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APPLICATION OF MDL CRITERION FOR MICROWAVE IMAGING BY MUSIC ALGORITHM

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Abstract—Multiple signal classification (MUSIC) algorithm has been applied to localize small scatterers for super-resolution imaging. A problem associated with this application is the estimation of the number of scatterers in presence of noise and multiple scattering between targets. In this paper, we show that the mathematical model behind the scattering from the small objects is well compatible with the minimum description length (MDL) model. This leads us to use the MDL so as to estimate the number of scatterers before application of the MUSIC algorithm. As the MDL assumes the sources are independent, the nearby wave sources are grouped together to improve the independency criterion. The application of MDL to synthetic and experimental data verifies accurate estimation of the target number with low complexity, even if the data embodies significant noise and multiple scattering.

1. INTRODUCTION

Microwave imaging is a new technology to image interior of objects. It solves an inverse problem, in which the profiles of objects are reconstructed from the electromagnetic scattering data. Two approaches, microwave tomography and time reversal (TR), have been proposed to solve such an inverse problem. Microwave tomography methods are based on optimization and needs to be solved iteratively that make them very time consuming. Some methods such as using phaseless data [1]; the artificial bee colony optimizer [2] and the Forward-Backward Time-Stepping (FBTS) technique [3] that uses a general formulation of the time domain scattering problem; and multi scaling particle swarm optimization [4] are proposed to partly solve this constraint.

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Time reversal (TR) method [5] has attracted increasing interest recently with broad applications; including underwater acoustics, radar, detection of defects in metals, communications and breast cancer detection (see [2, 5–8]), due to its low computational complexity over microwave tomography. It involves physical or synthetic backpropagation of signals received by a sensor array (TR antenna array) in a time-reversed fashion (first-in-last-out sequence) for detection and localization of targets. Based on physics of wave propagation in a reciprocal medium, in time reversal, the wave is traced back to the origin of the signal and the location of the scatterers or wave sources is obtained (see [5, 9, 10]). An improvement to TR is the multiple signal classification (MUSIC) algorithm that exploits the orthogonality of the scatterer and noise subspaces. In this way, the MUSIC algorithm significantly improves the resolution of the time reversal [11], as it is named super-resolution imaging (see [12–16]).

The resolution of the backpropagation algorithm is degraded with noise and multiple scattering between targets, whereas this is not the case for the MUSIC [17]. However, the noise and multiple scattering increase the error in estimating the number of targets, what is required in the MUSIC before running the imaging procedure. One way to solve this problem is based on hypothesis testing using the eigenvalues of the covariance matrix of the observed vector. Some proposals exist in the literatures to model this order detection problem. Anderson [18] proposes a hypothesis testing procedure based on the confidence interval of the noise eigenvalue, in which a threshold value must be assigned subjectively. Information theoretic criteria such as the AIC derived by Akaike [19], the minimum description length (MDL) given by Rissanen [20], and the Φ criterion proposed by Hannan [21] were developed so that the setting of the subjective threshold can be avoided. Wax and Kailath in [22] propose an approach based on the MDL criterion in which the number of signals is determined by the value for which the MDL criterion is minimized. The MDL is a low complexity information theoretic criterion, which does not require any subjective threshold setting usual in detection theoretic criteria. It is shown that the statistical performance of the MDL is approximately the same under both deterministic and stochastic signal models [23].

Conventional MDL methods work with time independent components; however we employ a modified version of MDL algorithm that works with independent sources obtained by a multistatic data matrix. We apply this method to determine the number of targets (i.e., the number of signal eigenvalues) and then apply MUSIC procedure to form pseudospectrum image. This paper compares the mathematical model of the scattering phenomenon of small object with that of the

MDL and shows they are the same. Therefore, the MDL can be used to determine the number of scatterers. The paper, also, demonstrates that grouping the nearby wave sources enhances the independency of the sources and thus, the estimation error of the MDL is decreased. The use of MDL indicates even in the presence of multiple scattering and noise, the MDL provides satisfactory results and afterward the MUSIC yields accurate estimates of the target locations.

The remaining of the paper is organized as follows. After the statement and formulation of the problem in Section 2, we introduce fundamental time reversal concepts and theory. Then, we review the theory of subspace signal processing first applied to time reversal imaging. In this section, we develop a generalized version of the MUSIC algorithm. Section 3 introduces the information theoretic criteria for model selection and discusses the application of these criteria to the problem of detecting the number of targets. Simulation results that illustrate the performance of the MDL for both synthetic and real experimental data are described in Section 4.

2. PROBLEM FORMULATION

We present a theoretic model of the multistatic response matrix that is obtained from the measurements made by an array of antennas. The imaging algorithms employ this matrix to detect and localize the targets.

2.1. Multistatic Data Matrix

The imaging system consists of N_t transmitter and N_r receiver antenna arrays centered at known positions denoted by $\mathbf{R}_i^t$ and $\mathbf{R}_i^r$ for the ith ones, respectively. The transmitters are individually excited and generate incident wave fields that propagate into a background medium containing a number of discrete scatterers (targets). The incident wave field generated by the jth source interacts with the scatterers, generates a total wave field (incident plus scattered) and is measured at any frequency ω by the ith receiver as,

$$\psi_j(\mathbf{R}_i^r,\omega) = \psi_j^{inc}(\mathbf{R}_i^r,\omega) + k_0^2(\omega)\int_{\mathcal{R}} G(\mathbf{R}_i^r,\mathbf{r},\omega)O(\mathbf{r},\omega)\psi_j(\mathbf{r},\omega)d\mathbf{r}, \quad (1)$$

where, $k_0(\omega)$ is the wave number of the background medium, $G(\mathbf{r},\mathbf{r}',\omega)$ is the green function between locations $\mathbf{r}$, $\mathbf{r}'$; $\psi_j^{inc}(\mathbf{r},\omega)$ and $\psi_j(\mathbf{r},\omega)$ are respectively the incidence and total waves, measured at location $\mathbf{r}$, and are generated by the jth transmitter. The object is described by the object profile (also known as object distribution function)

$O(\mathbf{r},\omega) = \frac{k^2(\mathbf{r},\omega)}{k_0^2(\omega)} - 1$, where $k(\mathbf{r},\omega)$ is the wave number of the total medium (background plus targets [24]), and $\mathcal{R}$ is corresponded to the surface occupied by the object, where $\mathcal{R} = \{\mathbf{r} \in R^2\}$. From Equation (1), the scattered wave, that is generated by the interaction of the incident wave with the targets, can be written as

$$\begin{aligned}\psi_j^{\text{scatt}}(\mathbf{R}_i^r,\omega) &= \psi_j(\mathbf{R}_i^r,\omega) - \psi_j^{inc}(\mathbf{R}_i^r,\omega)\\ &= k_0^2(\omega)\int_{\mathcal{R}} G(\mathbf{R}_i^r,\mathbf{r},\omega)O(\mathbf{r},\omega)\psi_j(\mathbf{r},\omega)d\mathbf{r}. \qquad (2)\end{aligned}$$

By considering approximations suggested in [12, 25], we can write an $N_r \times N_t$ matrix $\mathbf{K}$, that is called multistatic data matrix, as below,

$$\mathbf{K}(\omega) = k_0^2(\omega)\int_{\mathcal{R}} \mathbf{g}_r(\mathbf{r},\omega)O(\mathbf{r},\omega)\mathbf{g}_t^T(\mathbf{r},\omega)d\mathbf{r}, \qquad (3)$$

whose the (i,j)th element is the ratio of $\psi_j^{\text{scatt}}(\mathbf{R}_i^r,\omega)$ to $e_j(\omega)$, the incident pulse (source excitation) spectrum of the jth source, in which $\psi_j(\mathbf{r},\omega) = e_j(\omega)G(\mathbf{r},\mathbf{R}_j^t,\omega)$. In Equation (3), $\mathbf{g}_t(\mathbf{r},\omega)$, corresponds to the transmitter array, represents a vector associated to the green functions from transmitters to any point in the medium, $\mathbf{r}$, (that known as the green function of the transmitter), formulated as $\mathbf{g}_t(\mathbf{r},\omega) = [G(\mathbf{r},\mathbf{R}_1^t,\omega), G(\mathbf{r},\mathbf{R}_2^t,\omega),\ldots,G(\mathbf{r},\mathbf{R}_{N_t}^t,\omega)]^T$ and the green function of the receiver is $\mathbf{g}_r(\mathbf{r},\omega) = [G(\mathbf{R}_1^r,\mathbf{r},\omega), G(\mathbf{R}_2^r,\mathbf{r},\omega),\ldots,G(\mathbf{R}_{N_r}^t,\mathbf{r},\omega)]^T$ corresponds to the receiver array, where the superscript $[\cdot]^T$ is the Transpose operation. The data matrix $\mathbf{K}$ is a key quantity that is employed to generate an image of the target (the object profile $O(\mathbf{r},\omega)$). We will not explicitly display the frequency variable ω in subsequent equations.

The object profile $O(\mathbf{r})$ consists of D disjoint profiles $O_m(\mathbf{r})$ each centered at a location $\mathbf{X}_m$ and each having an effective size that is small relative to the wave length [12, 24], i.e.,

$$O(\mathbf{r}) = \sum_{m=1}^{D} O_m(\mathbf{r}-\mathbf{X}_m) = \sum_{m=1}^{D} \tau_m\delta(\mathbf{r}-\mathbf{X}_m), \qquad (4)$$

where $\tau_m = \int_{\mathcal{R}} O_m(\mathbf{r})d\mathbf{r}$ is the scattering coefficient of the object.

Substitution of Equation (4) into Equation (3) provides,

$$\begin{aligned}\mathbf{K} &= k_0^2(\omega)\sum_{m=1}^{D}\mathbf{g}_r(\mathbf{X}_m)\mathbf{g}_t^T(\mathbf{X}_m)\int_{\mathcal{R}} O_m(\mathbf{r}-\mathbf{X}_m)d\mathbf{r}\\ &= k_0^2(\omega)\sum_{m=1}^{D}\tau_m\mathbf{g}_r(\mathbf{X}_m)\mathbf{g}_t^T(\mathbf{X}_m). \qquad (5)\end{aligned}$$

When there is multiple scattering between the targets, the jth element of the green function of the receiver, $\mathbf{g}_r(\mathbf{X}_m)$, can be formulated as $G(\mathbf{R}_j^r, \mathbf{X}_m) + \sum_{m'=1, m'\neq m}^{D} \tau_{m'} G(\mathbf{R}_j^r, \mathbf{X}_{m'}) G(\mathbf{X}_{m'}, \mathbf{X}_m)$. The purpose of the imaging algorithms is to investigate of Equation (5) and estimate $\mathbf{X}_m$ (location of the targets).

2.2. TR Imaging by MUSIC Method

The theory of MUSIC is rooted in eigenvector decomposition of the matrix $\mathbf{T} = \mathbf{K}^H\mathbf{K}$ as below [12],

$$\begin{aligned} \mathbf{K}^H\mathbf{K}\mathbf{u}_j &= \gamma_j^2\mathbf{u}_j, \\ \mathbf{K}\mathbf{K}^H\mathbf{w}_j &= \gamma_j^2\mathbf{w}_j, \end{aligned} \tag{6}$$

where $[\cdot]^H$ denotes the conjugate Transpose, $\mathbf{u}_j$, $\mathbf{w}_j$ are respectively the jth eigenvectors of matrices $\mathbf{T}$ and $\mathbf{T}^H$, and γ_j^2 is the jth eigenvalue of $\mathbf{T}$. The matrix $\mathbf{T}$ is known as the time reversal matrix [12], and it is shown [25] that for well-resolved scatterers (scatterers with long enough distance between them), those eigenvectors corresponding to nonzero eigenvalues, span the signal subspace and are associated in a one-to-one manner to the scatterer locations. The noise subspace is proportional to the other eigenvectors. The time reversal invariants can also be directly worked out from the singular value decomposition (SVD) of the matrix $\mathbf{K}$ [25]. Hence the singular vectors of the matrix $\mathbf{K}$ are the eigenvectors of the matrices $\mathbf{T}$ and $\mathbf{T}^H$ and its singular values γ_j, are the square root of the eigenvalues of $\mathbf{T}$.

The MUSIC algorithm is based on orthogonality of the signal and noise subspace spanned by $\mathbf{u}$ and $\mathbf{w}$ [24]. The MUSIC image (or MUSIC Pseudospectrum) at each point $\mathbf{r}$, can be defined as follows [25],

$$P(\mathbf{r}) = \frac{1}{\sum_{j=D+1}^{\min(N_t,N_r)} \left(\left| \mathbf{u}_j^T \mathbf{g}_t(\mathbf{r}) \right| + \left| \mathbf{w}_j^H \mathbf{g}_r(\mathbf{r}) \right| \right)}, \tag{7}$$

where, $\mathbf{g}_t(\mathbf{r})$, $\mathbf{g}_r(\mathbf{r})$ are respectively the green functions of transmitters and receivers at each point $\mathbf{r}$, and D is the number of targets which is proportional to the number of non-zero eigenvalues. The target locations are corresponded to the poles of $P(\mathbf{r})$, so it will have distinct peaks at the scatterer locations (i.e., $\mathbf{X}_m$).

3. STATISTICAL PROCESSING OF EIGENVALUE

The performance of the MUSIC will be degraded in environments with noise and multiple reflections between targets. In such a situation, the magnitudes of the eigenvalues are close to each other and it is difficult to separate signal and noise eigenvalues. Any mixing of the eigenvalues makes some targets are either missed or falsely added to the pseudospectrum. In order to solve this problem, we use a modified version of MDL algorithm to determine the number of targets (i.e., number of signal eigenvalues) and then apply MUSIC procedure to form pseudospectrum image. Conventional MDL methods work with time independent components [22, 26], whereas our MDL algorithm works with independent sources.

3.1. MDL Method

Consider a sensor array of N elements by which we acquire M observations $\{\mathbf{x}_m | m = 1, \ldots, M\}$ in which $\mathbf{x}_m$ is an N dimensional vector. Each $\mathbf{x}_m$ is a linear transformation of D dimensional source vector $\mathbf{s}_m$, plus noise vector $\mathbf{v}_m$, i.e.,

$$\mathbf{x}_m = \mathbf{A}\mathbf{s}_m + \mathbf{v}_m, \tag{8}$$

where $\mathbf{A} \in \mathcal{C}^{N\times D}$, the steering matrix, is composed of D linearly independent column vectors of array response $\{\mathbf{a}_k | k = 1, \ldots, D\}$ and $D < N$. It is assumed that the noise is white Gaussian.

In order to estimate the number of independent sources D, the eigenvalues of the correlation matrix $\mathbf{R} = E(\mathbf{x}\mathbf{x}^H)$ are used. This matrix is approximated as $\widehat{\mathbf{R}} = \frac{1}{M}\sum_{m=1}^{M} \mathbf{x}_m\mathbf{x}_m^H$. The eigen-decomposition of $\widehat{\mathbf{R}}$ is,

$$\widehat{\mathbf{R}}\mathbf{v}_i = \lambda_i \mathbf{v}_i. \tag{9}$$

This decomposition includes D larger eigenvalues $\lambda_1 > \ldots > \lambda_D$ and the remaining $N - D$ eigenvalues are theoretically equal, i.e., $\lambda_{D+1} = \ldots = \lambda_N = \sigma^2$, where σ^2 is the variance of the noise. The number of targets D, is determined as the value of $d \in \{0, \ldots, \min(N_t, N_r) - 1\}$ that minimizes the MDL criterion as below [23, 27],

$$\mathrm{MDL}(d) = M(N-d)\log\left(\frac{\alpha_d}{g_d}\right) + \frac{1}{2}d(2N-d)\log(M), \tag{10}$$

where,

$$\alpha_d = \frac{1}{N-d}\sum_{i=d+1}^{N}\lambda_i, \tag{11}$$

$$g_d = \prod_{i=d+1}^{N}\lambda_i^{\left(\frac{1}{N-d}\right)}. \tag{12}$$

The first term in Equation (10) is obtained from ML (maximum likelihood) criterion while the second one is a penalty function that is based on the number of free parameters in the model [22].

3.2. Estimating the Number of Scatterers Using MDL

In microwave imaging, the available data, at each frequency ω, is a noisy matrix $\mathbf{K}^n$ as follows,

$$\mathbf{K}^n(\omega) = \mathbf{K}(\omega) + \mathbf{N}(\omega), \tag{13}$$

in which $\mathbf{K}(\omega)$ is multistatic data matrix, and $N = [n_1, n_2, \ldots, n_{N_r}]^T$ is the additive noise. This model is well matched to MDL, as it can be inferred from Equation (5). Let $\mathbf{s}_j = [\tau_1\mathbf{G}(\mathbf{X}_1, \mathbf{R}_j^t, \omega), \tau_2\mathbf{G}(\mathbf{X}_2, \mathbf{R}_j^t, \omega), \ldots, \tau_D\mathbf{G}(\mathbf{X}_D, \mathbf{R}_j^t, \omega)]^T$, $j = 1, \ldots, N_t$ be a source signal (here is a scatterer signal). According to Equation (5), the jth column of $\mathbf{K}$ can be given by $\mathbf{k}_j = \mathbf{A}\mathbf{s}_j$, where $\mathbf{A}$ is an $N_r \times D$ matrix whose qth column is $a_q = k_0^2[\mathbf{G}(\mathbf{R}_1^r, \mathbf{X}_q), \mathbf{G}(\mathbf{R}_2^r, \mathbf{X}_q), \ldots, \mathbf{G}(\mathbf{R}_{N_r}^r, \mathbf{X}_q)]^T$, $q = 1, \ldots, D$. It is straightforward to extend this model when there is multiple scattering between the targets. This indicates that Equation (13) is compatible with the MDL model in Equation (8).

The above discussion suggests that the number of scatterers D might be estimated from $\mathbf{K}^n$ using MDL. In practice, the data (i.e., the column of $\mathbf{K}^n$) are available from different transmitters as well as different frequency components. This paper develops a version of MDL that uses the data due to different transmitters so as to decrease the error in the estimation of D. This is a correct idea if sources be far enough from each other. Because the signals at the scatterer locations are linear combination of the sources signals, separate transmitters will lead to independent ones and this results independent signals. To apply MDL on multistatic response matrix, it can rewrite as

$$\begin{aligned}\mathbf{K}^n &= \left[\mathbf{k}_1^n, \mathbf{k}_2^n, \ldots, \mathbf{k}_{N_t}^n\right],\\ \mathbf{k}_i^n &= \left[k_{i1}^n, k_{i2}^n, \ldots, k_{iN_r}^n\right]^T,\end{aligned} \tag{14}$$

where, $\mathbf{k}_i^n$ is a vector with dimension, $N_r \times 1$.

The noise is zero mean and independent of the signals, so the covariance matrix of $\mathbf{K}^n$, can be written as

$$\mathbf{R} = \Psi + \sigma^2 \mathbf{I}, \tag{15}$$

where $\mathbf{I}$ is the identity matrix and Ψ denoting the covariance matrix of the signals, i.e., $\Psi = E[\mathbf{K}\mathbf{K}^H]$.

Denoting the parameter vector of the model by θ, it follows from [22] that unknown parameters of θ are given by,

$$\theta = \left[\lambda_1, \ldots, \lambda_d, \sigma^2, \mathbf{v}_1^T, \ldots, \mathbf{v}_d^T\right]^T, \tag{16}$$

where $\lambda_1, \ldots, \lambda_d$ and $\mathbf{v}_1, \ldots, \mathbf{v}_d$ are the eigenvalues and eigenvectors of $\mathbf{R}$, respectively, and $d \in \{0, 1, \ldots, N_r - 1\}$ is the rank of $\mathbf{R}$.

Since the observations are regarded as statistically independent Gaussian random vectors with zero mean, their joint probability density function for independent transmitters is given by,

$$f\left(\mathbf{K}_1^n, \mathbf{K}_2^n, \ldots, \mathbf{K}_{N_t}^n | \theta\right) = \prod_{i=1}^{N_t} \frac{1}{\pi^{N_r} \det(\mathbf{R})} \exp\left(-(\mathbf{K}_i^n)^H \mathbf{R}^{-1} \mathbf{K}_i^n\right). \tag{17}$$

With this parametrization, the log-likelihood function $L(\theta)$ is given by,

$$L(\theta) = -N_t \log\left(\det(\mathbf{R})\right) - \operatorname{tr}\left(\mathbf{R}^{-1}\widehat{\mathbf{R}}\right), \tag{18}$$

where tr denotes the trace of a matrix and $\widehat{\mathbf{R}}$ is the sample covariance matrix defined by

$$\widehat{\mathbf{R}} = \frac{1}{N_t} \sum_{i=1}^{N_t} \mathbf{K}_i^n (\mathbf{K}_i^n)^H, \tag{19}$$

The maximum likelihood estimation is then the value of θ which maximizes Equation (18). Following Wax [22], we obtain

$$L(\theta) = \log \left(\frac{\prod_{i=d+1}^{N_r} \widehat{\lambda}_i^{\frac{1}{N_r - d}}}{\frac{1}{N_r - d} \sum_{i=d+1}^{N_r} \widehat{\lambda}_i} \right)^{(N_r - d)N_t}, \tag{20}$$

where $\widehat{\lambda}_1 > \ldots > \widehat{\lambda}_{N_r}$ are the eigenvalues of the sample covariance matrix $\widehat{\mathbf{R}}$. Applying penalty function, the form of MDL for this problem is therefore given by,

$$\mathrm{MDL}(d) = -L(\theta) + \frac{1}{2} d(2N_r - d)\log(N_t). \tag{21}$$

The estimate of D is the value of d that minimizes Equation (21).

4. APPLICATION EXAMPLES

The algorithm is applied to both synthetic and real experimental data so as to investigate its performance.

4.1. Results Employing Synthetic Data

The synthetic data is generated by method of moments (MOM) for 2-D TM (transverse magnetic) electromagnetic wave incident. The simulated ensemble employs a uniform linear array consisting of 12 transmitters and 22 receivers with equal space of 0.5λ (that λ represent wavelength). The probing environment is an air-filled $100 \times 100\,\text{cm}$ rectangular area in which different configurations of scatterers will

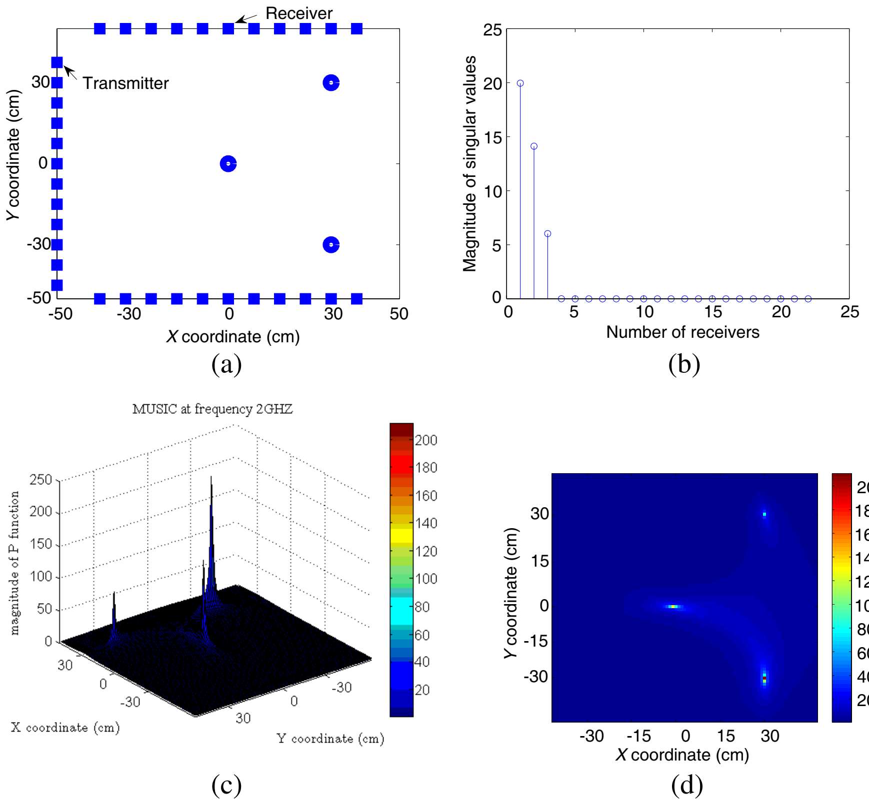

Figure 1. (a) Simulated ensemble, (b) singular values, (c) pseudospectrum and (d) tomography image, obtained from noise free data at 2 GHz.

be arranged. The dielectric targets are filled dielectric cylinders, with circular cross section of radius $a = 15\,\text{mm}$ with the relative permittivity of $\varepsilon_r = 3$. Infinite unit current lines serve as the transmitters, which radiate wave into the environment. The data collection scenario to form each column of $\mathbf{K}$ matrix is as follows: a transmitter corresponding to a specific column radiates wave and the scattering electric fields at the receivers form that column.

For the first configuration, we insert 3 point targets in the environment, which are centered at [0 0], $[2\lambda\ 2\lambda]$ and $[2\lambda\ -2\lambda]$ in rectangular coordinates, as shown in Figure 1(a) and obtain multistatic response matrix at 2 GHz. When there is no noise, Figure 1(b) illustrates the existence of 3 non-zero singular values. Running the MUSIC for three targets, Figures 1(c), 1(d) show pseudospectrum and

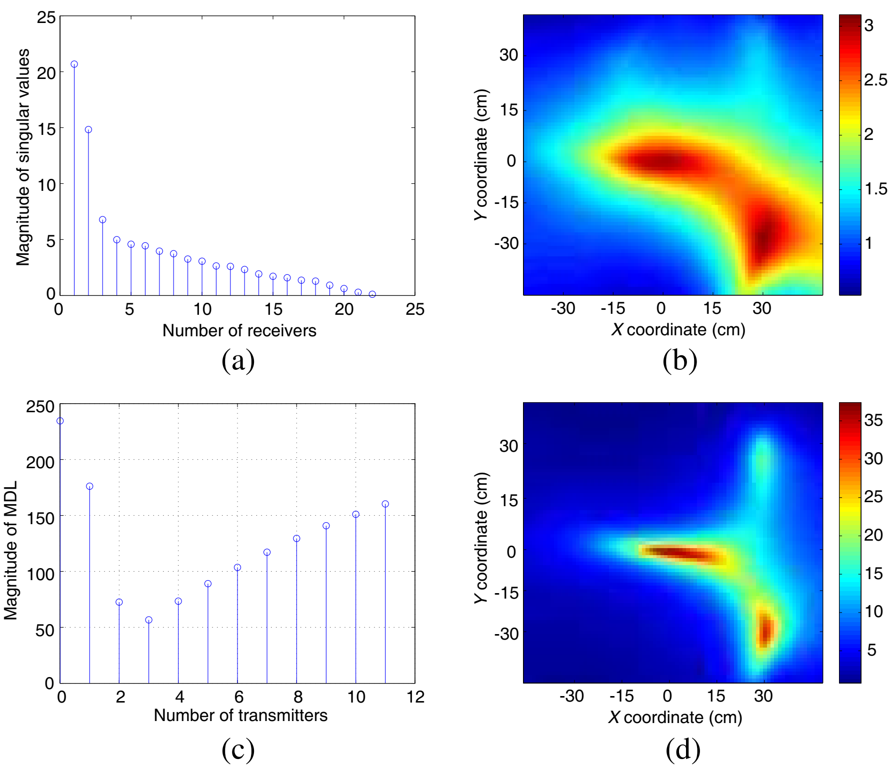

Figure 2. Results of the data with $SNR = 4\,\text{dB}$. (a) Singular values and (b) tomography image when MUSIC is run for the number targets = 2, (c) the MDL criterion for various number of targets, detecting 3 targets and (d) tomography image after applying the MDL algorithm.

tomography image of the received signal. It can be seen that MUSIC can exactly detect all positions if the number of targets is known in advance. Figure 2(a) shows that the performance of MUSIC can be degraded, when we have noise in the system. In this case, the noise is additive Gaussian with $SNR = 4\,\mathrm{dB}$. The singular values are close to each other and hence, different target numbers can be inferred. In Figure 2(b), this number is set to 2 (according to Figure 2(a)) and MUSIC can detect only 2 target positions. Applying MDL to the singular values of multistatic response matrix, the MDL criterion (Equation (21)) reaches the minimum at $d = 3$ (Figure 2(c)) and using this in MUSIC will locate the targets reasonably (Figure 2(d)).

The performance of the estimation algorithms should be analyzed in different noisy situations. This can be done by applying various noisy matrix, $\mathbf{K}^n$, to the algorithms. Because the added noise ought to be modeled as a stochastic process, the noisy matrix, $\mathbf{K}^n$, is generated randomly several times and the probability of correct detection of sources, p_D, (which is the percentage of the experiments in which the number of targets are correctly estimated) is obtained at each SNR. The usual estimation of the target numbers in MUSIC is according to major eigenvalues [12]. These are the eigenvalues that the least of them is greater than the twice of the next one and there is no other eigenvalues with this feature after it. Figure 3 indicates the variation of p_D against SNR and illustrates the superiority of MDL over Usual estimation.

To study the performance of the algorithms with respect to scatterer distances, the simulations are carried out for two targets with

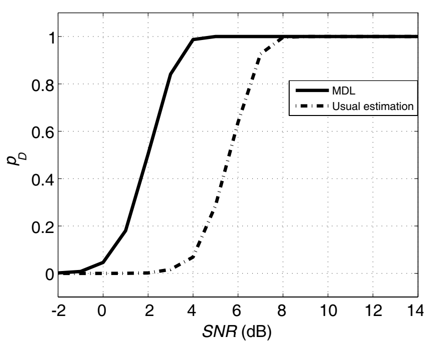

Figure 3. Probability of correct estimation of the targets by usual estimation and MDL at 2 GHz.

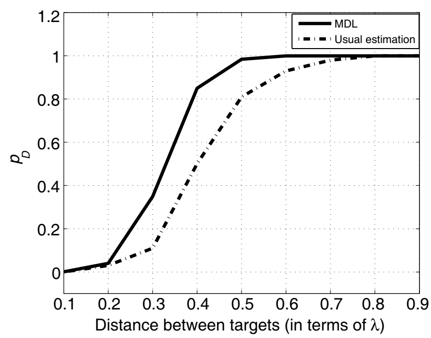

Figure 4. Probability of correct estimation as a function of distance between the targets at 2 GHz and $SNR = 5\,\mathrm{dB}$.

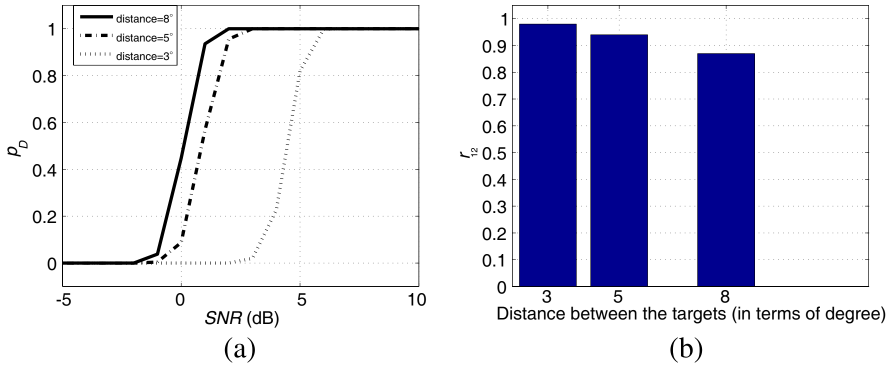

Figure 5. (a) Probability of correct estimation as a function of distance between the transmitters at 2 GHz, (b) correlation coefficients.

different separations. Figure 4 shows p_D as a function of the distance in terms of wavelength (λ) at $SNR = 5\,\text{dB}$. It is observed that the increase of the distance makes the reduction of the multiple reflections between targets and thus, the signal eigenvalues separate well from the noise ones and two estimation algorithms behave the same at large distances. However, the MDL algorithm can resolve the location of the targets better than usual estimation, at short distances.

As it mentioned, independence between the transmitters is a fundamental requirement in MDL algorithm. But in the most data gathering systems, because of small space between the transmitters, this constraint may not be established, exactly. In this case, performance of MDL will impair for low SNRs. This can be seen in Figure 5(a) for a system with 20 transmitters on a circle with different space between them in terms of degree. It is evidence that transmitters with larger space have better performance with MDL algorithm. We solve the constraint of non-independent transmitters by grouping them. It means that signals of some near transmitters (columns of matrix $\mathbf{K}^n$) are replaced by their average. It can also reduce somewhat the noise of the data. We do grouping by averaging of any two adjacent columns of the matrix $\mathbf{K}^n$. To prove this, we compare correlation coefficients of two adjacent columns of $\mathbf{K}^n$ (that are proportional to two adjacent transmitters) in both nongrouped and grouped modes. the correlation coefficient can be calculated for the ith and jth columns as below,

$$r_{i,j} = \frac{(\widetilde{\mathbf{k}}_i^n)^H \widetilde{\mathbf{k}}_j^n}{\|\widetilde{\mathbf{k}}_i^n\| \, \|\widetilde{\mathbf{k}}_j^n\|}, \tag{22}$$

in which $\|\cdot\|$ is the norm operation and $\widetilde{\mathbf{k}}_i^n = \mathbf{k}_i^n - \text{mean}(\mathbf{k}_i^n)$. Smaller amount of $r_{i,j}$ represent fewer correlation between the ith and jth sources.

In Figure 5(b) we show $r_{1,2}$ (the correlation coefficient between columns 1 and 2) for three states of transmitters. It is clear that larger distance between the transmitters results in smaller correlation coefficient, which means that they are more independent. In Figure 6, performance of MDL in normal and grouped modes is compared for different SNRs, when the distance between the transmitters is 3°. As we expected, MDL with grouped transmitters has better operation even at low SNRs, whereas in normal mode this algorithm has no precise decisions up to 6 dB.

4.2. Results Employing Experimental Data

Experimental data are gathered from CCRM lab. at Marseille, France [28]. The experimental setup consists of a large anechoic chamber, 14.50 m long, 6.50 m wide and 6.50 m hight, with a set of three positioners to adjust antennas or target positions [28]. In this construction, the number of transmitters and receivers are equal to 36 and 49, respectively, so **K** is an 49×36 matrix. Transmitters are on a circle with radius of 720 mm and rotate from 0° to 350° in steps of 10°, so the receivers rotate from 60° to 300° in steps of 5° on a circle with radius of 760 mm. The frequency ranges from 1 GHz to 4 GHz with step of 1 GHz. The dielectric targets are two filled dielectric cylinders, with $\epsilon_r = 3 \pm 0.3$, having circular cross section of radius $a = 15$ mm

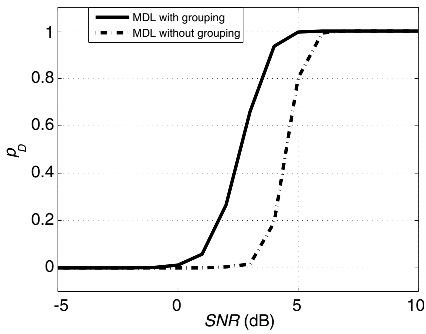

Figure 6. Probability of correct estimation when the distance between the transmitters is 3° with grouping and without grouping (at 2 GHz).

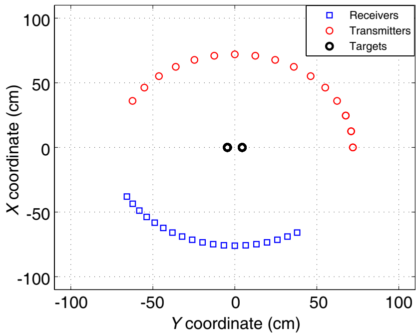

Figure 7. True position of transmitters, receivers and targets.

and placed about 45 mm from the center of axis.

The columns of matrix **K** correspond to the received signals due to different transmitters so that the receiver set does not change from column to column, i.e., from transmitter to another transmitter. This is not compatible exactly with the data collection described above [28]. For our purpose, the data of the transmitters that are corresponded to common receivers are used to form **K**. In this way, several **K** matrices can be generated with respect to different view angles, e.g., Figure 7 shows the transmitter-receiver structure for 75° view angle. We employ the 19×16 matrices with four view angles of 75°, 165°, 225° and 315° and compute the MDL in Equation (21) for them separately and, then, the candidate d is the one that minimizes the sum of four MDLs.

In Figure 8, we compare MDL operation for raw and common data. Figures 8(a), 8(b) show the magnitude of MDL for these data.

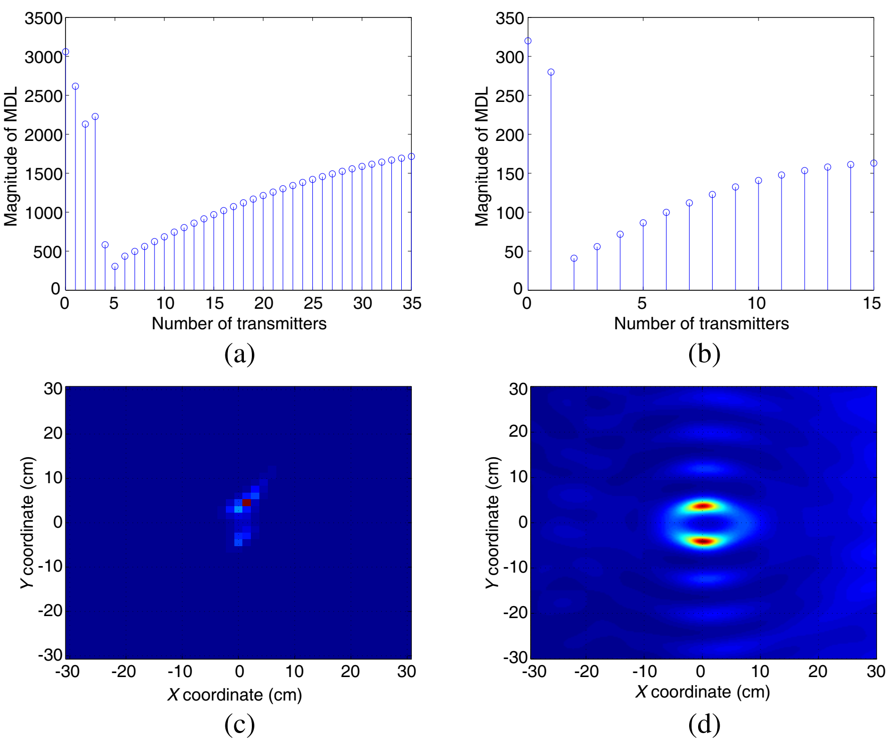

Figure 8. Experimental data: the MDL criterion for (a) raw data, (b) common data, and tomography image for (c) raw data, (d) common data, all at 2 GHz.

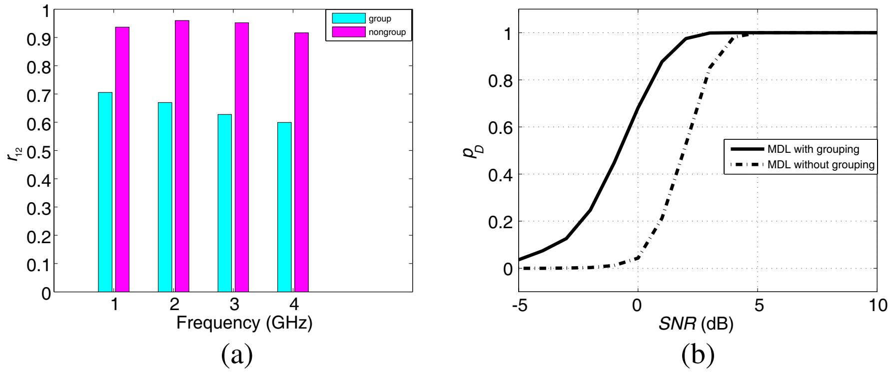

Figure 9. Experimental data: (a) correlation coefficients, (b) probability of correct estimation of the targets.

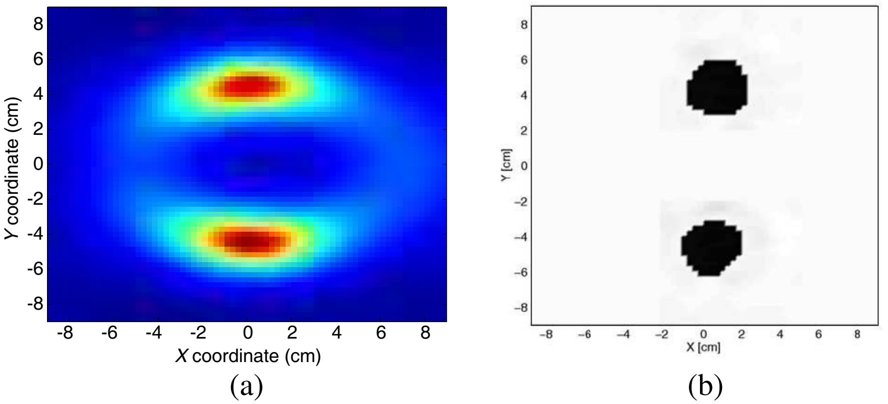

Figure 10. A comparing between (a) our results and (b) results in [28].

The result of applying the estimated numbers to the MUSIC algorithm is shown in Figures 8(c), 8(d). As it expected, the target locations accurately have been determined by the common data.

The correlation coefficients of the grouped **K** which is a 19×8 matrix is shown in Figure 9(a) as a function of frequency. It is clear that correlation coefficients for grouped mode are smaller than the normal state. In Figure 9(b), performance of MDL in normal and grouped modes is compared for different SNRs. As we expected, MDL with grouped transmitters has better operation even at low SNRs.

Figures 10(a), 10(b) compare the image of MUSIC+MDL method with what is obtained in [28] from the same real data. The images are, nearly, the same; nevertheless, the MUSIC+MDL method needs about 50 seconds that indicates it can do the microwave imaging in real time.

5. CONCLUSIONS

The MUSIC algorithm can significantly improve the resolution of the time reversal. A problem associated with this application is the estimation of the number of scatterers in presence of noise and multiple scattering between targets. In this paper, we show that the mathematical model behind the scattering from the small objects is well compatible with the minimum description length (MDL) model. This leads us to use the MDL so as to estimate the number of scatterers before application of the MUSIC algorithm. The paper, also, demonstrates that grouping the nearby wave sources enhances the independency of the sources and thus, the estimation error of the MDL is decreased. The use of MDL indicates even in the presence of multiple scattering and noise, the MDL provides satisfactory results and afterward the MUSIC yields accurate estimates of the target locations. Future work will be devoted to extending the method not only in freespace imaging, but also to subsurface imaging and through wall imaging problems.

REFERENCES

1. Massa, A., G. Franceschini, M. Donelli, and R. Azaro, "Inversion of phaseless total field data using a two step strategy based on the iterative multi scaling approach," *IEEE Trans. on Geoscience and Remote Sensing*, Vol. 44, No. 12, 3527–3539, 2006.
2. Donelli, M., I. J. Craddock, D. Gibbins, and M. Sarafianou, "A three-dimensional time domain microwave imaging method for breast cancer detection based on an evolutionary algorithm," *Progress In Electromagnetics Research M*, Vol. 18, 179–195, 2011.
3. Takenaka, T., H. Jia, and T. Tanaka, "Microwave imaging of electrical property distributions by a forward-backward time-stepping method," *Journal of Electromagnetic Waves and Applications*, Vol. 14, No. 12, 1609–1626, 2000.
4. Massa, A., D. Franceschini, M. Donelli, and P. Rocca, "Three dimensional microwave imaging problems solved through an efficient multi scaling particle swarm optimization," *IEEE Trans. on Geoscience and Remote Sensing*, Vol. 47, No. 5, 1467–1481, 2009.

5. Weedon, W. H., W. C. Chew, and P. E. Mayes, "A step-frequency radar imaging system for microwave nondestructive evaluation," *Progress In Electromagnetics Research*, Vol. 28, 121–146, 2000.
6. Liu, D. H., J. Krolik, and L. Carin, "Electromagnetic target detection in uncertain media: Time-reversal and minimum variance algorithms," *IEEE Trans. on Geoscience and Remote Sensing*, Vol. 45, 934–944, 2007.
7. Prada, C., S. Mannevile, D. Spoliansky, and M. Fink, "Decomposition of the time reversal operator: Detection and selective focusing on two scatterers," *J. Acoust. Soc. Amer.*, Vol. 99, 2067–2076, 1996.
8. Chambers, D. H. and J. G. Berryman, "Analysis of the time-reversal operator for a small spherical scatterer in an electromagnetic field," *IEEE Trans. Antennas Propag.*, Vol. 52, 1729–1738, 2004.
9. Zhu, X., Z. Zhao, W. Yang, Y. Zhang, Z.-P. Nie, and Q. H. Liu, "Iterative time-reversal mirror method for imaging the buried object beneath rough ground surface," *Progress In Electromagnetics Research*, Vol. 117, 19–33, 2011.
10. Marengo, E. A., F. K. Gruber, and F. Simonetti, "Time-reversal MUSIC imaging of extended targets," *IEEE Trans. Image Proc.*, Vol. 16, 1967–1984, 2007.
11. Zhang, W., A. Hoorfar, and L. Li, "Through-the-wall target localization with time reversal music method," *Progress In Electromagnetics Research*, Vol. 106, 75–89, 2010.
12. Lehman, S. K. and A. J. Devaney, "Transmission mode time-reversal super resolution imaging," *J. Acoust. Soc. Amer.*, Vol. 113, 2742–2752, 2003.
13. Lev-Ari, H. and A. J. Devaney, "The time reversal techniques reinterpreted: Subspace-based signal processing for multistatic target location," *IEEE Sensor Array Multichannel Signal Proc.*, Workshop, 509–513, 2000.
14. Yavuz, M. E. and F. L. Teixeira, "Full time-domain DORT for ultrawideband fields in dispersive, random inhomogeneous media," *IEEE Trans. Antennas Propag.*, Vol. 54, 2305–2315, 2006.
15. Zhao, H., "Analysis of the response matrix for an extended target," *SIAM J. Appl. Math.*, Vol. 64, 725–745, 2004.
16. Yavuz, M. E. and F. L. Teixeira, "On the sensitivity of time-reversal imaging techniques to model perturbations," *IEEE Trans. Antennas Propag.*, Vol. 56, 834–843, 2008.

17. Davy, F., J.-G. Minonzio, J. de Rosny, C. Prada, and M. Fink, "Influence of noise on subwavelength imaging of two close scatterers using time reversal method: Theory and experiments," *Progress In Electromagnetics Research*, Vol. 98, 333–358, 2009.
18. Anderson, T. W., "Asymptotic theory for principal component analysis," *Ann. J. Math. Stat.*, Vol. 34, 122–148, 1963.
19. Akaike, H., "A new look at the statistical model identification," *IEEE Trans. Automat. Contr.*, Vol. 19, No. 6, 716–723, 1974.
20. Rissanen, J., "Modeling by shortest data description," *Automatira*, Vol. 14, 465–471, 1978.
21. Hannan, E. J., "The determination of the order of an autoregression," *J. Roy. Srar. Soc. Bvol.*, Vol. 41, No. 2, 190–195, 1979.
22. Wax, W. and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. on Acoustic, Speech, and Signal Proc.*, Vol. 33, 387–392, 1985.
23. Haddadi, F., "Statistical performance analysis of mdl source enumeration in array processing," *IEEE Trans. on Signal Proc.*, Vol. 58, 452–457, 2010.
24. Devaney, A. J., "Super-resolution processing of multi-static data using time reversal and MUSIC," Northeastern University Report, available at http://www.ece.neu.edu/faculty/devaney/ajd/preprints.htm.
25. Marengo, E. A. and F. K. Gruber, "Subspace-based localization and inverse scattering of multiply scattering point targets," *EURASIP Journal on Advances in Signal Processing*, 1–16, 2007.
26. Fishler, E. and H. V. Poor, "Estimation of the number of sources in unbalanced array via information theoretic criteria," *IEEE Trans. on Signal Proc.*, Vol. 53, 3543–3553, 2005.
27. Fishler, E., M. Grossmann, and H. Messer, "Detection of signals by information theoretic criteria: General asymptotic performance analysis," *IEEE Trans. on Signal Proc.*, Vol. 50, 1027–1036, 2002.
28. Belkebir, K. and M. Saillard, "Special section: Testing inversion algorithms against experimental data," *Inverse Problems*, Vol. 17, 1565–1571, 2001.