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THE ANALYSIS OF THE INFLUENCE OF RANDOM ARRAY ERROR TO WIDEBAND SIGNAL DOA ESTIMATION REITERATIVE ALGORITHM

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Abstract—This paper proposed a new approach, which is based on minimum mean-square error (MMSE) criterion, for wideband signal spatial direction-of-arrival (DOA) estimation when there is array error, and the impact of random array error to the new algorithm is analyzed in this paper. Pass the wideband signal mixed with array error through a bank of narrowband filters to obtain narrowband signals, then recover the sparse representation of the narrowband signals by re-iterative method in the MMSE frameworks, and estimate the number and DOA of sources from the sparse representation. The new method does not require the number of sources for direction finding, furthermore, it can estimate the DOA of coherent signals and the robustness of new algorithm to array error is better than coherent subspace algorithms. The simulated results confirmed the effectiveness and robustness of the new method.

1. INTRODUCTION

Direction-of-arrival (DOA) estimation for wideband signal is an important branch of array signal processing and there has been growing interest in it recently. Coherent subspace method (CSM) [1–3] is the traditional method to deal with wideband signal DOA estimate, and it can separate signals whose DOAs are close in ideal condition. But the random array error always exists, which due to antenna element location uncertainty, realistic calibration tolerances, mutual coupling effects between elements, etc., and it will decrease its performance and

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even can't work, so array calibration [4] is the necessary procedure when there is array error. Another way to deal with wideband signal DOA is finding a robust algorithm that is not sensitive to array error [5].

This paper proposes an algorithm called wideband signal DOA estimation reiterative algorithm, which is denoted as WSERA and develops from [6], and it is not sensitive to array error. Besides, the impact of random array error on the new algorithm is also discussed in this paper. This algorithm considers random array error and recovers the sparse representation [7–9] of signals to estimate the number of signals and DOAs. Compared with subspace algorithms, the proposed algorithm does not require the priori number of sources for direction finding, and it can also work without decorrelation processing [10–13] when signals are coherent. Furthermore, the robustness of WSERA to array error is better than subspace algorithms.

2. SIGNAL MODEL

Assume that K far-field stationary and wideband signals corrupted by additive Gaussian white noise impinge on a linear array of N ($> K$) sensors from direction angles $\theta = [\theta_1, \ \ldots, \ \theta_K]$ and that the distance of adjacent elements is less than half wavelength of the highest frequency. Passing single wideband signal snapshot through J narrowband filters to obtain narrowband data at frequencies $\{f_j\}_{j=1}^{J}$ [1, 2], the signal $Y_j \in \mathbb{R}^{N\times 1}$ at frequency f_j can be represented in vector notation as the $N \times 1$ vector

$$Y_j = A(f_j, \theta)S(f_j) + V_j \tag{1}$$

$$A(f_j, \theta) = [\ a(f_j, \theta_1) \quad a(f_j, \theta_i) \quad \ldots \quad a(f_j, \theta_K)\ ] \tag{2}$$

$A(f_j, \theta) \in \mathbb{R}^{N\times K}$ is manifold matrix, whose kth column is the kth signal array steering vector $a(f_j, \theta_i) = [1 \ \ e^{-j2\pi f_j \sin(\theta_i)/c} \ \ \ldots \ \ e^{-j2\pi f_j (N-1)\sin(\theta_K)/c}]^T \in \mathbb{R}^{N\times 1}$, $S(f_j) \in \mathbb{R}^{K\times 1}$ is the signal complex vector which represents a stationary, zero mean random process uncorrelated with noise, and $V_j \in \mathbb{R}^{N\times 1}$ is the additive white noise.

Given the overcomplete basis $\{a(m\theta_\Delta)\}_{m=0}^{M-1}$ ($M \gg N$), where $\theta_\Delta = \Psi/M$ is the quantification when quantizing the interested spatial angle Ψ into M discrete values and M the spatial sample factor. According to sparse signal representation [7–9], the array received signals can be represented as

$$Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_J \end{bmatrix} \approx \tilde{Y} = \begin{bmatrix} B_1 X_1 \\ \vdots \\ B_J X_J \end{bmatrix} + \begin{bmatrix} V_1 \\ \vdots \\ V_J \end{bmatrix} \tag{3}$$

$$B_j = [\ a(f_j, 0) \quad a(f_j, \theta_\Delta) \quad \dots \quad a(f_j, (M-1)\theta_\Delta)\ ] \tag{4}$$
$$X_j = [\ x_{j1} \quad \dots \quad x_{jM}\ ]^T \tag{5}$$

The vector $B_j \in \mathbb{R}^{N\times M}$ is the array manifold at frequency f_j and the vector $X_j \in \mathbb{R}^{M\times 1}$, which called amplitude vector, contains the complex amplitude value associated with each of the M steering vectors in B_j. If θ_Δ is enough fine, X_j will be the vector with all elements zeros except for K elements associated with the K sources. Thus DOA estimation is reformulated as the estimation of the parameterized vector X_j.

The array error at f_j is denoted as $Z_j = [z_{j1}\ \dots\ z_{jn}\ \dots\ z_{jN}]^T \in \mathbb{R}^{N\times 1}$, where

$$z_{jn} = (1 + \Delta_{jan}) \cdot \exp(j\Delta_{j\varphi n}) \tag{6}$$

Δ_{jan} and $\Delta_{j\varphi n}$ are the random amplitude deviation of arbitrary distribution and random phase deviation of arbitrary distribution, respectively. Assume that the distribution of Δ_{jan} and $\Delta_{j\varphi n}$ are independent and identically distributed (IID) for each antenna element and are zero mean, the variance of z_{jn} is δ_z^2; array error, signals and additive noise are also IID. The array received signals should be donated as

$$Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_J \end{bmatrix} \approx \tilde{Y} = \begin{bmatrix} B_1X_1 \\ \vdots \\ B_JX_J \end{bmatrix} \odot \begin{bmatrix} Z_1 \\ \vdots \\ Z_J \end{bmatrix} + \begin{bmatrix} V_1 \\ \vdots \\ V_J \end{bmatrix} \tag{7}$$

$\odot$ denotes Hadamard product. Approximate Z_j as an additive noise, then (7) should be denoted as

$$Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_J \end{bmatrix} \approx \tilde{Y} = \begin{bmatrix} B_1X_1 \\ \vdots \\ B_JX_J \end{bmatrix} + \begin{bmatrix} V_{z1} \\ \vdots \\ V_{zJ} \end{bmatrix} + \begin{bmatrix} V_1 \\ \vdots \\ V_J \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} V_{z1} \\ \vdots \\ V_{zJ} \end{bmatrix} = \left(\begin{bmatrix} Z_1 \\ \vdots \\ Z_J \end{bmatrix} - \begin{bmatrix} 1_{N\times 1} \\ \vdots \\ 1_{N\times 1} \end{bmatrix} \right) \odot \begin{bmatrix} B_1X_1 \\ \vdots \\ B_JX_J \end{bmatrix} \tag{9}$$

According to the assumptions, it is evident that the mean of V_{zj} is zero.

3. WSERA ALGORITHM

Obtain the signal amplitude matrix $X = [X_1^H\ \dots\ X_J^H]^H$ that is composed of amplitude vector X_j by using reiterative algorithm.

Compute adaptive filter bank W to minimize the MMSE cost function

$$J\left\{\left\|X - W^H Y\right\|^2\right\} \tag{10}$$

$\{\cdot\}^H$ denotes the complex-conjugate transpose. Minimization of (10) yields the well-known MMSE filter structure

$$W = (E\left\{YY^H\right\})^{-1} E\left\{YX^H\right\} \tag{11}$$

According to the assumptions and (8), it can be known that

$$\begin{aligned} W =& \left(\begin{bmatrix} B_1 E\left\{X_1 X_1^H\right\} B_1^H & \dots & B_1 E\left\{X_1 X_J^H\right\} B_J^H \\ \vdots & \ddots & \vdots \\ B_J E\left\{X_J X_1^H\right\} B_1^H & \dots & B_J E\left\{X_J X_J^H\right\} B_J^H \end{bmatrix} \right. \\ & \left. + \begin{bmatrix} R_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R_J \end{bmatrix} + \begin{bmatrix} R_{z1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R_{zJ} \end{bmatrix} \right)^{-1} \\ & \cdot \begin{bmatrix} B_1 E\left\{X_1 X_1^H\right\} & \dots & B_1 E\left\{X_1 X_J^H\right\} \\ \vdots & \ddots & \vdots \\ B_J E\left\{X_J X_1^H\right\} & \dots & B_J E\left\{X_J X_J^H\right\} \end{bmatrix} \end{aligned} \tag{12}$$

where $R_j = E\{V_j V_j^H\} = \delta_v^2 I_{N \times N}$ and $R_{zj} = E\{V_{zj} V_{zj}^H\}$ are additive white noise covariance matrix and array noise covariance matrix, respectively. Enforcing signals at different frequency are uncorrelated temporally and the spatial power distribution matrix is defined as

$$\begin{aligned} P =& E\left\{XX^H\right\} \odot I_{J \cdot M \times J \cdot M} \\ =& \begin{bmatrix} E\left\{X_1 X_1^H\right\} \odot I_{M \times M} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & E\left\{X_J X_J^H\right\} \odot I_{M \times M} \end{bmatrix} \\ =& \begin{bmatrix} P_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & P_J \end{bmatrix} \end{aligned} \tag{13}$$

$I_{J \cdot M \times J \cdot M} \in \mathbb{R}^{J \cdot M \times J \cdot M}$ and $I_{M \times M} \in \mathbb{R}^{M \times M}$ are identity matrixes. The spatial power distribution is composed of the diagonal elements of $\mathbf{P}$.

Substituting (13) into (12) yields

$$W = \left(\begin{bmatrix} B_1 P_1 B_1^H & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_J P_J B_J^H \end{bmatrix} + \begin{bmatrix} R_{z1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R_{zJ} \end{bmatrix} + \begin{bmatrix} R_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R_J \end{bmatrix} \right)^{-1} \begin{bmatrix} B_1 P_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & B_J P_J \end{bmatrix} \tag{14}$$

$$\begin{aligned} R_{zj} &= E\left\{V_{zj} V_{zj}^H\right\} = E\left\{(Z_j - 1_{N\times1}) \odot (B_j X_j)(B_j X_j)^H \odot (Z_j - 1_{N\times1})^H\right\} \\ &= E\left\{\widehat{Z}_j (B_j X_j)(B_j X_j)^H \widehat{Z}_j^H\right\} \end{aligned} \tag{15}$$

where $\widehat{Z}_j = \mathrm{diag}\{z_{j1}, \dots, z_{jN}\} - I_{N\times N}$, and $I_{N\times N} \in \mathbb{R}^{N\times N}$ is identity matrix. Using assumptions, it can be shown that (15) simplifies to

$$R_{zj} = \delta_z^2 I_{N\times N} \odot \left(E\left\{(B_j X_j)(B_j X_j)^H\right\}\right) = \delta_z^2 I_{N\times N} \odot \left(B_j P_j B_j^H\right) \tag{16}$$

Set $\hat{X}^0 = [Y_1^H B_1 \dots Y_J^H B_J]^H$ be the initial estimate of signal amplitude matrix $X = [X_1^H \dots X_J^H]^H$, then the initial spatial power distribution $\hat{P}^0 = E\{\hat{X}^0 (\hat{X}^0)^H\} \odot I_{J\cdot M\times J\cdot M}$ can be obtained. Consider (14) and update $\hat{W}^i$ with $\hat{P}^{i-1}$, subsequently, compute signal amplitude matrix $\hat{X}^i = (W^i)^H Y$. The recursion can stop when $\|\hat{X}^i - \hat{X}^{i-1}\|^2 \le \varepsilon$, where ε is a predetermined small value that decides the error range.

When L snapshots signals are received, passing wideband signals through J narrowband filters to obtain $\bar{Y} = [\bar{Y}_1^H \dots \bar{Y}_J^H]^H$, then signal amplitude matrix

$$\bar{X} = (\bar{W})^H \bar{Y} \tag{17}$$

where $\bar{X}_j = [\bar{x}_j(1) \dots \bar{x}_j(L)] \in \mathbb{R}^{M\times L}$, the element of $\bar{X} = [\bar{X}_1^T \dots \bar{X}_J^T]^T \in \mathbb{R}^{J\cdot M\times L}$, is comprised of the spatial complex amplitude estimation $\bar{x}_j(\ell) = [x_{j1}(\ell) \dots x_{jM}(\ell)]^T$ for the ℓth snapshots at frequency point f_j. $\bar{Y}_j = [\bar{y}_j(1) \dots \bar{y}_j(L)] \in \mathbb{R}^{N\times L}$, where $\bar{y}_j(\ell) = [y_1(\ell) \dots y_N(\ell)]^T$ is the ℓth snapshots at frequency point f_j and $\{\cdot\}^T$ is the transpose operation. Then the spatial power distribution should be denoted as

$$\bar{P} = \left[\frac{1}{L}\sum_{\ell=0}^{L-1} \bar{X}(\ell)\left(\bar{X}(\ell)\right)^H\right] \odot I_{J\cdot M\times J\cdot M} \tag{18}$$

Substituting $\bar{\mathbf{P}}$ of (18) into (14) to estimate the MMSE filter bank $\bar{\mathbf{W}}^i$ for ith recursion.

$$\bar{W}^i = \left(\begin{bmatrix} B_1\bar{P}_1^{i-1}B_1^H & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_J\bar{P}_J^{i-1}B_J^H \end{bmatrix} + \begin{bmatrix} R_{z1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R_{zJ} \end{bmatrix} \begin{bmatrix} R_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R_J \end{bmatrix} \right)^{-1} \begin{bmatrix} B_1\bar{P}_1^{i-1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_J\bar{P}_J^{i-1} \end{bmatrix} \tag{19}$$

The complete procedures of WSERA are given as follows,

Step 1: Initialization. Passing L snapshots wideband signals through J narrowband filters to obtain $\bar{Y} = [\bar{Y}_1^H \ \dots \ \bar{Y}_J^H]^H$, set initial signal amplitude distribution as $\hat{X}^0 = [\bar{Y}_1^H B_1 \ \dots \ \bar{Y}_J^H B_J]^H$, compute initial spatial power distribution estimate $\bar{P}^0 = [\frac{1}{L}\sum_{\ell=0}^{L-1} \bar{X}^0(\ell)(\bar{X}^0(\ell))^H] \odot I_{J\cdot M\times J\cdot M}$. Compute white noise covariance matrix R_j and array noise covariance matrix R_{zj}, then set ε.

Step 2: Update the MMSE filter bank

$$\bar{W}^i = \left(\begin{bmatrix} B_1\bar{P}_1^{i-1}B_1^H & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_J\bar{P}_J^{i-1}B_J^H \end{bmatrix} + \begin{bmatrix} R_{z1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R_{zJ} \end{bmatrix} + \begin{bmatrix} R_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R_J \end{bmatrix} \right)^{-1} \begin{bmatrix} B_1\bar{P}_1^{i-1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_J\bar{P}_J^{i-1} \end{bmatrix}.$$

Step 3: Estimate the signal amplitude matrix $\bar{X}^i = (\bar{W}^i)^H\bar{Y}$.

Step 4: Update the spatial power distribution estimation using

$$\bar{P}^i = \left[\frac{1}{L}\sum_{\ell=0}^{L-1} \bar{X}^i(\ell)\left(\bar{X}^i(\ell)\right)^H \right] \odot I_{J\cdot M\times J\cdot M}$$

Step 5: Judge whether $\|\bar{X}^i - \bar{X}^{i-1}\|^2 \le \varepsilon$ is feasible or not, if it's feasible, the recursion is halted, or $i = i+1$, and jump to step 2.

When the recursion is halted, the spatial amplitude distribution $\widehat{X} = \mathrm{diag}\{\sqrt{\bar{P}^i}\}$ is obtained, then compute the mean value $\breve{X}$ of signal amplitude at all frequency points. Number of sources, source locations can be determined via the peaks in $\breve{X}$.

4. SIMULATION RESULTS

In this section, the performance of WSERA is discussed. The performance of WSERA is assessed as a function of 1) sample support size L, 2) the number of array elements N, 3) Signal to Noise Ratio (SNR), 4) the degree of separation of two closely-spaced sources, and 5) random array error. All simulations are obtained via 50 Monte-Carlo tests. Assume that two linear frequency modulation (LFM) signals located at $-20°$, $20°$, and their frequency distribute over the interval [200, 220] MHz. In order to reduce the amount of computation, set the interested spatial angle $\Psi = [-40°, 40°]$, and spatial sample factor $M = 81$, the received signal is sampled at 800 MHz. The filter bank is composed of 10 filters whose center frequencies uniformly distribute over the interval [200, 220] MHz. The uncertain array error vector $Z_j = [z_{j1} \ldots z_{jn} \ldots z_{jN}] \in R^{N\times 1}$ at frequency point f_j, where $z_{jn} = [1 + \frac{\rho}{100}N(0,1)]\exp\{j\pi\frac{\rho}{100}N(0,1)\}$, and $N(0,1)$ means Gaussian distribution with zero mean and unit variance, besides, $\frac{\rho}{100}$ is the percent error in terms of the standard deviation. Compute the estimated root mean square error (RMS) with (20), where θ_k is the real DOA of the kth source and $\hat{\theta}_k$ is the estimate of θ_k.

$$RMS = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left|\theta_k - \hat{\theta}_k\right|^2} \tag{20}$$

Before consider the impact of parameterization, the effectiveness of WSERA is examined. Set the number of snapshot L and array elements N is 200 and 12, respectively, $SNR = 10\,\text{dB}$, and $\rho = 10$. In Figure 1 we can see that WSERA is effective when sources are uncorrelated and coherent.

Subsequently, for the comparison sample support, values of $L = 5$, 20, 250 snapshots are examined. Set $\rho = [0, 20]$, $N = 12$ and $SNR = 10\,\text{dB}$. Figure 2 illustrates the performance of WSERA as a function of ρ for the different numbers of snapshots. It can be observed that as the ρ increases, the number of sources estimated error and the DOA estimated RMS becomes greater, and the performance of WSERA is better with greater number of snapshots.

Now, consider the impact of the number of sensors N on the performance of WSERA. Set $\rho = [0, 20]$, $L = 20$ and $SNR = 10\,\text{dB}$, values for N of 8, 14 and 20. In Figure 3 it can be found that as the ρ increases, the number of sources estimated error and the DOA estimated RMS becomes greater, and the performance of WSERA is better with greater value of N.

We now consider the impact of SNR on the performance of

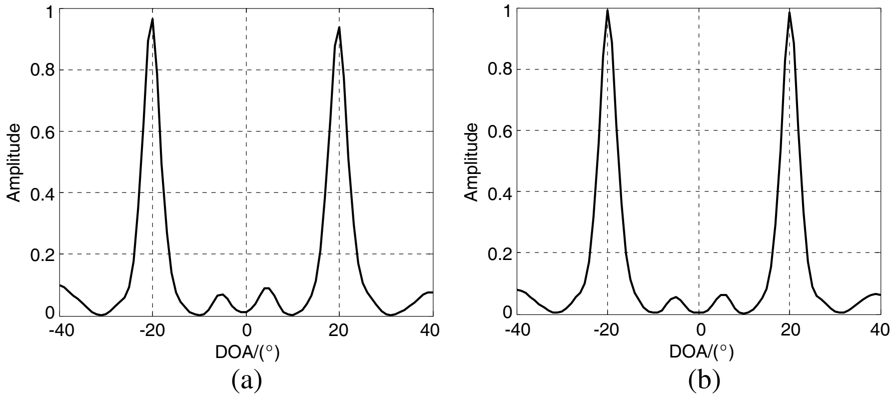


Figure 1. The performance of WSERA for uncorrelated and coherent wideband signals. (a) Sources are uncorrelated. (b) Sources are coherent.

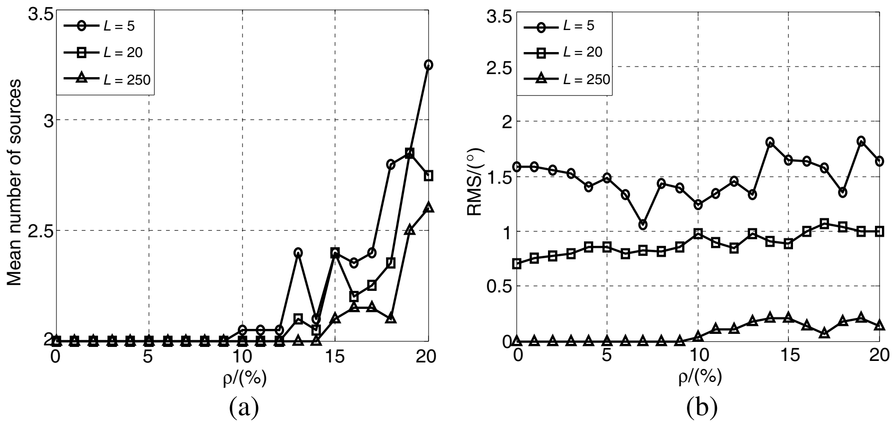


Figure 2. The performance of WSERA as a function of ρ for different numbers of snapshots L. (a) The mean number of sources versus L and ρ. (b) The RMS error versus L and ρ.

WSERA Set $\rho = [0, 20]$, $L = 20$, $N = 12$, values for SNR of 0 dB, 10 dB and 20 dB. It can be found in Figure 4 that as the ρ increases, the number of sources estimated error and the DOA estimated RMS becomes greater, and the ability to estimate the number of sources is better with greater value of SNR, the performances of RMS are mixed with each other when $\rho > 4$, so we can be informed that the impact of SNR to RMS is much smaller than the impact of array error to RMS

when ρ becomes greater.

Now the bias of WSERA is considered in terms of the angular separation between two signals. Assume two uncorrelated signals whose DOA is $\theta_1 = -20^\circ$ and $\theta_2 = -20^\circ + \Delta\theta$, respectively, where $\Delta\theta$ is varied from 1° to 40° in 1° steps. Set $SNR = 10\,\text{dB}$, the number of snapshots $L = 32$, $\rho = 10$ and $N = 15$. As what can be seen in Figure 5, that the mean number of sources approximate and even equal

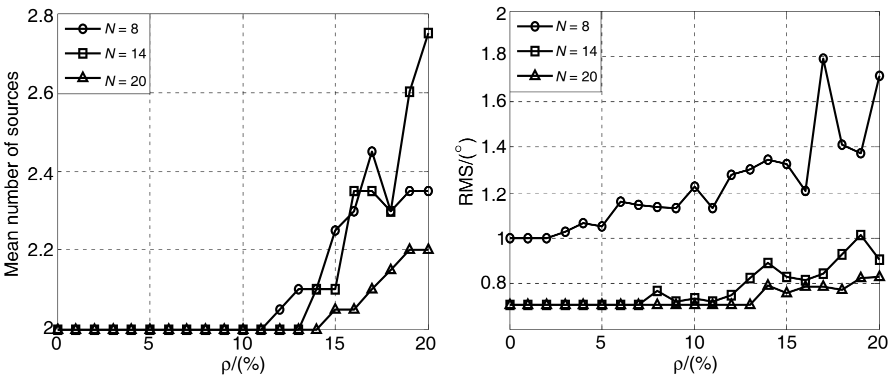


Figure 3. The performance of WSERA as a function of ρ for the number of sensors N. (a) The mean number of sources versus N and ρ. (b) The RMS error versus N and ρ.

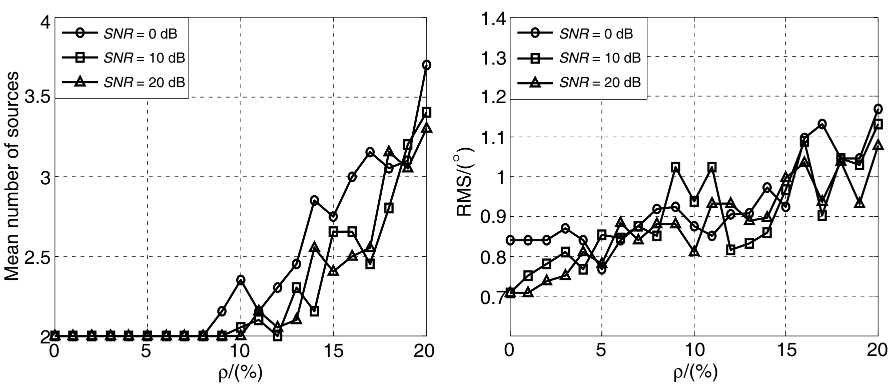
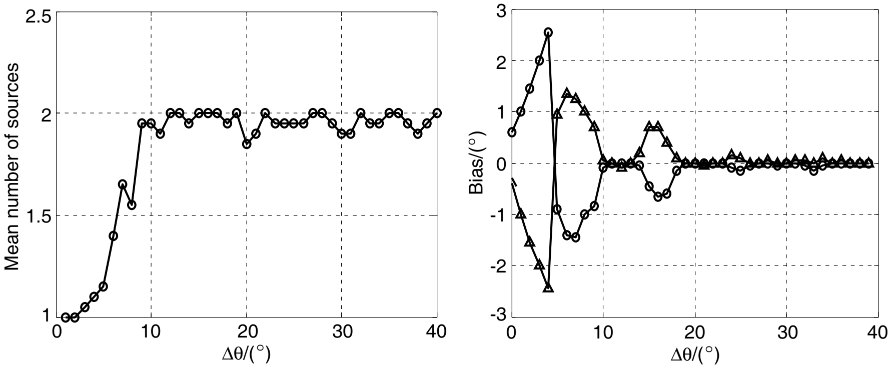
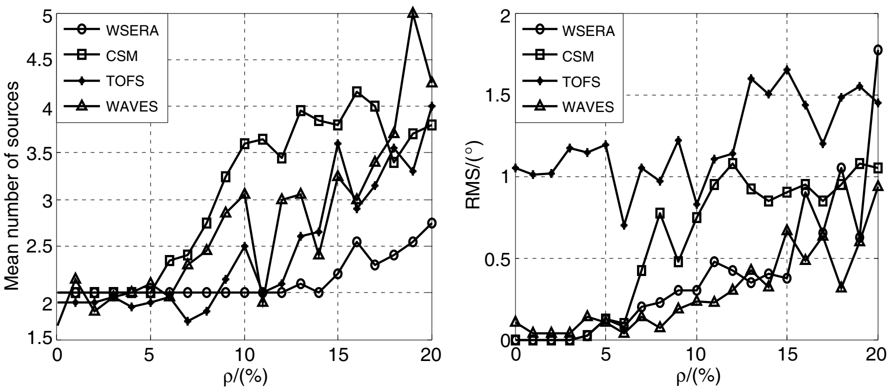


Figure 4. The performance of WSERA as a function of ρ for the number of sensors SNR. (a) The mean number of sources versus SNR and ρ. (b) The RMS error versus L and ρ.



(a) (b)

Figure 5. The performance of WSERA as a function of the angular separation $\Delta\theta$ between two signals. (a) The mean number of sources versus $\Delta\theta$. (b) The bias versus $\Delta\theta$.



(a) (b)

Figure 6. The performance of WSERA, CSM, WAVES and TOFS as a function of ρ. (a) The mean number of sources versus ρ. (b) The RMS error versus ρ.

to the real value with $\Delta\theta$ increasing, and as the $\Delta\theta$ increases, the bias is smaller and converges to be zero.

Finally, WSERA is compared with coherent subspace method (CSM), WAVES [2] and TOFS [14]. Set $\rho = [0, 20]$, $L = 8$ and $SNR = 10\,\text{dB}$, values for N of 12. In Figure 6 it is observed that as ρ increases, the number of sources estimate obtained by using WSERA is not correct when $\rho > 12$, the error becomes greater with about $\rho > 5$

when other algorithms are applied; furthermore, the *RMS* of WSERA is smaller than CSM and TOFS with ρ increasing, and is very close to WAVES that gets the best result. Then it can be known that the performance of WSERA is better than other methods mentioned when all results are taken into consideration comprehensively.

5. CONCLUSIONS

This paper proposes a new algorithm called WSERA for wideband signal DOA estimation, based on MMSE. It is different from methods that require peak searching and recovers the sparse representation of the signals received to realize wideband signal DOA estimate. Furthermore, it naturally estimates the number of sources, their locations, and their power incident on the array, regardless of the temporal correlation of the sources. The structure for model error discussed in Section 2 is incorporated into WSERA to account for unknown gain and phase errors that are present in all array elements in practice, and its robustness to array error is better than CSM, WAVES, and TOFS that are based on the eigendecomposition of a sample covariance matrix (SCM). The simulated results confirm the effectiveness of WSERA. The new method and simulated results provide a useful reference for wideband signal DOA estimate in practical engineering.

REFERENCES

1. Wang, H. and M. Kaveh, "Coherent signal-subspace processing for the detection and estimation of angles arrival of multiple wide-band sources," *IEEE Trans. on Acoustics Speech and Signal Processing*, Vol. 33, No. 4, 823–831, 1985.
2. Claudio, E. D. and R. P. Arisi, "WAVES: Weighted average of signal subspaces for robust wideband direction finding," *IEEE Trans. on Signal Processing*, Vol. 49, No. 10, 2179–2190, 2001.
3. Yu, H.-Q., J. Liu, and Z.-T. Huang, "A wideband DOA estimation algorithm based on signal subspace focusing," *System Engineering and Electronic Technology*, Vol. 30, No. 4, 609–612, 2008.
4. Tuan, D. H. and P. Russer, "An analysis of wideband direction of arrival estimation for closely-spaced sources in the presence of array model errors," *IEEE Trans. Microwave and Wireless Components*, Vol. 13, No. 8, 314–316, 2003.
5. Velni, J. M. and K. Khorasani, "A robust state-space approach for

localizing wideband sources in sensor arrays," *IEEE Sensor Array and Multichannel Signal Processing Workshop*, 298–302, 2004.

6. Blunt, S. D., T. Chan, and K. Gerlach, "Robust DOA estimation: The reiterative superresolution (RISR) algorithm," *IEEE Trans. Aerospace and Electronic Systems*, Vol. 47, No. 1, 332–346, 2011.
7. Chen, S. S., D. L. Donoho, and M. A. Saunders, "Atomic decomposition by basis pursuit," *SIAM J. Scientif. Comput.*, Vol. 20, 3–61, 1999.
8. Donoho, D. L., M. Elad, and V. Temlyakvo, "Stable recovery of sparse overcomplete representations in the presence of noise," *IEEE Trans. Inf. Theory*, Vol. 52, No. 1, 6–18, 2006.
9. Zhang, C., Z. Yin, X. Chen, and M. Xiao, "Signal overcomplete representation and sparse decomposition based on redundant dictionaries," *Chinese Sci. Bull.*, Vol. 50, No. 23, 2672–2677, 2005.
10. Shan, T. J., M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," *IEEE Trans. on Acoustics Speech and Signal Processing*, Vol. 33, No. 4, 806–811, Aug. 1985.
11. Pillai, S. U. and B. H. Kwon, "Forward/backward spatial smoothing techniques for the coherent signal identification," *IEEE Trans. on Acoustics Speech and Signal Processing*, Vol. 37, No. 1, 8–15, Jan. 1989.
12. Williams, R. T., S. Prasad, and A. K. Mahalanabis, "An improved spatial smoothing technique for bearing estimation in a multipath environment," *IEEE Trans. on Acoustics Speech and Signal Processing*, Vol. 36, No. 4, 425–432, Apr. 1988.
13. Choi, Y.-H., "On conditions for the rank restoration in forward/back ward spatial smoothing," *IEEE Trans. SP*, Vol. 50, No. 11, 2900–2901, Nov. 2002.
14. Yu, H., J. Liu, and Z. Huang, "A new method for wideband DOA estimation," *IEEE Wireless Communications, Networking and Mobile Computing*, 598–601, 2007.