Progress In Electromagnetics Research M, Vol. 24, 141–155, 2012

OPTIMAL SYNTHESIS OF THINNED ARRAYS USING BIOGEOGRAPHY BASED OPTIMIZATION

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Abstract—Thinning of large arrays in order to produce low side lobes is a difficult task. Conventional gradient methods often stuck in local minima and hence are not capable of obtaining optimum solutions. As a result, global optimization methods are required to thin large antenna arrays. In this paper, a global evolutionary method, Biogeography Based Optimization (BBO) is introduced as a new tool for thinning large linear and planar antenna arrays of uniformly excited isotropic antennas. The aim is to synthesize linear arrays so as to yield the maximum relative sidelobe level equal to or below a desired level while also keeping the percentage of thinning equal to or above the desired level. The results obtained by BBO are compared with the previous published results of Genetic Algorithm (GA), Ant Colony Optimization (ACO), Immunity Genetic Algorithm (IGA) and Binary Particle Swarm Optimization (BPSO).

1. INTRODUCTION

Thinning of an array means to selectively switch off certain elements of the array of a uniform spaced array to achieve a pattern with low sidelobes. Thinning a large array for low sidelobes involves checking a rather large number of possibilities and these increases exponentially with number of array elements. Hence checking every possible combination to find the optimum one is nearly impossible. Classical optimization methods are not well suited for thinning of arrays because they often get stuck in local minima and are capable of optimizing only a few continuous variables. Hence global optimization tools are a good option to solve these problems. Different global

Received 5 February 2012, Accepted 17 April 2012, Scheduled 19 April 2012
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optimization methods such as Genetic Algorithm (GA) [1–8], Particle Swarm Optimization (PSO) [9], simulated annealing [10], Differential Evolution (DE) [11, 12], Ant Colony Optimization (ACO) [13] etc. have already been utilized for thinning of arrays and have proven to provide high-quality results. Unlike classical methods, global methods do not require initial guess, are capable of escaping the local minima and are able to find the global minima. However, due to their global nature, these are slow in converging as compared to local gradient methods.

Haupt [1] used GA in process of thinning a linear array of 200 elements, and resulting sidelobe levels were lower than −18 dB in all cases shown. Weile and Michielssen have employed a Pareto Genetic Algorithm (PGA) for the thinning of linear arrays [2]. Johnson and Rahmat-Samii [3] used GA to thin a 40 elements linear array and achieved sidelobe levels of somewhat lower than −20 dB. Mahanti et al. [4] used Real-coded Genetic Algorithm (RGA) to thin a large linear array of uniformly excited isotropic elements to yield side lobe level (SLL) equal to or below a fixed level, while keeping the percentage of thinning equal to or above a fixed value. Hamici and Ismail have used Immunity Genetic Algorithm (IGA) based on stochastic crossover evolution to solve the synthesis problem of thinned arrays and have obtained good results [5]. Fernández-Delgado et al. [6] have proposed a simple and fast method which accelerates the calculation of the far-field pattern and consequently the evaluation of the fitness function in the global optimization methods used in array thinning. They have reduced the search time of algorithm by 90%. Zhang et al. have employed Orthogonal Genetic Algorithm (OGA) for thinning of planar arrays [8]. Jin and Rahmat-Samii [9] have used Binary Particle Swarm Optimization (BPSO) for the thinning of linear arrays. Quevedo-Teruel and Rajo-Iglesia have applied ACO for the thinning of linear and planar arrays for different scenarios [13]. Razavi and Forooraghi have employed pattern search algorithm for synthesis of linear arrays [14]. Wang et al. have utilised modified iterative Fourier technique for thinning of linear arrays [15].

The purpose of this paper is to use an algorithm based on another global search method known as Biogeography Based Optimization (BBO) to synthesize thinned arrays with low SLL and desired level of thinning. BBO has been applied for the design of linear antenna arrays for obtaining the maximum SLL reduction and null placement in desired directions in [16]. Results obtained using BBO for the linear arrays are encouraging. The BBO method produced a lower value of SLL and better null placement as compared to PSO [17]. BBO has also been used for the optimization of Yagi-Uda and circular antennas [18, 19]. This algorithm has also been employed to solve different problems in different areas such as the power flow problem [20], optimization of gear trains [21], and satellite image classification problems [22]. However, it has not been used never before for thinned array synthesis of linear antennas.

2. THINNED ARRAY SYNTHESIS

Thinning an array means turning off some elements in a uniformly spaced or periodic array to generate a pattern with low SLL. Thinning an array to produce low sidelobes is much simpler than unequally spacing the elements for generating pattern with low SLL [1]. Moreover, thinning of array results in reduction in cost, weight and power consumption. In this work, the elements positions are assumed to be fixed. All the elements have two states either "on" (if the element is fed) or "off" (if the element is passively terminated to a matched load). The linear antenna having $2N$ elements placed symmetrically along the z-axis is given in Figure 1. The array factor (AF) for this antenna can be written as [13]:

$$\mathrm{AF}(\theta) = 2\sum_{n=1}^{N} I_n \cos[\pi \cdot (2n-1) \cdot \cos\theta] \tag{1}$$

where n is the element number, I_n is the excitation amplitude of the n-th element. In our case, I_n is 0 if the state of the n-th element is "off" and 1 if it is "on". The distance between the elements is $\lambda/2$ and all the elements have same excitation phase. The elements are numbered from the array center and array center is assumed to be at the origin.

Figure 2 shows a planar array structure $2N \times 2M$ of elements. The array factor for this structure is given by (assuming the same

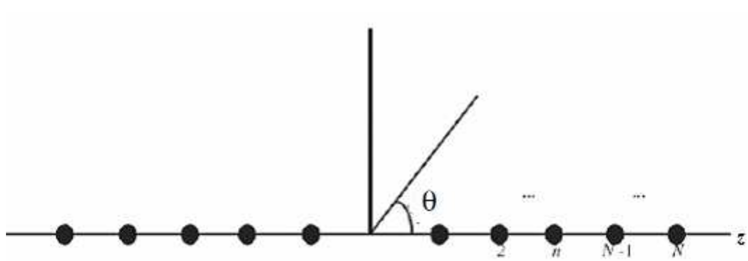

Figure 1. Geometry of a $2N$-element symmetric linear array along the z-axis.

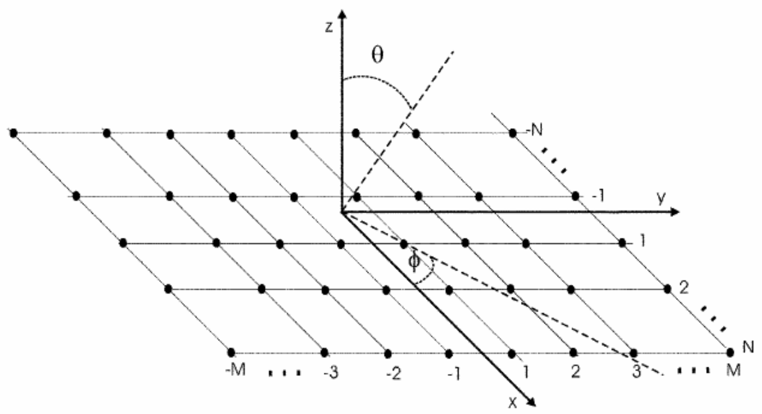

Figure 2. Geometry of a $2N \times 2M$-element symmetric planar array.

considerations as in the linear array) [13]:

$$\mathrm{AF}(\theta,\phi) = 4\sum_{n=1}^{N}\sum_{m=1}^{M} I_{nm}\cos[\pi(2n-1)\sin\theta\cos\phi] \cdot\cos[\pi(2m-1)\sin\theta\sin\phi] \quad (2)$$

Therefore, the aim of optimization is to find out which array elements should be enabled or disabled ($I_{nm} = 1$ or $I_{nm} = 0$) to get the desired radiation pattern characteristics.

3. BIOGEOGRAPHY THEORY

Biogeography is the study of distribution of biodiversity over space and time. The science of biogeography was sown by naturalists like Alfred Wallace and Charles Darwin. Till 1967, biogeography was mainly descriptive study. But the work carried out by MacAurthur and Wilson [23] changed this perception. They were able to construct a mathematical model for biogeography and made it feasible to predict the number of species in a habitat.

In the science of biogeography, a habitat is an ecological area that is inhabited by a particular plant or animal species and which is geographically isolated from other habitats. Each habitat is classified by Habitat Suitability Index (HSI). Areas or habitats which are well suited as living places for biological species have high HSI while habitats that are not good have low HSI. The value of HSI depends upon many features of habitat like rainfall, temperature, diversity of vegetation, land area, safety and security. If each of the features is assigned a value, HSI is a function of these values. Each of these features that characterize habitability is known as Suitability Index Variables (SIV). SIVs are the independent variables while HSI are the dependent variables.

Habitats with high HSI have large population, high emigration rate μ, simply by virtue of large number of species that migrate to other habitats. The immigration rate η is low for these habitats as these are already saturated with species. On the other hand, habitats with low HSI have high immigration rate η, low emigration rate μ because of sparse population. The value of HSI of low HSI habitat may increase with the influx of species from other habitats as suitability of a habitat is function of its biological diversity. But if HSI does not increase and remains low, species in that habitat go extinct and this leads to additional immigration. For sake of simplicity, it is safe to assume a linear relationship between a habitat HSI and its immigration and emigration rate and also that the rates are same for all the habitats. The immigration and emigration rate depends upon the number of species in the habitats. These relationships are shown in Figure 3.

The values of emigration and immigration rates are given as:

$$\eta = I\left(1 - \frac{k}{n}\right) \tag{3}$$

$$\mu = \frac{E}{n} \tag{4}$$

where I is the maximum possible immigration rate; E is the maximum possible emigration rate; k is the number of species of the k-th individual and n is the number of species.

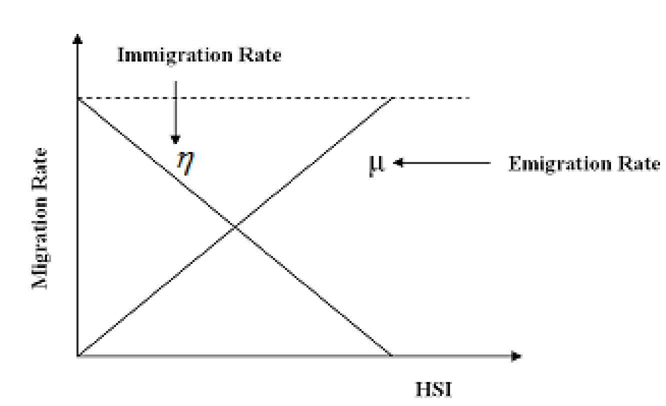

Figure 3. Habitat migration rate vs. habitat suitability index.

4. BIOGEOGRAPHY-BASED OPTIMIZATION

BBO is novel population based global optimization algorithm stimulated by science of biogeography. The candidate solutions for a problem are considered as habitats. Each solution is associated with the fitness which is analogous to HSI of a habitat. A good solution is analogous to a habitat with high HSI and a poor solution represents a habitat with a low HSI. Good solutions share their features with poor solutions by means of migration. Good solutions have more resistance to change than poor solutions. On the other hand poor solutions are more dynamic and accept a lot of new features from good solutions.

Consider a global optimization problem and population of possible solutions. In an N_{var} — dimensional optimization problem, a habitat is a $1 \times N_{var}$ array. Each solution is represented by N-dimensional integer vector as $[\mathrm{SIV}_{1\ldots},\mathrm{SIV}_{N_{var}}]$. In BBO, an SIV is a solution feature corresponding to "gene" while habitat is similar to "chromosome" in other population-based optimization algorithm like GA. The variable values or SIVs in the habitat are represented as binary numbers. The set of all such vectors is the search space from which the optimum solutions are to be found. The value of HSI of a habitat is value of objective function associated with that solution. The value of HSI is found by evaluating the cost of function at the variables $[\mathrm{SIV}_{1,\ldots}\mathrm{SIV}_{N_{var}}]$. Therefore,

$$\mathrm{HSI} = f(\mathrm{Habitat}) = f(\mathrm{SIV}_1, \ldots \mathrm{SIV}_{N_{var}}) \tag{5}$$

These solutions are made to share features among themselves by applying migration operator. For each SIV, in each solution, it is decided probabilistically whether or not to immigrate. If immigration is selected for a given solution feature, the emigrating habitat is selected for a given solution probabilistically using roulette wheel normalized by μ. The mutation operator is probabilistically applied to the habitat which tends to increase the biological diversity of the population. The mutation rate m is inversely proportional to the solution probability which is given by:

$$m = m_{\max}\left(1 - \frac{P}{P_{\max}}\right) \tag{6}$$

where $m_{\max}$ is a user-defined parameter.

As in other population-based optimization algorithms, elitism is introduced so that the best solutions are retained in the population from one generation to the next. The BBO algorithm is shown in Figure 4.

Mutation and migration operators in BBO are similar to GA and PSO and therefore it is also applicable to same type of problems as GA

```
Create a random set of habitats (population) H1, H2,.... Hn ;
Calculate SI values;
While the halting criterion is not satisfied do
    Compute immigration rate η and emigration rate μ for each habitat based on SI;
    For each habitat (solution)
        For each SIV (solution feature)
            Select habitat Hi with probability α ηi;
            If Hi is selected then
                Select Hj with probability α μj;
                If Hj is selected then
                    Hi (SIV) ← Hj (SIV);
                End if
            End if
            Select Hi (SIV) based on mutation probability mi;
            If Hi (SIV) is selected then
                Replace Hi (SIV) with randomly generated SIV;
            End if
        Next for
        Recalculate SI values;
    Next for
End while
```

Figure 4. The BBO algorithm.

and PSO are used for. BBO is different in some respects with the other global optimization techniques, e.g., as compared with GA it does not involve reproduction and it keeps the solution set while moving from one iteration to the next [24].

5. DESIGN EXAMPLES

5.1. Linear Array Thinning

In this section, three different examples of linear array thinning by BBO are presented. The objective of all the three examples is to minimize the SLL by thinning the linear array to a desired level. The fitness function used for this purpose is given as:

$$\text{Fitness} = (\text{SLL}_o - \text{SLL}_d)^2 H(X) + (\text{TH}_o - \text{TH}_d)^2 H(Y) \tag{7}$$

where SLL_o and SLL_d are obtained, and desired values of sidelobe level in dB, TH_o and TH_d, are obtained and desired values of thinning. $H(X)$ and $H(Y)$ are Heaviside step functions given by

$$X = (\text{SLL}_o - \text{SLL}_d), \quad Y = (\text{TH}_o - \text{TH}_d) \tag{8}$$

$$[H(X), H(Y)] = \begin{Bmatrix} [1,0], & \text{if} \quad X \geq 0,\ Y > 0 \\ [0,1], & \text{if} \quad X < 0,\ Y \leq 0 \end{Bmatrix} \tag{9}$$

For the first example, consider a linear array of 100 isotropic antennas symmetrically spaced 0.5λ apart along z-axis with its center at the origin. The aim is to generate a symmetric pattern with desired SLL of -20 dB or below and percentage of thinning equal to 22 or above. Since it is a uniform linear array, the current excitations of elements on the left hand and right hand sides of the origin will be same or are symmetric. Therefore, only 50 amplitudes are to be optimized. The following BBO parameters are used for this optimization:

- Number of islands or population: $N_{pop} = 100$.
- Iterations or Generations = 100.
- Island modification probability = 1.
- Mutation probability: $m_{\max} = 0.005$.
- Elitism parameter $p = 2$.

The results of the BBO optimization are shown in Table 1. The maximum SLL obtained by BBO is -20.84 dB. For comparison, the results of RGA [4], ACO [13] and IGA [5] thinned linear array are also shown in the same table. The maximum SLL obtained by RGA, ACO and IGA algorithms is -20.56 dB and -20.50 dB and -20.24 dB respectively. Hence, the SLL of BBO thinned array is lower than RGA,

Table 1. Element excitations of a 100-element thinned linear antenna array.

Method	Element No.	Array Weights	SLL (dB)	% of Thinning
RGA [4]	$n = 1, 2, \ldots, 50$	11111111111111111111111111101111001110100001011001	-20.56	22
ACO [13]	$n = 1, 2, \ldots, 50$	11111111111111111111111111011110010100111110010011	-20.50	22
IGA [5]	$n = 1, 2, \ldots, 50$	11111111111111111111111111111011011101000011101011	-20.24	20
BBO	$n = 1, 2, \ldots, 50$	11111111111111111111111111101111010110000100110110	-20.84	22

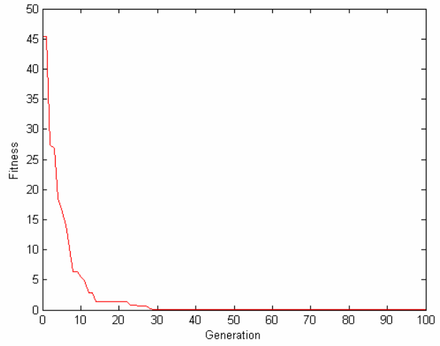

Figure 5. Convergence characteristics of BBO for thinning of a 100-element linear antenna array.

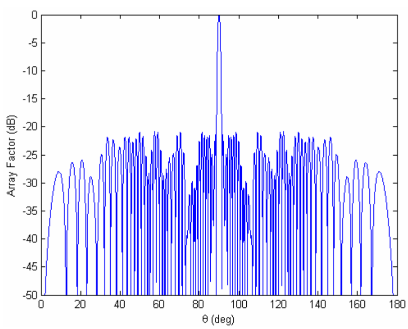

Figure 6. Radiation pattern of a 100-element thinned linear antenna array.

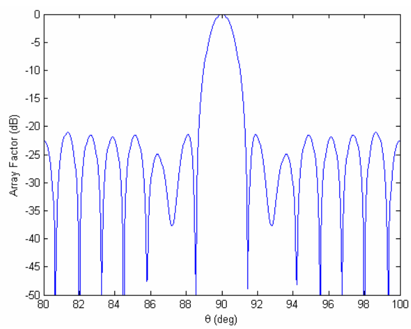

Figure 7. Zoomed radiation pattern with phi angle from 80° to 100° for the radiation pattern of linear array shown in Figure 6.

ACO and IGA thinned linear arrays by 0.28 dB, 0.34 dB and 0.60 dB respectively. The convergence characteristics for BBO are shown in Figure 5. It is to be noted that the BBO converges to best solution very quickly after 40 generations only. The radiation pattern of BBO optimized array is shown in Figures 6 and 7.

In the next example, a 200-element symmetric linear array is synthesised for the same objective as in the above example. As it is symmetric array, hence number of elements amplitudes optimized is only 100. The desired level of SLL and thinning are taken as -23 dB and 22% respectively. The optimized results of BBO are given in Table 2 along with the results of the GA [1], the IGA [5], the BPSO [9] and the PGA [2] algorithms. The maximum SLL obtained by BBO

Table 2. Element excitations of a 200-element thinned linear antenna array.

Method	Element No.	Array Weights	SLL (dB)	% of Thinning
GA [1]	$n = 1, 2, ..., 50$ $n = 51, ..., 100$	11111111111111111111111111111101111110111111111011 10110111110011101110110010110110100010111111111111	-22.09	23
IGA [5]	$n = 1, 2, ..., 50$ $n = 51, ..., 100$	11111111111111111111111111111111111111110111111101 01001111111011111000110100111001010001111011001101	-22.4	20
BPSO [9]	$n = 1, 2, ..., 50$ $n = 51, ..., 100$	11111111111111111111111111111111111111111111011111 10110101110111101011001011010101000001011100110011	-22.4	23
PGA [2]	$n = 1, 2, ..., 50$ $n = 51, ..., 100$	11111111111111111111111111111111111111111111111111 10011110111101101110000110100000110011001001010111	-22.79	23
BBO	$n = 1, 2, ..., 50$ $n = 51, ..., 100$	11111111111111111111111111111111111111111111111110 11110110111001110001110110100100000101010100011011	-23.20	24

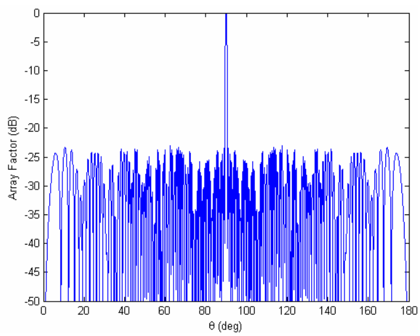

Figure 8. Radiation pattern of a 200-element thinned linear antenna array.

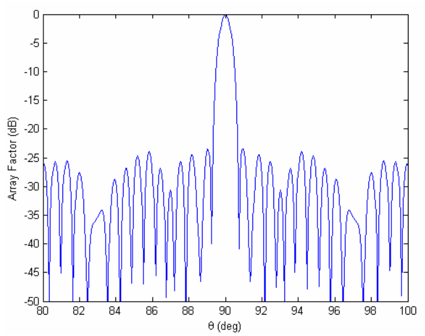

Figure 9. Zoomed radiation pattern with phi angle from 80° to 100° for the radiation pattern of linear array shown in Figure 8.

is -23.20 dB and that by GA, IGA, BPSO and PGA is -22.09 dB, -22.4 dB, -22.4 dB, and 22.79 dB respectively. Clearly, BBO has obtained better results than GA, IGA, BPSO and PGA optimized antennas. The radiation pattern of BBO optimized array is shown in Figures 8 and 9.

For some applications, the region of interest is only near the main beam of the radiation pattern. So in this case, SLL in the region around the main beam is to be reduced, i.e., $\pm 30°$ on both sides of the main beam (from 60° to 120°). Again for this example, a 100-element linear antenna array is considered. The desired level of SLL

Table 3. Element excitations of a 100-element thinned linear antenna array optimized for SLL in the region 60° to 120°.

Method	Element No.	Array Weights	SLL (dB)	% of Thinning
ACO [13]	$n = 1, 2, \ldots, 50$	11111111111111111111101110110101101010110010001001	-25.17	30
BBO	$n = 1, 2, \ldots, 50$	11111111111111111101110110110101101010010010001001	-26.15	32

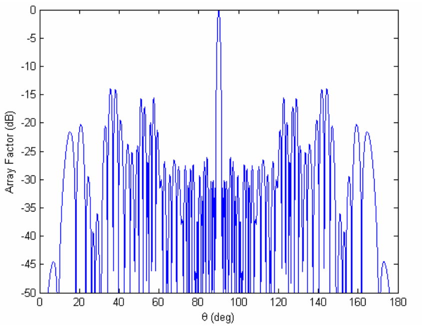

Figure 10. Radiation pattern of a 100-element thinned linear antenna array optimized for SLL in the region 60° to 120°.

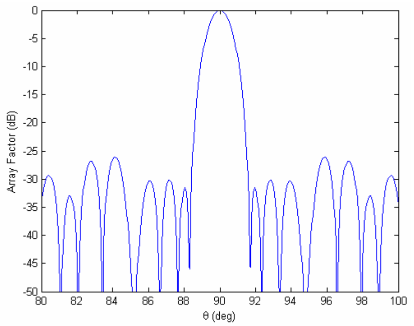

Figure 11. Zoomed radiation pattern with phi angle from 60° to 120° for the radiation pattern of linear array shown in Figure 10.

is taken as -25 dB with the desired level of thinning equal to 22%. The optimized results obtained by BBO are shown in Table 3. For comparison purpose, the results of ACO [13] optimized arrays are also given. The maximum SLL obtained by BBO is lower than achieved by ACO in the desired region. The SLL obtained is -26.15 dB which is lower by 1.41 dB than ACO optimized antenna array. The radiation pattern of the thinned antenna array obtained after optimization is shown in Figures 10 and 11.

5.2. Planar Array Thinning

In this section, a planar array with 20×10 elements is taken for thinning. The aim is to minimize the SLL in all ϕ planes of the array. For this the fitness function is taken as follows:

$$\text{Fitness} = \min\left(|\text{SLL (dB)}|\right)_{\text{for all}\,\phi\,\text{planes}} \tag{10}$$

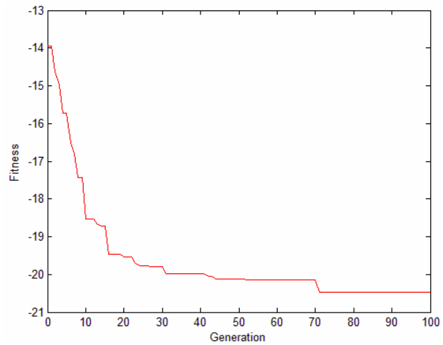

Figure 12. Convergence characteristics of BBO for thinning of a 20×10-element planar array optimized in all the planes.

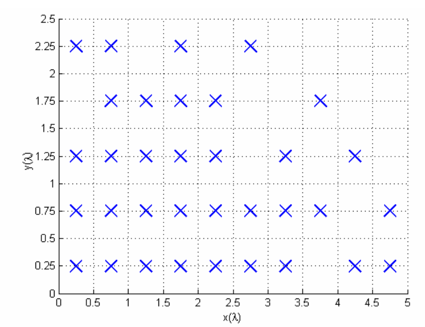

Figure 13. Element configuration for one quadrant of a 20×10-element planar array optimized for SLL in all the planes; X indicates the presence of element.

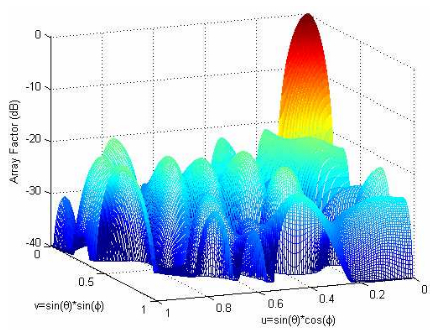

Figure 14. Radiation pattern of a 20×10 element thinned planar antenna array optimized using BBO in all the planes.

The same problem has been dealt with OGA [8]. As it is a symmetric array, there are only 50 current amplitudes that are to be optimized using BBO. The convergence graph of the BBO is shown in Figure 12. The maximum SLL achieved by the BBO algorithm for the planar array for all ϕ planes is $-20.49\,\text{dB}$ while that achieved by OGA is $-19.44\,\text{dB}$. Hence, the SLL is reduced by $1.05\,\text{dB}$ for BBO optimized planar array as compared to OGA array. The element configuration of one quadrant of planar array optimized by BBO is given in Figure 13. The radiation pattern of the optimal array having maximum SLL of $-20.49\,\text{dB}$ in ϕ all planes is shown in Figure 14.

6. CONCLUSIONS

This paper presents a novel algorithm for designing a thinned linear and planar antenna array with fixed percentage of thinning using BBO. Results clearly show a very good agreement between the desired and synthesized specifications. The results obtained by BBO are better than other global optimization techniques, such as GA, RGA, PGA, IGA and BPSO, which have been employed for thinning. The BBO method is an alternative to existing popular global algorithms. The main advantage of this method is its simplicity that provides an easy, quick and effective resolution of medium/large problems. BBO is applied to the thinning of arrays in this work, but it can also be applied to other antenna problems which include other parameters such as spacing, amplitudes, and phase of the elements.

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