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QUASI-ISOTROPIC APPROXIMATION OF GEOMETRICAL OPTICS METHOD WITH APPLICATIONS TO DENSE PLASMA POLARIMETRY

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Abstract—Basic equations of quasi-isotropic approximation (QIA) of geometrical optics method are presented, which describe electromagnetic waves propagation in weakly inhomogeneous and weakly anisotropic media. It is shown that in submillimiter range of electromagnetic spectrum plasma in all modern thermonuclear reactors, both acting and under construction, manifest properties of weakly inhomogeneous and weakly anisotropic medium, even for extreme electron density $N_e \sim 10^{14}\,\mathrm{cm}^{-3}$ and magnetic field $B_0 \sim 5\,\mathrm{T}$ accepted for project ITER. In these conditions QIA serves as natural theoretical basis for plasma polarimetry in tokamaks and stallarators. It is pointed out that Stokes vector formalism (SVF), widely used in polarimetry, can be derived from QIA in a generalized form, admitting the rays to be curvilinear and torsiened. Other important result of QIA is development of angular variables technique (AVT), which deals directly with angular parameters of polarization ellipse and operates with the system of two differential equations against three equations in form of SVF.

1. INTRODUCTION

Polarimetry of dense plasma in modern thermonuclear reactors, like largest working tokamak JET (Joint European Tor, Culham, UK), stellarator W7-X, which is under construction in Greifswald, Germany, and tokamak ITER (International Thermonuclear Experimental Reactor, Cadarache, France) uses, or will use, electromagnetic waves of submillimeter wavelength range: $\lambda \sim 0.1$ mm [1–3]. Plasma in projected thermonuclear reactor ITER will have electron density

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$N_e \sim 10^{14}\,\mathrm{cm}^{-3}$ and magnetic field about 5 T [3, 4]. In submillimeter range of electromagnetic spectrum such plasma will demonstrate properties of weakly inhomogeneous and weakly anisotropic medium. Electromagnetic wave propagation of this kind of media might be adequately described by a quasi-isotropic approximation (QIA) of geometrical optics method. QIA differs from traditional method of geometrical optics by introducing additional small parameter μ_A which characterizes degree of anisotropy [5]. Very brief outline of QIA one can found in the book [6]. More deep and more wide analysis of QIA is presented in the review paper [7] and in the book [8], where also are discussed some applications of QIA in different physics fields (e.g., radio wave propagation through Earth ionosphere and extraterrestrial moving plasma, phenomenon of normal modes conversion, acoustic phenomena in weakly anisotropic elastic media, birefringence of spinor wave function in a magnetic field (The Stern-Gerlach effect)).

This paper pursue two aims. First of all we would like to show that in submillimeter range of wavelengths plasma in large modern thermonuclear reactors possesses properties of weakly inhomogeneous and weakly anisotropic medium, so that QIA may serve as adequate theoretical basis for plasma polarimetry. We intend also to describe new approach in plasma polarimetry, which deals directly with evolution of angular parameters of polarization ellipse, omitting analysis of Stokes vector evolution.

The paper is organized as follows. Section 2 outlines basic elements of QIA according to [6–8]. Section 3 approves properties of weak inhomogeneity and weak anisotropy of thermonuclear plasma in submillimeter range of wavelengths and specifies QIA equations for plasma medium. Section 4 develops angular variables technique for two sets of angular variables — "azimuth-ellipticity" and "amplitude ratio-phase difference" and analyzes advantages and shortcomings of AVT as compared with Stokes vector formalism [9–11]. Section 5 summarizes the main results.

2. BASIC EQUATIONS OF QIA

Traditional geometrical optics method provides approximate *diffractionless* solution of Maxwell equations in weakly inhomogeneous media. Assuming harmonic dependence $\exp(-i\omega t)$, considering nonmagnetic media with $\mathbf{B} = \mathbf{H}$ and excluding magnetic field we reduce Maxwell equations to the form

$$\nabla \times \nabla \times \mathbf{E} - k^2\mathbf{D} = 0, \quad D_i = \varepsilon_{ik}E_k, \quad k = \omega/c. \tag{1}$$

When electrical intensity $\mathbf{E}$ is determined from Equation (1), magnetic field $\mathbf{H}$ can be found by relation $\mathbf{H} = -\frac{i}{\omega}\nabla \times \mathbf{E}$. Weak inhomogeneity implies that the wavelength $\lambda = 2\pi/k$ is short as compared with characteristic length L of medium parameters changeability, so that we are able to involve *geometrical* small parameter

$$\mu_{GO} = \frac{\lambda\!\!\!^{-}}{L} = \frac{1}{kL} \quad \lambda\!\!\!^{-} \equiv 1/k \tag{2}$$

Basic equations of geometrical optics approximation can be obtained by two methods. Rytov approach [5] deals with expansion of the ware amplitude in powers μ_{GO}^m of dimensionless small parameter (2), whereas Debye procedure operates with series in inverse powers of dimensional parameter wave number k

$$\mathbf{E} = \left(\mathbf{A}_0 + \frac{1}{ik}\mathbf{A}_1 + \frac{1}{(ik)^2}\mathbf{A}_2\right)\exp(ik\Psi) \tag{3}$$

here $k\Psi$ is a phase, and Ψ is an eiconal of the wave field.

Quasi-isotropic approximation (QIA) of geometrical optics method deals with wave propagation in weakly anisotropic media, whose dielectric tensor ε_{ik} consists of large isotropic part $\varepsilon_0\delta_{ik}$ and small anisotropy tensor

$$\nu_{ik} = \varepsilon_{ik} - \varepsilon_0\delta_{ik} \tag{4}$$

Thus, QIA involves the second — anisotropic — small parameter

$$\mu_A = \frac{1}{\varepsilon_0}\max|\nu_{ik}| \ll 1 \tag{5}$$

Keeping Debye procedure, we should artificially ascribe to small terms ν_{ik} the first order in inverse wave number. Assuming

$$\nu_{ik} = \frac{N_{ik}}{k}, \tag{6}$$

substituting asymptotic expansion (3) into Maxwell Equation (1) and equaling coefficients of powers k^m, $m = 2, 1, 0, -1, \ldots$ to zero, we arrive to a system of equations for vector amplitudes $\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2, \ldots$.

The terms of the second order in k lead to a system of linear homogeneous equations for components $A_{0\beta}$ of the vector $\mathbf{A}_0$:

$$q_{\alpha\beta}A_{0\beta} = 0 \tag{7}$$

where

$$q_{\alpha\beta} = \left(p^2 - \varepsilon_0\right)\delta_{\alpha\beta} - p_\alpha p_\beta \tag{8}$$

and $\mathbf{p} = \nabla\Psi$.

System of Equation (7) admits nontrivial solution

$$\det\|q_{\alpha\beta}\| = \left(p^2 - \varepsilon_0\right)^2\varepsilon_0 = 0 \tag{9}$$

what leads to the eiconal equation:

$$p^2 - \varepsilon_0 \equiv (\nabla\Psi)^2 - \varepsilon_0 = 0 \tag{10}$$

Eiconal Equation (10) can be solved by the method of characteristics. In a given case characteristic functions $\mathbf{p}(\tau)$ and $\mathbf{r}(\tau)$ satisfy the equations [5]:

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}\tau} = \mathbf{p}, \quad \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}\tau} = \frac{1}{2}\nabla\varepsilon_0 \tag{11}$$

which are equation for ray trajectories $\mathbf{r}(\tau)$ and for momentum $\mathbf{p} = \nabla\Psi$. Here parameter τ along the ray is connected with the ray arc length σ by the relation $\mathrm{d}\tau = \mathrm{d}\sigma\big/\sqrt{\varepsilon_0}$. Thus, the rays in a weakly isotropic medium coincide with the rays in the background isotropic medium.

When eiconal Equation (10) is satisfied, matrix $q_{\alpha\beta}$ in Equation (7) degenerates into matrix $\|p_\alpha p_\beta\|$, and Equation (7) takes a form

$$\mathbf{p}(\mathbf{p}\mathbf{A}_0) = 0 \tag{12}$$

It follows from (12) that field vector $\mathbf{A}_0$ is orthogonal to the ray tangent $\mathbf{p} = \mathrm{d}\mathbf{r}/\mathrm{d}\tau$, so that in the zero approximation we deal with transverse electromagnetic wave. In this case vector $\mathbf{A}_0$ can be presented as superposition

$$\mathbf{A}_0 = \Phi_1\mathbf{f}_1 + \Phi_2\mathbf{f}_2 \tag{13}$$

where vectors $\mathbf{f}_{1,2}$ are orthogonal to the ray $(\mathbf{f}_{1,2}\mathbf{l}) = 0$, and to each other: $(\mathbf{f}_1\mathbf{f}_2) = 0$. Here $\mathbf{l} = p\big/\sqrt{\varepsilon_0}$ is a unit vector, tangent to the ray.

The simplest choice for vectors $\mathbf{f}_{1,2}$ is normal, $\mathbf{n}$, and binormal $\mathbf{b}$, to the ray, so that

$$\mathbf{A}_0 = \Phi_n\mathbf{n} + \Phi_b\mathbf{b} \tag{14}$$

Corresponding leading term $\mathbf{H}_0$ in the series $\mathbf{H} = (\mathbf{H}_0 + \frac{1}{ik}\mathbf{H}_1 + \ldots)\exp(ik\Psi)$ for magnetic field takes the form:

$$\mathbf{H}_0 = (\mathbf{p}\times\mathbf{A}_0)\exp(ik\Psi) = \sqrt{\varepsilon_0}\,(\Phi_n\mathbf{b} - \Phi_b\mathbf{n}) \tag{15}$$

Scalar amplitudes $\Phi_{n,b}$, can be determined, using consistency condition for the first order equations

$$p^2\mathbf{A}_1 - \mathbf{p}(\mathbf{p}A_1) - \varepsilon_0\mathbf{A}_1 = -k^2\hat{\nu}\mathbf{A}_0 - \nabla\times(\mathbf{p}\times\mathbf{A}_0) - (\mathbf{p}\times\nabla\times\mathbf{A}_0) \equiv \mathbf{Z} \tag{16}$$

Thus, the components of the first order vector $\mathbf{A}_1$ obeys to a system of linear inhomogeneous equations with zero determinant. This system can be solved, when consistency conditions are fulfilled: right hand vector $\mathbf{Z}$ in Equation (15) should be orthogonal to eigenvectors of transposed homogeneous equations. In a given case these eigenvectors coincide with vectors of normal and binormal, so that consistency conditions take the form

$$\mathbf{Z}\cdot\mathbf{n}=0,\quad \mathbf{Z}\cdot\mathbf{b}=0 \tag{17}$$

Equations (15) can be presented in the explicit form:

$$\begin{cases} 2\mathbf{l}\sqrt{\varepsilon_0}\nabla\Phi_n+\Phi_n\mathbf{l}\nabla\sqrt{\varepsilon_0}+\Phi_n\sqrt{\varepsilon_0}\nabla\mathbf{l}+2\kappa\sqrt{\varepsilon_0}\Phi_b-ik(\nu_{nn}\Phi_n+\nu_{nb}\Phi_b)=0\\ 2\mathbf{l}\sqrt{\varepsilon_0}\nabla\Phi_b+\Phi_b\mathbf{l}\nabla\sqrt{\varepsilon_0}+\Phi_b\sqrt{\varepsilon_0}\nabla\mathbf{l}-2\kappa\sqrt{\varepsilon_0}\Phi_n-ik_0(\nu_{bn}\Phi_n+\nu_{bb}\Phi_b)=0 \end{cases} \tag{18}$$

Here $2\kappa=\mathbf{n}\nabla\times\mathbf{n}+\mathbf{b}\nabla\times\mathbf{b}$ is a torsion, which determines rate of vectors $\mathbf{n}$ and $\mathbf{b}$ rotation around the ray, and

$$\begin{aligned}\nu_{nn}&=\mathbf{n}\hat{\nu}\mathbf{n},\quad \nu_{nb}=\mathbf{n}\hat{\nu}\mathbf{b}\\ \nu_{bn}&=\mathbf{b}\hat{\nu}\mathbf{n},\quad \nu_{bb}=\mathbf{b}\hat{\nu}\mathbf{b}\end{aligned} \tag{19}$$

It follows from Equations (17) and (18) that squared modulus

$$|A_0|^2=|\Phi_n|^2+|\Phi_b|^2 \tag{20}$$

of the amplitude (14) satisfies energy flux conservation law

$$\nabla\left(\mathbf{p}\,|A_0|^2\right)=0 \tag{21}$$

It is convenient to present zero order amplitude in the form

$$\mathbf{A}_0=|A_0|\,\mathbf{\Gamma}\equiv|\mathbf{A}_0|\,(\Gamma_n\mathbf{n}+\Gamma_b\mathbf{b}) \tag{22}$$

where $\mathbf{\Gamma}$ is a unit vector: $|\mathbf{\Gamma}|=1$.

According to Equations (18) and (21), components Γ_n and Γ_b of this vector obey the equations:

$$\begin{cases} \dfrac{\mathrm{d}\Gamma_n}{\mathrm{d}\sigma}=\dfrac{ik}{2\sqrt{\varepsilon_0}}\left(\nu_{nn}\Gamma_n+\nu_{nb}\Gamma_b\right)-\kappa\Gamma_b\\ \dfrac{\mathrm{d}\Gamma_b}{\mathrm{d}\sigma}=\dfrac{ik}{2\sqrt{\varepsilon_0}}\left(\nu_{bn}\Gamma_n+\nu_{bb}\Gamma_b\right)+\kappa\Gamma_n \end{cases} \tag{23}$$

This system can be reduced to a single equation of Ricatti type for complex polarization angle (CPA) $\theta=\theta'+i\theta$", defined as

$$\tan\theta=\frac{\Gamma_b}{\Gamma_n} \tag{24}$$

It follows from Equation (23), that equation for CPA has a form

$$\frac{\mathrm{d}\theta}{\mathrm{d}\sigma}=\kappa+\frac{ik}{4\sqrt{\varepsilon_0}}\left[(\nu_{bn}-\nu_{nb})+(\nu_{bn}+\nu_{bn})\cos 2\theta-(\nu_{nn}-\nu_{bb})\sin 2\theta\right] \tag{25}$$

As shown in [12], the real part of CPA, $\mathrm{Re}\theta=\theta'$ coincides with the azimuthal angle ψ polarization ellipse (Fig. 1)

$$\mathrm{Re}\theta=\theta'=\psi \tag{26}$$

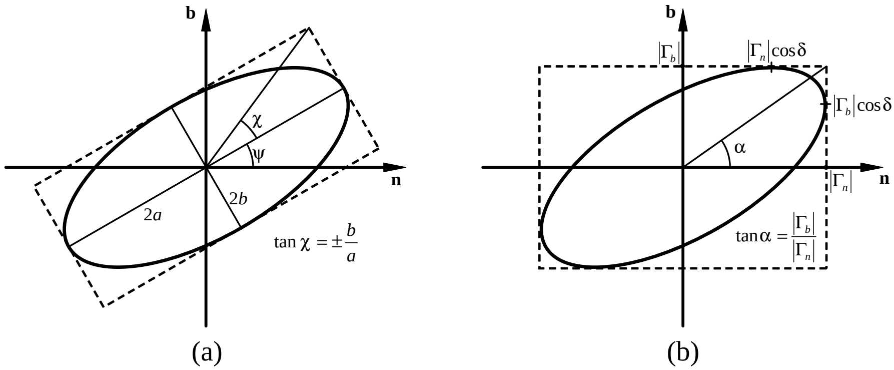

Figure 1. Angular parameters of polarization ellipse: (a) azimuthal angle ψ and ellipticity angle χ; (b) amplitude ratio α and phase difference δ.

whereas hyperbolic tangent of the imaginary part $\theta" = \mathrm{Im}\theta$ equals to tangent of ellipticity angle χ:

$$\tanh\left(\mathrm{Im}\theta\right) = \tan\chi \tag{27}$$

We use these relation for derivation of differential equation in Section 4.

In a limit case of isotropic medium, when all the terms ν_{ik} tend to zero, we arrive to the Rytov law [5, 7],

$$\frac{\mathrm{d}\theta}{\mathrm{d}\sigma} = \kappa \tag{28}$$

Therefore Equation (25) can be considered as Rytov law generalization for weakly anisotropic media.

Equations (21) and (23) and (25) are basic equations of QIA: Equation (21) describes energy evolution along the ray, whereas Equation (23) and (25) determines polarization properties of electromagnetic wave.

3. QIA EQUATION FOR ELECTROMAGNETIC WAVES IN PLASMA

Polarimetry systems in modern plasma devices — tokamaks and stellarators, deal with electromagnetic waves of submillimeter (for infrared) range of wavelengths: $\lambda = 0.01 \div 0.2\,\mathrm{mm}$ (corresponding to frequencies $1.5 \div 30\,\mathrm{THz}$). For instance, the largest working tokamak JET (Joint European Tor, Calhan/Oxford, UK) uses wavelengths $\lambda_1 = 195\,\mu\mathrm{m}$ ($f_1 = 1.54\,\mathrm{THz}$) and $\lambda_2 = 119\,\mu\mathrm{m}$ ($f_2 = 2.52\,\mathrm{THz}$). Even shorter wavelengths $\lambda \approx 50\,\mu\mathrm{m}$ and $\lambda \approx 10\,\mu\mathrm{m}$ are envisaged for

Project ITER, (International Thermonuclear Experimental Reactor), which is realized now in Cadarache, France.

Electron density in the future tokamak ITER is anticipated to be as large as $N_e = 10^{14}\,\mathrm{cm}^{-3}$. Plasma frequency

$$\omega_{pl} = \left(\frac{4\pi e^2 N_e}{m}\right)^{1/2} \tag{29}$$

for such a density will be about 0.56 THz, (here e and m are respectfully electron charge and mass). In these conditions standard plasma parameter

$$X = \left(\frac{\omega_{pl}}{\omega}\right)^2 \tag{30}$$

will be small enough. For instance, for frequency $f = 3\,\mathrm{THz}$ we have

$$X \approx 0.001 \tag{31}$$

The other important plasma parameter

$$Y = \frac{\omega_{ce}}{\omega} = \frac{eB_0}{mc\omega} \tag{32}$$

which characterizes the ratio of electron cyclotron frequency $\omega_{ce} = eB_0/mc$ to working frequency ω, also happens to be small even for extremely high static magnetic field $B_0 \approx 5\,\mathrm{T}$. For instance, at $f = 3\,\mathrm{THz}$

$$Y \approx 0.05 \tag{33}$$

We shall see below that smallness of parameters X and Y in submillimeter range of wavelengths warrants the plasma to be weakly anisotropic medium. We also can ascertain that in submillimeter range tokamak plasma belongs to a class of weakly inhomogeneous media and geometric optics approximations can be used.

Indeed, in modern tokamaks, where the plasma is the size of few meters, the shortest scale of plasma inhomogeneity is the lower turbulent scale $L_{\min} \approx 0.5\,\mathrm{cm}$ [4]. It means that outmost value of geometrical small parameter $\mu_{GO} = 1/kL$ is about

$$\max \mu_{GO} = \frac{1}{kL_{\min}} = \frac{\lambda}{2\pi L_{\min}} \approx 0.03 \ll 1 \tag{34}$$

Thus, quasi-isotropic approximation of the geometrical optics method has all the prerequisites to serve as theoretical basis of plasma polarimetry in tokamaks.

Let us write down QIA Equations (23) and (25) in cold plasma approximation. First of all, isotropic component of dielectric tensor equals $\varepsilon_0 = 1 - X$. In the frame of this model the components ν_{nn}, ν_{nb}, ν_{bn} and ν_{bb} of anisotropy tensor are of the form

$$\begin{cases} \nu_{nn} = -\dfrac{XY^2}{1-Y^2}\left(\sin^2\alpha_{||}\sin^2\alpha_\perp + \cos^2\alpha_{||}\right) \\ \nu_{nb} = \nu_{bn}^* = \dfrac{XY}{1-Y^2}\left(i\cos\alpha_{||} + Y\sin^2\alpha_{||}\sin\alpha_\perp\cos\alpha_\perp\right) \\ \nu_{bb} = -\dfrac{XY^2}{1-Y^2}\left(\sin^2\alpha_{||}\cos^2\alpha_\perp + \cos^2\alpha_{||}\right) \end{cases} \tag{35}$$

Expression (34) is presented in coordinate system $(\mathbf{n}, \mathbf{b}, \mathbf{l})$ (natural trihedral), where unit vector $\mathbf{l}$ points out the direction of the ray. Static magnetic vector $\mathbf{B}_0$ is supposed to form angle $\alpha_{||}$ with the ray direction, whereas transverse magnetic component $\mathbf{B}_{0\perp} = \mathbf{B}_0 - \mathbf{l}\,(\mathbf{l}\mathbf{B}_0)$ forms angle $\alpha_\perp$ with normal $\mathbf{n}$ (Fig. 2).

In fact, angle $\alpha_\perp$ is an angle between the "ray plane" $(\mathbf{n}, \mathbf{l})$, the plane tangent to the ray and containing normal $\mathbf{n}$, and the "magnetic plane", the plane containing vector $\mathbf{l}$ and magnetic vector $\mathbf{B}_0$.

Substituting anisotropy tensor in Equation (23), we obtain

$$\begin{cases} \dfrac{d\Gamma_n}{d\sigma} = -\dfrac{i}{2}\left(2\Omega_0 - \Omega_\perp - \Omega_1\right)\Gamma_n + \dfrac{i}{2}\left(\Omega_2 + i\Omega_3\right)\Gamma_b - \kappa\Gamma_b \\ \dfrac{d\Gamma_b}{d\sigma} = \dfrac{i}{2}\left(\Omega_2 - i\Omega_3\right)\Gamma_n - \dfrac{i}{2}\left(2\Omega_0 - \Omega_\perp + \Omega_1\right)\Gamma_b + \kappa\Gamma_n \end{cases} \tag{36}$$

Here $\Omega_{1,2,3}$ are the components of vector $\boldsymbol{\Omega}$

$$\boldsymbol{\Omega} = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix} = \frac{k}{2\sqrt{1-X}}\frac{X}{1-Y^2}\begin{pmatrix} Y^2\sin^2\alpha_{||}\cos 2\alpha_\perp \\ Y^2\sin^2\alpha_{||}\sin 2\alpha_\perp \\ 2Y\cos\alpha_{||} \end{pmatrix} \tag{37}$$

widely used in plasma polarimetry and $\Omega_\perp$ with Ω_0 are auxiliary parameters

$$\Omega_\perp = \sqrt{\Omega_1^2 + \Omega_2^2} = \frac{k}{2\sqrt{1-X}}\frac{XY^2}{1-Y^2}\sin^2\alpha_{||} = \Omega_0\sin^2\alpha_{||}. \tag{38}$$

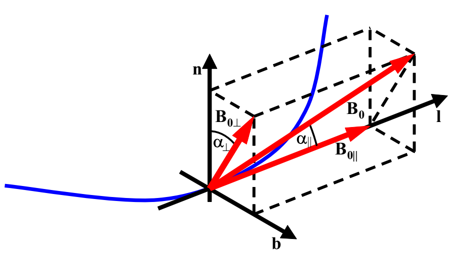

Figure 2. Magnetic vector $\mathbf{B}_0$ in a coordinate system $(\mathbf{n}, \mathbf{b}, \mathbf{l})$.

Parameters Ω_1 and Ω_2 characterize Cotton-Mouton effect, and Ω_3 corresponds to Faraday phenomenon. It is worth noticing that torsion κcould be entered in QIA Equation (36) jointly with Faraday parameter Ω_3, so that sum $\kappa + \Omega_3$ serves as effective parameter of vector $\mathbf{E}$ rotation around the ray. We shall denote this sum as

$$\Omega_3^{eff} = \Omega_3 + \kappa \tag{39}$$

Using (39), we rewrite Equation (36) as

$$\begin{cases} \dfrac{\mathrm{d}\Gamma_n}{\mathrm{d}\sigma} = -\dfrac{i}{2}\left(2\Omega_0 - \Omega_\perp - \Omega_1\right)\Gamma_n + \dfrac{i}{2}\left(\Omega_2 + i\Omega_3^{eff}\right)\Gamma_b \\ \dfrac{\mathrm{d}\Gamma_b}{\mathrm{d}\sigma} = \dfrac{i}{2}\left(\Omega_2 - i\Omega_3^{eff}\right)\Gamma_n - \dfrac{i}{2}\left(2\Omega_0 - \Omega_\perp + \Omega_1\right)\Gamma_b \end{cases} \tag{40}$$

Substituting Equations (35) and (39) in Equation (25), we arrive to the following equation for complex polarization angle $\theta = \theta' + i\theta$":

$$\frac{\mathrm{d}\theta}{\mathrm{d}\sigma} = \frac{1}{2}\Omega_3^{eff} - \frac{i}{2}\left(\Omega_1 \sin 2\theta - \Omega_2 \cos 2\theta\right) \tag{41}$$

As it was shown in [13, 14], QIA equations allow readily obtaining equation for Stokes vector evolution. Here we generalize these results for the case of the ray torsion.

There component Stokes vector $\mathbf{s}$, given in $(\mathbf{n}, \mathbf{b}, \mathbf{l})$ coordinate system, we defined as

$$\begin{cases} s_1 = |\Gamma_n|^2 - |\Gamma_b|^2 \\ s_2 = 2\mathrm{Re}\left(\Gamma_n^*, \Gamma_b\right) \\ s_3 = 2\mathrm{Im}\left(\Gamma_n^*, \Gamma_b\right) \end{cases} \tag{42}$$

Differentiating Equation (42) and using derivatives from Equation (36), we readily arrive to generalized equation for Stokes vector evolution:

$$\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}\sigma} = \mathbf{\Omega}^{eff} \times \mathbf{s} \tag{43}$$

Here modified vector $\mathbf{\Omega}^{eff}$ differs from standard vector $\mathbf{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$ by changing Ω_3 in effective value (39): $\mathbf{\Omega}^{eff} = (\Omega_1, \Omega_2, \Omega_3^{eff})$.

It is worth to emphasis principal distinction of presented here derivation of Equation (43) for Stokes vector evolution from derivation in [9–11] in frame of Stokes vector formalism (SVF). Equation (43) is derived here in a consequent way on the basis of expansion of the wave field into asymptotic series in frame of quasi-isotropic approximation of geometrical optics method. This derivation takes into account both curvature and torsion of the ray. Stokes vector formalism completely omits curvilinear rays, dealing only with plane layered plasma, which does not admit side refraction as well as torsion of the rays. Besides SVF tries to create impression that equation for Stokes vector evolution is valid even in strongly anisotropic plasma, where intensity vector **E** may be have significant longitudinal component. This does not allow introducing Stokes vector, defined only for transverse wave.

Presence of torsion κ in QIA equations is very important from general point of view, because the terms with κ provide smooth transition to isotropic medium. In practice the role of torsion depends on the ray position relative inhomogeneous plasma. Influence of torsion is not significant, when

$$\kappa \ll \Omega_3 \approx \frac{2\pi}{\lambda} XY, \tag{44}$$

or when geometrical small parameter $\mu_{GO} \sim \lambda/a$ does not exceed anisotropic small parameter $\mu_A \sim XY$:

$$\mu_{GO} \sim \frac{\lambda}{a} < \mu_A \sim XY \tag{45}$$

This condition should be tested every time, when the ray is not perpendicular to magnetic lines of poloidal magnetic field.

4. ANGULAR VARIABLES TECHNIQUE (AVT)

Separating the real and imaginary parts of Equation (41) for CPA and using relations ($\theta' = \psi$) and ($\tanh \theta'' = \tan \chi$) we arrive to the following equations for angular parameters of polarization ellipse:

$$\begin{cases} \dfrac{\mathrm{d}\psi}{\mathrm{d}\sigma} = \dfrac{1}{2}\Omega_3^{\mathit{eff}} - \dfrac{1}{2}(\Omega_1 \cos 2\psi + \Omega_2 \sin 2\psi)\, \tan 2\chi \\ \dfrac{\mathrm{d}\chi}{\mathrm{d}\sigma} = \dfrac{1}{2}(\Omega_1 \sin 2\psi - \Omega_2 \cos 2\psi) \end{cases} \tag{46}$$

Similar equations can be obtained also for dual set of angular variables: "amplitude ratio angle" α (tangent of which is defined as the ratio of polarization sizes in **n** and **b** directions: $\tan \alpha = |\Gamma_b|/|\Gamma_n|$ and phase difference δ between oscillations in **n** and **b** directions. According to recent paper [15], angular parameters α and δ satisfy the system of equations

$$\begin{cases} \dfrac{\mathrm{d}\alpha}{\mathrm{d}\sigma} = \dfrac{1}{2}\left(-\Omega_2 \sin \delta + \Omega_3^{\mathit{eff}} \cos \delta\right) \\ \dfrac{\mathrm{d}\delta}{\mathrm{d}\sigma} = \Omega_1 - \left(\Omega_2 \cos \delta + \Omega_3^{\mathit{eff}} \sin \delta\right) \cot 2\alpha \end{cases} \tag{47}$$

Equations (46) and (47) from a basis for angular variables technique (AVT). The advantage of AVT against Stokes vector formalism is that

AVT deals with a system of two coupled equations, where SVF operates with a system of three differential equations. The other advantage of AVT is its ability to obtain analytical solutions in the case strong Faraday effect, when $\Omega_3 \gg \Omega_\perp$ [16] or strong Cotton-Mouton effect $\omega_\perp \ll \Omega_3$ [17]. AVT has also already demonstrated its efficiency in a problem of plasma model fitting to experimental data [18].

5. CONCLUSIONS

The basic elements of quasi-isotropic approximation QIA of geometrical optics method are presented, QIA deals with electromagnetic waves in weakly anisotropic and weakly inhomogeneous media. It is shown that in the zero order of quasi-isotropic approximation the wave field represents a transverse wave, propagating along isotropic rays, whereas polarization structure of transverse wave is described by equation for complex polarization angle. We have shown also that dense plasma in modern fusion reactors disposes properties of weakly anisotropic and weakly inhomogeneous medium, what allows using QIA as theoretical basis for dense plasma polarimetry in submillimeter range of wavelengths.

One of important results is derivation of equation for Stokes vector evolution along curvilinear ray, experienced torsion.

Author' important new result is the derivation of evolution equations directly for angular parameters of polarization ellipse. These equations form new instrument in polarization analysis — angular variables technique.

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