Equation (23), and the evolution of these ellipses versus the equivalent layer thickness increasing one can see in Figs. 5(b), (c). These ellipses are calculated directly at the gyrotropic-nihility frequency f_{gn}.

The transmitted, reflected spectra and the absorption coefficient calculated in the vicinity of the gyrotropic-nihility frequency f_{gn} are presented in Fig. 6. The curves are plotted in the circular polarization space for two different thicknesses of the equivalent layer. From these figures one can see that, in this band, the reflection of both the RCP and LCP waves is low, while their absorption inside the system is different. Such a low reflection is due to the peculiarities of the medium impedances ($Z^{\pm} = \sqrt{\mu^{\pm}/\varepsilon^{\pm}}$) related to the RCP and LCP waves. It is particularly remarkable that in the vicinity of the gyrotropic-nihility frequency f_{gn}, the parameters α_e and β_e are close in value to each other and their real parts approach to unit which can be clearly seen in Fig. 3. It leads to the fact that the medium is impedance matched to the free space. Directly at the gyrotropic-nihility frequency, the impedances related to the RCP and LCP waves are indistinguishable

$$Z = Z^{+} = Z^{-} \approx \sqrt{\frac{|\alpha_e'|}{|\beta_e'|}}. \tag{29}$$

Actually, from our calculations, these impedances are: $Z^{+} = 0.873 - i0.067$ and $Z^{-} = 0.875 + i0.00002$. As a result of this matching, the incident wave passes into the system with a small reflection, and, at the gyrotropic-nihility frequency, the equality of both the propagation constants and the impedances of the RCP and LCP eigenwaves gives the identity $|R^{+}| = |R^{-}|$ (Fig. 6).

Also, the polarization characteristics of the system in the vicinity of the gyrotropic-nihility frequency can be understood from Fig. 7, where the corresponding frequency dependences of the polarization azimuth (θ) and ellipticity angle (η) for the transmitted and reflected fields are plotted for two different thicknesses of the equivalent layer. According to the definition of the Stokes parameters, we introduce the ellipticity η so that the field is linearly polarized when $\eta = 0$, and $\eta = -\pi/4$ for LCP and $+\pi/4$ for RCP (note that in the latter cases the preferential azimuthal angle of the polarization ellipse θ becomes undefined). In all other cases ($0 < |\eta| < \pi/4$), the

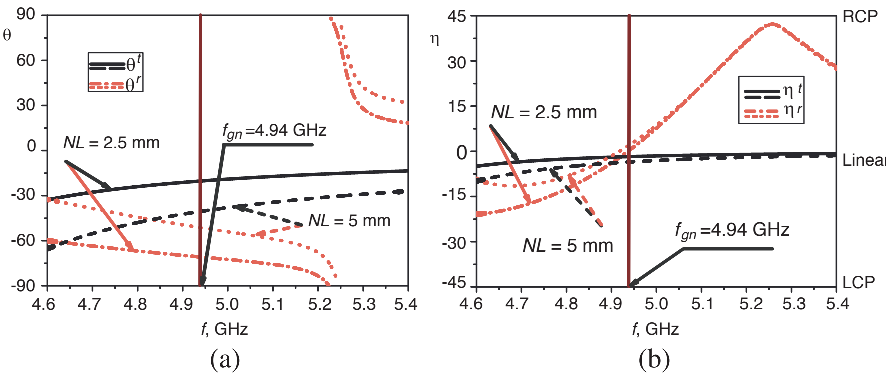


Figure 7. (a) The polarization azimuth and (b) ellipticity angle of the waves transmitted through (t) and reflected from (r) the equivalent gyrotropic layer with finite thickness. Parameters of the ferrite and semiconductor layers are the same as in Fig. 2; $d_1 = 0.05$ mm, $d_2 = 0.2$ mm.

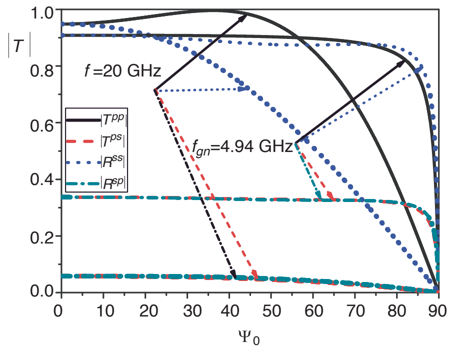


Figure 8. The angular dependences of the magnitudes of the (a) transmission and (b) reflection coefficients of the equivalent gyrotropic layer with finite thickness. Parameters of the ferrite and semiconductor layers are the same as in Fig. 2; $d_1 = 0.05\,\text{mm}$, $d_2 = 0.2\,\text{mm}$, $NL = 2.5\,\text{mm}$.

field is elliptically polarized. In the considered frequency band, the transmitted field is nearly linearly polarized and its polarization ellipse experiences clockwise rotation. As the equivalent layer thickness rises, the ellipticity of the transmitted field increases as a consequence of the circular dichroism, i.e., due to the stronger absorption of the RCP wave compared to the absorption of the LCP wave. On the contrary, the reflected field undergoes anticlockwise rotation and sequentially changes between LCP and RCP states and this transition occurs exactly at the gyrotropic-nihility frequency where the reflected field becomes linearly polarized ($\eta = 0$).

At the end of the paper we consider oblique wave incidence ($\psi_0 \neq 0$) on the equivalent gyrotropic layer of the finite thickness. We have calculated the transmission and reflection coefficients of the co-polarized and cross-polarized waves at two different frequencies. The first frequency is chosen to be far from the frequencies of the ferromagnetic and cyclotron resonances and the second one is selected to be at the gyrotropic-nihility frequency. The results of these calculations are presented in Fig. 8. One can see that in the first case, at the frequency of $f = 20.0\,\text{GHz}$, the curves of the magnitude of the transmission and reflection coefficients have typical form where the zero reflection and total transmission at a certain angle of incidence occur only for parallel polarization. It is consistent with the results related to the conventional free-space-double-positive dielectric interface. Note that at this frequency, the cross-polarized transmission and reflection are small. On the other hand, at the gyrotropic-nihility frequency, the curves of the transmission and reflection coefficients magnitudes are different drastically from that ones in the first case. Thus, the level of the transmission/reflection remains to be invariable almost down to the glancing angles. The cross-polarized transmission is considerable, and the conditions $|T^{pp}| \approx |T^{ss}|$ and $|T^{ps}| \approx |T^{sp}|$ are valid in the all range of angles. At the same time, both the co-polarized and cross-polarized reflection are small down to the glancing angles because the medium is impedance matched to the free space.

5. CONCLUSION

In this paper, we have investigated optical properties of a ferrite-semiconductor multilayer structure. In the long-wavelength limit, when the structure layers are optically thin, the effective medium theory is developed, and the effective constitutive parameters, index of refraction, wave impedances of the equivalent uniform anisotropic medium are obtained analytically. On the basis of these parameters the peculiarities of the eigenwaves propagation are studied and the possibility of achieving a double-negative condition is predicted.

The main part of our study is curried out in the frequency band where the real parts of both the effective permittivity and the effective permeability acquire a transition from negative values to positive ones. Such a transition is referred by us as a gyrotropic-nihility effect. The reflection, transmission, absorption and polarization transformation of waves in the system are studied in vicinity of the gyrotropic-nihility frequency. The effects of an enhancement of the polarization rotation, impedance matching, backward propagation are revealed. It is also shown that under the oblique wave incidence on the studied structure, the level of the transmission/reflection remains to be invariable almost down to the glancing angles when the gyrotropic-nihility condition is satisfied. This outcome can be of great interest in the problem of the transformation optics.

ACKNOWLEDGMENT

The authors are very grateful to Prof. Sergey Prosvirnin and Dr. Bogdan Kochetov for many fruitful discussions.

REFERENCES

1. Born, M. and E. Wolf, *Principles of Optics*, Pergamon Press, Oxford, 1968.

2. Rytov, S. M., "Electromagnetic properties of a finely stratified medium," *Sov. Phys. JETP*, Vol. 2, 466–475, 1956.
3. Sihvola, A., *Electromagnetic Mixing Formula and Applications*, IEE Press, London, 1999.
4. Agranovich, V. M., "Dielectric permeability and influence of external fields on optical properties of superlattices," *Solid State Commun.*, Vol. 78, 747–750, 1991.
5. Mosallaei, H. and K. Sarabandi, "Magneto-dielectrics in electromagnetics: Concept and applications," *IEEE Trans. Antennas Propag.*, Vol. 52, 1558–1567, 2004.
6. Kazanskiy, V. B. and V. R. Tuz, "The long-wave theory of N pairwise alternate homogeneous and heterogeneous layers diffraction," *Radioelectronics and Communications Systems*, Vol. 51, 16–23, 2008.
7. Tuz, V. and V. Kazanskiy, "Electromagnetic scattering by a quasiperiodic generalized multilayer Fibonacci structure with grates of magnetodielectric bars," *Waves in Random and Complex Media*, Vol. 19, No. 3, 501–508, 2009.
8. Prosvirnin, S. L. and S. Zouhdi, "On the effective constitutive parameters of metal-dielectric arrays of complex-shaped particles," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 5, 583–598, 2006.
9. Simovski, C. R., "On electromagnetic characterization and homogenization of nanostructured metamaterials," *J. Opt.*, Vol. 13, 013001, 2011.
10. Wu, R. X., "Effective negative refraction index in periodic metal-ferrite-metal film composite," *J. Appl. Phys.*, Vol. 97, 076105, 2005.
11. Tarkhanyan, R. H. and D. G. Niarchos, "Effective negative refractive index in ferromagnet-semiconductor superlattices," *Opt. Express*, Vol. 14, 5433–5444, 2006.
12. Wu, R. X., T. Zhao, and J. Q. Xiao, "Periodic ferrite-semiconductor layered composite with negative index of refraction," *J. Phys.: Condens. Matter.*, Vol. 19, 026211, 2007.
13. Shramkova, O. V., "Transmission spectra in ferrite-semiconductor periodic structure," *Progress In Electromagnetics Research M*, Vol. 7, 71–85, 2009.
14. Bulgakov, A. A., A. A. Girich, M. K. Khodzitsky, O. V. Shramkova, and S. I. Tarapov, "Transmission of electromagnetic waves in a magnetic fine-stratified structure," *J. Opt. Soc. Am. B*, Vol. 26, B156–B160, 2009.

15. Tarkhanyan, R. H., D. G. Niarchos, and M. Kafesaki, "Influence of external magnetic field on magnon-plasmon polaritons in negative-index antiferromagnet-semiconductor superlattices," *J. Magn. Magn. Mater.*, Vol. 322, 603–608, 2010.
16. Afanas'ev, S. A., D. G. Sannikov, and D. I. Sementsov, "'Left–handed' state and polarization characteristics of waves in 'semiconductor-magnet' superlattices," *Physics of the Solid State*, Vol. 54, 332–337, 2012.
17. Girich, A. A., M. K. Khodzitsky, and S. I. Tarapov, "Experimental investigation of left-handed medium properties of semiconductor-ferrite composite in millimetre waveband," *3rd International Congress on Advanced Electromagnetic Materials in Microwaves and Optics*, 590–592, London, UK, Aug. 30–Sep. 4, 2009.
18. Gurevich, A. G., *Ferrites at Microwave Frequencies*, Heywood, London, 1963.
19. Bass, F. G., and A. A. Bulgakov, *Kinetic and Electrodynamic Phenomena in Classical and Quantum Semiconductor Superlattices*, Nova Science, New York, 1997.
20. Alù, A. and N. Engheta, "Pairing an epsilon-negative slab with a mu-negative slab: Resonance, tunneling and transparency," *IEEE Trans. Antennas Propag.*, Vol. 51, 2558–2571, 2003.
21. Berreman, D. W., "Optics in stratified and anisotropic media: 4×4-matrix formulation," *J. Opt. Soc. Am.*, Vol. 62, 502–510, 1972.
22. Tuz, V. R., M. Y. Vidil, and S. L. Prosvirnin, "Polarization transformations by a magneto-photonic layered structure in the vicinity of a ferromagnetic resonance," *J. Opt.*, Vol. 12, 095102, 2010.
23. Wei Y., M. Yan, and M. Qiu, "Generalized nihility media from transformation optics," *J. Opt.*, Vol. 13, 024005, 2011.
24. Castaldi, G., I. Gallina, V. Galdi, A. Alù, and N. Engheta, "Transformation-optics generalization of tunnelling effects in bi-layers made of paired pseudo-epsilon-negative/mu-negative media," *J. Opt.*, Vol. 13, 024011, 2011.
25. Collin, R. E., *Foundation for Microwave Engineering*, Wiley-Interscience, New York, 1992.
26. Jakubovich, V. A. and V. H. Starzhinskij, *Linear Differential Equations with Periodic Coefficients*, Wiley, New York, 1975.
27. Lakhtakia, A. and C. M. Krowne, "Restricted equivalence of paired epsilon-negative and mu-negative layers to a negative phase-velocity materials (alias left-handed material)," *Optik*,

Vol. 114, 305–307, 2003.

28. Orfanidis, S. J., *Electromagnetic Waves and Antennas*, Chapter 4, Propagation in Birefringent Media, 2008, www.ece.rutgers.edu/∼orfanidi/ewa.
29. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ," *Sov. Phys. Usp.*, Vol. 10, 509–514, 1968.
30. Shevchenko, V. V., "Forward and backward waves: Three definitions and their interrelation and applicability," *Phys.-Usp.*, Vol. 50, 287, 2007.
31. Tretyakov, S., I. Nefedov, A. Sihvola, S. Maslovski, and C. Simovski, "Waves and energy in chiral nihility," *Journal of Electromagnetic Waves and Applications*, Vol. 17, No. 5, 695–706, 2003.
32. Qiu, C.-W., N. Burokur, S. Zouhdi, and L.-W. Li, "Chiral nihility effects on energy flow in chiral materials," *J. Opt. Soc. Am. A*, Vol. 25, 55–63, 2008.
33. Tuz, V. R. and C.-W. Qiu, "Semi-infinite chiral nihility photonics: Parametric dependence, wave tunneling and rejection," *Progress In Electromagnetics Research*, Vol. 103, 139–152, 2010.
34. Shen, J. Q., "Negative refractive index in gyrotropically magnetoelectric media," *Phys. Rev. B*, Vol. 73, 045113, 2006.