OPTIMUM DESIGN OF MODIFIED SCHIFFMAN MULTI-SECTION WIDE BAND DIFFERENTIAL PHASE SHIFTER WITH IMPEDANCE MATCHING

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Abstract—In this paper, a new circuit configuration composed of two main and reference paths is proposed for the modified multisection Schiffman phase shifter, which improves its bandwidth. The method of least squares (MLS) is developed for its design, based on a circuit model including dispersion relations and dissipation effects. The differential phase shift and reflection coefficients from the input ports are obtained by conversions among the impedance, admittance, ABCD and scattering matrices of the circuit model. Then, an error function is constructed, whose minimization is performed by the combination of genetic algorithm and conjugate gradient method, which gives the optimum values of the physical dimensions of the metallic strips. The impedance matching function of the proposed phase shifter circuit makes it a dual action device, leading to some circuit miniaturization. Three phase shifter circuits with single, double and quadruple sections are designed and one prototype model is fabricated and measured. The MLS. full-wave simulation softwares and measurement results agree very well, which validate the proposed circuit configuration and MLS design procedure. The multi-section phase shifters have actually increased the frequency bandwidth.

1. INTRODUCTION

Before the advent of variable electronic phase shifters in 1950 [1], all the practical fixed and variable phase shifters were mainly mechanical devices [2]. Then both passive and active phase shifter configurations

Received 27 February 2012, Accepted 6 April 2012, Scheduled 18 April 2012

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were devised for strip line and microstrip circuits [3]. There have been some attempts to devise phase shifters with wide bandwidths around a center frequency. Schiffman proposed a microstrip circuit configuration for phase shifters, with relatively flat bandwidth, where the lengths of strips were equal to $\lambda/2$ at the center frequency.

In this paper, we proposed a design procedure based on the method of least squares for a modified wide band Schiffman phase shifter, which also incorporates input to output impedance matching. Since the same microstrip structure is used for the realization of both phase shifting and impedance matching functions, the overall size of the circuit will be drastically reduced. We use the circuit model of the modified Schiffman phase shifter with the application of dispersion relations to construct an error function. Its minimization is performed by the combination of genetic algorithm (GA) and conjugate gradient (CG) method. Then, the optimum design values for the lengths and widths of strip lines are determined. The accuracy of the proposed design procedure for Schiffman phase shifters is verified by the available full-wave simulation softwares and actual fabrication and measurement of a prototype device.

2. NUMERICAL PROCEDURE

We develop a numerical procedure for the optimum design of a modified Schiffman multi-section differential phase shifter with the implementation of impedance matching, which is based on the evenand odd-mode analysis. We first obtain the impedance matrix of the Schiffman phase shifter composed of short circuited coupled lines. The total impedance matrix of the N-section phase shifter is obtained step by step by first starting from the single section phase shifter and subsequently adding a new section to the former circuit. The n the scattering matrix of the phase shifter is derived from the impedance matrix by the appropriate transformation relations. Then, the method of list squares is used to construct an error function from the scattering parameters over the desired frequency bandwidth, including the terms concerning the specified phase shift and the required impedance matching functions. Finally, the error function is minimized by a hybrid procedure as the combination of the genetic algorithm and conjugate gradient method to benefit from the distinct advantages of each algorithm, which leads to the determination of microstrip widths and lines. The proposed configuration for the phase shifter implements impedance matching as well as broadband phase shifting characteristics.

3. SINGLE SECTION PHASE SHIFTER

The basic Schiffman phase shifter composed of two coupled lines is shown in Fig. 1. Ports 1 and 2 are the input and output ports, respectively and the other two ports are connected together [4]. Its impedance matrix is obtained by the even-and odd-mode analysis [5]

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{-j(Z_{0e} \cot g\theta_e - Z_{0o} \tan \theta_o)}{2} & \frac{-j(Z_{0e} \cot g\theta_e + Z_{0o} \tan g\theta_o)}{2} \\ \frac{-j(Z_{0e} \cot g\theta_e + Z_{0o} \tan g\theta_o)}{2} & \frac{-j(Z_{0e} \cot g\theta_e - Z_{0o} \tan g\theta_o)}{2} \end{bmatrix}$$
(1)

where Z_{0o} and Z_{0e} are the odd- and even-mode characteristic impedances, and the electrical lengths for the odd-and even-modes are

$$\theta_e = \beta_e(l_i + \Delta l_i) = 2\pi f \sqrt{\mu_0 \varepsilon_{effe}(f) \varepsilon_0 (l_i + \Delta l_i)}$$

$$\theta_o = \beta_o(l_i + \Delta l_i) = 2\pi f \sqrt{\mu_o \varepsilon_{effo}(f) \varepsilon_0 (l_i + \Delta l_i)}$$
(2)

The dispersion relations for permittivity are also used [6]. The ABCD and scattering matrices are then obtained [7,8]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{Z_{11}Z_{22}-Z_{21}Z_{12}}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$
(3)

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{AZ_L + B + CZ_L Z_S^* - DZ_S^*}{AZ_L + B + CZ_L Z_S + DZ_S} & \frac{2\sqrt{R_S R_L}}{AZ_L + B + CZ_L Z_S + DZ_S} \\ \frac{2\sqrt{R_S R_L}}{AZ_L + B + CZ_L Z_S + DZ_S} & \frac{AZ_L + B + CZ_L Z_S^* - DZ_S^*}{AZ_L + B + CZ_L Z_S + DZ_S} \end{bmatrix} (4)$$

where Z_s and Z_l are the source and load impedances, respectively. The modified Schiffman phase shifter composed of a main path (from port 1 to port 2) and reference path (from port 3 to port 4) is shown in Fig. 2. The differential phase difference between the main and reference paths are:

$$\Delta \varphi = \angle S_{12} - \angle S_{34} \tag{5}$$



Figure 1. Single section Schiffman phase shifter.



Figure 2. Dimensions of the configuration of the proposed single section Schiffman phase shifter.

Now, for the design of modified Schiffman phase shifter we construct the following error function

$$error = W_{1} \sum_{f_{k}=f_{1} \text{ GHz}}^{f_{2} \text{ GHz}} |\Delta \varphi_{f_{k}} - \varphi_{g}|^{2} + W_{2} \sum_{f_{k}=f_{1} \text{ GHz}}^{f_{2} \text{ GHz}} |S_{11f_{k}} - S_{11g}|^{2} + W_{3} \sum_{f_{k}=f_{1} \text{ GHz}}^{f_{2} \text{ GHz}} |S_{33f_{k}} - S_{33g}|^{2}$$
(6)

where the first term is for the realization of specified phase shift φ_g , and the second and third terms are for the minimization of reflection coefficients (S_{11} and S_{33}) at the input and output ports, which ensure the impedance matching of the circuit. The weighting factors are W_1 , W_2 and W_3 for the relative enhancement of the three terms in the error function. The required bandwidth between frequencies f_1 and f_2 is divided into K discrete frequencies. The minimization of error function gives the dimensions of phase shifter geometry, namely W_{p1} , S_{p1} , L_{p1} , W_{r1} , S_{r1} and L_{r1} as indicated in Fig. 2.

4. DESIGN EXAMPLE 1

The design specifications of a single section phase shifter are: $f_1 = 6 \text{ GHz}$, $f_2 = 8 \text{ GHz}$, $\varphi_g = 50^\circ$, $S_{11} = 10 \text{ dB}$, $S_{33} = 10 \text{ dB}$, K = 201, $W_1 = 1$, $W_2 = 1$, $W_3 = 1$, $Z_s = 75 \Omega$, $Z_l = 100 \Omega$. The substrate

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selected for the microstrip circuit is Rogers Ro5880, with $\varepsilon_r = 2.2$, $h = 0.7874 \,\mathrm{mm}$ and loss tangent = 0.0009. Some constraints are also specified on the lengths and widths of strips, so that their fabrications are possible by available technology. That is, $1 \,\mathrm{mm} < L < 100 \,\mathrm{mm}$, $3 \,\mu\mathrm{m} < w < 10 \,\mathrm{mm}$ and $3 \,\mu\mathrm{m} < s < 10 \,\mathrm{mm}$. The optimum design values for lengths and widths of strips are listed in Table 1. The frequency responses of the phase shift achieved by the MLS algorithm and that obtained by the HFSS full-wave simulation software are drawn

Table 1. The optimum values of the geometry of modified Schiffman phase shifter.

After Optimization	L (optimum)	S (optimum)	W (optimum)
r1	$10.129\mathrm{mm}$	$1194.6\mu\mathrm{m}$	$635.669\mu\mathrm{m}$
<i>p</i> 1	$16.7398\mathrm{mm}$	$1009.64\mu\mathrm{m}$	$502.218\mu\mathrm{m}$



Figure 3. Frequency response of phase shift $\Delta \varphi$ of example 1.



Figure 4. Scattering parameters S_{11} and S_{33} of example 1.



Figure 5. Scattering parameters S_{12} and S_{34} of example 1.

in Fig. 3. Any discrepancy between the two curves is due to the approximations assumed in the numerical procedure and the ideal short circuits and neglecting the bends in strips. The amplitudes of reflection coefficients at ports 1 and 3, namely $|S_{11}|$ and $|S_{33}|$, are drawn in Fig. 4, as obtained by the MLS design and HFSS, with good agreement and also good impedance matching.

The amplitudes of transmission coefficients, namely $|S_{12}|$ and $|S_{34}|$, as obtained by MLS algorithm and HFSS are drawn in Fig. 5, which indicate low transmission losses.

5. DOUBLE TWO-SECTION PHASE SHIFTER

The circuit configuration of a two-section phase shifter is shown in Fig. 6, which is composed of the main and reference paths.

The analysis of this circuit proceeds by first obtaining the admittance matrix of the simple single section Schiffman phase shifter

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{Z_{11}Z_{22}-Z_{12}Z_{21}} & \frac{-Z_{12}}{Z_{11}Z_{22}-Z_{12}Z_{21}} \\ \frac{-Z_{21}}{Z_{11}Z_{22}-Z_{12}Z_{21}} & \frac{Z_{11}}{Z_{11}Z_{22}-Z_{12}Z_{21}} \end{bmatrix}$$
(7)

Then the admittance matrix of the two section circuit configuration as the main path between ports 1 and 2 is

$$[Y_P] = [Y_{P1}] + [Y_{P2}] \tag{8}$$

where $[Y_{p1}]$ is the admittance matrix of the upper branch of the main path from input to output ports 1 and 2, with the set of parameters W_{p1} , S_{p1} , L_{p1} , and $[Y_{p2}]$ is the admittance matrix of the lower branch of the main path between ports 1 and 2 with the set of dimensions



Figure 6. Dimensions of the configuration of the proposed double two-section Schiffman phase shifter.

 W_{p2} , S_{p2} and L_{p2} . Then the impedance matrix of the main path is

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{Y_{11}Y_{22} - Y_{12}Y_{21}} & \frac{-Y_{12}}{Y_{11}Y_{22} - Y_{12}Y_{21}} \\ \frac{-Y_{21}}{Y_{11}Y_{22} - Y_{12}Y_{21}} & \frac{Y_{11}}{Y_{11}Y_{22} - Y_{12}Y_{21}} \end{bmatrix}$$
(9)

Now, the scattering parameters $(S_{11} \text{ and } S_{21})$ may be obtained directly from Y, Z or ABCD matrices, by the relationships among them [9].

6. DESIGN EXAMPLE 2

The design specifications of a modified double two-section Schiffman phase shifter are: $f_1 = 5 \text{ GHz}$, $f_2 = 7.5 \text{ GHz}$, $\varphi_g = 50^\circ$, $S_{11} = 10 \text{ dB}$, $S_{33} = 10 \text{ dB}$, K = 301, $W_1 = 1$, $W_2 = 1$, $W_3 = 1$, $Z_s = 50 \Omega$, $Z_l = 50 \Omega$. The substrate selected for the microstrip circuit is Rogers Ro5880, with $\varepsilon_r = 2.2$, h = 0.7874 mm and loss tangent = 0.0009. Some constraints are also specified on the lengths and widths of strips, so that their fabrication is possible by the available technology. That is, 1 mm < L < 100 mm, $3 \mu \text{m} < w < 10 \text{ mm}$ and $3 \mu \text{m} < s < 10 \text{ mm}$. The initial values for various lengths are selected randomly. The optimum design values for lengths are listed in Table 2. The frequency response of the phase shift obtained by the MLS design procedure and HFSS are drawn in Fig. 7, for comparison [10, 11]. The amplitude of reflection

Table	2.	Optimum	values	of	the	dimensions	of	modified	Schiffman
phase s	hift	er in exam	ple 2.						

After Optimization	L (optimum)	S (optimum)	W (optimum)
P1	$13.1529\mathrm{mm}$	$292.714\mu\mathrm{m}$	$1121.65\mu\mathrm{m}$
P2	$14.6973\mathrm{mm}$	$292.714\mu\mathrm{m}$	$1121.65\mu\mathrm{m}$
R1	$10.6481\mathrm{mm}$	$596.483\mu\mathrm{m}$	$712.07\mu\mathrm{m}$
R2	$13.1621\mathrm{mm}$	$596.483\mu\mathrm{m}$	$712.07\mu\mathrm{m}$



Figure 7. Frequency response of phase shift $\Delta \varphi$ of example 2.

coefficients, $|S_{11}|$ and $|S_{33}|$, as obtained by MLS and HFSS are drawn in Figs. 8 and 9.

The signal transmission from port 1 to port 2 as represented by S_{12} and from port 3 to port 4 as represented by S_{34} are calculated by MLS and HFSS and drawn in Figs. 10 and 11, respectively. Any discrepancies between MLS and HFSS results are due to the approximations in the development of the MLS design procedure, as mentioned earlier.

7. DOUBLE FOUR-SECTION PHASE SHIFTER

The double four-section phase shifter is shown in Fig. 12, where the path from port 1 to port 2 is the main circuit and the path from port 3 to port 4 is the reference circuit. The differential phase difference between the main reference paths is the resultant phase difference obtained from the device [12].

First, the ABCD matrix of the upper section of the main path is



Figure 8. Scattering parameter S_{11} of example 2.



Figure 9. Scattering parameter S_{33} of example 2.

converted to its admittance matrix

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{D}{B} & \frac{BC-AD}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$$
(10)

The admittance matrix of main path is obtained as

$$[Y_P] = [Y_{P1u}] + [Y_{P2d}]$$
(11)



Figure 10. Scattering parameter S_{12} of example 2.



Figure 11. Scattering parameter S_{34} of example 2.

where $[Y_{P1u}]$ is the admittance matrix of the upper section with parameters $(W_{p1}, S_{p1}, L_{p1}, W_{p2}, S_{p2}, L_{p2})$, and $[Y_{P2d}]$ is the admittance matrix of the lower section with parameters $(W_{p3}, S_{p3}, L_{p3}, W_{p4}, S_{p4}, L_{p4})$, as shown in Fig. 12. The impedance matrix of the main path is obtained as

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{Y_{11}Y_{22} - Y_{12}Y_{21}} & \frac{-Y_{12}}{Y_{11}Y_{22} - Y_{12}Y_{21}} \\ \frac{-Y_{21}}{Y_{11}Y_{22} - Y_{12}Y_{21}} & \frac{Y_{11}}{Y_{11}Y_{22} - Y_{12}Y_{21}} \end{bmatrix}$$
(12)



Figure 12. Dimensions of the configuration of the proposed double four-section Schiffman phase shifter.



Figure 13. A photograph of the fabricated modified double foursection Schiffman phase shifter.

Similarly, the impedance matrix of the reference path is obtained. As before, the ABCD and scattering matrices of the circuit are then derived. Therefore, the phase shift φ and error function are obtained by (5) and (6).

8. DESIGN EXAMPLE 3

The design specifications of a double four-section phase shifter are: $f_1 = 4 \text{ GHz}$ and $f_2 = 9 \text{ GHz}$, $\varphi_g = 50^\circ$, $S_{11} = 10 \text{ dB}$, $S_{33} = 10 \text{ dB}$, K = 501, $W_1 = 1$, $W_2 = 1$, $W_3 = 1$, $Z_s = 50 \Omega$ and $Z_l = 50 \Omega$. The substrate selected for the microstrip circuit is Rogers Ro5880, with $\varepsilon_r = 2.2$, h = 0.7874 mm and loss tangent = 0.0009. The optimum values of parameters of the phase shifter are listed in Table 3. This double four-section phase shifter is fabricated according to the data in Table 3 and measured for the values of φ , S_{11} , S_{33} , S_{21} and S_{43} . A

After Optimization	L (optimum)	S (optimum)	W (optimum)
<i>p</i> 1	$8.27\mathrm{mm}$	$527.47\mu\mathrm{m}$	$961.92\mu\mathrm{m}$
<i>p</i> 2	$7.01\mathrm{mm}$	$527.47\mu\mathrm{m}$	$1178.71\mu\mathrm{m}$
p3	$4.38\mathrm{mm}$	$527.47\mu\mathrm{m}$	$1193.64\mu\mathrm{m}$
<i>p</i> 4	$2.54\mathrm{mm}$	$527.47\mu\mathrm{m}$	$961.92\mu\mathrm{m}$
r1	$2.02\mathrm{mm}$	$344.51\mu\mathrm{m}$	$814.04\mu\mathrm{m}$
r2	$3.14\mathrm{mm}$	$344.51\mu\mathrm{m}$	$1191.5\mu\mathrm{m}$
r3	$11.52\mathrm{mm}$	$344.51\mu\mathrm{m}$	$1198.1\mu\mathrm{m}$
r4	$5.75\mathrm{mm}$	$344.51\mu\mathrm{m}$	$814.07\mu\mathrm{m}$

Table 3. Optimum values of the geometrical dimensions of modified double four-section Schiffman phase shifter in example 3.



Figure 14. Frequency response of phase shift $\Delta \varphi$ of example 3.

photograph of the fabricated phase shifter is shown in Fig. 13. The frequency responses of all these parameters as obtained by MLS, HFSS and measurements are drawn in Figs. 14, 15, 16, 17 and 18.

Figure 14 shows that the frequency response of phase difference, namely the φ -f curve, is quite flat at $\varphi = 50^{\circ}$ in the bandwidth of 5 GHz. Fig. 15 shows the frequency responses of the reflection coefficient at the input port 1, namely the S_{11} -f curve, which is relatively lower than the specified -10 dB line.

Figure 16 shows the S_{33} -f curve for the reflection coefficient at port 3, which is significantly lower than the specified value -10 dB in the 5 GHz bandwidth.



Figure 15. Scattering parameter S_{11} of example 3.



Figure 16. Scattering parameter S_{33} of example 3.

The S_{11} -f and S_{33} -f curves indicate that good matching is achieved among the input and output ports. The transmission signal amplitudes from the input to output ports through the main and reference paths, namely $|S_{21}|$ and $|S_{43}|$, are drawn in Fig. 17 and Fig. 18, respectively. The S_{21} -f and S_{43} -f curves are quite higher than the -0.5 dB line in the 5 GHz bandwidth, except at higher frequencies approaching



Figure 17. Scattering parameter S_{21} of example 3.



Figure 18. Scattering parameter S_{43} of example 3.

9 GHz. In all the frequency response curves in Figs. 14 to 18, the MLS, HFSS and measurement results are in good agreement, which attest to the effectiveness of the proposed circuit configurations for the modified Schiffman phase shifters and the MLS design procedures presented for their design. The aforementioned circuit configurations for single section, double two-section and double four-section phase

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shifters and the associated design examples indicate that by increasing the number of sections of the phase shifters, its frequency response improves significantly and becomes quite wide band.

9. CONCLUSION

The original Schiffman phase shifter is modified to make multi-section differential phase shifter devices, having two main and reference paths. The method of least squares is used to develop a design procedure for the phase shifter, which combines both functions of producing a specified differential phase shift and impedance matching leading to some circuit miniaturization. The multi-section configuration provided in the phase shifter circuit results in a great increase of the bandwidth of phase shifting and impedance matching frequency responses. The full-wave simulation, fabrication and measurement results indicate the efficacy of the proposed circuits for multi-section phase shifter and also MLS design and optimization procedure.

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