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ELECTROMAGNETIC FIELD OF A HORIZONTAL INFINITELY LONG MAGNETIC LINE SOURCE OVER THE EARTH COATED WITH A DIELECTRIC LAYER

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Abstract—In this paper, the electromagnetic field of a horizontal infinitely long magnetic line source over the dielectric-coated earth is treated analytically, and the complete approximate solution for the radiated field under the far-field conditions is outlined. The total field is composed of four modes: the direct wave, the ideal reflected wave or image wave, the trapped surface wave, and the lateral wave. In particular, the complete analytical formulas are obtained for both the trapped surface wave and the lateral wave. The trapped surface wave is determined by the sum of residues of the poles. When the infinitely long magnetic line source or the observation point is away from the planar surface of the dielectric-coated earth, the trapped surface wave decreases exponentially with height, and the total field is determined primarily by the lateral wave. When the conductivity of the earth is large, and both the infinitely long magnetic line source and the observation point are on or close to the air-dielectric boundary, the total field is determined primarily by the trapped surface wave.

1. INTRODUCTION

The electromagnetic field generated by a horizontal infinite wire has been intensively investigated in the past century. The earliest analytical solution for the electromagnetic field of a horizontal infinite wire was formulated by Carson [1]. Lately, with the extension of the Carson's theory, the problem "a wire above earth" was treated by

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other pioneers [2–8]. In particular, it should be mentioned that an important progress was made by Wait [9]. In the well-known work, an exact modal equation was presented for the wave propagation of a thin wire above the earth. In 1977, Wait extended his work on this problem to the general case of an arbitrary number of wires above the earth with arbitrary number of layers [10]. The subsequent contributions on the problem "a wire above earth" were carried out by many investigators and a good summary of the research findings can be found in the invited paper by Olsen et al. [11].

For the analogical problem "a dipole above earth", the electromagnetic field of a horizontal or vertical electric dipole above the earth media was also intensively investigated for over a century. Many papers on this problem were published and the details were well summarized in the three classic books [12–14]. Nevertheless, the existence or nonexistence of the trapped surface wave for the problem "a dipole above layered media" was still the controversial issue in a series of papers. The new developments, which were summarized in the book by Li [15], rekindled the interest in the study on the problem "a wire above multi-layered earth".

In the parallel paper [16], the electromagnetic field of a horizontal infinitely long wire over the dielectric-coated earth is treated analytically. In this paper, the electromagnetic field of a horizontal infinitely long magnetic line source over the dielectric-coated earth is treated analytically, and the complete approximate solution for the electromagnetic field under the far-field conditions is outlined.

2. ELECTROMAGNETIC FIELD OF A HORIZONTAL INFINITELY LONG MAGNETIC LINE SOURCE OVER THE DIELECTRIC-COATED EARTH

The geometry and notation underlying the analysis are shown in Fig. 1. It consists of a horizontal infinitely long magnetic line source at the height h above the air-dielectric boundary with its axis parallel to the $\hat{y}$-axis. The air (Region 0, $z \geq 0$) is characterized by the permeability μ_0, uniform permittivity ε_0, and conductivity $\sigma_0 = 0$. The dielectric layer (Region 1, $-l \leq z \leq 0$) is characterized by the permeability μ_0, relative permittivity ε_{r1}, and conductivity σ_1. The earth (Region 2, $z \leq -l$) is characterized by the permeability μ_0, relative permittivity ε_{r2}, and conductivity σ_2. Then, the wave numbers in the three-layered region can be written in the following forms:

$$k_j = \omega\sqrt{\mu_0\left(\epsilon_0\epsilon_{rj} + i\sigma_j/\omega\right)}; \quad j = 0, 1, 2. \tag{1}$$

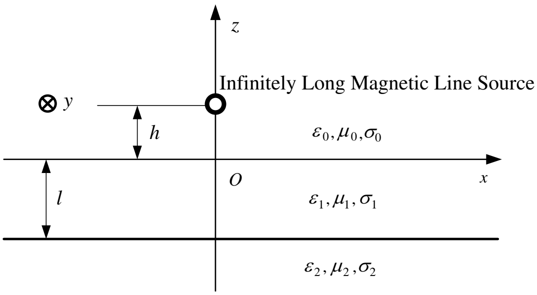

Figure 1. The geometry of a horizontal infinitely long magnetic line source at the height h above the dielectric-coated earth.

With a time dependence $e^{-i\omega t}$, the integrated formulas of the Hertz potential in the air for the electromagnetic field radiated by a horizontal infinitely long magnetic line source over a dielectric-coated earth are obtained in the classic book by Wait [13]. By using the relations between the electromagnetic field components and the Hertz potential, the magnetic field H_{0y} is expressed as follows:

$$H_{0y} = H_{0y}^{(1)} + H_{0y}^{(2)} + H_{0y}^{(3)}, \tag{2}$$

The first and second terms in (2), which were evaluated in the classic book by Wait [13], stand for the direct wave and the ideal reflected wave (or image field), respectively. They are written in the following forms:

$$\begin{aligned} H_{0y}^{(1)} &= -\frac{k_0^2}{4\omega\mu_0\pi}\int_{-\infty}^{\infty}\frac{e^{i\lambda x}}{\gamma_0}e^{i\gamma_0|z-h|}d\lambda \\ &= -\frac{k_0^2}{2\omega\mu_0\pi}K_0\left[ik_0\sqrt{x^2+(z-h)^2}\right], \end{aligned} \tag{3}$$

$$\begin{aligned} H_{0y}^{(2)} &= \frac{k_0^2}{4\omega\mu_0\pi}\int_{-\infty}^{\infty}\frac{e^{i\lambda x}}{\gamma_0}e^{i\gamma_0(z+h)}d\lambda \\ &= \frac{k_0^2}{2\omega\mu_0\pi}K_0\left[ik_0\sqrt{x^2+(z+h)^2}\right]. \end{aligned} \tag{4}$$

In (3) and (4), K_0 is the modified Bessel function. The third term in (2) is written in the form of

$$H_{0y}^{(3)} = k_0^2\frac{1}{4\omega\mu_0\pi}\int_{-\infty}^{\infty}\frac{e^{i\lambda x}}{\gamma_0}\left((1-R_{||})e^{i\gamma_0(z+h)}\right)d\lambda, \tag{5}$$

where

$$1 - R_{||} = 2 \cdot \frac{\gamma_0 - i\tan(\gamma_1 l)\left(\frac{\gamma_0\gamma_2}{\gamma_1}\right)}{(\gamma_2 + \gamma_0) - i\tan(\gamma_1 l)\left(\frac{\gamma_0\gamma_2}{\gamma_1} + \gamma_1\right)}, \tag{6}$$

$$\gamma_j = \sqrt{k_j^2 - \lambda^2}; \quad j = 0, 1, 2. \tag{7}$$

In the next step, we will attempt to evaluate the above integral in (5) by using analytical techniques. In order to evaluate the integral in (5), we shift the contours around the branch cuts at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. The poles of the integrand in (5) satisfy the following equation

$$q(\lambda) = (\gamma_2 + \gamma_0) - i\tan(\gamma_1 l)\left(\frac{\gamma_0\gamma_2}{\gamma_1} + \gamma_1\right) = 0. \tag{8}$$

For analyzing the characteristics of the pole Equation (8), a special case, when the earth is regarded as a perfect conductor and the dielectric is lossless, is considered. Let $k_2 \to \infty$, the pole equation is simplified as

$$q(\lambda) = \gamma_1 - i\tan(\gamma_1 l) \cdot \gamma_0 = 0. \tag{9}$$

Evidently, it is seen that, when $0 < \sqrt{k_1^2 - k_0^2}\, l < \pi$, the pole Equation (9) has one root. Correspondingly, there is one pole for the integrand function. In general, when $n\pi < \sqrt{k_1^2 - k_0^2}\, l < (n+1)\pi$, the pole Equation (9) has $(n+1)$ roots. Correspondingly, there are $(n+1)$ poles for the integrand function.

Considering $k_0 < \lambda_j^* < k_1$, $\gamma_0(\lambda_j^*) = i\sqrt{\lambda_j^{*2} - k_0^2}$ is a positive imaginary number. It is seen that $e^{i\gamma_{0j}^*(z+h)}$ is an attenuation factor. When the infinitely long magnetic line source or the observation point is away from the air-dielectric boundary, the terms of the trapped surface wave which include the factor $e^{i\gamma_{0j}^*(z+h)}$ will decrease exponentially as $e^{-\sqrt{\lambda_j^{*2} - k_0^2}(z+h)}$ in the $\hat{z}$ direction.

In both cases dielectric layer and the earth being lossy media, besides the poles λ_j, three integrands have the branch cuts at $\lambda = k_j (j = 0, 1, 2)$. As shown in Fig. 2, there are three branch cuts at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. The poles, which are the roots λ_j of the pole Equation (8), can be obtained readily by using Newton's iteration method or the numerical integration method addressed in Section 2.4.2 in [15]. It is noted that the roots λ_j, which are the singular poles in the λ complex plane, are in the range from k_0 to k_1. The poles, which are determined by the wave numbers k_0, k_1, and k_2, vary with the thickness of the dielectric layer.

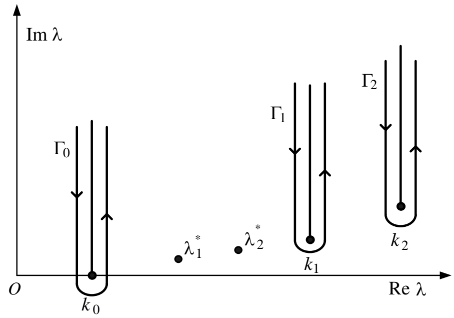

Figure 2. The configuration of the poles and branch lines.

It is well known that the evaluation is contributed by the sums of residues of the poles and the integrations along the branch cuts. The term of the trapped surface wave is contributed by the sums of residues of the poles. The integration along the branch cuts Γ_0 and Γ_1 is contribution to the term of the lateral wave. Thus, the trapped surface waves can be expressed in the following term:

$$H_{0y}^{S}=\frac{2}{\omega\mu_0}\sum_j e^{ix\lambda_j^*+i\gamma_0\left(\lambda_j^*\right)(z+h)}\cdot\frac{\gamma_0\left(\lambda_j^*\right)-i\tan\left[\gamma_1\left(\lambda_j^*\right)l\right]\frac{\gamma_0\left(\lambda_j^*\right)\gamma_2\left(\lambda_j^*\right)}{\gamma_1\left(\lambda_j^*\right)}}{q'\left(\lambda_j^*\right)},\quad(10)$$

where

$$\begin{aligned}q'(\lambda) \;=\; & -\frac{\lambda}{\gamma_2}-\frac{\lambda}{\gamma_0}+i\tan(\gamma_1 l)\left(\frac{\lambda}{\gamma_1}+\frac{\gamma_0\lambda}{\gamma_1\gamma_2}+\frac{\gamma_2\lambda}{\gamma_0\gamma_1}-\frac{\gamma_0\gamma_2\lambda}{\gamma_1^3}\right)\\ & +i\frac{l\lambda}{\gamma_1}\sec^2(\gamma_1 l)\left(\frac{\gamma_0\gamma_2}{\gamma_1}+\gamma_1\right).\end{aligned}\quad(11)$$

From (10), it is seen that the trapped surface wave is determined by the sums of residues of the poles. The wave numbers λ_j of the trapped surface wave, which are the roots of the pole Equation (8), are shown in Fig. 2. It is seen that $e^{i\gamma_{0j}^*(z+h)}$ has an attenuation factor for a positive imaginary number in γ_{0j}^*.

Because the wavefront of the trapped surface wave radiated by a horizontal infinitely long magnetic long wire over dielectric-coated earth is always a plane, the amplitude of the trapped surface wave has no dispersion losses, which is difficulty to be understood. This characteristic is like that of the propagation of the electromagnetic wave in a rectangle waveguide. It is well known that the amplitude of the electromagnetic wave in a rectangle waveguide has no dispersion losses. This characteristic is different from that of the electromagnetic field excited by a dipole source in the presence of a three-layered region.

For the case of an electric or magnetic dipole source, the wavefront is enlarged as the dispersion of the trapped surface, the amplitude of the trapped surface wave attenuates as $\rho^{-1/2}$ in the $\hat{\rho}$ direction.

By examining the integral in (5) and the pole Equation (8), it is seen that, besides the poles λ_j, there are three branch lines at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. Because the integrand $q(\lambda)$ is even function to γ_1, the evaluation of the integration along the branch line Γ_1 is zero. Considering that the wave number k_2 in the earth has a positive imaginary part, the integral in (5) along the branch line Γ_2, which means the lateral wave traveling in Region 2 (Earth) along the earth-dielectric boundary, can be neglected. In what follows, we will evaluate the contributions of the integrations along the branch line Γ_0.

$$H_{0y}^{L} = -k_0^2 \frac{1}{4\omega\mu_0\pi} \int_{\Gamma_0} \frac{e^{i\lambda x}}{\gamma_0} \cdot \left(1 - R_{||}\right) \cdot e^{i\gamma_0(z+h)} d\lambda. \tag{12}$$

Taking into account the far-field conditions of $k_0 x \gg 1$ and $z + h \ll x$, the dominant contribution of the integration in (12) along the branch line Γ_0 comes from the vicinity of k_0. On the left and right sides of the branch line Γ_0, the phases of γ_0 are $e^{-i\frac{\pi}{4}}$ and $e^{i\frac{3\pi}{4}}$, respectively. Let $\lambda = k_0(1 + i\tau^2)$, γ_0, γ_1, and γ_2 are approximated as follows:

$$\gamma_0 = \sqrt{k_0^2 - \lambda^2} \approx k_0\sqrt{2}e^{i\frac{3\pi}{4}}\tau, \tag{13}$$

$$\gamma_1 = \sqrt{k_1^2 - \lambda^2} \approx \sqrt{k_1^2 - k_0^2} = \gamma_1^*, \tag{14}$$

$$\gamma_2 = \sqrt{k_2^2 - \lambda^2} \approx \sqrt{k_2^2 - k_0^2} = \gamma_2^*. \tag{15}$$

Then, the result becomes

$$\begin{aligned} H_{0y}^{L} &= -\frac{i}{\sqrt{2}\omega\mu_0\pi} e^{ik_0x - ik_0x\Delta^2 + i\frac{k_0(z+h)^2}{2x}} \\ &\quad \cdot \left(\sqrt{\frac{\pi}{k_0 x}} + \frac{iB\pi}{\sqrt{2}Ak_0}\right) \cdot \operatorname{erfc}\left(\sqrt{-ik_0x\Delta^2}\right), \end{aligned} \tag{16}$$

where the error function is defined by

$$\operatorname{erfc}(x) = -\frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt, \tag{17}$$

and

$$A = -i\tan(\gamma_1 l) \cdot \frac{\gamma_2}{\gamma_1} + 1, \tag{18}$$

$$B = -i\tan(\gamma_1 l) \cdot \gamma_1 + \gamma_2, \tag{19}$$

$$\Delta = \frac{1}{2}\left(\frac{z+h}{x} - \frac{B}{Ak_0}\right). \tag{20}$$

By using the above derivations for the terms of the trapped surface wave and lateral wave, and the results of the direct and ideal reflected waves addressed in the book by Wait [13], the magnetic field H_{0y} can be expressed in the following form:

$$H_{0y} = -k_0^2 \frac{1}{2\omega\mu_0\pi} K_0\left[ik_0\sqrt{x^2+(z-h)^2}\right] + k_0^2 \frac{1}{2\omega\mu_0\pi} K_0\left[ik_0\sqrt{x^2+(z+h)^2}\right]$$
$$+\frac{2}{\omega\mu_0}\sum_j e^{ix\lambda_j^*+i\gamma_0\left(\lambda_j^*\right)(z+h)} \cdot \frac{\gamma_0\left(\lambda_j^*\right) - i\tan\left[\gamma_1\left(\lambda_j^*\right)l\right]\frac{\gamma_0\left(\lambda_j^*\right)\gamma_2\left(\lambda_j^*\right)}{\gamma_1\left(\lambda_j^*\right)}}{q'\left(\lambda_j^*\right)}$$
$$+i\frac{1}{\sqrt{2}\omega\mu_0\pi} e^{ik_0x-ik_0x\Delta^2+i\frac{k_0(z+h)^2}{2x}} \cdot \left(\sqrt{\frac{\pi}{k_0x}} + \frac{iB\pi}{\sqrt{2}Ak_0}\right) \mathrm{erfc}\left(\sqrt{-ik_0x\Delta^2}\right). \quad (21)$$

With similar procedure, the approximate formulas for the other two components E_{0x} and E_{0z}, can also be obtained readily. From the above derivations, it is seen that the total field is composed of the direct wave, the ideal reflected wave or image wave, the trapped surface wave, and the lateral wave.

3. COMPUTATIONS AND DISCUSSIONS

With $z = h = 0\,\mathrm{m}$, $f = 30\,\mathrm{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r2} = 8.0$, and $\sigma_2 = 0.4\,\mathrm{S/m}$, for the magnetic field H_{0y}, magnitudes of the total field, the trapped surface wave, and the "DRL waves", which is composed of the direct wave, the ideal reflected wave, and the lateral wave, are computed at $k_1l = 2.08\pi$, respectively. Those results are shown in Fig. 3.

With $z = h = 0\,\mathrm{m}$, $f = 30\,\mathrm{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r2} = 80.0$, $\sigma_2 = 4.0\,\mathrm{S/m}$, and $k_1l = 2.08\pi$, for the magnetic field H_{0y}, magnitudes of the total field, the trapped surface wave, and the DRL waves, are computed and plotted in Fig. 4, respectively. On contrast of Figs. 3 and 4, it is seen that the trapped surface wave is affected significantly by the conductivity of the earth. When the conductivity of the earth is low, the trapped surface wave attenuates rapidly. When the conductivity of the earth is large or $k_2 \to \infty$, and both the infinitely long magnetic line source and the observation point are on or close to the air-dielectric boundary, the total field is determined primarily by the trapped surface wave.

Similar to those in Figs. 3 and 4, the corresponding results are computed at $z = h = 3\,\mathrm{m}$ and shown in Fig. 5. It is concluded that the total field is dominated by the DRL waves when the infinitely long magnetic line source or the observation point is away from the air-dielectric boundary.

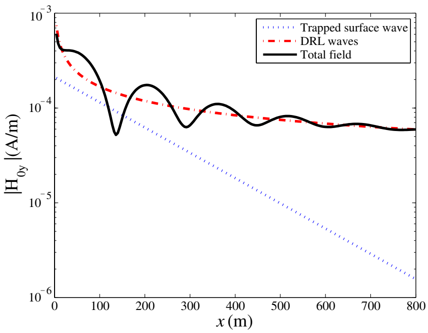

Figure 3. Magnitude of H_{0y} in A/m versus the propagating distances at $k_1 l = 2.08\pi$ with $f = 30\,\text{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r2} = 8.0$, $\sigma_2 = 0.4\,\text{S/m}$, and $z = h = 0\,\text{m}$.

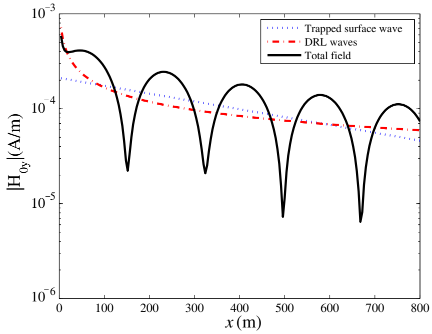

Figure 4. Magnitude of H_{0y} in A/m versus the propagating distances at $z = h = 0\,\text{m}$ with $f = 30\,\text{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r2} = 80.0$, $\sigma_2 = 4.0\,\text{S/m}$, and $k_1 l = 2.08\pi$.

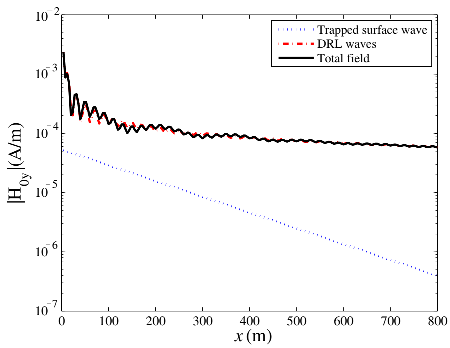

Figure 5. Magnitude of H_{0y} in A/m versus the propagating distances at $z = h = 3\,\text{m}$ with $f = 30\,\text{MHz}$, $\varepsilon_{r1} = 2.65$, $\varepsilon_{r2} = 8.0$, $\sigma_2 = 0.4\,\text{S/m}$, and $k_1 l = 2.08\pi$.

Generally speaking, for the problem "a dipole above multi-layered earth", because of wide applications of the radiation of line antenna, the study is usually demonstrated on the electromagnetic field radiated by an electric dipole. For the problem "a wire above multi-layered earth", each line can be regarded as a magnetic line source. So that the electromagnetic field radiated by a magnetic line source above multi-layered earth is investigated widely in available works, especially including those in the famous book by Wait [13]. In spite of the analytical solution of the electromagnetic field radiated by an electric line source on the dielectric-coated earth was given in the parallel paper [16], the study on the electromagnetic field by a magnetic line source is also important. In [16], some characteristics, which include no dispersion losses for the amplitude of the trapped surface wave and the affect by the conductivity of the earth, were not addressed clearly as those in the present papers.

4. CONCLUSIONS

Following the above derivations and analysis, it is seen that the total field consists of the direct wave, the ideal reflected wave or image wave, the trapped surface wave, and the lateral wave. In particular, both the trapped surface wave and the lateral wave are treated analytically. The wave numbers of the trapped surface wave, which are determined by the sum of residues of the poles, are in the range from k_0 to k_1. The wavefront of the trapped surface wave over dielectric-coated earth is always a plane, which is not enlarged in the propagation, and the amplitude of the trapped surface wave has no dispersion losses. The trapped surface wave is affected significantly by the conductivity of the earth. When the conductivity of the earth decreases, the trapped surface wave attenuates rapidly. When the infinitely long magnetic line source or the observation point is away from the air-dielectric boundary, the trapped surface wave decreases exponentially in the $\hat{z}$ direction, and the total field is determined primarily by the lateral wave. When the conductivity of the earth is large or $k_2 \to \infty$, and both the infinitely long magnetic line source and the observation point are on or close to the air-dielectric boundary, the total field is determined primarily by the trapped surface wave.

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