

## ELECTROMAGNETIC WAVES SCATTERING AND RADIATION BY VIBRATOR-SLOT STRUCTURE IN A RECTANGULAR WAVEGUIDE

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**Abstract**—A problem of electromagnetic waves scattering and radiation by a structure, consisting of a narrow transverse slot in broad wall of rectangular waveguide and a vibrator with variable surface impedance, located inside the waveguide and interacting with one another, is solved. A solution of integral equations for electric current on the vibrator and equivalent magnetic current in the slot is derived by the generalized method of induced electro-magneto-motive forces. Conditions necessary for achievement of maximal slot radiation coefficient are defined. Effectiveness of impedance vibrators application to ensure required level of radiation by vibrator-slot structure in low profile rectangular waveguides is shown. Calculated and experimental plots of energy characteristics of the vibrator-slot structure for different vibrator placement relative to the slot and for various surface impedance dependencies upon the vibrator length are presented.

### 1. INTRODUCTION

The problem of electromagnetic waves excitation by arbitrarily shaped holes in adjacent walls of various electromagnetic volumes such as half-space over perfectly conducting plane, a waveguide, a resonator etc. in the presence of conducting bodies is one of the key objectives of macroscopic electrodynamics. The most widespread elements for practical applications in antenna and waveguide technology are resonant holes and wires, narrow slots and thin vibrators

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are predominantly used as the most technologically advanced in manufacturing. These structures have been studied since the middle of last century and are continued intensively up to now (see, for example [1–15]). However, in these and other publications on the subject the authors assumed that vibrators are perfectly conducting or are made of metal with finite conductivity. To study the possible range of vibrator influence upon the characteristics of slot radiating element there arises necessity to investigate the vibrator with complex surface impedance, including cases of impedance variable along the axis of the vibrator.

In the paper [16], we have formulated in a rigorous self-consistent approach a problem of electromagnetic fields excitation by a material body of finite dimensions in presence of coupling hole between two arbitrary electrodynamic volumes. The problem was reduced to two-dimensional integral equations for surface electric current at a material body and equivalent magnetic currents in a coupling hole. We have also present a physically-based validation of transition from integral equations to the one-dimensional equations for currents in a narrow slot and a thin impedance vibrator which, in the general case, may have irregular geometric parameters. As a fairly simple and obvious example in [16] we have found a problem solution for a transverse slot, cut in broad wall of an infinite rectangular waveguide, and radiating into half space above a perfectly conducting screen and a scattering vibrator with variable surface impedance, by generalized method of induced electro-magneto-motive forces (EMMF). The axes of the vibrator and the slot are in the same cross-sectional plane of the waveguide. In this case, the problem solution is considerably simplified since interaction between the vibrator and the slot is absent due to polarization decoupling. We have shown that such structure provides a possibility to control the matching factor of combined vibrator-slot inhomogeneity inside the waveguide by varying value of vibrator surface impedance. However, the slot radiation coefficient could not be controlled since the interaction between the slot and monopole is absent.

In the present paper we will solve the above problem taking into account the interaction between the vibrator and slot, i.e., by shifting the longitudinal axis of the vibrator along that of waveguide relative to the slot axis. Such configuration when used with vibrators, having variable surface impedance, allows to widen the band of system electrodynamic characteristics as compared with that of single slot or a structure without interaction between slot and vibrator.

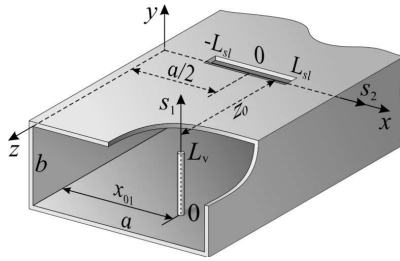


Figure 1. The geometry of vibrator-slot structure and notations.

## 2. PROBLEM FORMULATION AND SOLUTION OF INTEGRAL EQUATIONS

Let a thin asymmetric vibrator (monopole) with variable surface impedance is placed in cross-sectional plane of a hollow infinite rectangular waveguide (index  $Wg$ ) having perfectly conducting walls where fundamental wave  $H_{10}$  is propagated from the area  $z = -\infty$ . The waveguide size is  $\{a \times b\}$ , vibrator radius  $r$  and length  $2L_v$  satisfy inequalities  $[r/(2L_v)] \ll 1$ ,  $[r/\lambda] \ll 1$ , where  $\lambda$  is the free space wavelength. A narrow transverse slot is cut in the broad wall (with thickness  $h$ ) of the waveguide symmetrically relative to its longitudinal axis. The slot radiates in free half-space over infinite perfectly conducting plane (index  $Hs$ ). The slot width  $d$  and length  $2L_{sl}$  satisfy the inequalities  $[d/(2L_{sl})] \ll 1$ ,  $[d/\lambda] \ll 1$ .  $z_0$  is the distance between the axes of the vibrator and the slot (Fig. 1).

If time  $t$  dependence of electromagnetic fields is given by the factor  $e^{i\omega t}$ ,  $\omega$  is the circular frequency, the system of integral equations relative to electrical current at the vibrator  $J_v(s_1)$  and equivalent magnetic current in the slot  $J_{sl}(s_2)$  may be represented [16] as

$$\left(\frac{d^2}{ds_1^2} + k^2\right) \int_{-L_v}^{L_v} J_v(s'_1) \tilde{G}_{s_1}^{Wg}(s_1, s'_1) ds'_1 - ik \int_{-L_{sl}}^{L_{sl}} J_{sl}(s'_2) \tilde{G}_{s_2}^{Wg}(s_1, s'_2) ds'_2 = -i\omega E_{0s_1}(s_1) + i\omega z_i(s_1) J_v(s_1), \quad (1a)$$

$$\left(\frac{d^2}{ds_2^2} + k^2\right) \int_{-L_{sl}}^{L_{sl}} J_{sl}(s'_2) [G_{s_2}^{Wg}(s_2, s'_2) + G_{s_2}^{Hs}(s_2, s'_2)] ds'_2 - ik \int_{-L_v}^{L_v} J_v(s'_1) \tilde{G}_{s_1}^{Wg}(s_2, s'_1) ds'_1 = -i\omega H_{0s_2}(s_2). \quad (1b)$$

Here  $s_1$  and  $s_2$  are the local coordinates, associated with the axes of the vibrator and the slot, respectively,  $z_i(s_1)$  is the internal impedance per unit length of the vibrator ([Ohm/m]),  $E_{0s_1}(s_1)$  and  $H_{0s_2}(s_2)$  are projections of impressed sources fields on the vibrator and the slot axis,  $G_{s_1}^{Wg}(s_1, s'_1)$ ,  $G_{s_2}^{Wg}(s_2, s'_2)$  and  $G_{s_2}^{Hs}(s_2, s'_2)$  are respective components of Green's functions of rectangular waveguide and half-space over a plane [17, 18],  $-L_v$  is the end coordinate of the mirror image of the vibrator, counted off from the waveguide broad wall [18], ( $J_v(\pm L_v) = 0$ ,  $J_{sl}(\pm L_{sl}) = 0$ ),  $k = 2\pi/\lambda$ ,  $\tilde{G}_{s_1}^{Wg}(s_2, s'_1) = \frac{\partial}{\partial z} G_{s_1}^{Wg}[x(s_2), 0, z; x'(s'_1), y'(s'_1), z_0]$  and  $\tilde{G}_{s_2}^{Wg}(s_1, s'_2) = \frac{\partial}{\partial z} G_{s_2}^{Wg}[x(s_1), y(s_1), z; x'(s'_2), 0, 0]$  are expressions derived by substitution  $z = 0$  into  $\tilde{G}_{s_1}^{Wg}$  and  $z = z_0$  into  $\tilde{G}_{s_2}^{Wg}$  is made after derivation.

If interaction between the vibrator and slot is absent ( $z_0 = 0$ )  $\tilde{G}_{s_1}^{Wg} = \tilde{G}_{s_2}^{Wg} = 0$ , and the system of coupled Equations (1) is reduced to two independent equations

$$\left(\frac{d^2}{ds_1^2} + k^2\right) \int_{-L_v}^{L_v} J_v(s'_1) G_{s_1}^{Wg}(s_1, s'_1) ds'_1 = -i\omega E_{0s_1}(s_1) + i\omega z_i(s_1) J_v(s_1), \quad (2a)$$

$$\left(\frac{d^2}{ds_2^2} + k^2\right) \int_{-L_{sl}}^{L_{sl}} J_{sl}(s'_2) [G_{s_2}^{Wg}(s_2, s'_2) + G_{s_2}^{Hs}(s_2, s'_2)] ds'_2 = -i\omega H_{0s_2}(s_2). \quad (2b)$$

The solution of system (1) will be sought by the generalized induced EMMF method [17–19]. The functions  $J_v(s_1) = J_{0v} f_v(s_1)$  and  $J_{sl}(s_2) = J_{0sl} f_{sl}(s_2)$  are used as approximating functions for currents. Here  $f_v(s_1)$  and  $f_{sl}(s_2)$  are preassigned currents distributions functions,  $J_{0v}$  and  $J_{0sl}$  are unknown current amplitudes which can be determined from solution of Equation (2) by asymptotic averaging method [17, 18, 20]. For arbitrary vibrator-slot structures and coupled electrodynamic volumes the distribution functions may have both symmetric ( $f_v^s(s_1)$ ,  $f_{sl}^s(s_2)$ ) and antisymmetric ( $f_v^a(s_1)$ ,  $f_{sl}^a(s_2)$ ) components relative to vibrator ( $s_1 = 0$ ) and slot ( $s_2 = 0$ ) centers. The expressions for  $f_v^{s,a}(s_1)$  and  $f_{sl}^{s,a}(s_2)$  can be derived from

the relations [17, 18, 20]

$$f_v^{s,a}(s_1) \sim \left\{ \begin{aligned} & \sin \tilde{k}(L_v - s_1) \int_{-L_v}^{s_1} E_{0s_1}^{s,a}(s'_1) \sin \tilde{k}(L_v + s'_1) ds'_1 \\ & + \sin \tilde{k}(L_v + s_1) \int_{s_1}^{L_v} E_{0s_1}^{s,a}(s'_1) \sin \tilde{k}(L_v - s'_1) ds'_1 \end{aligned} \right\}, \quad (3a)$$

$$f_{sl}^{s,a}(s_2) \sim \left\{ \begin{aligned} & \sin k(L_{sl} - s_2) \int_{-L_{sl}}^{s_2} H_{0s_2}^{s,a}(s'_2) \sin k(L_{sl} + s'_2) ds'_2 \\ & + \sin k(L_{sl} + s_2) \int_{s_2}^{L_{sl}} H_{0s_2}^{s,a}(s'_2) \sin k(L_{sl} - s'_2) ds'_2 \end{aligned} \right\}, \quad (3b)$$

where  $E_{0s_1}^{s,a}(s_1)$  and  $H_{0s_2}^{s,a}(s_2)$  are the projections of symmetrical and antisymmetrical components of impressed sources fields at the vibrator and slot axes, respectively. The sign  $\sim$  means that in (3) only multipliers, depending upon coordinates  $s_1$  and  $s_2$ , are left after integrations. For the vibrator-slot structure with symmetrical slot, excited by the fundamental wave  $H_{10}$ , we have

$$f_v(s_1) = f_v^s(s_1) = \cos \tilde{k}s_1 - \cos \tilde{k}L_v, \quad (4a)$$

$$f_{sl}(s_2) = f_{sl}^s(s_2) = \cos ks_2 - \cos kL_{sl}, \quad (4b)$$

where  $\tilde{k} = k - \frac{i2\pi z_i^{av}}{Z_0\Omega}$ ,  $z_i^{av} = \frac{1}{2L_v} \int_{-L_v}^{L_v} z_i(s_1) ds_1$  is the mean value of internal impedance along the vibrator length [16, 18],  $Z_0 = 120\pi$  [Ohm], and the origin of the local axis  $\{0s_2\}$  is  $a/2$ .

Let us multiply the Equations (1a) and (1b) by  $f_v(s_1)$  and  $f_{sl}(s_2)$ , respectively, and integrate the resulting equations over the vibrator and slot lengths. Thus we get a system of linear equations

$$\begin{aligned} J_{0v} \left[ Z_{11}(kr, \tilde{k}L_v) + F_z(\tilde{k}r, \tilde{k}L_v) \right] + J_{0sl} Z_{12}(z_0, \tilde{k}L_v, kL_{sl}) &= -\frac{i\omega}{2k} E_1(\tilde{k}L_v), \\ J_{0sl} Z_{22}^\Sigma(kd_e, kL_{sl}) + J_{0v} Z_{21}(z_0, kL_{sl}, \tilde{k}L_v) &= -\frac{i\omega}{2k} H_2(kL_{sl}). \end{aligned} \quad (5)$$

Here

$$\begin{aligned}
Z_{11}(kr, \tilde{k}L_v) &= \frac{4\pi}{ab} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{\varepsilon_n (k^2 - k_y^2) \tilde{k}^2}{kk_z (\tilde{k}^2 - k_y^2)^2} e^{-k_z r} \sin^2 k_x x_{01} \\
&\quad \times \left[ \sin \tilde{k}L_v \cos k_y L_v - \left( \tilde{k}/k_y \right) \cos \tilde{k}L_v \sin k_y L_v \right]^2, \\
Z_{12}(z_0, \tilde{k}L_v, kL_{sl}) &= \frac{4\pi}{ab} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{\varepsilon_n k \tilde{k} e^{-k_z z_0}}{i(k^2 - k_x^2) (\tilde{k}^2 - k_y^2)} \sin k_x x_{01} \sin k_x \frac{a}{2} \\
&\quad \times \left[ \sin \tilde{k}L_v \cos k_y L_v - \left( \tilde{k}/k_y \right) \cos \tilde{k}L_v \sin k_y L_v \right] \\
&\quad \times \left[ \sin kL_{sl} \cos k_x L_{sl} - (k/k_x) \cos kL_{sl} \sin k_x L_{sl} \right], \\
Z_{21}(z_0, kL_{sl}, \tilde{k}L_v) &= Z_{12}(z_0, \tilde{k}L_v, kL_{sl}), \tag{6} \\
Z_{22}^{Wg}(kd_e, kL_{sl}) &= \frac{8\pi}{ab} \sum_{m=1,3,\dots}^{\infty} \sum_{n=0,1,\dots}^{\infty} \frac{\varepsilon_n k}{k_z (k^2 - k_x^2)} e^{-k_z \frac{d_e}{4}} \\
&\quad \times \left[ \sin kL_{sl} \cos k_x L_{sl} - (k/k_x) \cos kL_{sl} \sin k_x L_{sl} \right]^2, \\
Z_{22}^{Hs}(kd_e, kL_{sl}) &= (\text{Si}4kL_{sl} - i\text{Cin}4kL_{sl}) - 2 \cos kL_{sl} \\
&\quad \times \left[ 2(\sin kL_{sl} - kL_{sl} \cos kL_{sl}) \left( \ln \frac{16L_{sl}}{d_e} \right. \right. \\
&\quad \left. \left. - \text{Cin}2kL_{sl} - i\text{Si}2kL_{sl} \right) + \sin 2kL_{sl} e^{-ikL_{sl}} \right], \\
Z_{22}^{\Sigma}(kd_e, kL_{sl}) &= Z_{22}^{Wg}(kd_e, kL_{sl}) + Z_{22}^{Hs}(kd_e, kL_{sl}), \\
E_1(\tilde{k}L_v) &= 2H_0 \frac{k}{k_g \tilde{k}} \sin \frac{\pi}{a} x_{01} e^{-ik_g z_0} f(\tilde{k}L_v), \\
f(\tilde{k}L_v) &= \sin \tilde{k}L_v - \tilde{k}L_v \cos \tilde{k}L_v, \\
H_2(kL_{sl}) &= 2H_0 \frac{1}{k} f(kL_{sl}), \quad f(kL_{sl}) = \sin kL_{sl} - kL_{sl} \cos kL_{sl}, \\
F_z(\tilde{k}r, \tilde{k}L_v) &= -\frac{i}{r} \int_0^{L_v} f_v^2(s_1) \bar{Z}_S(s_1) ds_1. \tag{7}
\end{aligned}$$

In the above formulas  $\varepsilon_n = \begin{cases} 1, & n = 0 \\ 2, & n \neq 0 \end{cases}$ ,  $k_x = \frac{m\pi}{a}$ ,  $k_y = \frac{n\pi}{b}$ ,  $k_z = \sqrt{k_x^2 + k_y^2 - k^2}$ ,  $m$  and  $n$  are integers,  $k_g = 2\pi/\lambda_g = \sqrt{k^2 - (\pi/a)^2}$ ,  $\text{Si}$  and  $\text{Cin}$  are integral sine and cosine functions,  $\bar{Z}_S(s_1) = \bar{R}_S + i\bar{X}_S \phi(s_1)$

is the complex distributed surface impedance, normalized by  $Z_0$ , ( $\bar{Z}_S(s_1) = 2\pi r z_i(s_1)/Z_0$ ,  $\phi(s_1)$  is the given function),  $d_e = de^{-\frac{\pi h}{2a}}$  is equivalent slot width which takes into account a wall thickness  $h$  of the waveguide [17, 20].

The solution of equations system (5) is

$$J_{0v} = -\frac{i\omega}{2k} \frac{\begin{bmatrix} E_1(\tilde{k}L_v) Z_{22}^\Sigma(kd_e, kL_{sl}) \\ -H_2(kL_{sl}) Z_{12}(z_0, \tilde{k}L_v, kL_{sl}) \end{bmatrix}}{\begin{bmatrix} [Z_{11}(kr, \tilde{k}L_v) + F_z(\tilde{k}r, \tilde{k}L_v)] Z_{22}^\Sigma(kd_e, kL_{sl}) \\ -Z_{21}(z_0, kL_{sl}, \tilde{k}L_v) Z_{12}(z_0, \tilde{k}L_v, kL_{sl}) \end{bmatrix}} = -\frac{i\omega}{2k} \tilde{J}_{0v}, \tag{8}$$

$$J_{0sl} = -\frac{i\omega}{2k} \frac{\begin{bmatrix} H_2(kL_{sl}) [Z_{11}(kr, \tilde{k}L_v) + F_z(\tilde{k}r, \tilde{k}L_v)] \\ -E_1(\tilde{k}L_v) Z_{21}(z_0, kL_{sl}, \tilde{k}L_v) \end{bmatrix}}{\begin{bmatrix} [Z_{11}(kr, \tilde{k}L_v) + F_z(\tilde{k}r, \tilde{k}L_v)] Z_{22}^\Sigma(kd_e, kL_{sl}) \\ -Z_{21}(z_0, kL_{sl}, \tilde{k}L_v) Z_{12}(z_0, \tilde{k}L_v, kL_{sl}) \end{bmatrix}} = -\frac{i\omega}{2k} \tilde{J}_{0sl}.$$

Using (4) and (8), we obtain expressions for the currents in the vibrator and slot

$$J_v(s_1) = -H_0 \frac{i\omega}{k} \tilde{J}_{0v} (\cos \tilde{k}s_1 - \cos \tilde{k}L_v), \tag{9}$$

$$J_{sl}(s_2) = -H_0 \frac{i\omega}{k} \tilde{J}_{0sl} (\cos ks_2 - \cos kL_{sl}),$$

where  $H_0$  is amplitude of  $H_{10}$  wave.

Energy characteristics of the vibrator-slot structure: reflection coefficient  $S_{11}$ , transmission coefficients  $S_{12}$ , and radiation coefficient  $|\Sigma_\Sigma|^2$  are defined as

$$S_{11} = \frac{4\pi i}{abk k_g} \left\{ \frac{2k_g^2}{k} \tilde{J}_{0sl} f(kL_{sl}) - \frac{k k_g}{\tilde{k}} \tilde{J}_{0v} \sin(\pi x_{01}/a) e^{-ik_g z_0} f(\tilde{k}L_v) \right\} e^{2ik_g z}, \tag{10}$$

$$S_{12} = 1 + \frac{4\pi i}{abk k_g} \left\{ \frac{2k_g^2}{k} \tilde{J}_{0sl} f(kL_{sl}) + \frac{k k_g}{\tilde{k}} \tilde{J}_{0v} \sin(\pi x_{01}/a) e^{-ik_g z_0} f(\tilde{k}L_v) \right\}, \tag{11}$$

$$|\Sigma_\Sigma|^2 = 1 - |S_{11}|^2 - |S_{12}|^2. \tag{12}$$

Let us consider, as in [16], functions:  $\phi_0(s_1) = 1$ ,  $\phi_1(s_1) = 2[1 - (s_1/L_v)]$  and  $\phi_2(s_1) = 2(s_1/L_v)$ , defining distribution of imaginary part of the surface impedance along vibrator, i.e., the constant distribution, the distribution, decreasing to the vibrator end linearly and the linearly

increasing distribution. Then in concordance with (7) expression for  $F_{z0}(\tilde{k}r, \tilde{k}L_v)$  are

$$F_{z0}(\tilde{k}r, \tilde{k}L_v) = -\frac{2i(\bar{R}_S + i\bar{X}_S)}{\tilde{k}^2 L_v r} \left[ \left( \frac{\tilde{k}L_v}{2} \right)^2 (2 + \cos 2\tilde{k}L_v) - \frac{3}{8} \tilde{k}L_v \sin 2\tilde{k}L_v \right] \\ = \tilde{F}_z(\bar{R}_S + i\bar{X}_S) \Phi(\tilde{k}L_v) \quad (13)$$

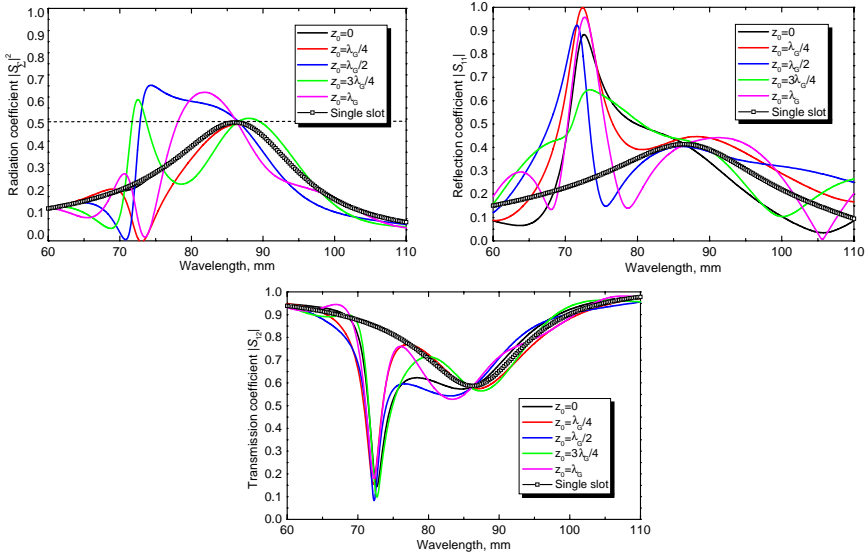
for constant distribution,

$$F_{z1}(\tilde{k}r, \tilde{k}L_v) = \tilde{F}_z \left\{ \bar{R}_S \Phi(\tilde{k}L_v) + i\bar{X}_S \left[ \left( \frac{\tilde{k}L_v}{2} \right)^2 (2 + \cos 2\tilde{k}L_v) - \frac{7}{4} \sin^2 \tilde{k}L_v - 2(\cos \tilde{k}L_v - 1) \right] \right\} \quad (14)$$

for linearly decreasing distribution,

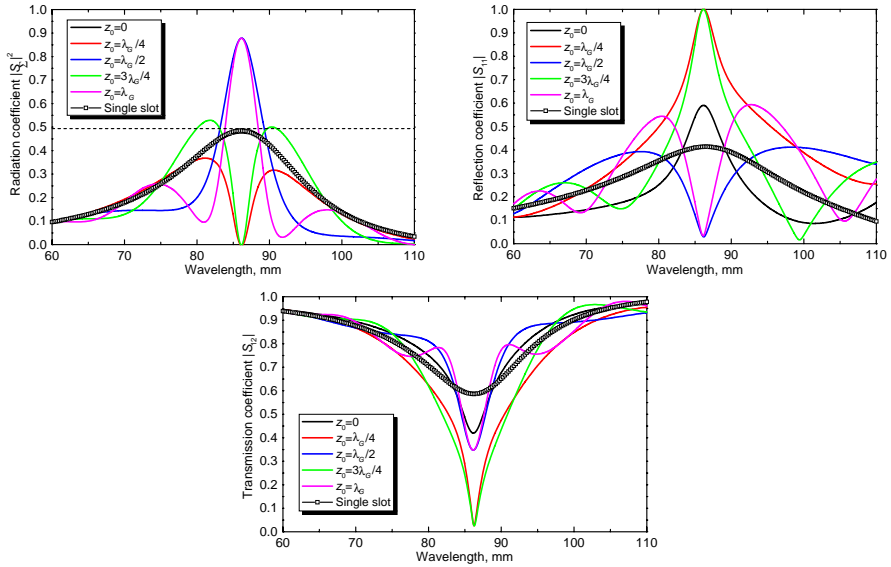
$$F_{z2}(\tilde{k}r, \tilde{k}L_v) = \tilde{F}_z \left\{ \bar{R}_S \Phi(\tilde{k}L_v) + i\bar{X}_S \left[ \left( \frac{\tilde{k}L_v}{2} \right)^2 (2 + \cos 2\tilde{k}L_v) + \frac{7}{4} \sin^2 \tilde{k}L_v - \frac{3}{4} \tilde{k}L_v \sin 2\tilde{k}L_v + 2(\cos \tilde{k}L_v - 1) \right] \right\} \quad (15)$$

for linearly increasing distribution.



**Figure 2.** Energy characteristics of vibrator-slot system versus wavelength at  $x_{01} = a/8$ ,  $\bar{Z}_S = 0$ .

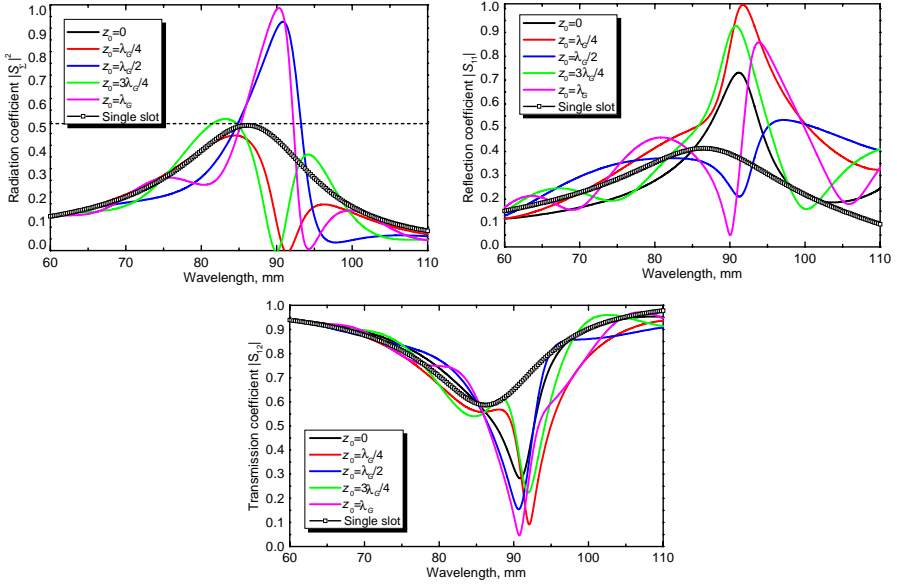




**Figure 3.** Energy characteristics of vibrator-slot system versus wavelength at  $x_{01} = a/8$ ,  $Z_S = ikr \ln(5.5)$ .

### 3. NUMERICAL RESULTS

Based on the above mathematical model for the vibrator-slot structure we have carried out the numerical analysis to determine a range of possible changes in the energy characteristics of the structure as compared with the single radiating slot, i.e., slot without vibrator. Figs. 2–6 present radiation coefficient  $|S_{\Sigma}|^2(\lambda)$ , reflection  $|S_{11}|(\lambda)$  and transmission  $|S_{12}|(\lambda)$  coefficients in the single-mode band of the waveguide. The geometrical parameters of the problem are as follows  $a = 58.0$  mm,  $b = 25.0$  mm,  $h = 0.5$  mm,  $r = 2.0$  mm,  $L_v = 15.0$  mm,  $d = 4.0$  mm,  $2L_{sl} = 40.0$  mm. The slot and monopole dimensions were chosen so that the slot natural resonance (without the vibrator) at  $\lambda_{sl}^{res} = 86.0$  mm and monopole natural resonance  $\lambda_v^{res}$  were within the operating range of the waveguide. The displacement  $z_0$  of vibrator longitudinal axis relative longitudinal slot axis is expressed in fractions of the slot resonant wavelength in the waveguide  $\lambda_{sl}^{gres} = \frac{2\pi}{\sqrt{(2\pi/\lambda_{sl}^{res})^2 - (\pi/a)^2}} = \lambda_G$  since the energy characteristic (10)–(12) contain periodic functions of wave number  $k_g$ . Below we present calculation results only for monopoles with inductive-type impedance since it increases the electrical length of vibrator while the gap between



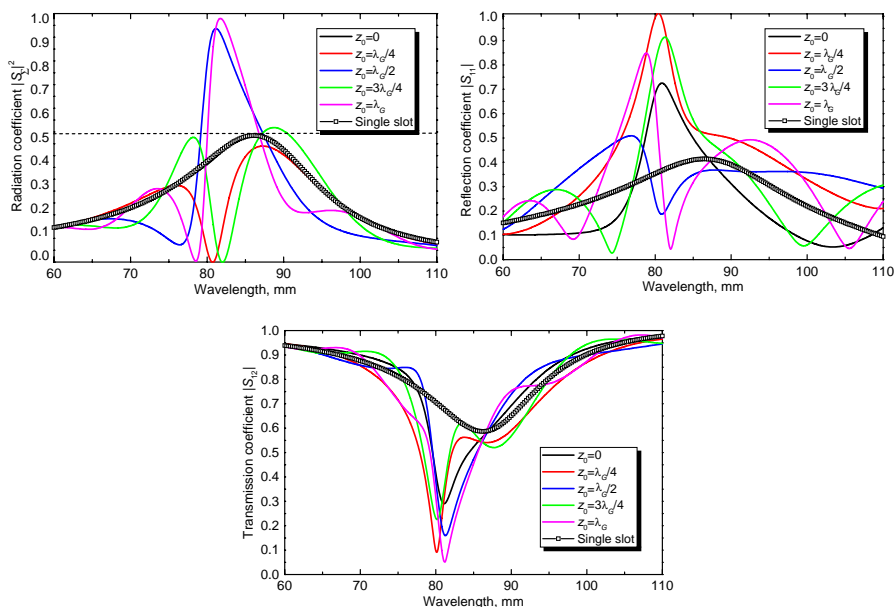
**Figure 4.** Energy characteristics of vibrator-slot system versus wavelength at  $x_{01} = a/8$ ,  $\bar{Z}_S(s_1) = ikr \ln(5.5)\phi_1(s_1)$ .

its butt end and upper wall of waveguide remains constant and thus waveguide breakdown power is increased.

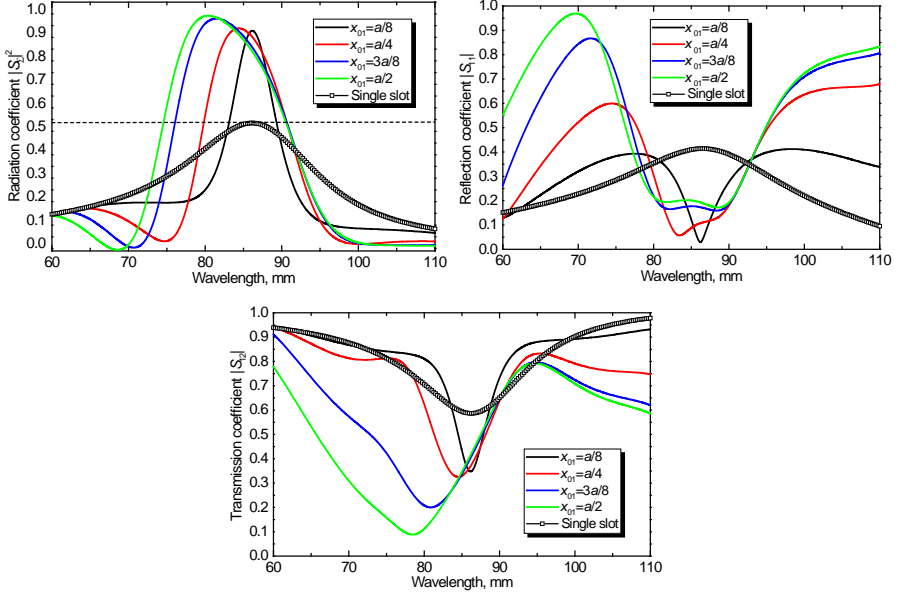
As it follows from the data, presented in the figures, both perfectly conducting vibrator and impedance monopoles may change the radiation coefficient of vibrator-slot system. This effect is more significant for impedance vibrators since impedance allows to bring close together the values of their natural resonant wavelength to the value of the slot resonant wavelength. However, as is seeing from Fig. 3, a simple rapprochement of these two resonances is not optimal to maximize their interaction. To achieve maximal slot radiation coefficient equal to unity which according to the theory of slotted waveguide radiator is unattainable in principle for a hollow infinite waveguide, monopoles with variable along their axis inductive impedance must be used (Figs. 4–5). We may observe that natural resonant wavelength of monopole and slot are slightly spaced relative to each other and resulting resonance of a vibrator-slot system is shifted to the resonant wavelength for a monopole. As expected from physical considerations, the displacement  $z_0$  to achieve maximal mutual influence between elements of vibrator-slot structure are multiples of  $\lambda_G/4$ . And maximal slot radiation coefficient values close to unity are attained for  $z_0$  which are multiples of  $\lambda_G/2$ .

If a vibrator is shifted in transverse direction to the longitudinal axis of the waveguide (Fig. 6) at  $z_0 = \lambda_G/2$  a slight increase in the maximum system radiation coefficient and significant increase of operating band  $|S_{\Sigma}|^2(\lambda)$  are observed. Thus for  $x_{01} = a/2$  bandwidth of radiation coefficient of vibrator-slot structure at the level  $|S_{\Sigma}|^2 = 0.6$  is increased by 3 times as compared to its value for  $x_{01} = a/8$ . It is interesting to note the maximal value of the system radiation coefficient occurs at a wavelength which does not coincide with the natural resonance wavelengths of both the slot and vibrator.

Low-profile waveguides which height  $b$  is much smaller than that for waveguides with the standard cross-section are often used in microwave devices. Of course, such waveguides impose even greater restrictions on the vibrator length. For example, if  $b = 12.5$  mm a suitable length of the vibrator  $L_v$  is 10.0 mm,  $\lambda_{sl}^{res} = 83.0$  mm and  $\lambda_{sl}^{gres} = 118.0$  mm. Our calculations have shown that a perfectly conducting vibrator with such dimensions can not influence the slot characteristics and significant increase of slot radiation coefficient  $|S_{\Sigma}|^2$  could not be achieved due to large separation between the natural resonant wavelengths of system elements. The electrical length of the vibrator may be increased, as before, through the use of a



**Figure 5.** Energy characteristics of vibrator-slot system versus wavelength at  $x_{01} = a/8$ ,  $\bar{Z}_S(s_1) = ikr \ln(5.5)\phi_2(s_1)$ .

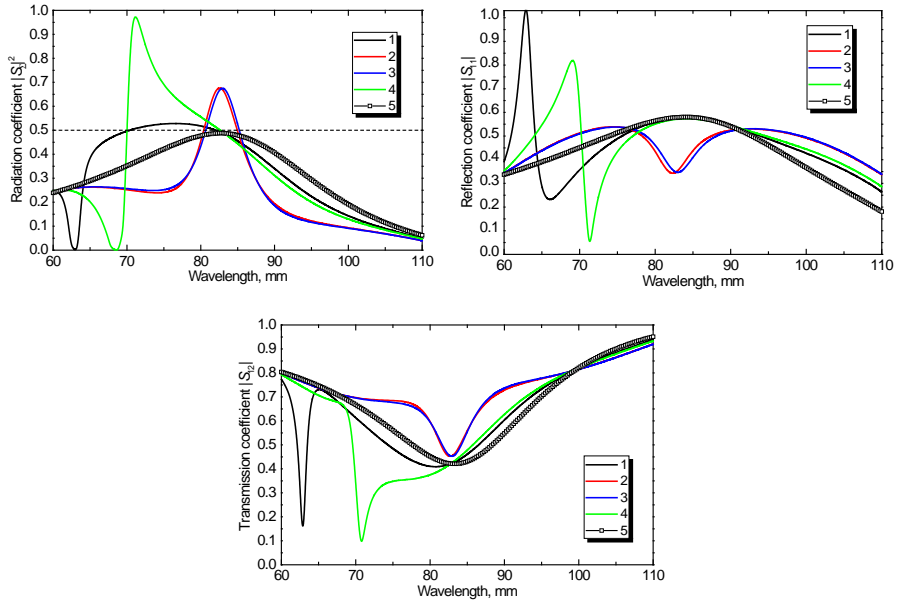


**Figure 6.** Energy characteristics of vibrator-slot system versus wavelength at  $z_0 = \lambda_G/2 = 64.0$  mm,  $\bar{Z}_S = ikr \ln(5.5)$ .

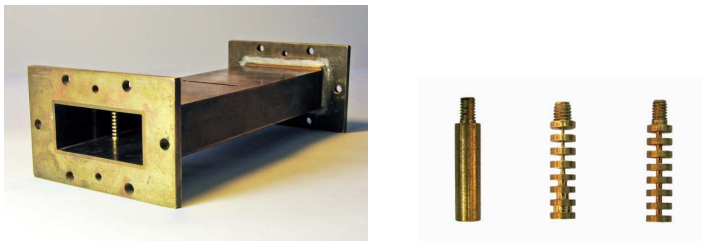
monopole with inductive impedance. The effectiveness of impedance monopoles application for the low-profile waveguide can be estimated from the numerical results, presented in Fig. 7. As can be seen, if resonant wavelengths of vibrator and slot coincide, a monopole with constant impedance increases radiation coefficient upto 0.7 and at the same time narrows the operating band of  $|S_\Sigma|^2(\lambda)$ . However, required value of impedance  $\bar{Z}_S = ikr \ln(51.0)$  is difficult to realize in practice. A monopole with variable impedance, defined by the function  $\phi_1(s_1)$ , have almost the same energy characteristics as a vibrator with a constant impedance, but the logarithmic factor  $\ln(17.0)$  is more acceptable now. If impedance varies as function  $\phi_2(s_1)$  the value of  $|S_\Sigma|^2$  approaches very close to 1.0.

The validity of numerical simulations have been tested, using an experimental observational dummy shown at Fig. 8 for perfectly conducting vibrator ( $\bar{Z}_S = 0$ ), for vibrator with constant impedance  $\bar{Z}_S(s_1) = ikr \ln(4.0)$ , and vibrator with variable impedance  $\bar{Z}_S(s_1) = ikr \ln(4.0)\phi_1(s_1)$ . Calculated and experimental wavelength dependencies of energy characteristics of the vibrator-slot structure with  $z_0 = \lambda_G/2$  for various impedance vibrators are shown at Fig. 9. Suffice satisfactory agreement between the calculated and

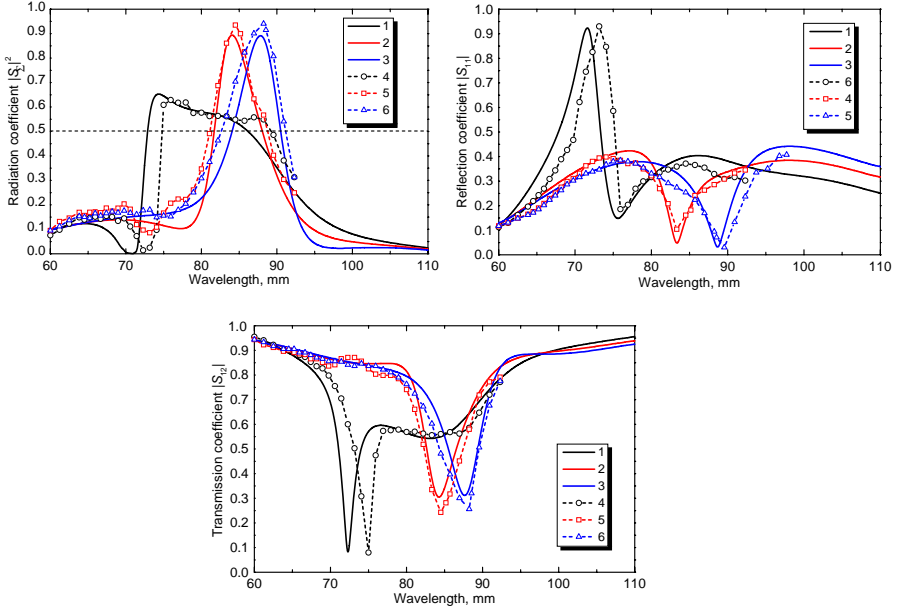
experimental data confirms the physical adequacy of our mathematical model, correctness of electrodynamic problem solution and validity of numerical calculations.



**Figure 7.** Energy characteristics of vibrator-slot system versus wavelength at  $a = 58.0$  mm,  $b = 12.5$  mm,  $2L_{sl} = 40.0$  mm,  $L_v = 10.0$  mm,  $x_{01} = a/8$ ,  $z_0 = \lambda_{sl}^{gres}/2 = 59.0$  mm: 1 —  $\bar{Z}_S = 0$ ; 2 —  $\bar{Z}_S = ikr \ln(51.0)$ ; 3 —  $\bar{Z}_S(s_1) = ikr \ln(17.0)\phi_1(s_1)$ ; 4 —  $\bar{Z}_S(s_1) = ikr \ln(17.0)\phi_2(s_1)$ ; 5 — single slot.



**Figure 8.** The experimental observational dummy and the samples of impedance vibrators.



**Figure 9.** Energy characteristics of vibrator-slot system versus wavelength at  $a = 58.0$  mm,  $b = 25.0$  mm,  $h = 0.5$  mm,  $2L_{sl} = 40.0$  mm,  $d = 4.0$  mm,  $L_v = 15.0$  mm,  $r = 2.0$  mm,  $x_{01} = a/8$ ,  $z_0 = 64.0$  mm: 1, 4 —  $\bar{Z}_S = 0$ ; 2, 5 —  $\bar{Z}_S = ikr \ln(4.0)$ ; 3, 6 —  $\bar{Z}_S(s_1) = ikr \ln(4.0)\phi_1(s_1)$ ; 4, 5, 6 — experimental data.

#### 4. CONCLUSION

The problem of electromagnetic waves scattering and radiation by the structure, consisting of the slot in broad wall of a rectangular waveguide and a vibrator with variable surface impedance interacting with one another over internal space of waveguide is solved. The solution has been derived by the generalized method of induced EMMF with approximated functions for slot and vibrator currents, defined by averaging method. The numerical analysis of energy characteristics of vibrator-slot structure in the single-mode waveguide with a standard and low-profile cross-sections has been carried out. The possibility and the conditions to achieve maximal slot radiation coefficient, approaching close to unity, for vibrator-slot system has been shown. Note that in accordance with the theory such possibility is principally unachievable for hollow rectangular waveguides (without vibrator). The efficiency of impedance monopole application in vibrator-slot structures for radiation level control in low-

profile rectangular waveguides has been proved. These results may be used to enhance the control of energy parameters in vibrator-slot structure, including a narrow transverse slot and impedance vibrator in a rectangular waveguide. The results may also be used in the designing of single-slot or multiple-slot radiators.

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