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MOLDING THE FLOW OF MAGNETIC FIELD WITH METAMATERIALS: MAGNETIC FIELD SHIELDING

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Abstract—In this paper, it is demonstrated how anisotropic and inhomogeneous magnetic metamaterials may be used for molding the flow of the magnetic field, considering magnetic field shielding as the main application of practical interest. It is shown that using anisotropic materials, magnetic field shielding may be improved, and this anisotropy can be realized by metamaterials. Introducing additional inhomogeneity in the metamaterial can increase the shielding performance even more. The required parameters for inhomogeneity may be obtained by representing the shielding problem in matrix form, using a quasi-static magnetic field approximation. Finally, some comments on the practical implementation of the metamaterial and comparisons with the standard shielding techniques are given.

1. INTRODUCTION

The electromagnetic properties of natural material are usually caused by interactions of the electromagnetic field with electrons, atoms, and molecules. In rather rare cases, much larger structures are involved, namely in opal [1]. By engineering larger structures, i.e., "artificial atoms" (AAs), "artificial materials" (AMs) are obtained, which may have very special and strong properties. AMs were frequently fabricated in the 20th century under various names, e.g., frequency selective surfaces and photonic crystals, but the term AMs was never widely used and such media were usually studied without using the term "artificial" e.g., AMs with strong chirality [2]. In the extreme case, AMs may even have properties that are not observed in nature. One then calls them metamaterials (MMs − meta =

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beyond) [3]. In 2000, MMs with both negative permittivity and negative permeability attracted much interest because they provide negative refraction [4]. Over several years, the research was focused on one main application: the "superresolution lens" or "Pendry lens" consisting of a homogeneous, isotropic metamaterial slab with negative refraction [5].

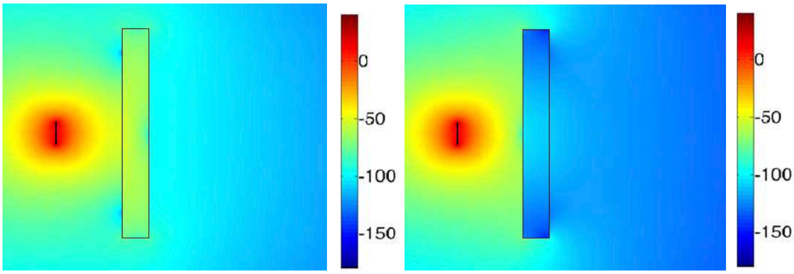
In general, linear, homogeneous, isotropic material properties simplify the analysis and the design considerably. For this reason, one often tries to exclusively work with such materials. At the same time, these simplifications exclude attractive effects that might be exploited. For example, when admitting anisotropic material properties, the quality of electromagnetic absorbers may be improved, which is well-known from the design of the so-called Uniaxial Perfectly Matched Layers (UPMLs) for absorbing outgoing waves in finite differences, finite elements and other methods for computational electromagnetic [6, 7]. In the last few years, metamaterial research focused much more on transformation optics for molding the flow of light and so-called cloaking applications which are based on inhomogeneous MMs [8]. It should be mentioned that inhomogeneous AMs and MMs are easily obtained when the size and geometry of the AAs depend on the location in space. Furthermore, AMs and MMs usually become anisotropic when all of the AAs are oriented in one and the same direction. Thus, it is reasonable to take advantage of anisotropy and inhomogeneity in the design of AMs and MMs for specific applications as demonstrated in this paper.

For both MMs with negative refraction and MMs for cloaking, first proofs of concept were done in the microwave regime because scaling down from optical to microwave frequencies allows one to work with considerably larger AAs, which are much easier to manufacture [9]. Working on cloaking at microwaves also has some practical value, i.e., radar cloaking.

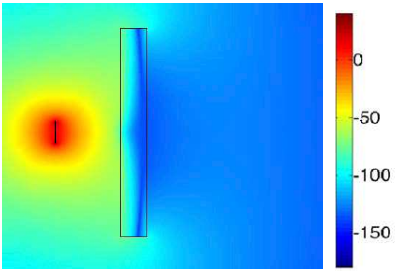
It is highly important to notice that strong electromagnetic effects in the AAs are only obtained in the regime where resonances occur [10]. As a consequence the extraordinary metamaterial properties are usually only observed in a narrow frequency band and the MMs tend to be highly dispersive and lossy [10, 11]. A promising approach to widen the bandwidth and compensate for the losses is embedding electronic components in the AAs (for example, see [12]). It is obvious that this approach becomes increasingly demanding with increasing frequency, i.e., decreasing size of the AAs.

At low frequencies one may easily construct AAs using lumped elements (capacitors, inductors, and resistors) and electronic circuits (transistors, operational amplifiers, etc.) which are much smaller than the wavelength and provide an extremely high degree of freedom in the design of the AAs. Then, the main remaining issue is to obtain a reasonable coupling of the AAs with the electromagnetic field (while especially electronic parts might need to be shielded). Here, the coupling of the magnetic field through magnetic induction is relatively easy. Therefore one may easily engineer the magnetic material properties and mold the flow of magnetic field in a way similar to molding the flow of light by means of optical metamaterials.

The aim of this paper is to demonstrate how anisotropic and inhomogeneous magnetic MMs may be used for molding the flow of the magnetic field. Furthermore, magnetic field shielding as an application of strong practical interest is considered. We will demonstrate that shielding by means of MMs has some advantage over traditional shielding with high μ materials and active magnetic field shielding.



(a) Magnetic field shielding with high μ (b) Magnetic field shielding with low μ



(c) Magnetic field shielding with anisotropic permeability

Figure 1. Shielding with different materials. Magnitude of magnetic flux density B in dB scale at $z = 0$ plane. x axis is the horizontal axis in the figures and y axis is the vertical axis. The magnetic field source is a coil in the center whose projection is depicted as a black line in the figure and the material is given in a rectangular prism shape, whose projection is the black rectangle in the figure. (a) Isotropic case with low μ. (b) Isotropic case with high μ. (c) Anisotropic case with low μ in x direction and high μ in y, z directions.

2. ISOTROPIC VS. ANISOTROPIC MATERIALS

First, we demonstrate that magnetic field shielding can be improved by anisotropic materials considering hypothetical blocks of high permeability μ, low permeability μ, and anisotropic material, which has low permeability in one direction. In Figures 1(a) and 1(b), we see the shielding performance of isotropic homogenous materials, with a high permeability and a low permeability. Relative permeability values $\mu_r = 100$ and $\mu_r = 0.01$ were used to exemplify high permeability and low permeability materials respectively.

Magnetic field shielding by materials with high permeability is based on trapping or guiding the magnetic field in the shield, whereas with low permeability, the material repulses the magnetic field [13]. The anisotropic material used has a low permeability in x direction (See Figure 1). When an anisotropic material with $\mu_{rx} = 0.01$, $\mu_{ry} = 1$ and $\mu_{rz} = 1$ is introduced, we see that shielding may be improved (Figure 1(c)).

3. HOMOGENOUS ANISOTROPIC METAMATERIAL

An approach to manufacture the above mentioned anisotropic low or high μ in practice is to use loaded conductor loops (inductors) to form MMs [14–17]. Anisotropy than comes naturally due to the orientation of the loops in space. Different magnetic properties can be obtained, depending on the resonance frequency of the AAs, which are modeled by RLC circuits. If the frequency of the harmonic magnetic field is lower than the resonance frequency of the RLC circuit, it behaves like a paramagnetic material whereas it behaves like a diamagnetic material above the resonance. Thus, by changing the resonance frequency, one can have different magnetic features, i.e., different effective values of the permeability. An illustration of a "naturally" anisotropic metamaterial is shown in Figure 2.

Assuming that the magnetic field caused by the source and other elements on a RLC resonator is uniform through the resonator coil, i.e., the inductor L, the current induced in the loop is given by [18]:

$$I = -j\omega S B_n / \left(Z_{loop} + Z_{load}\right) \tag{1}$$

where ω is 2π times the frequency of the source magnetic field, B_n the axial component of magnetic flux density produced by the source and the other loops on the resonator loop, S the area of the loop, $Z_{loop} + Z_{load}$ the total impedance of the RLC resonator.

We can write this as:

$$I = k \cdot B_n \tag{2}$$

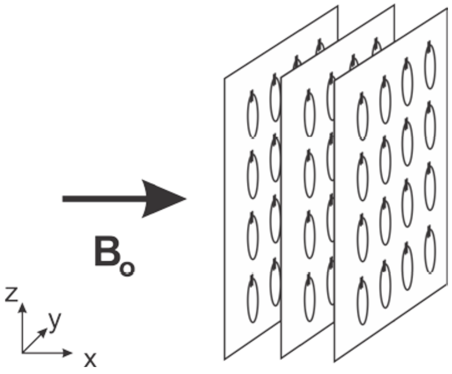


Figure 2. An illustration of a simple metamaterial that becomes "naturally" anisotropic. B_o represents the magnetic flux density to be shielded. Unit elements are capacitor loaded inductors with axes in x direction, i.e., parallel to B_o. In this metamaterial, the permittivity μ_x may be higher or lower than 1, depending on the selected capacitors and inductors. For obtaining strong shielding, a low μ_x is desired here. Additional loaded inductors with axes in y and z directions may be created for obtaining high μ_y and μ_z, which further improves the shielding.

where $k = -j\omega S / \left(Z_{loop} + Z_{load}\right)$.

We can write the axial component of external magnetic field on the loop number j by:

$$B_{n,j} = B_{o,n}\left(x_j, y_j, z_j\right) + \sum_{i \neq j}^{N} m_{ij} I_i \tag{3}$$

where $B_{o,n}$ is the axial component of magnetic flux density produced by the source, $(x_j,\ y_j,\ z_j)$ the location of the center of the jth loop, m_{ij} the axial magnetic field produced by the ith loop with unit current on the jth loop, I_i the current in the ith loop, and N the number of loops.

By (1), (2) and (3), we can write the current in the jth element by:

$$I_j = k_j \left(B_{o,n}\left(x_j, y_j, z_j\right) + \sum_{i \neq j}^{N} m_{ij} I_i \right) \tag{4}$$

After putting the terms with currents on the left and the source term on the right, we obtain the matrix equation:

$$\begin{pmatrix} 1 & -k_1 m_{21} & -k_1 m_{31} & \cdots & -k_1 m_{N1} \\ -k_2 m_{12} & 1 & -k_2 m_{32} & \cdots & -k_1 m_{N2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -k_N m_{1N} & \cdots & \cdots & \cdots & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{pmatrix}$$

$$= \begin{pmatrix} k_1 B_{o,n}(x_1, y_1, z_1) \\ k_2 B_{o,n}(x_2, y_2, z_2) \\ \vdots \\ k_N B_{o,n}(x_N, y_N, z_N) \end{pmatrix} \tag{5}$$

In Figure 3, we see the shielding performance of an array of such loops. In this simulation, magnetic polarizability of loops was optimized to have a good shielding in a certain region. Having a constant magnetic polarizability only in one direction for all loops, we have created a homogenous anisotropic metamaterial. The magnetic field source is a current loop with a radius of 8 cm, the unit elements of metamaterial are current loops with radii of 0.8 cm and the distance between elements is 7 cm. Assume that the aim is to shield the region where $x > 132$ cm. With the normalized current value in the source loop giving a maximum magnetic flux density $B = 100\,\mu\mathrm{T}$ in this region, the maximum magnetic flux density in this region is reduced to approximately 12 μT with the metamaterial. The effective relative permeability calculated for the magnetic polarizability used in Figure 3 is approximately 0.09 [18]. For the calculation of the total magnetic field, we calculate the magnetic field analytically for circular loops, assuming a quasi-static problem [19].

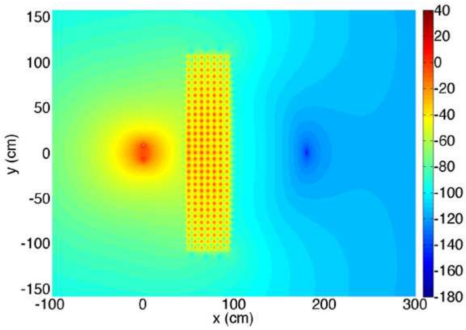


Figure 3. Shielding effect of a homogenous metamaterial. Color shows the magnitude of magnetic flux density B in the $z = 0$ plane in dB scale.

4. INHOMOGENEOUS ANISOTROPIC METAMATERIAL

To improve shielding, we introduce inhomogeneity additional to anisotropy. In this step, instead of coming from a continuous medium approach, we directly solve for the current values in loops, which is different from the approach in [8] and [9]. The current values in the metamaterial loops are unknowns to be solved and equating magnetic flux density at some test points, we obtain a linear matrix equation.

At all test points, the total magnetic flux density shall be minimized in order to obtain strong shielding. If the number of test points is equal to 1/3 of the number of AAs, we may set the flux densities in the test points equal to zero and express it in a linear matrix equation form. For the ith test point we have

$$\vec{B}_{oi} + \sum_{j=1}^{N} I_j \cdot \vec{B'}_{ji} = 0 \tag{6}$$

where $\vec{B}_{oi}$ is the source magnetic flux density vector at the ith test point, $\vec{B'}_{ji}$ the magnetic flux density produced by the jth element carrying unit current at the ith test point, and I_j the current in the jth element. Writing this equation for 3 components of B for all the test points, we have the following matrix equation:

$$\underbrace{\begin{pmatrix} B'_{11,x} & B'_{21,x} & \cdots & B'_{N1,x} \\ B'_{11,y} & B'_{21,y} & \cdots & B'_{N1,y} \\ B'_{11,z} & B'_{21,z} & \cdots & B'_{N1,z} \\ \vdots & \vdots & \vdots & \vdots \\ B'_{1M,y} & \cdots & \cdots & B'_{NM,y} \\ B'_{1M,z} & \cdots & \cdots & B'_{NM,z} \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} I_1 \\ I_2 \\ \\ \vdots \\ \\ I_N \end{pmatrix}}_{I} = \underbrace{\begin{pmatrix} -B_{o1,x} \\ -B_{o1,y} \\ \\ \vdots \\ \\ -B_{oM,y} \end{pmatrix}}_{D} \tag{7}$$

N is the number of loops and M the number of test points.

We used more test points than necessary for a linear matrix equation because using more test points helps delocalize shielding effect more. Hence, the matrix equation was overdetermined. In addition, we put a maximum limit for the current values to prevent too high values and jumps in currents within the current loops of metamaterial. Thus, the problem turned into the following constrained linear least squares problem [20]:

$$\min_{I} \frac{1}{2} \left\|CI - D\right\|_2^2 \text{ such that } -I_{\max} < I < I_{\max}$$

where $I_{\max}$ is the maximum limit for the current magnitude in metamaterial loops and the problem was solved in MATLAB [20].

The new shielding effect can be seen in Figure 4. The maximum magnetic field in the region to be shielded is approximately 0.78 μT and the maximum current in the small loops is less than the 62.5% of the maximum current in the case in Figure 3. Thus, having inhomogeneity additional to anisotropy improves shielding more than 10 times although current values in the metamaterial loops decrease.

Inhomogeneity can be obtained by tuning the resonance frequencies of elements separately for obtaining the desired current values. This can be achieved 1) by changing the capacitance values, 2) by changing the inductors, or 3) by using two inductors in series

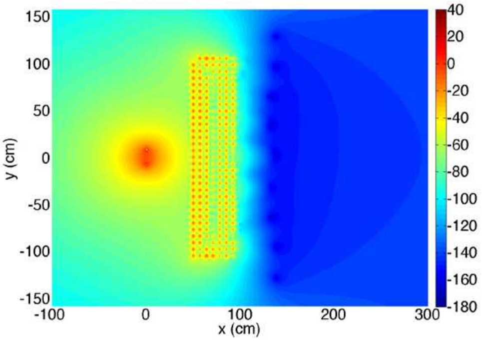


Figure 4. Shielding effect of inhomogeneous metamaterial.

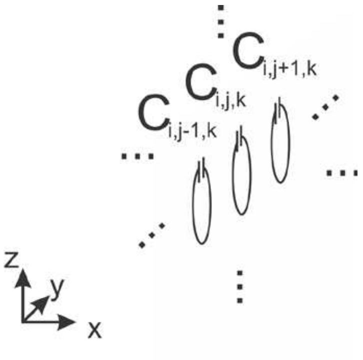


Figure 5. An illustration of an inhomogeneous anisotropic metamaterial built by capacitance values that depend on the location. $C_{i,j,k}$ corresponds to the capacitance in the element having indices i, j, and k, where indices i, j and k run in x, y and z directions respectively.

and changing the mutual inductance between these two by changing the distance between the inductors, as mentioned in the next section. Figure 5 depicts an inhomogeneous metamaterial obtained by changing capacitance values.

5. PRACTICAL ASPECTS

The stray magnetic field produced by power transformer stations in residential areas is subject to regulations and shielding is needed to satisfy the requirements [13]. In some countries, these regulations became stricter a few years ago. For example, in Switzerland the 100 μT limit for 50 Hz was reduced to 1 μT, which leads to the need of additional shielding in building near power transformers. Since the traditional shielding technique by means of μ-metal may become very expensive, the shielding technique by metamaterials proposed in this paper is attractive.

In theory, the magnetic flux density phasor of an LC loop is in the same direction or opposite direction as the external magnetic flux density phasor in the complex phasor plane. However, in practice, resistance introduces reduction in current values and a phase shift. As seen in Figure 6, a single RLC loop is never able to shield perfectly with

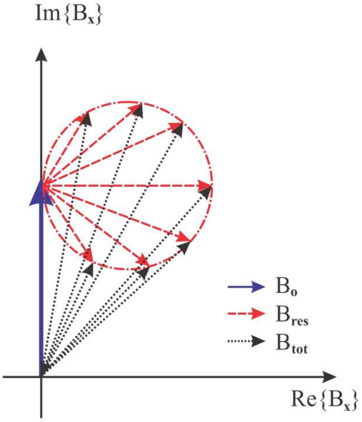


Figure 6. Response of an RLC loop: Phasors of the source (blue, continuous), resonator (red, dashed), and total(black, dotted) magnetic flux density. The circle shows the trace of resonator magnetic flux density for frequencies from 0 to infinity. The total magnetic flux density can never reach 0 if there is a resistance R in the circuit, which implies a finite radius of the circle.

a nonzero resistance R, because the total magnetic field phasor cannot be zero. Adding a series capacitance to a shielding loop and a similar diagram can be found in [21]. The polarizabilities of the LC loops in the preceding sections are possible only with zero resistance. However, since the method aims very low frequencies (for example, power frequency magnetic field shielding is for 50–60 Hz), compensation of the resistance is possible with some additional electronic circuits. Using a negative impedance converter, the resistance of the coil can be compensated or the circuit can be converted to a Non-Foster circuit which has more bandwidth and a gyrator would allow to replace the capacitor by a second inductor (which would also couple to the magnetic field) [22–24].

The metamaterial unit element which has an inductor connected to a capacitor is limited by commercially available components and does not provide a fine tuning after assembling. Using more than one inductor enables tuning of inductance by mutual coupling, thus tuning of resonance frequency and current in a unit element, i.e., an AA. The total inductance of two inductors connected in series depends on the mutual inductance between coils as well as individual self inductances [25]. By changing the relative positioning of two coils connected in series, the total inductance in the circuit, hence the resonance frequency can be adjusted to the desired value.

Instead of using mechanical tuning to tune resonance frequency by changing mutual inductance, the active circuits mentioned above to compensate resistance may also be designed in a way to tune the impedance/current values electronically.

As shown in Figure 7, a field shielding device can be converted to a field focusing device by controlling the current/impedance values in elements. Thus, electronic control of currents would make electronic steering of magnetic field possible.

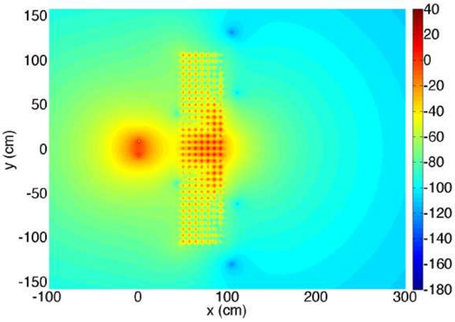


Figure 7. Focusing the magnetic field by a metamaterial block.

6. COMPARISON WITH COMMONLY USED SHIELDING TECHNIQUES

Magnetic field shielding techniques can be categorized in two main groups: passive and active shielding [13]. In passive shielding, no external power sources are added into the shielding system and shielding is done by material properties only. There are two traditional shielding mechanisms for passive shielding. One is to trap/guide the magnetic field using materials with high relative permeability and the second mechanism is eddy current cancellation by highly conductive materials, where source magnetic field induces some current loops in the conductive material which oppose the source magnetic field. The use of materials with very high permeability (mu-metal is a typical example) provides better shielding performance than eddy current cancellation at low field values in general. However, permeability and shielding performance at high field strengths decreases [26, 27]. Additional to this, mu-metal comes with the disadvantage of high cost [26].

Active shielding is a suitable method for local shielding in general. It is based on applying some controlled currents in the system to cancel the source magnetic field [11, 28]. The metamaterial shielding is similar to passive shielding when there is only RLC resonator and no external circuit, because the shielding is done in principle by effective material properties. When some active components are added to the RLC circuits to control the resonance frequency or to provide some external power to the circuits for loss compensation, the metamaterial starts resembling an active shield. However, it is fundamentally different from active shielding techniques, which are continuously monitoring the magnetic field at some test points and provide current values in the loops with a reasonable reaction time to cancel the magnetic field [28]. The metamaterial method presented here, requires the source magnetic field profile but not a continuous monitoring of it. If the strength of the magnetic source varies, the electronic steering of the active shielding must react and tune the currents in the loops. No such steering is required in the metamaterial approach because the AAs react automatically on variations of the field strength.

7. CONCLUSION

In this work, we showed how metamaterials can be designed to mold the flow of magnetic field, focusing on shielding magnetic field in the quasi-static limit. It was demonstrated that it is possible to shield the magnetic field with improved performance by designing anisotropic, inhomogeneous metamaterials. This method may be an alternative to standard passive shielding and to active shielding techniques. Moreover, the possibility of electronic steering of the magnetic field may be interesting for some applications other than shielding.

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